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STRESSES IN WARM IRON SUPERCONDUCTING DIPOLES UTILIZING GENERAL ELASTIC CONSTANTS

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SUMMARY

Calculations are presented which are suitable for estimating mechanical and thermal stresses in superconducting dipoles being built at Fermilab.[1] For the geometrical structure chosen the calculations are analytically exact. However, the structure only approximates the physical structure of the Doubler dipole[2] and the beam-line dipoles.[3] The stresses calculated include those resulting from prestrain, cooldown, and magnetic-field excitation.

Earlier calculations on the same structural composite assumed media possessing isotropic symmetry.[4] The present calculation utilizes elastic constants possessing locally orthotropic symmetry (reflection symmetry about three orthogonal planes).[5] This permits in particular a more realistic representation of the subdivision of superconductor into cables. A solution to the elastic equilibrium equations has been found utilizing these elastic constants in a composite consisting of three nested hollow cylinders, an innermost cylinder to represent the bore tube region, a middle cylinder to model the region of superconductor, and an outermost region to provide structural support for resisting magnetic forces. These three regions are shown in Fig. 1 in relation to a warm iron shield. In keeping with the orthotropic symmetry used to characterize the elastic constants, three thermal strain constants are used in each region, one for each principal axis of symmetry.[6] Under zero stress a distribution of current is chosen to give a pure dipole field. Subsequent effects of prestrain, cooldown, and excitation on the stress distribution are found. Numerical results are presented for the Doubler.[1]

EQUILIBRIUM

The condition for static equilibrium in an elastic body subjected to Lorentz forces is

$$\nabla \cdot \boldsymbol{\sigma} + \mathbf{J} \times \mathbf{B} = 0, \qquad \text{(emu)} \tag{1}$$

where $\boldsymbol{\sigma}$ is the elastic stress tensor, $\mathbf{J}$ is the current density and $\mathbf{B}$ the magnetic induction.

COORDINATES AND LONG BODY APPROXIMATION

Since most dipole magnets are long compared to their transverse dimensions a complicated three dimensional calculation may be avoided. The end effects on the body can be handled in an integral manner through use of the virial theorem.[7] In elasticity language this is designated as a generalized plane strain[8] approximation. As for the coordinates to be used it seems most appropriate to use cylindrical coordinates. Hence if $\mathbf{u}$ is the elastic displacement vector associated with each point in the body and generalized plane strain is invoked the functional dependence is limited to:

$$u_r(r,\theta) \quad u_\theta(r,\theta) \quad u_z = z\varepsilon_{zz}. \quad (\varepsilon_{zz} = \text{constant}) \tag{2}$$

RELATION OF STRESS TO STRAIN

Since the ends are far removed from the region of interest the shear stresses σ_{rz} and $\sigma_{\theta z}$ may be neglected. Consequently two of the nine elastic constants representing orthotropic symmetry are not needed. Thus[5]

$$\begin{aligned}
\sigma_{rr} &= C_{11}(\varepsilon_{rr}-k_1) + C_{12}(\varepsilon_{\theta\theta}-k_2) + C_{13}(\varepsilon_{zz}-k_3) \\
\sigma_{\theta\theta} &= C_{12}(\varepsilon_{rr}-k_1) + C_{22}(\varepsilon_{\theta\theta}-k_2) + C_{23}(\varepsilon_{zz}-k_3) \\
\sigma_{zz} &= C_{13}(\varepsilon_{rr}-k_1) + C_{23}(\varepsilon_{\theta\theta}-k_2) + C_{33}(\varepsilon_{zz}-k_3) \\
\sigma_{r\theta} &= C_{44}\varepsilon_{r\theta},
\end{aligned} \tag{3}$$

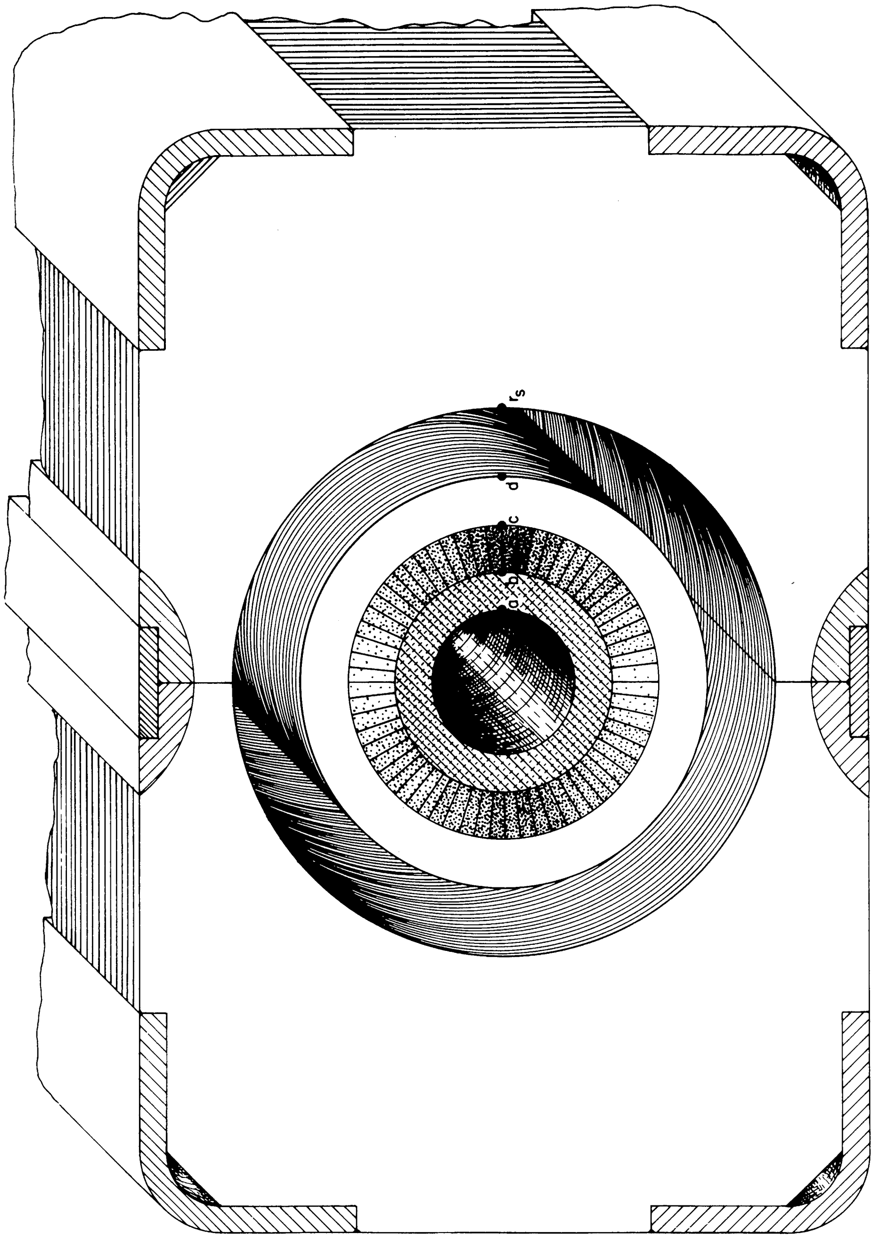

FIGURE 1 Model of Warm Iron Superconducting Dipole.

where k_1, k_2, k_3 represent the thermal strains from room temperature where they are zero to some other operating temperature. For orientation if isotropic symmetry had been evoked

$$C_{11} = C_{22} = C_{33} = \lambda + 2\mu$$
$$C_{12} = C_{13} = C_{23} = \lambda$$
$$C_{44} = 2\mu, \tag{4}$$

where λ, μ are the Lamé constants related to Young's modulus E and Poisson's ratio ν

$$\lambda = \frac{\nu}{(1+\nu)(1-2\nu)} E, \quad \mu = \frac{1}{2(1+\nu)} E. \tag{5}$$

RELATION OF STRAIN TO DISPLACEMENT

In cylindrical coordinates the strain tensor is given by[9]

$$\varepsilon_{rr} = \frac{\partial u_r}{\partial r}$$
$$\varepsilon_{\theta\theta} = \frac{u_r}{r} + \frac{1}{r}\frac{\partial u_\theta}{\partial \theta}$$
$$\varepsilon_{r\theta} = \tfrac{1}{2}\left(\frac{\partial u_\theta}{\partial r} - \frac{u_\theta}{r} + \frac{1}{r}\frac{\partial u_r}{\partial \theta}\right)$$
$$\varepsilon_{zz} = \frac{\partial u_z}{\partial z} \tag{6}$$

SUMMARY OF MAGNETIC QUANTITIES OF INTEREST

The variation of current density required to produce a uniform field is given by

$$J_z = J_0 \cos\theta \tag{7}$$

in the cylinder region assigned to current. Two components of the Lorentz force give

$$J_z B_\theta = \frac{\pi}{3} J_0^2 (4r - 3\lambda - b^3 r^{-2})(1 + \cos 2\theta), \tag{8}$$

$$J_z B_r = \frac{\pi}{3} J_0^2 (2r - 3\lambda + b^3 r^{-2}) \sin 2\theta, \tag{9}$$

where

$$\lambda = c + \tfrac{1}{3}(c^3 - b^3) r_s^{-2}. \tag{10}$$

As in Fig. 1 b is the inner radius of the cylinder containing the longitudinal current density J_z. The outer radius of this region is c and r_s is the inner radius of the circular iron shield.

The energy per unit length contained within the region bounded by the cylnder $r = r_s$ is

$$W_B = \tfrac{1}{3}\pi^2 J_0^2 [-\tfrac{1}{2}(c^4 - b^4) + \lambda(c^3 - b^3) - b^3(c - b)]. \quad \text{(emu)} \tag{11}$$

Finally one needs an integral containing the Maxwell stress tensor $\boldsymbol{\tau}$ which is defined such that its divergence is the Lorentz force. The quantity needed in the virial theorem to be invoked subsequently is[12]

$$\int_{r=r_s} \mathbf{r}\cdot\boldsymbol{\tau}\cdot\mathbf{n}\, r d\theta + W_B = \tfrac{1}{3}\pi^2 J_0^2 \{-\tfrac{1}{2}(c^4 - b^4) + [c + (c^3 - b^3) r_s^{-2}](c^3 - b^3) - b^3(c - b)\}, \tag{12}$$

where $\mathbf{n}$ is the outward normal at $r = r_s$.

DISPLACEMENT EQUATIONS

The equilibrium condition of Eq. (1) becomes[9]

$$\frac{\partial \sigma_{rr}}{\partial r} + \frac{1}{r}(\sigma_{rr} - \sigma_{\theta\theta}) + \frac{1}{r}\frac{\partial \sigma_{\theta\theta}}{\partial \theta} - J_z B_\theta = 0, \tag{13}$$

$$\frac{\partial \sigma_{r\theta}}{\partial r} + \frac{2}{r}\sigma_{r\theta} + \frac{1}{r}\frac{\partial \sigma_{\theta\theta}}{\partial \theta} + J_z B_r = 0, \tag{14}$$

under the functional limitations described in Eq. (2). Using Eq. (3) and remembering that ε_{zz} is a constant

$$\frac{\partial}{\partial r}(C_{11}\varepsilon_{rr} + C_{12}\varepsilon_{\theta\theta}) + \frac{1}{r}[(C_{11} - C_{12})(\varepsilon_{rr} - k_1) + (C_{12} - C_{22})(\varepsilon_{\theta\theta} - k_2) + (C_{13} - C_{23})(\varepsilon_{zz} - k_3)] + C_{44}\frac{1}{r}\frac{\partial \varepsilon_{r\theta}}{\partial \theta} - J_z B_\theta = 0, \tag{15}$$

$$C_{44}\frac{\partial \varepsilon_{r\theta}}{\partial r} + 2C_{44}\frac{\varepsilon_{r\theta}}{r} + \frac{1}{r}\frac{\partial}{\partial \theta}(C_{12}\varepsilon_{rr} + C_{22}\varepsilon_{\theta\theta}) + J_z B_r = 0. \tag{16}$$

Finally from Eq. (6)

$$C_{11}\frac{\partial^2 u_r}{\partial r^2} + (C_{11}-C_{12})\frac{1}{r}\frac{\partial u_r}{\partial r} + \left[C_{12}\frac{\partial}{\partial r} + (C_{12}-C_{22})\frac{1}{r}\right] \times \left(\frac{u_r}{r} + \frac{1}{r}\frac{\partial u_\theta}{\partial \theta}\right) + \tfrac{1}{2}C_{44}\frac{1}{r}\frac{\partial}{\partial \theta} \times \left(\frac{\partial u_\theta}{\partial r} - \frac{u_\theta}{r} + \frac{1}{r}\frac{\partial u_\theta}{\partial \theta}\right) + (C_{13}-C_{23})\frac{1}{r}\varepsilon_{zz} - [(C_{11}-C_{12})k_1 + (C_{12}-C_{22})k_2 + (C_{13}-C_{23})k_3]\frac{1}{r} - J_z B_\theta = 0, \quad (17)$$

and

$$C_{44}\left(\tfrac{1}{2}\frac{\partial}{\partial r} + \frac{1}{r}\right)\left(\frac{\partial u_\theta}{\partial r} - \frac{u_\theta}{r} + \frac{1}{r}\frac{\partial u_r}{\partial \theta}\right) + \frac{1}{r}\frac{\partial}{\partial \theta}\left[C_{12}\frac{\partial u_r}{\partial r} + C_{22}\left(\frac{u_r}{r} + \frac{1}{r}\frac{\partial u_\theta}{\partial \theta}\right)\right] + J_z B_r = 0. \quad (18)$$

FORM OF SOLUTION

The Lorentz force excitation given in Eqs. (8–9) indicates that the displacements in Eqs. (17–18) must have the form

$$u_r = P_0(r) + P_2(r)\cos 2\theta$$
$$u_\theta = Q_0(r)\theta + Q_2(r)\sin 2\theta. \quad (19)$$

Substituting these into Eqs. (17–18) and utilizing Eqs. (8–9) gives for the coefficients of θ-independent terms

$$C_{11}P_0'' + C_{11}\frac{1}{r}P_0' + C_{12}\frac{1}{r}Q_0' - C_{22}\frac{1}{r^2}(P_0 + Q_0) + \tfrac{1}{2}C_{44}\left(\frac{1}{r}Q_0' - \frac{1}{r^2}Q_0\right) = \frac{\pi}{3}J_0^2(4r - 3\lambda - b^3 r^{-2}) - (C_{13}-C_{23})\frac{1}{r}\varepsilon_{zz} + [(C_{11}-C_{12})k_1 + (C_{12}-C_{22})k_2 + (C_{13}-C_{23})k_3]\frac{1}{r}, \quad (20)$$

$$\tfrac{1}{2}\left(Q_0'' + \frac{1}{r}Q_0' - \frac{1}{r^2}Q_0\right) = 0. \quad (21)$$

And, for the coefficients of the θ-dependent terms

$$C_{11}P_2'' + C_{11}\frac{1}{r}P_2' - (C_{22}+2C_{44})\frac{1}{r^2}P_2 + (2C_{12}+C_{44})\frac{1}{r}Q_2' - (2C_{22}+C_{44})\frac{1}{r^2}Q_2 = \frac{\pi}{3}J_0^2(4r - 3\lambda - b^3 r^{-2}), \quad (22)$$

$$\tfrac{1}{2}C_{44}Q_2'' + \tfrac{1}{2}C_{44}\frac{1}{r}Q_2' - (4C_{22}+\tfrac{1}{2}C_{44})\frac{1}{r^2}Q_2 - (2C_{12}+C_{44})\frac{1}{r}P_2' - (2C_{22}+C_{44})\frac{1}{r^2}P_2 = -\frac{\pi}{3}J_0^2(2r - 3\lambda + b^3 r^{-2}). \quad (23)$$

SOLUTIONS OF HOMOGENEOUS EQUATIONS

Special solutions of Eqs. (20–21) related to a dislocation[13] ($Q_0 \neq 0$) are

$$P_0 = \frac{C_{12}-C_{22}}{C_{11}-C_{22}}\,Gr \qquad Q_0 = Gr. \quad (24)$$

The regular solutions of Eqs. (20–21) in which $Q_0 = 0$ then become

$$P_0 = Ar^p + Br^{-p} \qquad Q_0 = 0, \quad (25)$$

where

$$p = \sqrt{\frac{C_{22}}{C_{11}}}\,. \quad (26)$$

Solutions of Eqs. (22–23) are of the form

$$P_2 = Cr^s \qquad Q_2 = Dr^s. \quad (27)$$

Substituting these forms into Eqs. (22–23) gives

$$(C_{11}s^2 - C_{22} - 2C_{44})C + [(2C_{12} + C_{44})s - (2C_{22}+C_{44})]D = 0, \quad (28)$$

$$[-(2C_{12}+C_{44})s - (2C_{22}+C_{44})]C + (\tfrac{1}{2}C_{44}s^2 - 4C_{22} - \tfrac{1}{2}C_{44})D = 0. \quad (29)$$

For compatibility

$$s^2 = \tfrac{1}{2}\left(8\frac{C_{22}}{C_{44}} - 8\frac{C_{12}^2}{C_{11}C_{44}} - 8\frac{C_{12}}{C_{11}} + \frac{C_{22}}{C_{11}} + 1\right)$$

$$\pm \tfrac{1}{2}\sqrt{\left(8\frac{C_{22}}{C_{44}} - 8\frac{C_{12}^2}{C_{11}C_{44}} - 8\frac{C_{12}}{C_{11}} + \frac{C_{22}}{C_{11}} + 1\right)^2 - 36\frac{C_{22}}{C_{11}}} \qquad (30)$$

Let

$$s_1 = s^{(+)} \quad s_2 = -s^{(+)} \quad s_3 = s^{(-)} \quad s_4 = -s^{(-)} \qquad (31)$$

where $s^{(+)}$ is the solution of Eq. (30) using the positive root and $s^{(-)}$ is the solution using the negative root. From Eq. (29) the relation between the coefficients may be taken as

$$C_k = \frac{\tfrac{1}{2}(s_k^2-1)C_{44}-4C_{22}}{(2C_{12}+C_{44})s_k + 2C_{22}+C_{44}} \; D_k \equiv \gamma_k D_k. \qquad (32)$$

Then

$$P_2 = \gamma_1 D_1 r^{s1} + \gamma_2 D_2 r^{s2} + \gamma_3 D_3 r^{s3} + \gamma_4 D_4 r^{s4}, \qquad (33)$$

$$Q_2 = D_1 r^{s1} + D_2 r^{s2} + D_3 r^{s3} + D_4 r^{s4}. \qquad (34)$$

PARTICULAR SOLUTION

Since each term of the right-hand side of Eq. (20) has the form Br^q and since the right-hand side of Eq. (21) is zero, one may take $P_0 = Ar^p$, $Q_0 = 0$, where $p = q+2$ and

$$A = \frac{B}{C_{11}p^2-C_{22}} \,. \qquad (35)$$

Adding the contributions for $q = 1, 0, -1, -2$ gives

$$P_0 = \frac{4\pi}{3} J_0^2 \frac{r^3}{9C_{11}-C_{22}} - \pi J_0^2 \frac{\lambda r^2}{4C_{11}-C_{22}} + \frac{\pi}{3} J_0^2 \frac{b^3}{C_{22}} - \frac{C_{13}-C_{23}}{C_{11}-C_{22}} \varepsilon_{zz} r$$
$$+ \frac{(C_{11}-C_{12})k_1+(C_{12}-C_{22})k_2+(C_{13}-C_{23})k_3}{C_{11}-C_{22}} r, \qquad (36)$$

$$Q_0 = 0. \qquad (37)$$

Each term of the right-hand side of Eqs. (22–23) is of the form $\left(\frac{E}{F}\right)r^q$ which then admits solutions of the form $\left(\frac{C}{D}\right)r^p$ where $p = q+2$. Thus

$$\begin{pmatrix} p^2C_{11}-C_{22}-2C_{44} & (2C_{12}+C_{44})p - (2C_{22}+C_{44}) \\ -(2C_{12}+C_{44})p - (2C_{22}+C_{44}) & \tfrac{1}{2}(p^2-1)C_{44}-4C_{22} \end{pmatrix} \begin{pmatrix} C \\ D \end{pmatrix} = \begin{pmatrix} E \\ F \end{pmatrix} . \qquad (38)$$

The determinant of the coefficients is

$$\Delta(p) = \tfrac{1}{2}C_{11}C_{44}\left\{ p^4 - \left[8\frac{C_{22}}{C_{44}} - 8\frac{C_{12}^2}{C_{11}C_{44}} - 8\frac{C_{12}}{C_{11}} + \frac{C_{22}}{C_{11}} + 1\right] p^2 + 9\frac{C_{22}}{C_{11}} \right\}. \qquad (39)$$

Inversion of Eq. (38) gives

$$\begin{pmatrix} C \\ D \end{pmatrix} = \begin{pmatrix} \tfrac{1}{2}(p^2-1)C_{44}-4C_{22} & -(2C_{12}+C_{44})p+2C_{22}+C_{44} \\ (2C_{12}+C_{44})p+2C_{22}+C_{44} & p^2C_{11}-C_{22}-2C_{44} \end{pmatrix} \begin{pmatrix} E \\ F \end{pmatrix} \tag{40}$$

From Eqs. (22—23) one see that

$$\begin{array}{cccc} q & 1 & 0 & -2 \\ p & 3 & 2 & 0 \\ E & \frac{4\pi}{3}J_0^2 & -\pi J_0^2\lambda & -\frac{\pi}{3}J_0^2b^3 \\ F & -\frac{2\pi}{3}J_0^2 & \pi J_0^2\lambda & -\frac{\pi}{3}J_0^2b^3 \end{array} \tag{41}$$

DISPLACEMENT

The particular solution may be thought of as an incremental displacement to be added to the solution of the homogeneous equations. However, since ε_{zz} is an unknown this contribution of the particular solution will be transferred to the homogeneous category. After assembling all the contributions

$$\Delta u_r = \frac{\pi}{3}J_0^2\left[\frac{4r^3}{9C_{11}-C_{22}} - \frac{3\lambda r^2}{4C_{11}-C_{22})} + \frac{b^3}{C_{22}}\right]$$

$$+ \frac{(C_{11}-C_{12})k_1+(C_{12}-C_{22})k_2+(C_{13}-C_{23})k_3}{C_{11}-C_{22}}\, r$$

$$+ \frac{\pi J_0^2}{3\Delta(3)}\,[12C_{12}-20C_{22}+20C_{44}]r^3\cos 2\theta$$

$$+ \frac{\pi J_0^2}{3\Delta(2)}\left[-12C_{12}+18C_{22}-\frac{15}{2}C_{44}\right]\lambda r^2\cos 2\theta$$

$$+ \frac{\pi J_0^2}{3\Delta(0)}\,[2C_{22}-\tfrac{1}{2}C_{44}]b^3\cos 2\theta, \tag{42}$$

$$\Delta u_\theta = \frac{\pi J_0^2}{3\Delta(3)}\,[-18C_{11}+24C_{12}+10C_{22}$$

$$+ 20C_{44}]r^3\sin 2\theta$$

$$+ \frac{\pi J_0^2}{3\Delta(2)}\,[12C_{11}-12C_{12}-9C_{22}$$

$$-\ 15C_{44}]\lambda r^2\sin 2\theta$$

$$+ \frac{\pi J_0^2}{3\Delta(0)}\,[-C_{22}+C_{44}]b^3\sin 2\theta. \tag{43}$$

STRAIN

The incremental strain is given by applying Eq. (6) to Eqs. (42–43)

$$\Delta\varepsilon_{rr} = \frac{\pi J_0^2}{3}\left[\frac{12r^2}{9C_{11}-C_{22}} - \frac{6\lambda r}{4C_{11}-C_{22}}\right]$$

$$+ \frac{(C_{11}-C_{12})k_1+(C_{12}-C_{22})k_2+(C_{13}-C_{23})k_3}{C_{11}-C_{22}}$$

$$+ \frac{\pi J_0^2}{3\Delta(3)}\,[36C_{12}-60C_{22}+60C_{44}]r^2\cos 2\theta$$

$$+ \frac{\pi J_0^2}{3\Delta(2)}\,[-24C_{12}+36C_{22}-15C_{44}]\lambda r\cos 2\theta \tag{44}$$

$$\Delta\varepsilon_{\theta\theta} = \frac{\pi J_0^2}{3}\left[\frac{4r^2}{9C_{11}-C_{22}} - \frac{3\lambda r}{4C_{11}-C_{22}} + \frac{b^3r^{-1}}{C_{22}}\right]$$

$$+ \frac{(C_{11}-C_{12})k_1+(C_{12}-C_{22})k_2+(C_{13}-C_{23})k_3}{C_{11}-C_{22}}$$

$$+ \frac{\pi J_0^2}{3\Delta(3)}\,[-36C_{11}+60C_{12}+60C_{44}]r^2\cos 2\theta$$

$$+\ \frac{\pi J_0^2}{3\Delta(2)}\left[24C_{11}-36C_{12}-\frac{75}{2}\,C_{44}\right]\lambda r\cos 2\theta$$

$$+\ \frac{\pi J_0^2}{3\Delta(0)}\cdot {}^3/_2\, C_{44}\, b^3 r^{-1}\cos 2\theta, \qquad (45)$$

$$\Delta\varepsilon_{r\theta}=\frac{\pi J_0^2}{3\Delta(3)}\,[-18C_{11}+12C_{12}+30C_{22}]r^2\sin 2\theta$$

$$+\ \frac{\pi J_0^2}{3\Delta(2)}\left[6C_{11}+6C_{12}-\frac{45}{2}\,C_{22}\right]\lambda r\sin 2\theta$$

$$-\ \frac{\pi J_0^2}{3\Delta(0)}\cdot {}^3/_2\, C_{22}\, b^3 r^{-1}\sin 2\theta. \qquad (46)$$

STRESS

The incremental stress is given by applying Eq. (3) to Eqs. (44–46) except that ε_{zz} is set to zero since its contribution will be taken into the homogeneous portion of the solution.

$$\Delta\sigma_{rr}=\frac{\pi}{3}\,J_0^2\left[\frac{12C_{11}+4C_{12}}{9C_{11}-C_{22}}\,r^2\right.$$

$$\left.-\ \frac{6C_{11}+3C_{12}}{4C_{11}-C_{22}}\,\lambda r+\frac{C_{12}}{C_{22}}\,b^3r^{-1}\right]$$

$$-\ \frac{C_{11}C_{22}-C_{12}^2}{C_{11}-C_{22}}(k_2-k_1)-\left[C_{13}-\frac{C_{13}-C_{23}}{C_{11}-C_{22}}\right.$$

$$\left.\times\ (C_{11}+C_{12})\right]k_3+\frac{\pi J_0^2}{3\Delta(3)}\,[-60C_{11}C_{22}$$

$$+\ 60C_{12}^2+60C_{11}C_{44}+60C_{12}C_{44}]r^2\cos 2\theta$$

$$+\ \frac{\pi J_0^2}{3\Delta(2)}\,[36C_{11}C_{22}-36C_{12}^2-15C_{11}C_{44}$$

$$-\ \frac{75}{2}\,C_{12}C_{44}]\lambda r\cos 2\theta$$

$$+\ \frac{\pi J_0^2}{3\Delta(0)}\cdot\frac{3}{2}\,C_{12}C_{44}b^3r^{-1}\cos 2\theta \qquad (47)$$

$$\Delta\sigma_{\theta\theta}=\frac{\pi}{3}J_0^2\left[\frac{12C_{12}+4C_{22}}{9C_{11}-C_{22}}\,r^2-\frac{6C_{12}+3C_{22}}{4C_{11}-C_{22}}\right.$$

$$\left.\times\ \lambda r+b^3r^{-1}\right]-\frac{C_{11}C_{22}-C_{12}^2}{C_{11}-C_{22}}(k_2-k_1)$$

$$-\left[C_{23}-\frac{C_{13}-C_{23}}{C_{11}-C_{22}}\,(C_{12}+C_{22})\right]k_3$$

$$+\ \frac{\pi J_0^2}{3\Delta(3)}\,[-36C_{11}C_{22}+36C_{12}^2+60C_{12}C_{44}$$

$$+\ 60C_{22}C_{44}\Big]\ r^2\cos 2\theta$$

$$+\ \frac{\pi J_0^2}{3\Delta(2)}\,[24C_{11}C_{22}-24C_{12}^2-15C_{12}C_{44}$$

$$-\ \frac{75}{2}\,C_{22}C_{44}\Big]\ \lambda r\cos 2\theta+\frac{\pi J_0^2}{3\Delta(0)}$$

$$\cdot\ \frac{3}{2}\,C_{22}C_{44}b^3r^{-1}\cos 2\theta, \qquad (48)$$

$$\Delta\sigma_{zz}=\frac{\pi}{3}\,J_0^2\left[\frac{12C_{13}+4C_{23}}{9C_{11}-C'_{22}}\,r^2\right.$$

$$\left.-\ \frac{6C_{13}+3C_{23}}{4C_{11}-C_{22}}\,\lambda r+\frac{C_{23}}{C_{22}}\,b^3r^{-1}\right]$$

$$+\ \frac{C_{13}(C_{12}-C_{22})+C_{23}(C_{12}-C_{11})}{C_{11}-C_{22}}\,(k_2-k_1)$$

$$-\left[C_{33}-\frac{C_{13}^2-C_{23}^2}{C_{11}-C_{22}}\right]k_3+\frac{\pi J_0^2}{3\Delta(3)}\Big[36C_{12}C_{13}$$

$$-\ 36C_{11}C_{23}+60C_{12}C_{23}+60C_{13}C_{22}$$

$$+\ 60C_{13}C_{44}+60C_{23}C_{44}\Big]r^2\cos 2\theta$$

$$+\ \frac{\pi J_0^2}{3\Delta(2)}\,[-24C_{12}C_{13}+24C_{11}C_{23}$$

$$-\ 36C_{12}C_{23}+36C_{13}C_{22}-15C_{13}C_{44}$$

$$- \frac{75}{2} C_{23}C_{44}]\lambda r \cos 2\theta + \frac{\pi J_0^2}{3\Delta(0)} \cdot \frac{3}{2} C_{23}C_{44}b^3r^{-1} \cos 2\theta, \quad (49)$$

$$\Delta\sigma_{r\theta} = \frac{\pi J_0^2}{3\Delta(3)} [-18C_{11}C_{44}+12C_{12}C_{44} + 30C_{22}C_{44}]r^2 \sin 2\theta + \frac{\pi J_0^2}{3\Delta(2)} [6C_{11}C_{44} + 6C_{12}C_{44} - \frac{45}{2} C_{22}C_{44}]\lambda r \sin 2\theta - \frac{\pi J_0^2}{3\Delta(0)} \cdot \frac{3}{2} C_{22}C_{44}b^3r^{-1} \sin 2\theta. \quad (50)$$

COMPLETE SOLUTION

Collecting together the various partial solutions

$$u_r = Ar^p + Br^{-p} - \frac{C_{12}-C_{22}}{C_{11}-C_{22}} Gr - \frac{C_{13}-C_{23}}{C_{11}-C_{22}} \varepsilon_{zz}r + \Delta u_{r0} + (\gamma_1 Cr^{s_1} + \gamma_2 Dr^{s_2} + \gamma_3 Er^{s_3} + \gamma_4 Fr^{s_4} + \Delta u_{r2}) \cos 2\theta, \quad (51)$$

$$u_\theta = Gr\theta + (Cr^{s_1} + Dr^{s_2} + Er^{s_3} + Fr^{s_4} + \Delta u_{\theta 2}) \sin 2\theta, \quad (52)$$

where p is given by Eq. (26), Δu_{r0} is the term independent of theta in Eq. (42), Δu_{r2} is the coefficient of $\cos 2\theta$ in Eq. (42), and $\Delta u_{\theta 2}$ is the coefficient of $\sin 2\theta$ in Eq. (43).

The complete expressions for strain follow from Eq. (6)

$$\varepsilon_{rr} = pAr^{p-1} - pBr^{-p-1} - \frac{C_{12}-C_{22}}{C_{11}-C_{22}} G - \frac{C_{13}-C_{23}}{C_{11}-C_{22}} \varepsilon_{zz} + \Delta\varepsilon_{rr0} + (s_1\gamma_1 Cr^{s_1-1} + s_2\gamma_2 Dr^{s_2-1} + s_3\gamma_3 Er^{s_3-1} + s_4\gamma_4 Fr^{s_4-1} + \Delta\varepsilon_{rr2}) \cos 2\theta, \quad (53)$$

$$\varepsilon_{\theta\theta} = Ar^{p-1} + Br^{-p-1} + \frac{C_{11}-C_{12}}{C_{11}-C_{22}} G - \frac{C_{13}-C_{23}}{C_{11}-C_{22}} \varepsilon_{zz} + \Delta\varepsilon_{\theta\theta 0} + [(\gamma_1+2)Cr^{s_1-1} + (\gamma_2+2)Dr^{s_2-1} + (\gamma_3+2)Er^{s_3-1} + (\gamma_4+2)Fr^{s_4-1} + \Delta\varepsilon_{\theta\theta 2}] \cos 2\theta, \quad (54)$$

$$\varepsilon_{r\theta} = \{[½(s_1-1)-\gamma_1]Cr^{s_1-1} + [½(s_2-1)-\gamma_2]Dr^{s_2-1} + [½(s_3-1)-\gamma_3]Er^{s_3-1} + [½(s_4-1)-\gamma_4]Fr^{s_4-1} + \Delta\varepsilon_{r\theta 2}\} \sin 2\theta, (55)$$

where $\Delta\varepsilon_{rr0}$ is the term independent of theta in Eq. (44), $\Delta\varepsilon_{\theta\theta 0}$ is ther independent of theta in Eq. (45), and $\Delta\varepsilon_{\theta\theta 2}$, $\Delta\varepsilon_{r\theta 2}$ are coefficients of $\cos 2\theta$ in Eqs. (44–45) and of $\sin 2\theta$ in Eq. (46).

Finally the complete expressions for stress follow from Eq. (3)

$$\sigma_{rr} = (pC_{11}+C_{12})Ar^{p-1} + (-pC_{11}+C_{12})Br^{-p-1} + \frac{C_{11}C_{12}-C_{12}^2}{C_{11}-C_{22}} G + \left[C_{13} - \frac{C_{13}-C_{23}}{C_{11}-C_{22}} \times (C_{11}+C_{12})\right] \varepsilon_{zz} + \Delta\sigma_{rr0} + \{[C_{11}s_1\gamma_1+C_{12}(\gamma_1+2)]Cr^{s_1-1} + [C_{11}s_2\gamma_2+C_{12}(\gamma_2+2)]Dr^{s_2-1} + [C_{11}s_3\gamma_3+C_{12}(\gamma_3+2)]Er^{s_3-1} + [C_{11}s_4\gamma_4+C_{12}(\gamma_4+2)]Fr^{s_4-1} + \Delta\sigma_{rr2}\}\cos 2\theta, \quad (56)$$

$$\sigma_{\theta\theta} = (pC_{12}+C_{22})Ar^{p-1}+(-pC_{12}+C_{22})Br^{-p-1} + \frac{C_{11}C_{22}-C_{12}^2}{C_{11}-C_{22}} G + \left[C_{23} - \frac{C_{13}-C_{23}}{C_{11}-C_{22}} \times(C_{12}+C_{22})\right]\varepsilon_{zz}+\Delta\sigma_{\theta\theta 0}$$

$$+ \{[C_{12}s_1\gamma_1+C_{22}(\gamma_1+2)]Cr^{s_1-1}$$
$$+ [C_{12}s_2\gamma_2+C_{22}(\gamma_2+2)]Dr^{s_2-1}$$
$$+ [C_{12}s_3\gamma_3+C_{22}(\gamma_3+2)]Er^{s_3-1}$$
$$+ [C_{12}s_4\gamma_4+C_{22}(\gamma_4+2)]Fr^{s_4-1}$$
$$+ \Delta\sigma_{\theta\theta 2}\}\cos 2\theta, \quad (57)$$

$$\sigma_{zz} = (pC_{13}+C_{23})Ar^{p-1} + (-pC_{13}+C_{23})Br^{-p-1}$$
$$- \frac{C_{13}(C_{12}-C_{22}) + C_{23}(C_{12}-C_{11})}{C_{11}-C_{22}} G$$
$$+ \left[C_{33} - \frac{C_{13}-C_{23}}{C_{11}-C_{22}} (C_{13}+C_{23})\right] \varepsilon_{zz} + \Delta\sigma_{zz0}$$
$$+ \{[C_{13}s_1\gamma_1+C_{23}(\gamma_1+2)]Cr^{s_1-1}$$
$$+ [C_{13}s_2\gamma_2+C_{23}(\gamma_2+2)]Dr^{s_2-1}$$
$$+ [C_{13}s_3\gamma_3+C_{23}(\gamma_3+2)]Er^{s_3-1}$$
$$+ [C_{13}s_4\gamma_4+C_{23}(\gamma_4+2)]Fr^{s_4-1} + \Delta\sigma_{zz2}\}\cos 2\theta, \quad (58)$$

$$\sigma_{r\theta} = \{C_{44}[\tfrac{1}{2}(s_1-1)-\gamma_1]Cr^{s_1-1}$$
$$+ C_{44}[\tfrac{1}{2}(s_2-1)-\gamma_2]Dr^{s_2-1}$$
$$+ C_{44}[\tfrac{1}{2}(s_3-1)-\gamma_3]Er^{s_3-1}$$
$$+ C_{44}[\tfrac{1}{2}(s_4-1)-\gamma_4]Fr^{s_4-1} + \Delta\sigma_{r\theta 2}\} \sin 2\theta, \quad (59)$$

where $\Delta\sigma_{rr0}$, $\Delta\sigma_{\theta\theta 0}$, $\Delta\sigma_{zz0}$ are the theta-independent contributions from Eqs. (47–49) and $\Delta\sigma_{rr2}$, $\Delta\sigma_{\theta\theta 2}$, $\Delta\sigma_{zz2}$, $\Delta\sigma_{r\theta 2}$ are the coefficients of the theta-dependent contribution from Eqs. (47–50).

BOUNDARY CONDITIONS

The complete solution for displacement, Eqs. (51–52) describes a state of elasticity involving seven unknowns in each region of interest. In the present problem three regions are utilized as shown in Fig. 1. Region 1 is between $r = a$ and $r = b$. Region 2 is between $r = b$ and $r = c$. Region 3 is between $r = c$ and $r = d$. Thus the unknowns are $A_1B_1G_1C_1D_1E_1F_1A_2B_2G_2C_2D_2E_2F_2A_3B_3G_3C_3D_3E_3F_3$) and ε_{zz} which by assumption is common to all regions making twenty-two unknowns in all. The coefficient G is out of sequence only because pre-strain was the last effect to be included.

Beam-line dipoles generally make structural use of the inner bore tube, whereas the Doubler dipole makes no structural use of its vacuum chamber. The Doubler case may be calculated from the beam-line case by choosing sufficiently weak elastic constants in the bore-tube region.

Boundary conditions require the traction and displacement to be continuous. One exception is allowed however. The notion of a dislocation or discontinuity in displacement[13] is useful in characterizing pre-strain. Thus in detail:

At $r = a$, the innermost boundary

$$\sigma_{rr}^{(+)} = \sigma_{r\theta}^{(+)} = 0, \quad (60)$$

dislocation

$$u_\theta^{(+)}(2\pi) - u_\theta^{(+)}(0) = a\alpha_1. \quad (61)$$

At $r = b$,

$$\sigma_{rr}^{(+)} - \sigma_{rr}^{(-)} = \sigma_{r\theta}^{(+)} - \sigma_{r\theta}^{(-)} = 0, \quad (62)$$

dislocation

$$u_\theta^{(+)}(2\pi) - u_\theta^{(+)}(0) = b\alpha_2, \quad (63)$$

after deleting dislocation

$$u_r^{(+)} - u_r^{(-)} = u_\theta^{(+)} - u_\theta^{(-)} = 0. \quad (64)$$

At $r = c$

$$\sigma_{rr}^{(+)} - \sigma_{rr}^{(-)} = \sigma_{r\theta}^{(+)} - \sigma_{r\theta}^{(-)} = 0, \quad (65)$$

dislocation

$$u_\theta^{(+)}(2\pi) - u_\theta^{(+)}(0) = c\alpha_3, \quad (66)$$

after deleting dislocation

$$u_r^{(+)} - u_r^{(-)} = u_\theta^{(+)} - u_\theta^{(-)} = 0. \quad (67)$$

At $r = d$, the outermost boundary

$$\sigma_{rr}^{(-)} = \sigma_{r\theta}^{(-)} = 0. \quad (68)$$

APPLICATION OF BOUNDARY CONDITIONS

There are twenty-one relations obtainable from Eqs. (60–68). However, one global relation may be obtained from the virial theorem thereby providing as many conditions as unknowns. As regards notation all quantities in the general development will now have an additional subscript since it is necessary to distinguish each region. This additional subscript will be the last one, i.e., C_{121} will refer to the elastic constant C_{12} in region 1. Equation (60) gives three relations

$$(p_1C_{111}+C_{121})A_1a^{p_1-1} + (-p_1C_{111}+C_{121}) \times B_1a^{-p_1-1} + \frac{C_{111}C_{221}-C_{121}^2}{C_{111}-C_{221}} G_1 + \left[C_{131} - \frac{C_{131}-C_{231}}{C_{111}-C_{221}} (C_{111}-C_{121})\right] \varepsilon_{zz} = -\Delta\sigma_{rr0}(a^+), \quad (69)$$

$$[C_{111}s_{11}\gamma_{11}+C_{121}(\gamma_{11}+2)]C_1a^{s_{11}-1} + [C_{111}s_{21}\gamma_{21}+C_{121}(\gamma_{21}+2)]D_1a^{s_{21}-1} + [C_{111}s_{31}\gamma_{31}+C_{121}(\gamma_{31}+2)]E_1a^{s_{31}-1} + [C_{111}s_{41}\gamma_{41}+C_{121}(\gamma_{41}+2)]F_1a^{s_{41}-1} = -\Delta\sigma_{rr2}(a^+), \quad (70)$$

$$C_{441}[\tfrac{1}{2}(s_{11}-1)-\gamma_{11}]C_1a^{s_{11}-1} + C_{441}[\tfrac{1}{2}(s_{21}-1)-\gamma_{21}]D_1a^{s_{21}-1} + C_{441}[\tfrac{1}{2}(s_{31}-1)-\gamma_{31}]E_1a^{s_{31}-1} + C_{441}[\tfrac{1}{2}(s_{41}-1)-\gamma_{41}]F_1a^{s_{41}-1} = -\Delta\sigma_{r\theta 2}(a^+). \quad (71)$$

Equation (61) gives

$$2\pi G_1a = a\alpha_1. \quad (72)$$

Equation (62) gives three relations

$$(p_2C_{112}+C_{122})A_2b^{p_2-1} + (-p_2C_{112}+C_{122})B_2b^{-p_2-1} + \frac{C_{112}C_{222}-C^2_{122}}{C_{112}-C_{222}} G_2 - (p_1C_{111}+C_{121})A_1b^{p_1-1} - (-p_1C_{111}+C_{121})B_1b^{-p_1-1} - \frac{C_{111}C_{221}-C^2_{121}}{C_{111}-C_{121}} G_1$$
$$+ \left[C_{132} - \frac{C_{132}-C_{232}}{C_{112}-C_{222}} (C_{112}-C_{122})-C_{131} + \frac{C_{131}-C_{231}}{C_{111}-C_{221}} (C_{111}-C_{121})\right] \varepsilon_{zz} = -\Delta\sigma_{rr0}(b^+)+\Delta\sigma_{rr0}(b^-), \quad (73)$$

$$[C_{112}s_{12}\gamma_{12}+C_{122}(\gamma_{12}+2)]C_2b^{s_{12}-1} + [C_{112}s_{22}\gamma_{22}+C_{122}(\gamma_{22}+2)]D_2b^{s_{22}-1} + [C_{112}s_{32}\gamma_{32}+C_{122}(\gamma_{32}+2)]E_2b^{s_{32}-1} + [C_{112}s_{42}\gamma_{42}+C_{122}(\gamma_{42}+2)]F_2b^{s_{42}-1}$$
$$- [C_{111}s_{11}\gamma_{11}+C_{121}(\gamma_{11}+2)]C_1b^{s_{11}-1} - [C_{111}s_{21}\gamma_{21}+C_{121}(\gamma_{21}+2)]D_1b^{s_{21}-1} - [C_{111}s_{31}\gamma_{31}+C_{121}(\gamma_{31}+2)]E_1b^{s_{31}-1} - [C_{111}s_{41}\gamma_{41}+C_{121}(\gamma_{41}+2)]F_1b^{s_{41}-1} = - \Delta\sigma_{rr2}(b^+) + \Delta\sigma_{rr2}(b^-), \quad (74)$$

$$C_{442}[\tfrac{1}{2}(s_{12}-1)-\gamma_{12}]C_2b^{s_{12}-1} + C_{442}[\tfrac{1}{2}(s_{22}-1)-\gamma_{22}]D_2b^{s_{22}-1} + C_{442}[\tfrac{1}{2}(s_{32}-1)-\gamma_{32}]E_2b^{s_{32}-1} + C_{442}[\tfrac{1}{2}(s_{42}-1)-\gamma_{42}]F_2b^{s_{42}-1}$$
$$- C_{441}[\tfrac{1}{2}(s_{11}-1)-\gamma_{11}]C_1b^{s_{11}-1} - C_{441}[\tfrac{1}{2}(s_{21}-1)-\gamma_{21}]D_1b^{s_{21}-1} - C_{441}[\tfrac{1}{2}(s_{31}-1)-\gamma_{31}]E_1b^{s_{31}-1} - C_{441}[\tfrac{1}{2}(s_{41}-1)-\gamma_{41}]F_1b^{s_{41}-1} = -\Delta\sigma_{r\theta 2}(b^+) + \Delta\sigma_{r\theta 2}(b^-). \quad (75)$$

Equation (63) gives

$$2\pi G_2b = b\alpha_2. \quad (76)$$

Equation (64) gives three relations

$$A_2b^{p_2}+B_2b^{-p_2} - \frac{C_{122}-C_{222}}{C_{112}-C_{222}} bG_2 - \frac{C_{132}-C_{232}}{C_{112}-C_{222}} b\varepsilon_{zz} - A_1b^{p_1}-B_1b^{-p_1}$$

$$+ \frac{C_{121}-C_{221}}{C_{111}-C_{221}} bG_1 + \frac{C_{131}-C_{231}}{C_{111}-C_{221}} b\varepsilon_{zz} = -\Delta u_{r0}(b^+) + \Delta u_{r0}(b^-), \quad (77)$$

$$\gamma_{12}C_2b^{s_{12}} + \gamma_{22}D_2b^{s_{22}} + \gamma_{32}E_2b^{s_{32}} + \gamma_{42}F_2b^{s_{42}} - \gamma_{11}C_1b^{s_{11}} - \gamma_{21}D_1b^{s_{21}} - \gamma_{31}E_1b^{s_{31}} - \gamma_{41}F_1b^{s_{41}} = -\Delta u_{r2}(b^+) + \Delta u_{r2}(b^-), \quad (78)$$

$$C_2b^{s_{12}} + D_2b^{s_{22}} + E_2b^{s_{32}} + F_2b^{s_{42}} - C_1b^{s_{11}} - D_1b^{s_{21}} - E_1b^{s_{31}} - F_1b^{s_{41}} = -\Delta u_{\theta 2}(b^+) + \Delta u_{\theta 2}(b^-). \quad (79)$$

Equation (65) gives three relations

$$(p_3C_{113}+C_{123})A_3c^{p_3-1} + (-p_3C_{113}+C_{123})B_3c^{-p_3-1} + \frac{C_{113}C_{223}-C_{123}^2}{C_{113}-C_{223}} G_3 - (p_2C_{112}+C_{122})A_2c^{p_2-1} - (-p_2C_{112}+C_{122})B_2c^{-p_2-1} - \frac{C_{112}C_{222}-C_{122}^2}{C_{112}-C_{222}} G_2$$
$$+ \left[C_{133} - \frac{C_{133}-C_{233}}{C_{113}-C_{223}} (C_{113}+C_{123}) - C_{132} + \frac{C_{132}-C_{232}}{C_{112}-C_{222}} (C_{112}+C_{122})\right] \varepsilon_{zz} = -\Delta\sigma_{rr0}(c^+) + \Delta\sigma_{rr0}(c^-), \quad (80)$$

$$[C_{113}s_{13}\gamma_{13}+C_{123}(\gamma_{13}+2)]C_3c^{s_{13}-1} + [C_{113}s_{23}\gamma_{23}+C_{123}(\gamma_{23}+2)]D_3c^{s_{23}-1} + [C_{113}s_{33}\gamma_{33}+C_{123}(\gamma_{33}+2)]E_3c^{s_{33}-1} + [C_{113}s_{43}\gamma_{43}+C_{123}(\gamma_{43}+2)]F_3c^{s_{43}-1}$$
$$- [C_{112}s_{12}\gamma_{12}+C_{122}(\gamma_{12}+2)]C_2c^{s_{12}-1} - [C_{112}s_{22}\gamma_{22}+C_{122}(\gamma_{22}+2)]D_2c^{s_{22}-1} - [C_{112}s_{32}\gamma_{32}+C_{122}(\gamma_{32}+2)]E_2c^{s_{32}-1} - [C_{112}s_{42}\gamma_{42}+C_{122}(\gamma_{42}+2)]F_2c^{s_{42}-1} = -\Delta\sigma_{rr2}(c^+) + \Delta\sigma_{rr2}(c^-), \quad (81)$$

$$C_{443}[\tfrac{1}{2}(s_{13}-1)-\gamma_{13}]C_3c^{s_{13}-1} + C_{443}[\tfrac{1}{2}(s_{23}-1)-\gamma_{23}]D_3c^{s_{23}-1} + C_{443}[\tfrac{1}{2}(s_{33}-1)-\gamma_{33}]E_3c^{s_{33}-1} + C_{443}[\tfrac{1}{2}(s_{43}-1)-\gamma_{43}]F_3c^{s_{43}-1}$$
$$- C_{442}[\tfrac{1}{2}(s_{12}-1)-\gamma_{12}]C_2c^{s_{12}-1} - C_{442}[\tfrac{1}{2}(s_{22}-1)-\gamma_{22}]D_2c^{s_{22}-1} - C_{442}[\tfrac{1}{2}(s_{32}-1)-\gamma_{32}]E_2c^{s_{32}-1} - C_{442}[\tfrac{1}{2}(s_{42}-1)-\gamma_{42}]F_2c^{s_{42}-1} = -\Delta\sigma_{r\theta}(c^+) + \Delta\sigma_{r\theta}(c^-). \quad (82)$$

Equation (66) gives

$$2\pi G_3c = c\alpha_3. \quad (83)$$

Equation (67) gives three relations

$$A_3c^{p_3} + B_3c^{-p_3} - \frac{C_{123}-C_{223}}{C_{113}-C_{223}} cG_3 - \frac{C_{113}-C_{233}}{C_{113}-C_{223}} c\varepsilon_{zz} - A_2c^{p_2} - B_2c^{-p_2} + \frac{C_{122}-C_{222}}{C_{112}-C_{222}} cG_2 + \frac{C_{112}-C_{232}}{C_{112}-C_{222}} c\varepsilon_{zz} = -\Delta u_{r0}(c^+) + \Delta u_{r0}(c^-), \quad (84)$$

$$\gamma_{13}C_3c^{s_{13}} + \gamma_{23}D_3c^{s_{23}} + \gamma_{33}E_3c^{s_{33}} + \gamma_{43}F_3c^{s_{43}} - \gamma_{12}C_2c^{s_{12}} - \gamma_{22}D_2c^{s_{22}} - \gamma_{32}E_2c^{s_{32}} - \gamma_{42}F_2c^{s_{42}} = -\Delta u_{r2}(c^+) + \Delta u_{r2}(c^-), \quad (85)$$

$$C_3c^{s_{13}} + D_3c^{s_{23}} + E_3c^{s_{33}} + F_3c^{s_{43}} - C_2c^{s_{12}} - D_2c^{s_{22}} - E_2c^{s_{32}} - F_2c^{s_{42}} = -\Delta u_{\theta 2}(c^+) + \Delta u_{\theta 2}(c^-). \quad (86)$$

Equation (68) gives three relations

$$- (p_3C_{113}+C_{123})A_3d^{p_3-1} - (-p_3C_{113}+C_{123})B_3d^{-p_3-1} - \frac{C_{113}C_{223}-C_{123}^2}{C_{113}-C_{223}}G_3 - \left[C_{133} - \frac{C_{133}-C_{233}}{C_{113}-C_{223}}(C_{113}+C_{123})\right]\varepsilon_{zz} = \Delta\sigma_{rr0}(d^-), \quad (87)$$

$$- [C_{113}s_{13}\gamma_{13}+C_{123}(\gamma_{13}+2)]C_3d^{s_{13}-1} - [C_{113}s_{23}\gamma_{23}+C_{123}(\gamma_{23}+2)]D_3d^{s_{23}-1} - [C_{113}s_{33}\gamma_{33}+C_{123}(\gamma_{33}+2)E_3d^{s_{33}-1} - [C_{113}s_{43}\gamma_{43}+C_{123}(\gamma_{43}+2)]F_3d^{s_{43}-1} = \Delta\sigma_{rr2}(d^-), \quad (88)$$

$$-C_{443}[\tfrac{1}{2}(s_{13}-1)-\gamma_{13}]C_3d^{s_{13}-1} - C_{443}[\tfrac{1}{2}(s_{23}-1)-\gamma_{23}]D_3d^{s_{23}-1} - C_{443}[\tfrac{1}{2}(s_{33}-1)-\gamma_{33}]E_3d^{s_{33}-1} - C_{443}[\tfrac{1}{2}(s_{43}-1)-\gamma_{43}]F_3d^{s_{43}-1} = \Delta\sigma_{r\theta 2}(d^-) \quad (89)$$

INTEGRAL CONDITION SATISFIED BY STRESSES

The last condition needed to provide as many relations as unknowns may be obtained from the virial theorem. Under the restrictions assumed in Eq. (2) this theorem of the mean stress becomes[7]

$$\iint (\sigma_{rr}+\sigma_{\theta\theta}+\sigma_{zz})rdrd\theta = \int_{r=r_s} \mathbf{r}\cdot\boldsymbol{\tau}\cdot\mathbf{n}rd\theta+W_B. \quad (90)$$

The double integral is throughout the cross section of material under stress. Clearly only theta-independent terms in the stresses survive the integration. The left-hand side may be evaluated using Eqs. (47–49) and Eqs. (56–58), while the right-hand side may be evaluated using Eq. (12). However, in order to have unknowns on the left-hand side and knowns on the right-hand side transfer all excitation and thermal strain dependent contributions to the right-hand side. Hence the left-hand side $(A_1B_1G_1A_2B_2G_2A_3B_3G_3\varepsilon_{zz})$ will equal the right-hand side $(k_{11}k_{21}k_{31}J_0k_{12}k_{22}k_{32}k_{13}k_{23}k_{33})$.

The left-hand side is equal to

$$2\pi[p_1(C_{111}+C_{121}+C_{131})+C_{121}+C_{221}+C_{231}]A_1\frac{1}{1+p_1}(b^{1+p_1}-a^{1+p_1})$$
$$- 2\pi[-p_1(C_{111}+C_{121}+C_{131})+C_{121}+C_{221}+C_{231}]B_1\frac{1}{1-p_1}(b^{1-p_1}-a^{1-p_1})$$
$$+ \pi\frac{2(C_{111}C_{221}-C_{121}^2)-C_{131}(C_{121}-C_{221})-C_{231}(C_{121}-C_{111})}{C_{111}-C_{221}}G_1(b^2-a^2)$$
$$+ \pi\left[C_{131}+C_{231}+C_{331}-\frac{C_{131}-C_{231}}{C_{111}-C_{221}}(C_{111}+2C_{121}+C_{221}+C_{131}+C_{231})\right]\varepsilon_{zz}(b^2-a^2)$$
$$+ 2\pi[p_2(C_{112}+C_{122}+C_{132})+C_{122}+C_{222}+C_{232}]A_2\frac{1}{1+p_2}(c^{1+p_2}-b^{1+p_2})$$

$$- \quad 2\pi[-p_2(C_{112}+C_{122}+C_{132})+C_{122}+C_{222}+C_{232}]B_2 \frac{1}{1-p_2} (c^{1-p_2}-b^{1-p_2})$$

$$+ \; \pi \frac{2(C_{112}C_{222}-C^2_{122})-C_{132}(C_{122}-C_{222})-C_{232}(C_{122}-C_{112})}{C_{112}-C_{222}} \; G_2(c^2-b^2)$$

$$+ \; \pi[C_{132}+C_{232}+C_{332}- \frac{C_{132}-C_{232}}{C_{112}-C_{222}} (C_{112}+2C_{122}+C_{222}+C_{132}+C_{232})]\varepsilon_{zz}(c^2-b^2)$$

$$+ \quad 2\pi[p_3(C_{113}+C_{123}+C_{133})+C_{123}+C_{223}+C_{233}]A_3 \frac{1}{1+p_3} (d^{1+p_3}-c^{1+p_3})$$

$$- \quad 2\pi[-p_3(C_{113}+C_{123}+C_{133})+C_{123}+C_{223}+C_{233}]B_3 \frac{1}{1-p_3} (d^{1-p_3}-c^{1-p_3})$$

$$+ \; \pi \frac{2(C_{113}C_{223}-C^2_{123})-C_{133}(C_{123}-C_{223})-C_{233}(C_{123}-C_{113})}{C_{113}-C_{223}} \; G_3(d^2-c^2)$$

$$+ \; \pi[C_{133}+C_{233}+C_{333}- \frac{C_{133}-C_{233}}{C_{113}-C_{223}} (C_{113}+2C_{123}+C_{223}+C_{133}+C_{233})]\varepsilon_{zz}(d^2-c^2), \quad (91)$$

and the right-hand side is equal to

$$\pi \frac{2(C_{111}C_{221}-C^2_{121})-C_{131}(C_{121}-C_{221})-C_{231}(C_{121}-C_{111})}{C_{111}-C_{221}} (k_{21}-k_{11}) (b^2-a^2)$$

$$+ \; \pi \, [C_{131}+C_{231}+C_{331} \; - \; \frac{C_{131}-C_{231}}{C_{111}-C_{221}} (C_{111}+2C_{121}+C_{221}+C_{131}+C_{231})]k_{31}(b^2-a^2)$$

$$+ \; \tfrac{2}{3}\pi^2 J_0^2 \; \left\{ -\left[5 + \frac{12(C_{112}+C_{122}+C_{132})+4(C_{122}+C_{222}+C_{232})}{9C_{112}-C_{222}}\right] \tfrac{1}{4}(c^4-b^4) \right.$$

$$+ \; \left[\tfrac{9}{2} + \frac{6(C_{112}+C_{121}+C_{132})+3(C_{122}+C_{222}+C_{232})}{4C_{112}-C_{222}}\right] \frac{\lambda}{3} \; (c^3-b^3)$$

$$+ \left[\tfrac{1}{2} - \frac{C_{122}+C_{222}+C_{232}}{C_{222}}\right] b^3(c-b)$$

$$+ \; \pi \frac{2(C_{112}C_{222}-C^2_{122})-C_{132}(C_{122}-C_{222})-C_{232}(C_{122}-C_{112})}{C_{112}-C_{222}} (k_{22}-k_{12}) \; (c^2-b^2)$$

$$+ \; \pi \left[C_{132}+C_{232}+C_{332}- \frac{C_{132}-C_{232}}{C_{112}-C_{222}} (C_{112}+2C_{122}+C_{222}+C_{132}+C_{232}) \right] k_{32}(c^2-b^2)$$

$$+ \pi \frac{2(C_{113}C_{223}-C^2_{123})-C_{133}(C_{123}-C_{223})-C_{233}(C_{123}-C_{113})}{C_{113}-C_{223}} (k_{23}-k_{13}) (d^2-c^2)$$

$$+ \pi \left[C_{133}+C_{233}+C_{333}- \frac{C_{133}-C_{233}}{C_{113}-C_{223}} (C_{113}+2C_{123}+C_{223}+C_{133}+C_{233}) \right] k_{33}(d^2-c^2). \quad (92)$$

NUMERICAL RESULTS

Twenty-two algebraic relations resulting from the boundary conditions, pre-strain conditions, and the virial theorem have been given in Eqs. (69–92). These relations are used to obtain the twenty-two unknowns A_1B_1...etc. Hence the displacement, strain, and stress may be found at any point in the structure of the dipole model. For compactness in presentation, however, only the values at $r = a$, $r = b$, $r = c$, and $r = d$ are given for a few angles. It is usually clear whether a quantity is stress or strain. Otherwise *R* is radial, *T* is theta or azimuthal, *Z* is axial or longitudinal. To indicate the side of a point, *P* is used for positive and *M* for negative. Thus, for example, *RTBP* indicates the (r,θ) component at the positive side of the point $r = b$.

In order to enter the elastic constants used in Eq. (3) it seems preferable to utilize Young's modulus and Poisson's ratio for each axis. The compliance [*S*] rather than the stiffness [*C*] is easily expressed in terms of these more familiar constants. Thus[6]

$$[S] = \begin{bmatrix} \frac{1}{E_{11}} & -\frac{\nu_{12}}{E_{11}} & -\frac{\nu_{13}}{E_{11}} & 0 \\ -\frac{\nu_{12}}{E_{11}} & \frac{1}{E_{22}} & -\frac{\nu_{23}}{E_{22}} & 0 \\ -\frac{\nu_{13}}{E_{11}} & -\frac{\nu_{23}}{E_{22}} & \frac{1}{E_{33}} & 0 \\ 0 & 0 & 0 & \frac{1}{2G_{44}} \end{bmatrix}, \quad (93)$$

where, if needed,

$$\frac{\nu_{12}}{E_{11}} = \frac{\nu_{21}}{E_{22}} \quad \frac{\nu_{23}}{E_{22}} = \frac{\nu_{32}}{E_{33}} \quad \frac{\nu_{31}}{E_{33}} = \frac{\nu_{13}}{E_{11}}. \quad (94)$$

To represent the Doubler choose

	Conductor	Collar
E_{11}(psi)	4.000×10^6	30.000×10^6
E_{22}(psi)	0.500×10^6	29.999×10^6
E_{33}(psi)	8.000×10^6	2.35×10^6
ν_{12}	0.35	0.3333
ν_{13}	0.15	0.3333
ν_{23}	0.0156	0.3333
G_{44}(psi)	1.375×10^6	11.25×10^6

These constants were suggested by the work of Weston,[14] modifications being made as seemed appropriate keeping in mind that no Poisson's ratio was to exceed 0.5 and all roots of Eq. (30) were to be real. For the collar the desire was to have elastic constants representative of a structure laminated in the longitudinal direction. Thus E_{33} was chosen much less than E_{11} or E_{22}. Note however that E_{22} was chosen somewhat smaller than E_{11} in order to avoid a numerical problem that is evident in Eq. (36), for example, if C_{11} were to equal C_{22}. Inversion of the matrix [*S*] of Eq. (93) will give the [*C*] matrix used in Eq. (3).

The state of elasticity is presented in two runs. In the first run (Fig. 2), the structure is pre-strained and cooled down with the magnetic field off. In the second run (Fig. 3), the structure is pre-strained and cooled down with the magnetic field on. Note for instance the longitudinal expansion of the structure by $\Delta\varepsilon_{zz}$ = 0.000255 or 0.064 in/252 in. Furthermore, pre-strain brought about by assembling the structure on conductors that are azimuthally oversize by 65 mrad or 0.030 in/quadrant is sufficient to permit the conductor region to remain comfortably in compression after the magnetic field is turned on.

The change in displacement caused by turning on the magnetic field is:

at $r = b^+$

$$\Delta u_r = 0.0002 + 0.0027 \cos 2\theta \text{ (in.)}$$
$$\Delta u_\theta = -0.0029 \sin 2\theta \text{ (in.)} \quad (95)$$

```
                                 ELASTIC STRESS AND STRAIN IN ORTHOTROPIC STRUCTURE DOUBLER DIPOLE
CENTRAL MAGNETIC FIELD(KG)      =    0.0000 INNER BORE TUBE RADIUS(IN)       =     1.2500 INNER CONDUCTOR RADIUS(IN))   =     1.5000
OUTER CONDUCTOR RADIUS(IN))     =    1.9000 OUTER RADIUS OF BANDING(IN)      =     2.8125 RADIUS OF IRON SHIELD(IN)     =     3.7500
E111(LB/IN/IN)                  =        0. E112(LB/IN/IN)                   =   4000000. E113(LB/IN/IN)                = 30000000.
E221(LB/IN/IN)                  =        0. E222(LB/IN/IN)                   =    500000. E223(LB/IN/IN)                = 29999000.
E331(LB/IN/IN)                  =        0. E332(LB/IN/IN)                   =   8000000. E333(LB/IN/IN)                =  2350000.
P121                            =     .3333 P122                             =      .3500 P123                          =     .3333
P131                            =     .3333 P132                             =      .1500 P133                          =     .3333
P231                            =     .3333 P232                             =      .0156 P233                          =     .3333
G441(LB/IN/IN)                  =        0. G442(LB/IN/IN)                   =   1375000. G443(LB/IN/IN)                = 11250000.
THERMAL STRAIN(11)              =    0.0000 THERMAL STRAIN(12)               =     -.0046 THERMAL STRAIN(13)            =    -.0028
THERMAL STRAIN(21)              =    0.0000 THERMAL STRAIN(22)               =     -.0065 THERMAL STRAIN(23)            =    -.0028
THERMAL STRAIN(31)              =    0.0000 THERMAL STRAIN(32)               =     -.0026 THERMAL STRAIN(33)            =    -.0040
ALPHA1(MILLIRAD)                =     0.000 ALPHA2(MILLIRAD)                 =    -65.000 ALPHA3(MILLIRAD)              =     0.000
NUMBER OF STRUCTURAL SHELLS     =   2

ANGLE                                                     TRANSVERSE STRESS(LB/IN/IN)
(DEG)  RRAP   TTAP   RTAP   RRBM   TTBM   RTBM   RRBP   TTBP   RTBP   RRCM   TTCM   RTCM   RRCP   TTCP   RTCP   RRDM   TTDM   RTDM

90.0   -0.    -0.    0.     -0.    -0.    0.     -0. -3197.    0.   -698. -3422.    0.   -698.  1871.    0.     -0.   1172.    0.
80.0   -0.    -0.    0.     -0.    -0.    0.     -0. -3197.    0.   -698. -3422.    0.   -698.  1871.    0.     -0.   1172.    0.
70.0   -0.    -0.    0.     -0.    -0.    0.     -0. -3197.    0.   -698. -3422.    0.   -698.  1871.    0.     -0.   1172.    0.
60.0   -0.    -0.    0.     -0.    -0.    0.     -0. -3197.    0.   -698. -3422.    0.   -698.  1871.    0.     -0.   1172.    0.
50.0   -0.    -0.    0.     -0.    -0.    0.     -0. -3197.    0.   -698. -3422.    0.   -698.  1871.    0.     -0.   1172.    0.
40.0   -0.    -0.    0.     -0.    -0.    0.     -0. -3197.    0.   -698. -3422.    0.   -698.  1871.    0.     -0.   1172.    0.
30.0   -0.    -0.    0.     -0.    -0.    0.     -0. -3197.    0.   -698. -3422.    0.   -698.  1871.    0.     -0.   1172.    0.
20.0   -0.    -0.    0.     -0.    -0.    0.     -0. -3197.    0.   -698. -3422.    0.   -698.  1871.    0.     -0.   1172.    0.
10.0   -0.    -0.    0.     -0.    -0.    0.     -0. -3197.    0.   -698. -3422.    0.   -698.  1871.    0.     -0.   1172.    0.
  .0   -0.    -0.    0.     -0.    -0.    0.     -0. -3197.    0.   -698. -3422.    0.   -698.  1871.    0.     -0.   1172.    0.

ANGLE                                                     TRANSVERSE STRAIN (MUIN/IN)
(DEG)  RRAP   TTAP   RTAP   RRBM   TTBM   RTBM   RRBP   TTBP   RTBP   RRCM   TTCM   RTCM   RRCP   TTCP   RTCP   RRDM   TTDM   RTDM

90.0   0.     0.     0.     0.     0.     0. -4105.-12714.     0. -4249.-13096.    0. -2865. -2751.    0. -2834. -2782.    0.
80.0   0.     0.     0.     0.     0.     0. -4105.-12714.     0. -4249.-13096.    0. -2865. -2751.    0. -2834. -2782.    0.
70.0   0.     0.     0.     0.     0.     0. -4105.-12714.     0. -4249.-13096.    0. -2865. -2751.    0. -2834. -2782.    0.
60.0   0.     0.     0.     0.     0.     0. -4105.-12714.     0. -4249.-13096.    0. -2865. -2751.    0. -2834. -2782.    0.
50.0   0.     0.     0.     0.     0.     0. -4105.-12714.     0. -4249.-13096.    0. -2865. -2751.    0. -2834. -2782.    0.
40.0   0.     0.     0.     0.     0.     0. -4105.-12714.     0. -4249.-13096.    0. -2865. -2751.    0. -2834. -2782.    0.
30.0   0.     0.     0.     0.     0.     0. -4105.-12714.     0. -4249.-13096.    0. -2865. -2751.    0. -2834. -2782.    0.
20.0   0.     0.     0.     0.     0.     0. -4105.-12714.     0. -4249.-13096.    0. -2865. -2751.    0. -2834. -2782.    0.
10.0   0.     0.     0.     0.     0.     0. -4105.-12714.     0. -4249.-13096.    0. -2865. -2751.    0. -2834. -2782.    0.
  .0   0.     0.     0.     0.     0.     0. -4105.-12714.     0. -4249.-13096.    0. -2865. -2751.    0. -2834. -2782.    0.

ANGLE   LONG. STRESS(LB/IN/IN) AND STRAIN(MUIN/IN)                      TRANSVERSE DISPLACEMENT(IN)
(DEG)  ZZAP   ZZBM  ZZBP   ZZCM   ZZCP   ZZDM    EZZ      URAP  URBM  URBP  URCM  URCP  URDM    UTAP  UTBM  UTBP  UTCM  UTCP  UTDM

90.0   -0.    -0.-5752.-6018.  1866.  1866. -3219.     .0000 .0000-.0036-.0052-.0052-.0078 0.00000.0000-.0244-.0309 .0000 .0000
80.0   -0.    -0.-5752.-6018.  1866.  1866. -3219.     .0000 .0000-.0036-.0052-.0052-.0078 0.00000.0000-.0217-.0274 .0000 .0000
70.0   -0.    -0.-5752.-6018.  1866.  1866. -3219.     .0000 .0000-.0036-.0052-.0052-.0078 0.00000.0000-.0190-.0240 .0000 .0000
60.0   -0.    -0.-5752.-6018.  1866.  1866. -3219.     .0000 .0000-.0036-.0052-.0052-.0078 0.00000.0000-.0163-.0206 .0000 .0000
50.0   -0.    -0.-5752.-6018.  1866.  1866. -3219.     .0000 .0000-.0036-.0052-.0052-.0078 0.00000.0000-.0135-.0172 .0000 .0000
40.0   -0.    -0.-5752.-6018.  1866.  1866. -3219.     .0000 .0000-.0036-.0052-.0052-.0078 0.00000.0000-.0108-.0137 .0000 .0000
30.0   -0.    -0.-5752.-6018.  1866.  1866. -3219.     .0000 .0000-.0036-.0052-.0052-.0078 0.00000.0000-.0081-.0103 .0000 .0000
20.0   -0.    -0.-5752.-6018.  1866.  1866. -3219.     .0000 .0000-.0036-.0052-.0052-.0078 0.00000.0000-.0054-.0069 .0000 .0000
10.0   -0.    -0.-5752.-6018.  1866.  1866. -3219.     .0000 .0000-.0036-.0052-.0052-.0078 0.00000.0000-.0027-.0034 .0000 .0000
  .0   -0.    -0.-5752.-6018.  1866.  1866. -3219.     .0000 .0000-.0036-.0052-.0052-.0078 0.00000.0000-.0000-.0000 .0000 .0000
```

FIGURE 2 State of elasticity with prestrain, cooldown, and magnetic field off.

ELASTIC STRESS AND STRAIN IN ORTHOTROPIC STRUCTURE DOUBLER DIPOLE

CENTRAL MAGNETIC FIELD(KG)	= 45.0000	INNER BORE TUBE RADIUS(IN)	= 1.2500	INNER CONDUCTOR RADIUS(IN))	= 1.5000		
OUTER CONDUCTOR RADIUS(IN))	= 1.9000	OUTER RADIUS OF BANDING(IN)	= 2.8125	RADIUS OF IRON SHIELD(IN)	= 3.7500		
E111(LB/IN/IN)	= 0.	E112(LB/IN/IN)	= 4000000.	E113(LB/IN/IN)	= 30000000.		
E221(LB/IN/IN)	= 0.	E222(LB/IN/IN)	= 500000.	E223(LB/IN/IN)	= 29999000.		
E331(LB/IN/IN)	= 0.	E332(LB/IN/IN)	= 8000000.	E333(LB/IN/IN)	= 2350000.		
P121	= .3333	P122	= .3500	P123	= .3333		
P131	= .3333	P132	= .1500	P133	= .3333		
P231	= .3333	P232	= .0156	P233	= .3333		
G441(LB/IN/IN)	= 0.	G442(LB/IN/IN)	= 1375000.	G443(LB/IN/IN)	= 11250000.		
THERMAL STRAIN(11)	= 0.0000	THERMAL STRAIN(12)	= -.0046	THERMAL STRAIN(13)	= -.0028		
THERMAL STRAIN(21)	= 0.0000	THERMAL STRAIN(22)	= -.0065	THERMAL STRAIN(23)	= -.0028		
THERMAL STRAIN(31)	= 0.0000	THERMAL STRAIN(32)	= -.0026	THERMAL STRAIN(33)	= -.0040		
ALPHA1(MILLIRAD)	= 0.000	ALPHA2(MILLIRAD)	= -65.000	ALPHA3(MILLIRAD)	= 0.000		
NUMBER OF STRUCTURAL SHELLS	= 2						

TRANSVERSE STRESS(LB/IN/IN)

ANGLE (DEG)	RRAP	TTAP	RTAP	RRBM	TTBM	RTBM	RRBP	TTBP	RTBP	RRCM	TTCM	RTCM	RRCP	TTCP	RTCP	RRDM	TTDM	RTDM
90.0	-0.	-0.	0.	-0.	-0.	0.	-0.	-2065.	0.	-270.	-2965.	-0.	-270.	26616.	-0.	-0.	-12440.	0.
80.0	-0.	-0.	0.	-0.	-0.	0.	-0.	-2129.	-0.	-316.	-2991.	403.	-316.	25179.	403.	-0.	-11585.	0.
70.0	-0.	-0.	0.	-0.	-0.	0.	-0.	-2312.	-0.	-449.	-3065.	758.	-449.	21039.	758.	-0.	-9123.	0.
60.0	-0.	-0.	0.	-0.	-0.	0.	-0.	-2594.	-0.	-653.	-3179.	1021.	-653.	14697.	1021.	-0.	-5350.	0.
50.0	-0.	-0.	0.	-0.	-0.	0.	-0.	-2940.	-0.	-903.	-3319.	1161.	-903.	6917.	1161.	-0.	-723.	0.
40.0	-0.	-0.	0.	-0.	-0.	0.	-0.	-3308.	-0.	-1169.	-3468.	1161.	-1169.	-1363.	1161.	0.	4202.	0.
30.0	-0.	-0.	0.	-0.	-0.	0.	-0.	-3653.	-0.	-1420.	-3607.	1021.	-1420.	-9143.	1021.	0.	8830.	0.
20.0	-0.	-0.	0.	-0.	-0.	0.	-0.	-3935.	-0.	-1624.	-3721.	758.	-1624.	-15486.	758.	0.	12603.	0.
10.0	-0.	-0.	0.	-0.	-0.	0.	-0.	-4119.	-0.	-1757.	-3796.	403.	-1757.	-19625.	403.	0.	15065.	0.
.0	-0.	-0.	0.	-0.	-0.	0.	-0.	-4183.	-0.	-1803.	-3822.	0.	-1803.	-21063.	0.	0.	15920.	0.

TRANSVERSE STRAIN (MUIN/IN)

ANGLE (DEG)	RRAP	TTAP	RTAP	RRBM	TTBM	RTBM	RRBP	TTBP	RTBP	RRCM	TTCM	RTCM	RRCP	TTCP	RTCP	RRDM	TTDM	RTDM
90.0	0.	0.	0.	0.	0.	0.	-4291.	-10522.	0.	-4268.	-12290.	-0.	-3139.	-1944.	-0.	-2685.	-3238.	0.
80.0	0.	0.	0.	0.	0.	0.	-4285.	-10649.	-0.	-4276.	-12337.	147.	-3125.	-1991.	18.	-2695.	-3210.	0.
70.0	0.	0.	0.	0.	0.	0.	-4267.	-11016.	-0.	-4301.	-12472.	276.	-3082.	-2127.	34.	-2723.	-3129.	0.
60.0	0.	0.	0.	0.	0.	0.	-4239.	-11577.	-0.	-4339.	-12679.	371.	-3016.	-2334.	45.	-2766.	-3004.	0.
50.0	0.	0.	0.	0.	0.	0.	-4206.	-12266.	-0.	-4385.	-12933.	422.	-2936.	-2588.	52.	-2819.	-2851.	0.
40.0	0.	0.	0.	0.	0.	0.	-4170.	-12998.	-0.	-4434.	-13204.	422.	-2850.	-2859.	52.	-2875.	-2688.	0.
30.0	0.	0.	0.	0.	0.	0.	-4137.	-13687.	-0.	-4480.	-13458.	371.	-2770.	-3113.	45.	-2928.	-2535.	0.
20.0	0.	0.	0.	0.	0.	0.	-4110.	-14248.	-0.	-4518.	-13665.	276.	-2704.	-3320.	34.	-2971.	-2411.	0.
10.0	0.	0.	0.	0.	0.	0.	-4092.	-14614.	-0.	-4542.	-13801.	147.	-2661.	-3456.	18.	-2999.	-2329.	0.
.0	0.	0.	0.	0.	0.	0.	-4086.	-14742.	-0.	-4551.	-13848.	0.	-2646.	-3503.	0.	-3009.	-2301.	0.

LONG. STRESS(LB/IN/IN) AND STRAIN(MUIN/IN)

ANGLE (DEG)	ZZAP	ZZBM	ZZBP	ZZCM	ZZCP	ZZDM	EZZ
90.0	-0.	-0.	-3431.	-3737.	3122.	2109.	-2964.
80.0	-0.	-0.	-3447.	-3757.	3083.	2131.	-2964.
70.0	-0.	-0.	-3493.	-3815.	2972.	2196.	-2964.
60.0	-0.	-0.	-3563.	-3905.	2801.	2294.	-2964.
50.0	-0.	-0.	-3649.	-4015.	2591.	2415.	-2964.
40.0	-0.	-0.	-3741.	-4132.	2368.	2544.	-2964.
30.0	-0.	-0.	-3828.	-4242.	2158.	2664.	-2964.
20.0	-0.	-0.	-3898.	-4332.	1987.	2763.	-2964.
10.0	-0.	-0.	-3944.	-4390.	1876.	2827.	-2964.
.0	-0.	-0.	-3963.	-4411.	1837.	2850.	-2964.

TRANSVERSE DISPLACEMENT(IN)

ANGLE (DEG)	URAP	URBM	URBP	URCM	URCP	URDM	UTAP	UTBM	UTBP	UTCM	UTCP	UTDM
90.0	.0000	.0000	-.0061	-.0079	-.0079	-.0104	0.0000	0.0000	-.0244	-.0309	.0000	.0000
80.0	.0000	.0000	-.0060	-.0077	-.0077	-.0103	0.0000	0.0000	-.0227	-.0282	-.0007	-.0002
70.0	.0000	.0000	-.0055	-.0072	-.0072	-.0098	0.0000	0.0000	-.0208	-.0254	-.0013	-.0004
60.0	.0000	.0000	-.0048	-.0065	-.0065	-.0091	0.0000	0.0000	-.0188	-.0224	-.0018	-.0006
50.0	.0000	.0000	-.0039	-.0056	-.0056	-.0083	0.0000	0.0000	-.0164	-.0192	-.0020	-.0007
40.0	.0000	.0000	-.0030	-.0047	-.0047	-.0073	0.0000	0.0000	-.0137	-.0158	-.0020	-.0007
30.0	.0000	.0000	-.0021	-.0038	-.0038	-.0065	0.0000	0.0000	-.0107	-.0121	-.0018	-.0006
20.0	.0000	.0000	-.0014	-.0031	-.0031	-.0058	0.0000	0.0000	-.0073	-.0082	-.0013	-.0004
10.0	.0000	.0000	-.0009	-.0027	-.0027	-.0053	0.0000	0.0000	-.0037	-.0041	-.0007	-.0002
.0	.0000	.0000	-.0007	-.0025	-.0025	-.0051	0.0000	0.0000	-.0000	-.0000	-.0000	-.0000

FIGURE 3 State of elasticity with prestrain, cooldown, and magnetic field on.

at $r = c^-$

$$\begin{aligned}\Delta u_r &= 0.0027 \cos 2\theta \text{ (in.)}\\ \Delta u_\theta &= -0.0020 \sin 2\theta, \text{ (in.)}\end{aligned} \tag{96}$$

and at $r = d^-$

$$\begin{aligned}\Delta u_r &= 0.0027 \cos 2\theta \text{ (in.)}\\ \Delta u_\theta &= -0.0007 \sin 2\theta. \text{ (in.)}\end{aligned} \tag{97}$$

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