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A LASER-DRIVEN GRATING LINAC

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The fields induced over a grating exposed to plane parallel light are explored. It is shown that acceleration is possible if either the particles travel skew to the grating lines or if the radiation is falling at a skew angle onto the grating. A general theory of diffraction in this skew case is given, including an exact solution for the induced fields in the case of a shallow grating. In one particular case numerical solutions are worked out for some deep gratings. It is found that accelerating fields larger even than the initial fields can be obtained, the limit being set by resistive losses on the grating surface. Simple calculations are made to see what accelerating fields might be obtained using CO_2 lasers. Accelerations of 2 or 20 GeV per meter seem possible, depending on whether the grating is allowed to be destroyed or not. Power requirements, injection, and focusing are briefly discussed and no obvious difficulties are seen. It is concluded, therefore, that the proposed mechanism should be considered as a good candidate for the next generation of particle accelerators.

I. INTRODUCTION

The use of a laser to accelerate particles was first proposed by K. Shimoda[1] in 1962. He noted that high values of acceleration per meter could be obtained if velocity matching and mode selection were achieved. These requirements are, however, not easy to obtain.

Fields in free space, far from all sources, consist of a sum of all possible traveling electromagnetic waves. Provided the particles to be accelerated are traveling significantly less than the velocity of light, acceleration[2] can occur. Once the velocity approaches that of light, only waves traveling in the same direction as the particles remain in phase with the particles. Unfortunately, since free radiation is transversely polarized, no continuous acceleration is possible. Despite claims,[3] no juggling with holograms, phase plates or foci can change this. In the presence of a magnetic field, the particle's direction can be perturbed in such a way as to allow continuous acceleration,[4] but this too decreases as the particle's momentum increases and significant perturbations become impractical. In[5] or near[6] a dielectric, the inverse Ĉerenkov effect will accelerate, but the field that can be used is limited, because the dielectric cannot be allowed to break down. At high fields any dielectric becomes a plasma and the situation becomes very complicated. Acceleration within such a plasma is certainly possible[7] but the magnitude of such acceleration remains to be determined.

Acceleration has also been proposed in a vacuum close to a periodic structure. In particular, two papers have attempted[8] to employ the inverse Purcell effect[9] by illuminating a grating with plane parallel light and passing the particles over the surface of the grating at right angles to the lines (Fig. 1a). Unfortunately, it has been shown by Lawson and Woodward[6] that these geometries also fail to accelerate relativistic particles. In Lawson's paper consideration is also given to the field between two parallel plane structures. It is shown in this case finite acceleration of relativistic particles is possible. Such a geometry seems to be little more practical than a scaled down conventional Linac. It does, however, show that there is no fundamental reason why a solution cannot be found.

First I will restate Lawson's theorem and show that it applies only to the simple two-dimensional situation. I will then discuss the general nature of "skew" diffraction at a grating when this two-dimensionality is violated. I will give a general analytic solution for the case of a shallow sinusoidal grating, which will show that acceleration is indeed possible. In order to determine the magnitude of such acceleration, I will then give a few specific numerically calculated solutions.

In order to see whether such an accelerating mechanism might have practical application, I then attempt to estimate the actual field that might be obtained, the power necessary, and the injection and focusing requirements.

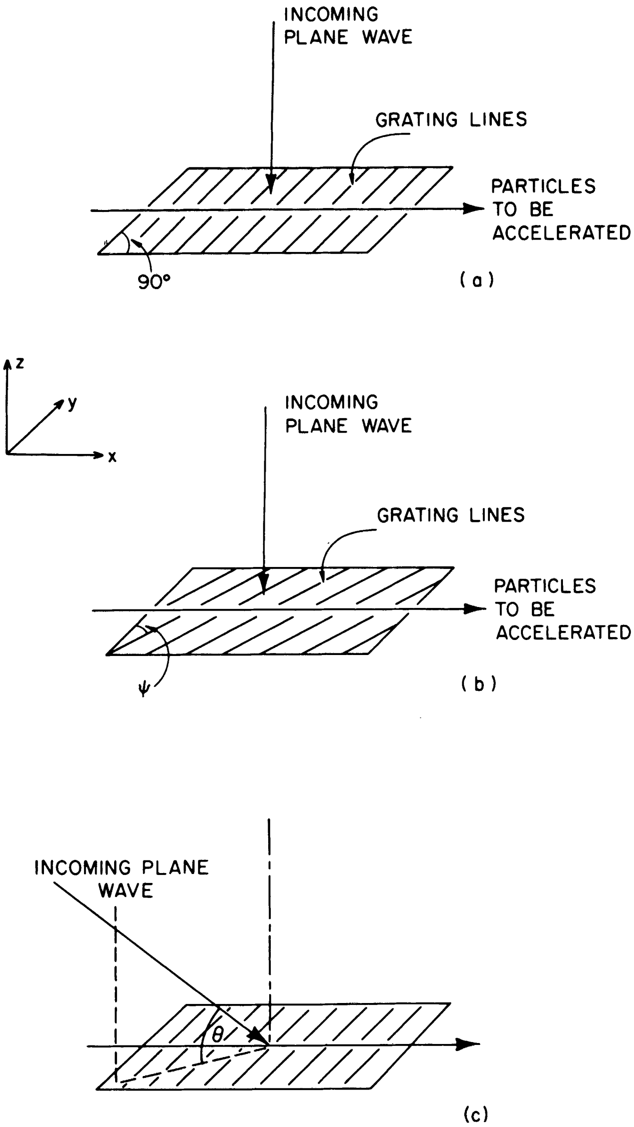

FIGURE 1 Geometries of Grating Accelerators: a) As proposed by Takeda and Matsui; b) with skew grating to allow acceleration of relativistic particles; c) with skew initial wave as alternative to b).

II. LAWSON AND WOODWARD'S THEOREM[6]

We are considering the acceleration in fields above a linear grating when that grating is exposed to a propagating or standing free wave. In the two papers[8] referred to above, this incoming radiation consisted of plane waves falling onto the grating with the rays perpendicular to the grating lines. The acceleration was of particles traveling across the surface, also perpendicular to the grating lines. Such geometries impose the symmetry condition

$$\frac{dE}{dy} = 0, \tag{1}$$

where y is the coordinate along the grating lines and at right angles to the particles. Let z be the coordinate perpendicular to the grating and x be along the particles' direction of motion. Let S be the grating spacing.

Given condition (1), the fields above a surface in the direction of motion x of the particle can always be given[10] as a sum of fields of the type

$$E_x = A_n e^{j(p_n z + K_n x - \omega t)}$$

and if $E_y = 0$, then

$$E_z = \frac{A_n K_n}{p_n} e^{j(p_n z + K_n x - \omega t)}, \tag{2}$$

where

$$p_n = \pm \sqrt{k_0^2 - K_n^2}$$

$$k_0 = 2\pi/\lambda.$$

The A_n are a set of complex constants describing the amplitude and phase of the different modes n.

When $K_n < k_o$, then p_n is real and the mode is a free propagating wave either approaching (p_n negative) or leaving (p_n positive) the surface. These waves are at an angle θ with respect to the normal given by $\sin\theta = K_n/k_o$. When $K_n > k_o$ and p_n is positive and complex, the mode is a surface or evanescent wave that falls off exponentially from the surface. Modes with the negative sign would rise exponentially from the surface and cannot be present.

The requirement that the field remain in phase with a particle of velocity βc is

$$K_n \beta = k_o.$$

Thus,

$$p_n = \sqrt{1 - 1/\beta^2} \tag{3}$$

As the momentum of the particles increases, β approaches 1 and from Eq. (3) we see that p_n approaches zero. From Eq. (2), we then see that E_x/E_z for that mode also approaches zero and there can be no net acceleration. The reason for this is that the only wave consistent with the symmetry, condition (1) that stays in phase with a particle traveling at the velocity of light is a simple propagating plane wave traveling in the direction of the particle. Such a wave is always transversely polarized and thus cannot accelerate in its direction of propagation. In order to overcome this restriction, we must break the symmetry condition (1) and consider waves traveling at an angle to the beam direction. If, for instance, we simply rotate the grating by an angle ψ with respect to the beam (see Fig. 1b), then the condition for synchronism becomes

$$K_n \beta \cos \psi = k_o \qquad (4)$$
$$p_n = \sqrt{1 - 1/(\beta \cos \psi)^2};$$

Now p_n and E_x no longer approach zero as β approaches unity. We thus see that Lawson's theorem, while showing that the proposed geometries do not work, does not rule out all acceleration in the fields above a grating. An alternative to a skew grating is to employ a skew initial wave (figure 2c). In this case, although the grating lines are perpendicular to the particle beam, nevertheless the induced surface waves can still be at an angle to the beam and Eqs. (4) still apply. In order to study this case, we need to know what waves will be induced by an initial wave that is "skew" to the grating.

III. SKEW DIFFRACTION

In order to consider diffraction when condition (1) does not apply, it will be convenient to introduce the following modified vector notation. Three-dimensional vectors (A_x, A_y, A_z) will be described by the two-dimensional vector (A_x, A_y) together with the z component A_z. The two-dimensional vectors will be shown $\mathbf{A}$, the corresponding z component would then be shown as A_z. We will be considering the fields above a grating placed nominally at $z = 0$. Any such fields can be parameterized by

$$\mathbf{E} = \sum_{n=-\infty}^{n=+\infty} \mathbf{A}_n \exp[j(p_n z + \mathbf{K}_n \cdot \mathbf{R} - \omega t)]$$

$$E_z = \sum_{n=-\infty}^{n=+\infty} \frac{-\mathbf{A}_n \cdot \mathbf{K}_n}{p_n} \exp[j(p_n z + \mathbf{K}_n \cdot \mathbf{R} - \omega t)] \qquad (5)$$

where

$$\mathbf{K}_n = \mathbf{K}_o + n\mathbf{G}$$

$$p_n = \pm\sqrt{-|\mathbf{K}_n|^2 + k_o^2}$$

$$|\mathbf{G}| = 2\pi/S.$$

Here n is the order of the diffracted wave, $\mathbf{A}_n$ is a set of two-dimensional complex vectors (A_x, A_y) describing the amplitudes of the modes polarized in the two directions, $\mathbf{G}$ is a vector pointing along the surface perpendicular to the grating lines, and whose amplitude is as given, $\mathbf{K}_n$ is a vector along the surface perpendicular to the wave fronts of the mode, and $\mathbf{K}_o$ is this same vector for the incoming wave.

When $|\mathbf{K}_n| < k_o$, then p_n is real and the mode is a free propagating wave either approaching (p_n negative) or leaving (p_n positive) the surface. Only the initial wave with $n = 0$ is incoming with p_n negative. All others have p_n positive and are the various diffracted modes. To distinguish between the amplitude of the single incoming (p negative) and outgoing (p_o positive) wave, the former will be given without subscript ($\mathbf{A}$) and the latter with subscript $\mathbf{A}_o$. The sum in equation 5 covers both the incoming $\mathbf{A}$ and the set of outgoing waves $\mathbf{A}_n$ ($n = -\infty$ to $+\infty$). When $|\mathbf{K}_n| > k_o$, then p_n is positive and complex and the mode is a surface or evanescent wave that falls off exponentially from the surface.

It is easy to see that given the condition (1), then the two-dimensional vectors $\mathbf{A}_n$, $\mathbf{K}_n$, $\mathbf{G}$ and $\mathbf{R}$ must point along the x direction and the Eqs. (5) reduce to the Eqs. (2). When condition (1) is not satisfied, the situation can be represented in graphical form as in Fig. 2. The various modes propagate along the surface in different directions, and the rays of the diffracted propagating waves do not lie in a plane. In the example given in the figure the incoming mode with amplitude $\mathbf{A}$ is,

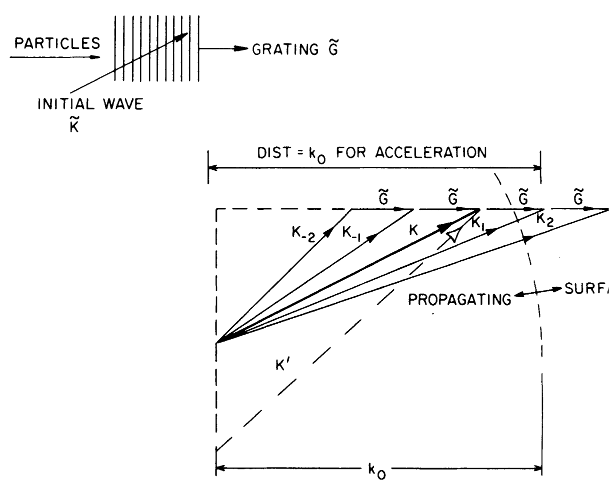

FIGURE 2 Graphical representation of diffracted waves (K_n) induced by a skew initial wave K.

of course, a free wave, as are the diffracted modes $\mathbf{A}_o$, $\mathbf{A}_{-1}$, $\mathbf{A}_{-2}$, etc, but, the modes with n greater than or equal to 1 have $|\mathbf{K}_n| > k_o$ and are surface waves. The surface velocity of these waves have magnitude $ck_o/|\mathbf{K}_n|$ and direction $\mathbf{K}$. If the particle has a direction and velocity $\boldsymbol{\beta}c$, then the condition that the particle remain in phase with the field is:

$$\mathbf{K}_n \cdot \boldsymbol{\beta} = k_o \tag{6}$$

The case illustrated in Figure 1c is when $\boldsymbol{\beta}$ is parallel to $\mathbf{G}$, i.e. when the particles are traveling perpendicular to the grating lines. For $\beta = 1$ this implies that the projection of the $\mathbf{K}_n$ vector onto the vector $\mathbf{G}$ have the length k_o. This condition is shown in Fig. 2 for $n = +1$. We may now note that there is an infinite set of initial waves $\mathbf{K}'$ whose first mode will satisfy the condition (6). It can then be shown that the angle ξ between such initial rays and the beam axis is given by

$$\beta \cos \xi = 1 - \frac{n\mathbf{G}\cdot\boldsymbol{\beta}}{k_o}. \tag{7}$$

The set of all such rays form a half cone about the beam axis analogous with a Ĉerenkov cone (see Fig. 3). This fact turns out to be very advantageous since the sum of all such waves will form a line image on the grating such that the direction of the line points along the particle direction. The narrowness of such a line image ($\sim\lambda$) will then assure the maximum local field for a given electromagnetic energy, and thus represents an efficient laser accelerator.

It should be noted that the one initial ray that is perpendicular to the grating lines (AO in Fig. 3) cannot, by Lawson's theorem, induce acceleration. In practice rays near to this case would probably be omitted.

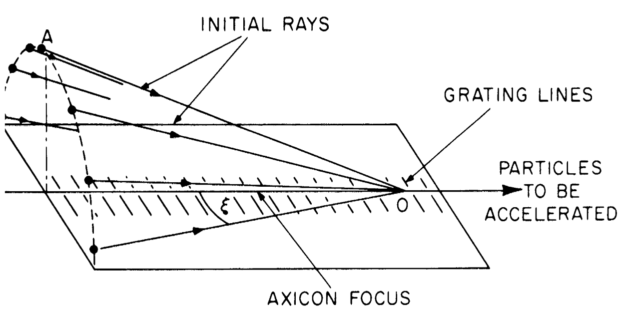

FIGURE 3 General geometry of initial waves inducing acceleration of particles over a grating.

It remains now to determine the actual magnitude of the acceleration for given incident electromagnetic energy. This I will first do for the general case but with a shallow grating, and then for a few particular but deep grating cases.

IV. SHALLOW GRATING

The initial wave is given by Eq. (5) with $n = 0$ and p negative. If the grating is shallow, then to first approximation it will behave as a plane mirror. Ignoring losses the primary outgoing wave will then be a simple reflected wave with the same amplitude as the incoming one. That is

$$\begin{aligned} \theta_{\text{out}} &= -\theta_{\text{in}} \\ p_{\text{out}} &= -p_{\text{in}} \\ \mathbf{K}_o &= \mathbf{K} \\ \mathbf{A}_o &= -\mathbf{A} \end{aligned} \tag{8}$$

The sum of the incoming and this reflected wave is given by

$$\mathbf{E} = \mathbf{A}2j\sin(pz)e^{j(\mathbf{K}\cdot\mathbf{R}-\omega t)}$$

For $pz \leqslant \pi/2$

$$\mathbf{E} \approx \mathbf{A}2jpze^{j(\mathbf{K}\cdot\mathbf{R}-\omega t)}. \tag{9}$$

Let the surface of the grating be described by

$$z = Z_0 \sin(\mathbf{G}\cdot\mathbf{R}), \tag{10}$$

where Z_o gives the amplitude of the bumps and dips, which we are here assuming to be small. The x, y fields at the surface of the grating from the initial and reflected waves are

$$\begin{aligned} \mathbf{E} &\simeq 2j\mathbf{A}pZ_o\sin(\mathbf{G}\cdot\mathbf{R})e^{j(\mathbf{K}\cdot\mathbf{R}-\omega t)} \\ &= \mathbf{A}pZ_o(e^{j\mathbf{G}\cdot\mathbf{R}} - e^{-j\mathbf{G}\cdot\mathbf{R}})e^{j(\mathbf{K}\cdot\mathbf{R}-\omega t)} \\ &= \mathbf{A}pZ_o(e^{j\mathbf{R}\cdot(\mathbf{K}+\mathbf{G})} - e^{j\mathbf{R}\cdot(\mathbf{K}-\mathbf{G})})e^{-j\omega t} \end{aligned} \tag{11}$$

We know that at the surface of the grating the field parallel to that surface must be zero. Other waves besides the reflected wave must be induced by the initial wave such as to satisfy this condition. Since the grating is shallow, and providing $p \neq 0$, the fields parallel to the surface are given approximately by the xy fields $\mathbf{E}$ at the grating surface, and these must be zero. It is noted that the form of Eq. (11) is that of the sum of two waves, and thus if we introduce two such waves with the opposite sign, this boundary condition will be satisfied. The two new waves are recognized to be those corresponding to $n = +1$ and $n = -1$:

$$\mathbf{K}_{+1} = \mathbf{K} + \mathbf{G} \qquad \mathbf{K}_{-1} = \mathbf{K} - \mathbf{G}$$
$$\mathbf{A}_{+1} = -\mathbf{A}pZ_o \qquad \mathbf{A}_{-1} = +\mathbf{A}pZ_o. \tag{12}$$

We thus have an eigensolution with the incoming wave $\mathbf{A}$ and three diffracted waves: $\mathbf{A}_0$, $\mathbf{A}_{+1}$ and $\mathbf{A}_{-1}$. If the condition for continuous acceleration (6) is satisfied for $n = +1$, then the mean acceleration is given by

$$\left\langle \frac{\partial \mathscr{E}}{\partial x} \right\rangle = A_x pZ_o = A_x 2\pi \frac{Z_o}{\lambda} \cos\theta, \tag{13}$$

where θ is the angle between the incoming ray and the normal to the surface. Since $pZ_o \ll \pi/2$ this acceleration is, of course, small. If we increase Z_o so that this condition is no longer satisfied, then the eigensolution will include more, in fact an infinite set of, modes. The situation then becomes much more complicated and an analytic solution is no longer possible. Numerical solutions can be obtained, but have to be worked out for particular individual cases.

V. DEEP GRATINGS

There are two fundamentally different approaches to obtaining numerical eigensolutions to the fields above a surface boundary condition. The first and more common is to define the boundary and then adjust the amplitudes of all possible modes until the boundary condition is satisfied. An alternative that I will follow here is to pick a suitable combination of modes and then find the boundary condition (i.e. grating shape) that is consistent with the resulting fields. This approach is particularly easy if incident waves are chosen such that all resulting modes form standing waves. The field lines for these waves can then be drawn and any surface that is perpendicular to these lines is an acceptable shape for the grating.

For simplicity, I will restrict myself to special cases with the following character: the incoming rays will be chosen to be perpendicular to the beam axis. In addition, the beam axis will be chosen to lie perpendicular to the grating lines. Such a case is illustrated in Figs. 4 and 5. The only variables in describing the incoming wave are its angle ϕ to the normal and its state of polarization, which will be taken to be in the beam direction (x). i.e.

$$\mathbf{K}_o \cdot \boldsymbol{\beta} = 0, \qquad \mathbf{K} \cdot \mathbf{G} = 0,$$
$$\mathbf{A} \times \boldsymbol{\beta} = 0. \tag{14a}$$

If we further require that the grating shape be symmetric with respect to a reversal of the beam direction then

$$\mathbf{A}_n = \mathbf{A}_{-n} \tag{14b}$$

and the number of free parameters is reduced by two. If we consider $\beta = 1$ and require acceleration for $n = 1$, then the condition for acceleration (Eq. 6) reduces to

$$n|\mathbf{G}|\beta = k_o \quad \text{or} \quad S = \lambda\beta n, \tag{14c}$$

but $\beta = 1$, $n = 1$, so $S = \lambda$, where S is the grating spacing. Finally, I will consider fields induced by

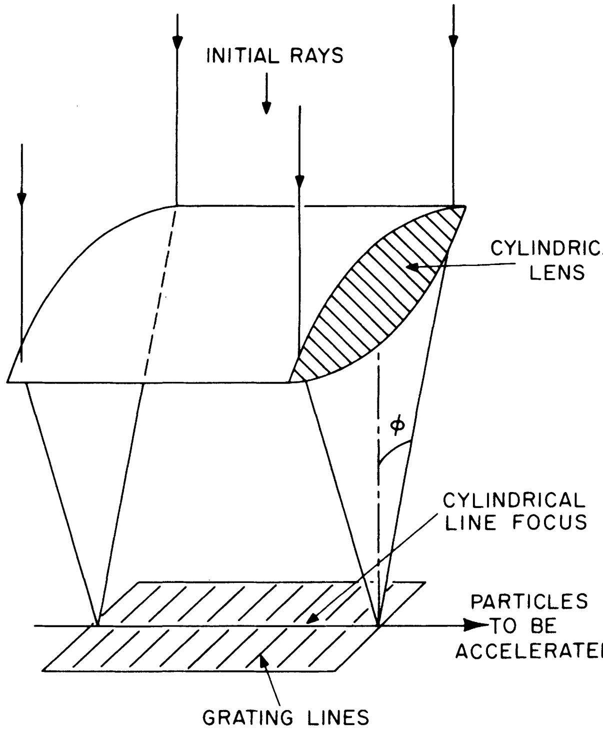

FIGURE 4 Special case of the geometry shown in Figure 3 where $\theta = 90°$.

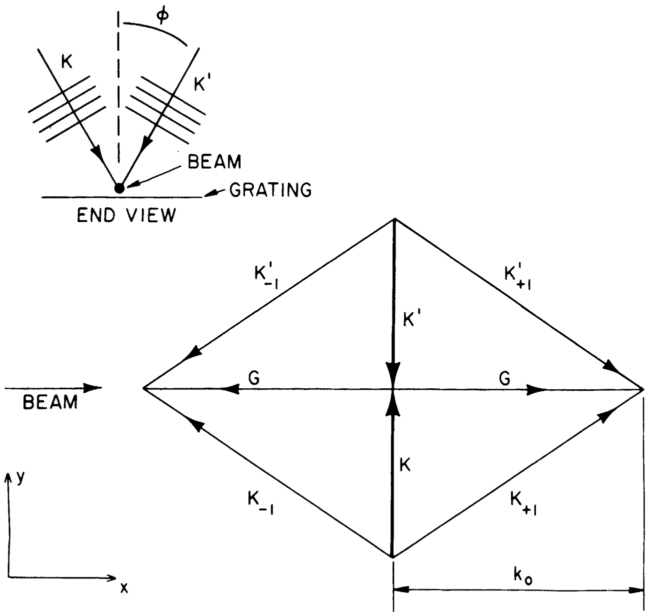

FIGURE 5 Graphical representation of first diffracted waves (K_1) induced by initial waves in the geometry of Figure 4.

two equal and simultaneous incoming waves, A and A', whose angles to the normal ϕ and ϕ' are equal and opposite. Two sets of induced waves will then be present denoted by $\mathbf{A}_n$ and $\mathbf{A}_n'$ with the condition that

$$\mathbf{A}_n = \mathbf{A}_n'. \tag{14d}$$

Condition 14c assures that all modes other than $n = 0$ are surface waves. Energy conservation thus requires that the amplitudes of the outgoing reflected waves ($n = 0$) be equal and opposite to the incoming waves. Then

$$\mathbf{A}_o = -\mathbf{A}, \qquad \mathbf{A}_o' = -\mathbf{A}'. \tag{14e}$$

With all these conditions (14a–e), the resulting fields are assured to be standing waves and the only variables are the set of scalar amplitudes A_n for $n = 1$ to ∞.

The fields are now given by

$$E_x = \cos \omega t \cos Ky \times \left\{ B \sin pz + \sum_{n=1}^{\infty} B_n e^{-q_n z} \cos nk_o x \right\}$$

$$E_y = 0 \tag{15}$$

$$E_z = -\cos \omega t \cos Ky \times \left\{ 0 + \sum_{n=1}^{\infty} B_n (nk_o/q_n) e^{-q_n z} \sin nk_o x \right\},$$

where

$$B = 4j|\mathbf{A}|$$
$$B_n = 4|\mathbf{A}_n|$$
$$p = +\sqrt{k_o^2 - K^2}$$
$$K = |\mathbf{K}|$$
$$q_n = -jp_n + \sqrt{K^2 + k_o^2(n-1)^2}$$

Note $q_1 = K$.

B and B_n are now real numbers. All waves vary in the same way with both time and y position. Clearly maximum acceleration is obtained at $y = 0$ and at values of y spaced at intervals of $2\pi/K$. The first term inside the curly bracket is that due to the incoming and outgoing waves. It is only in the x direction, varies sinusoidally with distance above the grating, and is constant along the direction of acceleration. The second term in the brackets includes all the surface waves that fall off exponentially with height above the grating and vary periodically with position in x. The average acceleration of a particle traveling in the x direction at a height h above the surface depends only on the mode $n = 1$ and is

$$\left\langle \frac{d\mathscr{E}}{dx} \right\rangle = \frac{B_1}{2} e^{-Kh}. \tag{16}$$

It is convenient to express this mean accelerating field as a fraction ϵ of the peak field $\frac{1}{2}B$ that would be present in the absence of the grating. Thus

$$\left\langle \frac{d\mathscr{E}}{dx} \right\rangle = \epsilon \frac{B}{2},$$

$$\epsilon = \frac{B_1}{B} e^{-Kh}. \tag{17}$$

All fields vary in the same way with y. Thus, if a line is found that is perpendicular to the fields at one y, the same line will be perpendicular at all other values of y. In other words, we will have found a surface with a cross section independent

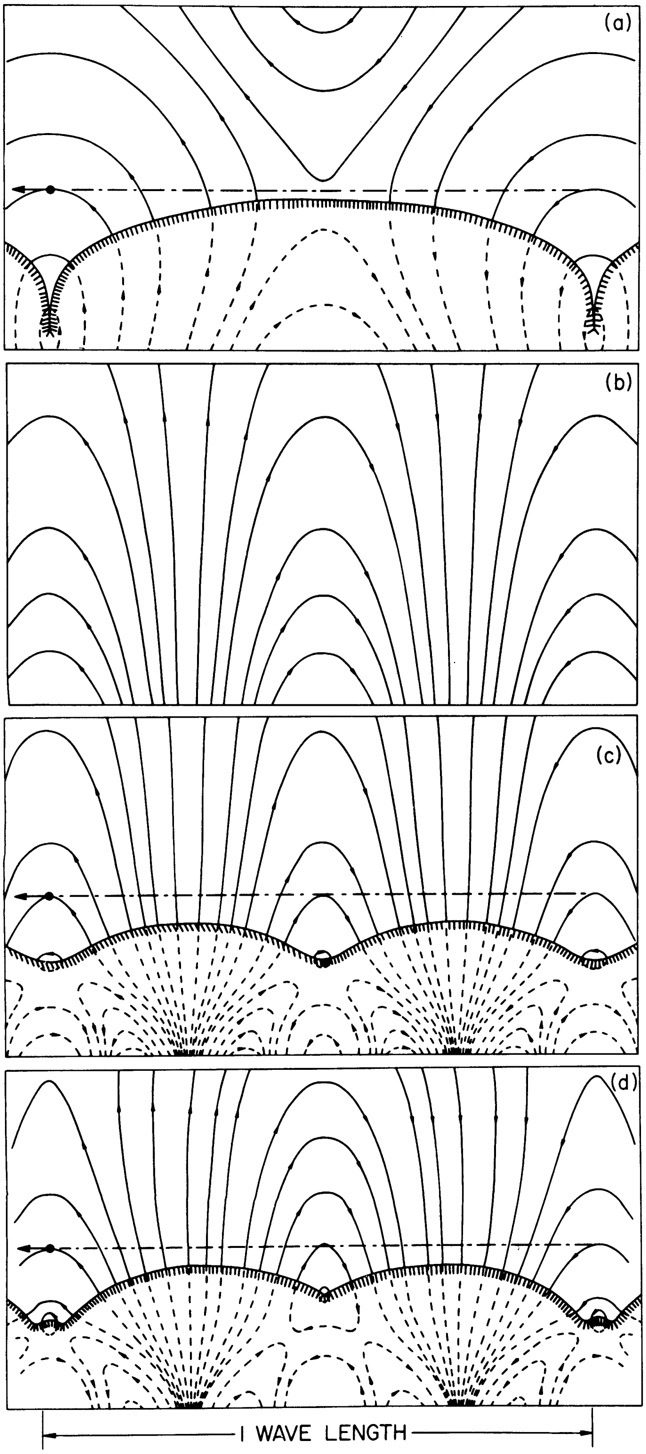

FIGURE 6 The electric field patterns produced by different combinations of modes, together with the shape of the grating surfaces that will support these combinations: a) Case with initial wave ($n = 0$) and the accelerating modes ($n = \pm 1$) only; b) Field lines for the accelerating ($n = \pm 1$) modes alone. There is no grating surface that will support this mode alone; c) Case with accelerating mode ($n = \pm 1$) and a small addition of the third mode ($n = \pm 3$); this solution does not couple to any initial wave; d) Case with a small initial wave ($n = 0$), a strong accelerating mode ($n = \pm 1$) and a small addition of the third ($n = \pm 3$), this solution couples to the initial wave and provides good acceleration.

of y, i.e. a grating. It remains then to consider some individual cases, examine the pattern of x, z fields at $y = 0$ and find lines perpendicular to these fields, thus defining eigensolutions to the problem.

a) Case with $n = 0$ and 1 only

The first example I will consider is when all amplitudes are zero except for $n = 0$ and $n = 1$. These are the same amplitudes present in the shallow grating case. I increase B_1 searching for a suitable boundary in each case. At first the boundary is nearly sinusoidal, but departs from it as the amplitude increases. Beyond a certain value, no boundary can be constructed. An example of the x, z field pattern and grating surface shape is shown in Fig. 6a. This example has the parameters: $K = 0.3k_o, B_1 = 0.6B$. The highest point h of the grating is at $z = 0.3$ and the magnitude of the acceleration is given by Eq. (17), which yields $\epsilon = 0.34$. In other words, we have a solution in which a particle passing just over the surface will be accelerated continuously by a field equal to 34% of the peak field present without the grating. This is already a substantial achievement. We note, however, that the shape of the grating contains sharp cuts that would be impractical to construct. Clearly, we should consider solutions that include more modes, have a more reasonable shape, and we hope, even better performance.

b) Cases with higher modes

We are searching for a solution in which the ratio of the accelerating mode to the incoming mode is as large as possible. It becomes relevant, therefore, to ask why this ratio cannot be infinite. In other words, ask whether there is an eigensolution with surface modes, including an accelerating mode, and no free propagating waves at all. In such a solution the grating is behaving like a cavity containing accelerating fields, which would, if there were no losses, remain identifinitely without the application of any external field. First we can examine the accelerating mode ($n = 1$) alone. This is shown in Fig. 6b. Any surface perpendicular to these field lines contain cuts that extend to infinite depth; clearly not a practical solution. If, however, we add the mode ($n = 3$) with opposite phase, then at once a solution becomes possible. Consider for instance $K = 0.2k_o$, $B_3 = -0.025B_1$. The field pattern obtained is shown in Fig. 4c where a surface perpendicular to all lines is indicated. This is then a solution that, if excited, would accelerate and would remain without radiating away its energy. But, since it is not coupled to the incoming wave, it would in fact not be excited in the first place. What is required is a solution similar to the above but with a small admixture of the incoming mode. For instance $K = 0.2k_o$, $B_3 = -0.025B_1$, $B = -0.5B_1$, which gives the field pattern of Fig. 6d. It may be noted that the uncoupled grating, Fig. 6c, had periodicity of half the wave length, and it can be shown that such a periodicity cannot couple to the incoming waves. This new solution is similar but has a small component of one-wavelength periodicity. It is this component that provides the coupling.

The acceleration at the surface of this grating is given by $\epsilon = 5.0$ and is thus considerably larger than the peak field present without the grating! This result is not surprising when compared to a conventional accelerating rf cavity. If the Q of the cavity is higher, then the accelerating fields for given rf power are also higher. The realizable accelerating field is set when the losses in the cavity approach the rf power applied.

In the grating case, the losses at the surface can be calculated if they are due purely to resistive effects. They are then given approximately by

$$f = \frac{P_{\text{losses}}}{P_{\text{incoming}}} \simeq \left(\frac{k_o}{K}\varepsilon\right)^2 \cdot \frac{1}{4}\left(\frac{c}{\lambda\sigma}\right)^{1/2}. \tag{18}$$

For a copper grating ($\sigma = 1/1.5\ 10^{-6}\ \Omega$ cm $\equiv 6\ 10^{17}$ sec^{-1}), wave length of 10μ, $K = 0.2k_o$, and $\epsilon = 5.0$, we obtain the fractional loss $f \simeq 100\%$. Thus the value of $\epsilon = 5$ represents the highest value possible. It would be more realistic to limit the fractional loss to approximately 25% and thus ϵ to 2.5. This value will be used for the following examples.

VI. PRACTICAL CONSIDERATIONS

a. *Grating Survival*

Two quite different limits must be considered here. First: up to what power level will the grating survive such that it can be used for subsequent pulses? Second: up to what power level will the grating survive in the sense that acceleration will still occur above its surface? The second limit is quite appropriate for a grating (whereas quite inappropriate for a conventional linac) since only a narrow band ($\sim 25\mu$ wide) would be destroyed and the grating could be displaced between pulses and eventually replaced. Alternatively, it is possible that one could use ripples on a liquid metal surface such as mercury or potassium.

The limits, and the pulse duration as well, clearly depend on the instantaneous power level and frequency. If a CO_2 laser were employed, then the shortest pulse obtainable would be about 30 psec and for such a pulse the first "few-pulse" limit is[11] about 10^{11} W per cm^2. This figure is extrapolated from experiments with copper gratings at Los Alamos. The second "one-pulse" limit is far harder to estimate but could be as high as 10^{13} W/cm^2. Experiment is required to determine this, but I will use these numbers for the subsequent discussion.

The relation between local power density in space and the local field is given by the Poynting Vector

$$E = 27.5\sqrt{P}, \tag{19}$$

where E is the maximum local field in V/cm and P is the power density in W per cm^2. The value of the accelerating field can, as we have seen, be higher than this by a factor ϵ depending on the losses in the grating. This factor I will take to be 2.5. The accelerations obtained are then 2 GeV/m and 20 GeV/m, for the "few-pulse" and "one-pulse" limits respectively (see Table I). We may

TABLE I

Example	S (Watts/cm^2)	l (km)	W (Watts)	J (joules)	$\delta\mathscr{E}/\delta l$ (GeV/m)	$\mathscr{E}$ (TeV)
a)	10^{11}	1	2.5 10^{13}	750	2	2
b)	10^{13}	1	2.5 10^{15}	75,000	20	20

Parameters of two speculative accelerators: a) in which the grating is not destroyed, and b) in which a disposable grating is used.

compare these figures with those obtained at SLAC (10 MeV/m) or with the highest obtained in test cavities (about 100 MeV/m). It appears that the grating has the potential for far higher accelerations than is possible with conventional linacs. In a length of 1 km, energies of 2 or 20 TeV would be obtained.

b. *Power Requirements*

I will now address the question of the power requirements of the driving laser. As has been shown above, the conditions for acceleration can be met at a line image from a cylindrical lens system (see Fig. 4). The aperture would consist of two bands, one either side of the vertical since by the Lawson theorem the vertical light cannot contribute. The width of the image will be given approximately by $\lambda/2\phi$, where ϕ is the half-angle between the two strips. Thus for solutions similar to that shown in Fig. 6d, the width w of the line image will be

$$w = \lambda/2\phi \simeq 2.5\lambda \qquad (20)$$

and thus the total power W required for a line image of length l is

$$W = Pwl,$$

and using Eqs. (17) and (19),

$$\left\langle \frac{d\mathscr{E}}{dx} \right\rangle = 27.5\epsilon\sqrt{Wl/2.5\lambda} \qquad (21)$$

For the two cases we have been considering, the total power for a length of 1 km would be 2.5 10^{13} and 2.5 10^{15} W, respectively. The energies in a 30-psec pulse would be 750 and 75,000 J respectively. The first specification is really quite modest by todays large CO_2 laser standards[12] and the second could probably be built for a reasonable cost if many separate beams were passed through the amplifiers, each delayed with respect to the former (multiplexed operation).

c. *Injection*

One of the objections raised to such laser-driven accelerators was that the phase space accepted was so small that a negligible number of particles could be accelerated. This appears not, in fact, to be the case when the phase-space density of the proposed SLAC single-pass collider[13] is considered. The beams in that proposal would contain 10^{11} particles in a 10-psec bunch that would be focused to about 2μ diameter for a length of 1 cm. The acceptance of the laser accelerator proposed here would be approximately 25μ wide and 6μ high. If suitably matched, the proposed SLAC beam would have a depth of focus of about 1 m and would be a suitable injector. An electron cooling ring with suitable rf might also be suitable, but this possibility has not been studied.

d. *Stability and Focusing*

It is found that vertical and horizontal stability are obtained if the bunches are slightly behind the phase of maximum acceleration. What is needed is a horizontal fixed magnetic field to counteract the rf electrostatic and magnetic forces tending to push the particles away from the surface. We also require the use of some initial radiation with polarization perpendicular to the particle direction applied with appropriate phase. An example has been worked out[14] that at 20 GeV/c provided a vertical betatron wavelength of 16 cm and a horizontal wavelength of 80 cm. These values are entirely compatible with the emittance of the SLAC beam discussed above.

Longitudinal stability is not obtained in this example but is found not to be needed since the synchrotron wave length is several kilometers. A special buncher would, however, be required at the front end that would employ magnetic fields to lower the synchrotron wavelength.

VII. CONCLUSION

The acceleration of particles in the fields above a grating exposed to coherent radiation has been studied. It has been shown that such acceleration is possible and that it can occur at a cylindrical or axicon focus over a linear grating. It has further been shown that the accelerating field can be larger than that present in the absence of the grating. The situation is analogous to conventional tuned cavities with finite Q. I have considered what limits practical considerations might place on the energies that could be obtained with such devices. The conclusion is that very high energies (2–20 TeV) might be possible. More detailed study, including experimental tests should be carried out, but unless such work reveals a major flaw the proposed scheme should be taken as a serious candidate for the next generation of accelerators.

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