

LANS—A NEW CODE FOR EVALUATION OF THE ELECTROMAGNETIC FIELDS AND RESONANCE FREQUENCIES OF AXISYMMETRICAL RF CAVITIES

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A new code for calculation of axisymmetric cavities is developed. The mathematical basis of this code is the method of inverse iterations with a shift, which is the most adequate for the problem of finding the eigenfrequencies and fields for the cavities. This code has some advantages compared with SUPERFISH; it requires a smaller number of operations necessary for calculations and it gives better resolution of resonance modes with close frequencies.

I. INTRODUCTION

RF cavities are important components of linear and cyclic particle accelerators, as well as of rf generators and amplifiers. Naturally, the possibility of using computers for calculations of the resonance frequencies and electromagnetic fields of cavities facilitates significantly the design and production of cavities with optimized characteristics. Of the known programs for calculations of axisymmetric cavities, the most perfect is apparently the program SUPERFISH.¹

We have developed a new program LANS² the mathematical basis of which is the so-called method of inverse iterations (with a shift),³ which is the most adequate for the problem of finding the eigenfrequencies and fields for the cavities. This program has some advantages compared with SUPERFISH; namely, it requires a smaller number of operations for calculations; it also gives better resolution of resonance modes with close frequencies.

A mathematical formulation of the method of inverse iterations (with a shift) in application to the evaluation of electromagnetic fields in cavities is given in Section 2. In Section 3 an algorithm is discussed for the calculation of the magnetic field and resonance frequencies. In Section 4 a direct noniterative method is described for solution of a set of inhomogeneous linear equations. The main features of the program LANS interesting for users are given in Section 5.

2. THE PROBLEM OF EIGENVALUES AND EIGENFUNCTIONS. THE METHOD OF INVERSE ITERATIONS.

Electromagnetic fields in a cavity are described by Maxwell's equations, which in the case of harmonic oscillations have the form

$$\text{curl } \mathbf{E} = -i\omega\mu\mathbf{H} - \mathbf{J}_m, \quad (1)$$

$$\text{curl } \mathbf{H} = i\omega\epsilon\mathbf{E} + \mathbf{J}_e,$$

where \mathbf{J}_m and \mathbf{J}_e are the densities of magnetic and electric currents, respectively. In addition to Eqs. (1), the fields \mathbf{E} and \mathbf{H} should also satisfy the boundary conditions. Usually, in these calculations, the boundary conditions are taken to be ideal. The boundary conditions on the ideal conducting surface (Neumann boundary) have the form

$$\mathbf{E} \times \mathbf{n} = 0, \quad \mathbf{H} \cdot \mathbf{n} = 0 \quad (2)$$

where \mathbf{n} is a vector normal to the surface. Another kind of boundary conditions (Dirichlet boundary), usually used on the symmetry plane of a cavity, is

$$\mathbf{E} \cdot \mathbf{n} = 0, \quad \mathbf{H} \times \mathbf{n} = 0 \quad (3)$$

Eliminating one of the fields, \mathbf{E} , from Eqs. (1), one can get a wave equation for another com-

ponent of the electromagnetic field

$$\text{curl curl } \mathbf{H} - K^2 \mathbf{H} = \text{curl } \mathbf{J}_e - iK \sqrt{\frac{\epsilon}{\mu}} \mathbf{J}_m$$

where $K^2 = \omega^2 \mu \epsilon = \omega^2 / e^2$

The right-hand side of this equation can be written in a simpler way

$$\text{curl curl } \mathbf{H} - K^2 \mathbf{H} = K \mathbf{J}, \quad (4)$$

\mathbf{J} being a vector proportional to the magnetic current density.

It is known that Eq. (4) with boundary conditions (2) and (3) has a unique solution if K^2 does not coincide with any of the eigenvalues K_n^2 of the homogeneous equation

$$\text{curl curl } \mathbf{H} - K^2 \mathbf{H} = 0 \quad (5)$$

with the same boundary conditions.

Calculation for a cavity consists first of defining resonant frequencies $\omega_n^2 = c^2 K_n^2$ and eigenfunctions \mathbf{H}_n i.e., the fields at resonance. Usually, one confines oneself to the solution of a partial problem of eigenvalues, finding only some of the eigenvalues and their corresponding eigenfunctions. For example, the design of accelerator cavities starts with the calculation of the mode with the lowest resonant frequency. Often, though, it is necessary to calculate in addition a number of higher modes.

The program LANS is based on the use of an inverse iteration method.³ The essence of the method is as follows. Instead of the homogeneous Eq. (5) one solves the inhomogeneous Eq. (4) with the boundary conditions (2) and (3). Let us assume, that (with any arbitrary method) we have obtained an approximation of the eigenvalues K_n^2 to be equal to K_{no}^2 and also a zeroth approximation of the field $\mathbf{H}^{(0)}$. In the right hand side of Eq. (4), let us take the current to be

$$\mathbf{J} = K_{no} \mathbf{H}^{(0)}. \quad (6)$$

Solving the equation

$$\text{curl curl } \mathbf{H}^{(1)} - K_{no}^2 \mathbf{H}^{(1)} = K_{no}^2 \mathbf{H}^{(0)} \quad (7)$$

we find the first iteration $\mathbf{H}^{(1)}$. Since by assumption, K_{no}^2 is close to K_n^2 , the field $\mathbf{H}^{(1)}$ will be close to the resonance field with a frequency ω_n . For the $(m + 1)$ -th iteration the equation has the form

$$\text{curl curl } \mathbf{H}^{(m+1)} - K_{no}^2 \mathbf{H}^{(m+1)} = K_{no}^2 \mathbf{H}^{(m)} \quad (8)$$

The fields $\mathbf{H}^{(m)}$ and $\mathbf{H}^{(m+1)}$ can be expanded into series over the eigenfunctions of Eq. (5)

$$\mathbf{H}^{(m)} = \sum_i A_i^{(m)} \mathbf{H}_i; \quad \mathbf{H}^{(m+1)} = \sum_i A_i^{(m+1)} \mathbf{H}_i \quad (9)$$

Substitution of Eq. (9) into Eq. (8) will give

$$A_i^{(m+1)} = \frac{K_{no}^2}{K_i^2 - K_{no}^2} A_i^{(m)} \quad (10)$$

Let us assume that the approximation K_{no}^2 is so selected that in absolute value the difference $K_n^2 - K_{no}^2$ is the smallest among the differences of the kind $K_i^2 - K_{no}^2$. Such a choice is illustrated by Fig. 1, which shows location of the eigenvalues neighbouring K_n^2 . Then with each succeeding iteration, the relative value of the factor A_n increases compared with the remaining coefficients of expansion. In other words, with the growth of iteration number, the field $\mathbf{H}^{(m)}$ tends to its eigenfunction \mathbf{H}_n . Since the absolute value of the field $\mathbf{H}^{(m)}$ increases because of a resonant factor $\gamma_n = K_{no}^2 / (K_n^2 - K_{no}^2)$, it is reasonable to normalize the field to unity after each iteration.

The number of iterations necessary for approximation to \mathbf{H}_n with a given accuracy depends, first, on how close the chosen approximation K_{no}^2 is to K_n^2 and, second, on the zeroth approximation of the field $\mathbf{H}^{(0)}$.

Note, however, that K_{no}^2 should not be too close to K_n^2 since, in this case, the method of finding a solution for Eq. (8) becomes incorrect due to approaching the resonance. After carrying out the required number of iterations the eigenvalue K_n^2 can easily be found from the relation

$$\frac{A_n^{(m+1)}}{A_n^{(m)}} = \gamma = \frac{K_{no}^2}{K_n^2 - K_{no}^2}, \quad (11)$$

whence

$$K_n^2 = \left(1 + \frac{1}{\gamma_n}\right) K_{no}^2 \quad (12)$$

Thus with this iteration process one can find all necessary characteristics in addition to the eigenfunction \mathbf{H}_n , i.e., the magnetic field and also the value of the resonance frequency ω_n .

Discrete approximation of Eq. (4) is performed

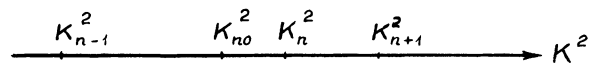


FIGURE 1 Location of the eigenvalues neighboring K_n^2 .

by a method similar to that given in Ref. 1. It is therefore not presented in this paper.

We should note that the only difference is that here Eq. (4) has a right-hand side. Therefore after integration of Eq. (4) over the internal area of a dodecahedron (see Fig. 1) and use of the Stokes' theorem, one obtains

$$\oint \text{curl } \mathbf{H}_\varphi \, de - K^2 \int \mathbf{H}_\varphi \, dS = K \int \mathbf{J}_\varphi \, dS \quad (13)$$

And, finally, taking into account the linear dependence on the coordinates inside every triangle, one has

$$\sum_{p=0}^b \mathcal{H}_{ip}(w_{ip} - K^2 v_{ip}) = \sum_{q=0}^o K \mathbf{J}_{iq} v_{iq}, \quad (14)$$

where the coefficients v_{ip} and w_{ip} depend on the coordinates of the triangular mesh.

3. MAGNETIC FIELD AND RESONANT FREQUENCY CALCULATION

If Eq. (14) is written for all mesh points, one can obtain the relation

$$B(K^2)\mathcal{H} = KV\mathbf{J} \quad (15)$$

where $B(K^2) = W - K^2V$, W and V are matrices formed of the coefficients w and v respectively, \mathcal{H} , \mathbf{J} are column vectors formed of values of the magnetic field and magnetic current densities in the mesh points having dimension $N = K_{\max} \times L_{\max}$ (the mesh contains K_{\max} points on the z coordinate and L_{\max} points on the R coordinate). A square matrix $B(K^2)$ is a three-diagonal matrix, the elements of which are small matrices of dimension $L_{\max} \times L_{\max}$. Small matrices on the diagonal of the matrix $B(K^2)$ have three nonzero diagonals but each of the remaining small matrices have two nonzero diagonals.

Let $\omega_n = K_n C$ be an eigenfrequency of the required mode with number n and $\omega_{no} = K_{no} C$ as its initial approximate value. Also, let the distribution of magnetic currents with density $\mathbf{J} = K_{no}\mathcal{H}^{(0)}$ be given in a cavity. Then from Eq. (15) follows

$$(W - K_{no}^2 V)\mathcal{H}^{(1)} = K_{no}^2 V\mathcal{H}^{(0)}. \quad (16)$$

Substituting the vector $\mathcal{H}^{(1)}$ obtained from Eq. (16) into the left-hand side of Eq. (15), one can continue the process of iteration. For the $(m + 1)$ -

th iteration we obtain

$$(W - K_{no}^2 V)\mathcal{H}^{(m+1)} = K_{no}^2 V\mathcal{H}^{(m)}. \quad (17)$$

Let \mathcal{H}_i be eigenvectors of the matrix $B(K^2)$ and let K_i^2 be its eigenvalues, i.e.,

$$(W - K_i^2 V)\mathcal{H}_i = 0 \quad (18)$$

The vectors $\mathcal{H}^{(m)}$ and $\mathcal{H}^{(m+1)}$ in Eq. (17) can be represented as expansions over eigenvectors of the matrix $B(K^2)$.

$$\begin{aligned} \mathcal{H}^{(m)} &= \sum_{i=1}^N A_i^{(m)} \mathcal{H}_i \\ \mathcal{H}^{(m)} &= \sum_{i=1}^N A_i^{(m+1)} \mathcal{H}_i. \end{aligned} \quad (19)$$

Substituting these expansions into Eq. (17), we obtain

$$A_i^{(m+1)} = \frac{K_{no}^2}{K_i^2 - K_{no}^2} A_i^{(m)} = \gamma_i A_i^{(m)}. \quad (20)$$

Let us assume that the vectors \mathcal{H}_i are normalized to unity, i.e.,

$$\|\mathcal{H}_i\| = \left(\sum_{s=1}^N \mathcal{H}_{is}^2 \right)^{1/2} = 1. \quad (21)$$

If the result of every iteration is normalized to unity, for the normalized vector $\mathcal{H}^{(m+1)}$ we obtain

$$\mathcal{H}^{(m+1)} = \sum_{i=1}^N a_i^{(m+1)} \mathcal{H}_i,$$

with coefficients $a_i^{(m+1)}$ equal to

$$a_i^{(m+1)} = \frac{A_i^{(m+1)}}{\left[\sum_{s=1}^N (A_s^{(m+1)})^2 \right]^{1/2}}. \quad (22)$$

Substituting the latter equality into the expression (20) for $A_i^{(m+1)}$ and squaring it, we obtain

$$[a_i^{(m+1)}]^2 = \frac{[A_i^{(m)}]^2}{\sum_{s=1}^N \left(\frac{K_i^2 - K_{no}^2}{K_s^2 - K_{no}^2} \right)^2 [A_s^{(m)}]^2}. \quad (23)$$

Assuming that normalization is carried out after every iteration, in eq. (23) one can substitute $a_i^{(m)}$

for $A_i^{(m)}$ to find

$$\left[\frac{a_i^{(m+1)}}{a_i^{(m)}} \right]^2 = \frac{1}{\sum_{s=1}^N \left(\frac{K_i^2 - K_{no}^2}{K_s^2 - K_{no}^2} \right)^2 [a_s^{(m)}]^2}. \quad (24)$$

In particular, at $i = n$

$$\left[\frac{a_n^{(m+1)}}{a_n^{(m)}} \right]^2 = \frac{1}{\sum_{s=1}^N \left(\frac{K_n^2 - K_{no}^2}{K_s^2 - K_{no}^2} \right)^2 [a_s^{(m)}]^2}. \quad (25)$$

Let us assume that the difference $|K_n^2 - K_{no}^2|$ is less than any of the similar differences $|K_i^2 - K_{no}^2|$ for $i \neq n$. Since

$$\sum_{s=1}^N [a_s^{(m)}]^2 = 1,$$

the denominator in eq. (25) is then less than unity. Thus

$$\sum_{s=1}^N \left(\frac{K_n^2 - K_{no}^2}{K_s^2 - K_{no}^2} \right)^2 [a_s^{(m)}]^2 < 1 \quad (26)$$

unless $[a_n^{(m+1)}] < 1$. Consequently, under this condition we have the inequality

$$\left[\frac{a_n^{(m+1)}}{a_n^{(m)}} \right]^2 > 1; \quad (27)$$

i.e., after every iteration, the coefficient $a_n^{(m+1)}$ increases in absolute value, remaining less than unity. Hence, one can draw the conclusion that

$$\lim_{m \rightarrow \infty} [a_n^{(m)}]^2 = 1 \quad (28)$$

$$\lim_{m \rightarrow \infty} [a_i^{(m)}]^2 = 0 \quad \text{at } i \neq n. \quad (29)$$

Thus, making a certain number of iterations one can, with sufficient accuracy, make the solution close enough to the eigenvector \mathcal{H}_n . After that, from the equality (20), one can find an ei-

genvalue K_n^2

$$(K_n^{(m)})^2 = K_{no}^2 \left(1 + \frac{1}{\gamma_n^{(m)}} \right). \quad (30)$$

To improve accuracy in the program LANS, the coefficient γ_n is evaluated with the aid of a norm

$$|\gamma_n^{(m)}| \cdot \frac{\|\mathcal{H}^{(m+1)}\|}{\|\mathcal{H}^{(m)}\|}, \quad (31)$$

taking into account that $\|\mathcal{H}^{(m+1)}\|$ is determined prior to normalizing $\mathcal{H}^{(m+1)}$ to unity. The sign for γ_n can be found from the equality (20).

Note that the accuracy of approximation of the final iteration $\mathcal{H}^{(m)}$ to the eigenvector \mathcal{H}_n does not determine entirely the accuracy of approximation of the obtained solution to the true eigenfunction \mathbf{H}_n . Indeed, the vector \mathcal{H}_n approaches the \mathbf{H}_n mode with an accuracy defined by the degree of approximation of the differential Eq. (4) by the set of difference Eqs. (15).

The result of the iteration process is a convenient sequence of vectors $\mathcal{H}^{(m)}$, the limit of which are the vectors \mathcal{H}_n . At the same time, a sequence $(K_n^{(m)})^2$ can be obtained with the formula (30). The convergence rate for sequence $(K_n^{(m)})^2$ is higher than that for $\mathcal{H}^{(m)}$ because small variations of field do not affect the eigenvalues according to their well-known property. Therefore, the calculation of eigenvalues with desired accuracy requires much smaller number of iterations than that needed for calculation of a field.

The program LANS includes the completion criterion for iterations which is based on approaching the limit over the field norm. The iteration process is considered to be completed when the following inequality is satisfied

$$\epsilon^{(m+1)} < \epsilon_0 [1 - \eta^{(m+1)}], \quad (32)$$

where

$$\eta^{(m+1)} = \left[\frac{\sum_i (\mathcal{H}_i^{(m+1)} - \mathcal{H}_i^{(m)})^2}{\sum_i (\mathcal{H}_i^{(m)} - \mathcal{H}_i^{(m+1)})^2} \right]^{1/2}$$

$$\epsilon^{(m+1)} = \left[\frac{\sum_i (\mathcal{H}_i^{(m+1)} - \mathcal{H}_i^{(m)})^2}{\sum_i (\mathcal{H}_i^{(m+1)})^2} \right]^{1/2}.$$

Summation is carried out over all the mesh points. Usually, the value ϵ_0 is selected in the range 10^{-6} to 10^{-7} .

4. SOLUTION OF A SET OF LINEAR EQUATIONS.

The iteration process described above provides a repeated solution of an inhomogeneous set of linear equations (15) with the matrix $B(K^2) = W - K^2V$ the same for all iterations and only the right-hand sides of the equations changed. Therefore, it seems more reasonable to transform the matrix $B(K^2)$ into a modified LU-form⁴ and then use it in further calculation rather than to solve the whole equation (15) many times.

Equation (15) can be written in the form

$$\begin{pmatrix} a_1 & c_1 & & & \\ b_2 & a_2 & c_2 & & \\ & & \dots & & 0 \\ 0 & & & \dots & \\ & & & & b_{K_{\max}} a_{K_{\max}} \end{pmatrix} \times \begin{pmatrix} \mathcal{H}_1 \\ \mathcal{H}_{K_{\max}} \end{pmatrix} = \begin{pmatrix} G_1 \\ G_{K_{\max}} \end{pmatrix}, \quad (33)$$

where a_i, b_i, c_i are matrices of dimension $L_{\max} \times L_{\max}$ and $\mathbf{G} = \mathbf{KVJ}$ is the right-hand side of Eq. (15). Following the Gaussian elimination algorithm, one can transform Eq. (33) into the form

$$\begin{pmatrix} a_1 & c_1 & & & \\ & a_2 & c_2 & & 0 \\ & & \dots & & \\ 0 & & & \dots & \\ & & & & a_{K_{\max}} \end{pmatrix} \times \begin{pmatrix} \mathcal{H}_1 \\ \mathcal{H}_{K_{\max}} \end{pmatrix} = \begin{pmatrix} F_1 \\ F_{K_{\max}} \end{pmatrix}, \quad (34)$$

where

$$d_1 = a_1 \quad d_j = a_j - b_j d_{j-1}^{-1} c_{j-1}$$

$$F_1 = G_1 \quad F_j = G_j - b_j d_{j-1}^{-1} F_{j-1},$$

or otherwise

$$F_j = G_j - e_j F_{j-1}$$

(here $e_j = b_j d_{j-1}^{-1}$, j varies within the limits 2 to

K_{\max}); can be transformed into the form

$$\begin{pmatrix} 1h_1 & & & & \\ 1 & h_2 & & & 0 \\ & & \dots & & \\ 0 & & & & 1h_{K_{\max}-1} \\ & & & & 1 \end{pmatrix} \times \begin{pmatrix} \mathcal{H}_1 \\ \mathcal{H}_{K_{\max}} \end{pmatrix} = \begin{pmatrix} P_1 \\ P_{K_{\max}} \end{pmatrix}, \quad (35)$$

where $h_j = d_j^{-1} c_j P_j = d_j^{-1} F_j = d_j^{-1} G_j - d_j^{-1} e_j F_{j-1}$, or otherwise

$$P_j = f_j G_j - g_j P_{j-1},$$

where $f_j = d_j^{-1}$, $g_j = f_j e_j$. Then $\mathcal{H}_{K_{\max}} = P_{K_{\max}}$; $\mathcal{H}_j = P_j - h_j P_{j-1}$ ($1 \leq j < K_{\max}$). Thus, \mathcal{H} is calculated through the matrices f_j, h_j, g_j . For finding these matrices, one should reverse the matrices d_j only once. These matrices can replace matrices a_j, b_j, c_j in the computer memory and then be used for solution of Eq. (15) with different right-hand sides. As can be seen, for storing matrix $B(n^2)$, a memory of $(3K_{\max} - 2) \times L_{\max}^2$ words is required.

5. CALCULATION OF ELECTRIC FIELD AND INTEGRAL CHARACTERISTICS OF CAVITIES

After the magnetic-field distribution is found and resonance frequency of the desired mode is evaluated, one can determine in every triangle the electric-field components (radial and axial) from Maxwell's equations $\text{curl } \mathbf{H}_n = i\omega_n \mathbf{E}_n$ by the assumption that the magnetic field inside every triangle depends linearly on the coordinates R and z . Electric fields at the mesh points are calculated as the average over the adjacent triangles.

In addition to electromagnetic fields and eigenvalues, the program provides for calculation of the following values, which characterize a cavity:

a) overvoltage, i.e., the ratio of maximum value of electric field on the cavity surface to the effective value of accelerating field on the axis:

$$\sigma = E_{\text{surf. max}}/E_{\text{eff}},$$

$$E_{\text{eff}} = \frac{2}{L} \int_0^{L/2} E_z(z) \cos(Kz) dz$$

where L is the length of accelerating gap and $E_z(z)$ is an axial component of electric field on the cavity axis.

b) transit time factor for relativistic particles

$$T = \int_0^{L/2} E_z(z) \cos(Kz) dz / \int_0^{L/2} E_z(z) dz$$

c) quality factor

$$Q = \frac{2}{\delta} \int_V |H|^2 dV / \int_S |H|^2 ds,$$

where δ is the skin-depth thickness.

d) characteristic impedance

$$\rho = \left| \int_0^{L/2} E_z(z) dz \right|^2 / \omega \mu \int_V |H|^2 dV$$

e) shunt impedance

$$R_{sh} = Q\rho$$

In the program LANS it is also possible to calculate the resonance frequency variation with small changes in shape, which is important for design purposes. To achieve this, the program LANS calculates the so-called influence factors that indicate the variation of resonance frequency due to unit wall deformation at a given point, the unit deformation being the axisymmetric deformation of a wall section with unit length (along a cross section) by a shift of unit length in the direction normal to the surface. The influence factors are calculated through differences of magnetic and electric energy densities in the vicinity of the surface.

6. ACCURACY AND OTHER CHARACTERISTICS OF THE PROGRAM

Errors in calculation of electromagnetic fields and eigenfrequencies for cavities depend on the number of mesh points as a/N^2 , where N is the total number of points. The coefficient "a" depends on the cavity geometry.

The more complex is the cavity geometry, the larger is the number of mesh points that are required for achieving the necessary accuracy. Experience shows that 500 to 2500 mesh points are required for achieving errors of the level $\sim 10^{-4}$ in the calculation of the eigenfrequency for the main mode. The error of magnetic-field evalua-

tion under this condition is greater and is about 10^{-3} . For testing the program and checking the accuracy of evaluation fields and resonance frequencies, as well as integral characteristics of cavities, calculations were performed for cylindrical and spherical cavities. The error in calculation of frequencies for modes E_{010} and E_{011} of a cylindrical resonator is about 10^{-4} , the number of mesh points being equal to 400. The error in calculation of magnetic fields is of order 10^{-3} . For the E_{010} mode of a spherical cavity, the error in calculation of frequency (at the same number of mesh points) is 4×10^{-4} , and that of magnetic field is 2×10^{-3} .

Time consumption is square dependent on the mesh-point number, but depends weakly on the number of iterations, since the major part of the time is spent in modifying the matrix $B(K^2)$ to the LU-form at a given approximate value K_{no}^2 . It requires αN^2 multiplicative operations. Every iteration requires $\beta N^{3/2} (P < \alpha)$ of such operations, i.e., at $N \gg 1$ a much smaller number than required for transformation of the matrix $B(K^2)$.

Note that the convergence rate of iterations is that of a geometric progression with the denominator $(K_n^2 - K_{no}^2)/(K_{n+1}^2 - K_{no}^2)$, where K_{n+1}^2 is the eigenvalue which is nearest to the eigenvalue K_n^2 that is sought. If the eigenvalue K_n^2 is close to K_{n+1}^2 then the denominator is close to unity. In this case, the solution of a problem will require a few tens of iterations, but, this leads to a small increase in time of calculation. If the eigenvalues K_n^2 and K_{n+1}^2 are well separated, then calculations require less than ten iterations.

The properties of the program mentioned above make the program LANS especially convenient for calculations of periodic accelerating structures having several close resonance frequencies due to coupling between neighbouring cavities.

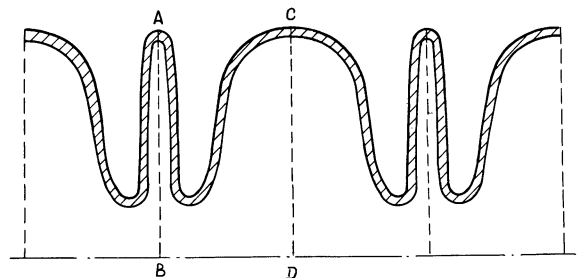


FIGURE 2 The configuration of the biperiodical accelerating structure.

Figure 2 shows schematically the configuration of a biperiodic accelerating structure for a superconducting accelerator which was calculated with the program LANS. The structure has accelerating and coupling cavities and it is designed for operation on a standing wave of $\pi/2$ type. The calculation comprises the evaluation of the frequency of the $\pi/2$ mode and also the coupling factor.

For determining the coupling factor it was necessary to calculate the frequencies of the 0 and π modes. For calculations of these frequencies, the cell shown in Fig. 3 was cut on the symmetry planes. Resonance frequencies were then calculated with Neumann conditions ($\text{curl}_L \mathbf{H} = 0$) being given on these planes of symmetry. Two resonance frequencies were found under these conditions, namely the frequencies of the 0 and π modes. For calculation of the $\pi/2$ mode frequency, Dirichlet boundary conditions ($H_\varphi = 0$) were given on the plane AB and Neumann conditions were given on the CD plane, as in the previous case. Checking on symmetry of location for frequencies of 0 and π modes with respect to a frequency of the $\pi/2$ mode allows one to make corrections for coupling cavity in such a way that its resonance frequency should be equal to the frequency of the accelerating cavity. The frequencies calculated for this example differ by 1 to 3%. The program LANS enabled us to calculate frequencies with an accuracy level of 10^{-4} . The number of iterations used was not very large due to an appropriate choice of initial approximation for the magnetic field $\mathcal{H}^{(0)}$. For example, for calculation of the 0-mode magnetic fields in adjacent cavities are selected with the

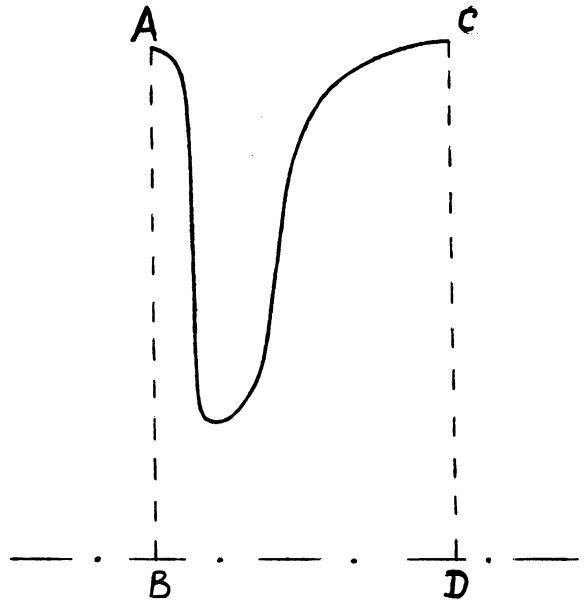


FIGURE 3 The cell of the accelerating structure.

same signs, but for the π mode with opposite signs.

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