Particle Accelerators
1979, Vol. 10, pp. 41-59
0031-2460/79/1001-0041$04.50/0

THE GYROCON—AN EFFICIENT RELATIVISTIC HIGH-POWER VHF GENERATOR

G. I. BUDKER, M. M. KARLINER, I.G. MAKAROV, S. N. MOROSOV, O. A. NEZHEVENKO, G. N. OSTREIKO, and I. A. SHEKHTMAN

Institute of Nuclear Physics, Siberean Division, USSR Academy of Sciences, Novosibirsk 90, USSR

(Received July 25, 1978)

The Gyrocon, an rf generator proposed by G. I. Budker, is described. Its operation principle is based on modulation by deflection of a high-power relativistic electron beam which is subsequently decelerated in the annular traveling wave resonator. The projection of the attainable parameters to higher efficiency shows that the Gyrocon can develop high-frequency power substantially exceeding the power of known VHF devices. We describe the results of an analysis of the excitation of the Gyrocon output resonator by a relativistic beam continuously changing its place of entrance to the resonator. We also consider effects on the system of a rotating magnetic field in the cylindrical resonator for circular scanning of the relativistic beam. Various possible designs of the Gyrocon are described, including those built at INP (Novosibirsk) for HF supply of the accumulating-accelerating facility VEPP-4. Some results of tests are given.

INTRODUCTION

The problem of obtaining high-power rf accelerating voltages for accelerators remains an urgent problem. In 1967, G. I. Budker proposed the Gyrocon,† a relativistic VHF generator with modulation by deflection, as one of the solutions.[1]

In the Gyrocon, the relativistic beam is modulated by an input VHF signal, and then it radiates its energy by interacting with an output resonator. In contrast to other high power VHF devices, the electron beam in the Gyrocon is not density modulated, but it continuously changes the place of its entrance to the resonator. This is achieved by circular scanning of the beam. A waveguide in which the electrons induce a traveling wave with a transverse electric field serves as the output resonator of the Gyrocon. Since the phase velocity of this wave is exactly equal to the shift velocity of the entrance place of the electron beam to the resonator, the beam is constantly at the same decelerating phase, while its energy is efficiently converted into energy of electromagnetic VHF oscillations.

The Gyrocon belongs to a class of electron-beam VHF devices in which the position of passage of the electron current is changeable in time. Devices of this class were invented about forty years ago[2-4] and improved in the 60's[5,6] when an annular traveling-wave resonator was proposed as an output resonator.

The devices described in [2-6] did not find any useful application because they could not operate either at high power or with high efficiency. The limitation arises from the nonrelativistic energies of the electron beam exciting the resonator. At low energies and when the accompanying magnetic field is absent, the transverse size of the beam is determined by space-charge effects and small currents are required to minimize the losses through the holes and slots of the resonators. Therefore, the beam power is small. In addition, the losses in the output resonator constitute an essential part of this power, which leads to a low overall efficiency of the device.

G. I. Budker was the first who paid attention to the possibilities offered by the use of relativistic energies of the electron beam in such a device. In the relativistic beam, the magnetic constriction forces attenuate the effect of the transverse space-charge effects by a factor γ^2 (where γ is the relative energy of electrons). This ensures the possibility of achieving the beam power and consequently the output VHF power which are not attainable in devices operating at nonrelativistic energies.[2-8] The losses in the output resonator increase with the electron energy substantially slower than the maximum beam power and they constitute only a small fraction at relativistic energies.

Because there is no density modulation of the electron flux in the Gyrocon, it is free from limitations of the electronic conversion efficiency which

† From "gyros" (Greek)—a circle and "continuum" (latin)—continuous; a circular deflection of the continuous (unbunched) beam.

are peculiar to high-power klystrons[7] and to VHF devices with grid control.[8] Indeed, the transverse dimensions of the relativistic beam are considerably smaller than a wave length of the wave traveling along the output resonator down to a range of $\lambda =$ 10 cm; therefore all the electrons are decelerated practically in the same manner. This gives the possibility of obtaining a conversion efficiency close to 100%.

In early devices with an annular resonator—Gyrocon prototypes[5,6]—such a possibility exists only at nonrelativistic energies. At relativistic levels, the traveling-wave magnetic-field effect appears. This will essentially change the beam direction at the output from the resonator. An electron moving along the decelerating electric field gains a velocity perpendicular to this direction. Part of the energy arising from this velocity cannot be converted into VHF oscillations, and the conversion efficiency becomes substantially lower than 100%. But the Gyrocon is free from this disadvantage because it is equipped with special devices providing an overall deceleration of the relativistic electrons.

In this paper, the description of operation principles of the Gyrocon and the possibilities of this new VHF generator are theoretically estimated, and information on the development of the first operating devices is also given.

I. DESIGN AND OPERATION PRINCIPLES OF THE GYROCON

The design of the simplest version of the Gyrocon is shown in Fig. 1. The continuous electron beam (2) from the high-voltage accelerator (1) is deflected by the rotating magnetic field of the resonator (3). This resonator is excited by a VHF signal (4). The deflected electrons travel along the level lines of a conic surface, on which they trace out a helical line with an end describing a circle. The electrostatic electrode system (5) directs the electrons to an annular slot of an output resonator (6). This is tuned at the scanning frequency to induce at resonance oscillations with a maximum of the electric field in the region of the annular slot through which the electron beam travels. The beam by continuously varying the place of its entrance into the resonator induces the traveling wave in it. The electric field of the wave decelerates the electrons converting the beam power which is taken out through the channel (7). The residual electron energy is dissipated in a collector (8).

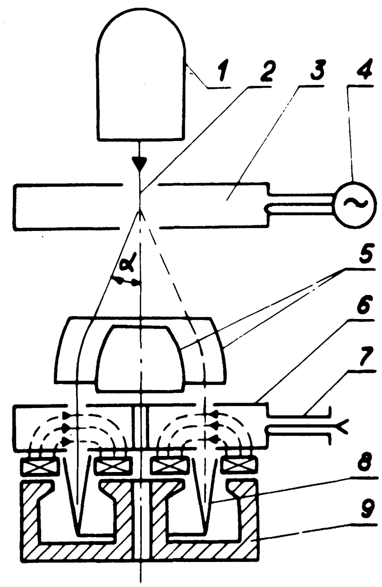

FIGURE 1 The design of the Gyrocon. 1) HV accelerator; 2) electron beam; 3) scanning resonator; 4) HF exciter; 5) electrostatic deflection system; 6) output resonator; 7) energy outputs; 8) collector; 9) compensating electromagnet. α–deflecting angle, — → — are the lines of static magnetic field.

The circular scanning of the Gyrocon beam is realized by the magnetic field of the cylindrical resonator with an E_{110} mode of oscillation[9] (Fig. 2). Two power inputs of this resonator are supplied with a phase shift of 90°, which provides a circular polarization of the magnetic field in the near-axis region where the deflected electrons travel.

In the output resonator (6) (Fig. 1) the electrons passing in the maximum of the electric field induce a traveling electromagnetic wave. The output resonator is essentially a rectangular toroidal waveguide with H_{10} mode. This resonator is tuned to the scanning frequency but it may be tuned as well to the frequencies that are multiples of it. In this case, the Gyrocon operates as a frequency multiplier. Two energy outputs (7) are shifted in the azimuthal direction by 90° with respect to each other, similar to the power inputs in the scanning resonator (Fig. 2). Being identically loaded, they provide the traveling-wave regime in the resonator (6) when the latter is excited by electron beam. This regime may be maintained also with a large number of energy outputs.[1]

The electrostatic deflecting system (5) is a part of the spherical capacitor. An electromagnet (9) generates a transverse static magnetic field in the flight

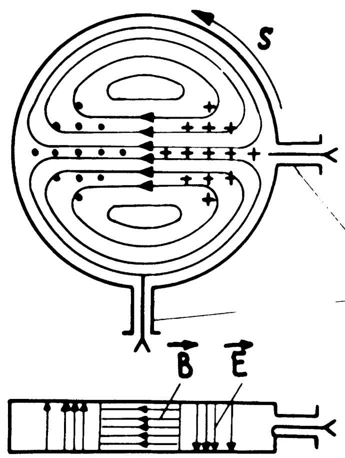

FIGURE 2 The scanning resonator. B are the lines of magnetic flux density, E are the lines of electric field strength, 1, 2) power outputs, S is the direction of field rotation.

gap to compensate for the traveling-wave magnetic-field effect which reduces the efficiency of deceleration of the relativistic particles.

In order to estimate the efficiency reduction of the electron deceleration and to explain the action of the devices used in the Gyrocon for the compensation of this effect, consider a motion of an electron in the traveling-wave field. To simplify the analysis, replace the annular resonator by an infinite waveguide with an H-type wave traveling in it, this wave having the maxima of electric field in E-planes of symmetry (Fig. 3).

The field components acting on the electron in this plane are of the form

$$E_y = E\cos(\omega t - \frac{\omega}{V}x)$$
$$B_z = B\cos(\omega t - \frac{\omega}{V}x), \qquad (1)$$

where E is the electric-field amplitude, B is the magnetic-induction amplitude, ω is the angular frequency of oscillations, V is the traveling wave phase velocity, x is the longitudinal coordinate (Fig. 3) along the wave-propagation direction, and t is time.

The ratio E/B is equal to the wave phase velocity and depends on transverse dimensions of the waveguide.

$$\frac{E}{B} = V = \frac{c}{\sqrt{1-(\lambda/\lambda_k)^2}}, \qquad (2)$$

where λ is the wavelength in free space, λ_k is the cutoff wavelength of the waveguide, and c is the velocity of light in vaccum. The ratio $V/c = \nu$ characterizes the annular traveling wave resonator and is equal to $\nu \approx 1.84$ for a resonator of type (6) shown in Fig. 1.†

The relativistic equations of electron motion in the traveling wave field may be integrated and from the integral of motion given in Appendix I, a relation may be found between the parameter ν and the

† The value $\nu = 1.84$ is calculated for the resonator with field distribution shown in Fig. 2. The central pivot of small diameter in the resonator (6) (in Fig. 1) has not been taken into account.

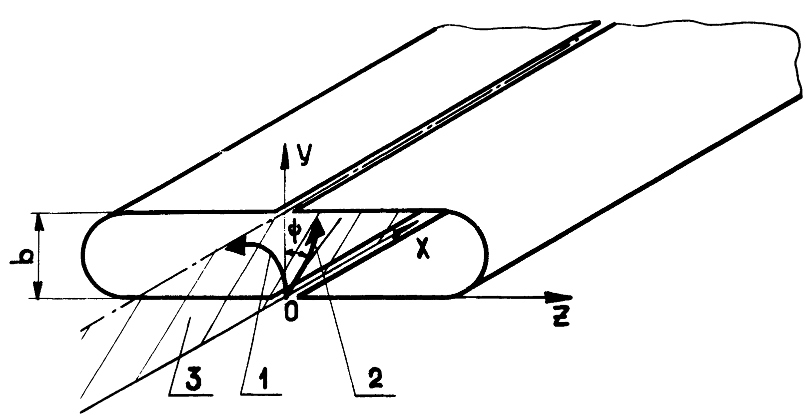

FIGURE 3 The output resonator (in the approximation of an infinite waveguide). 1) electron trajectory in the device with annular traveling wave resonator; 2) trajectory of the electron in the Gyrocon; 3) E-plane symmetry.

electron energy at the point where the deceleration is completed. If the initial electron velocity is directed along the electric field, then this relation is of the form

$$-\nu\sqrt{\gamma_b^2-1}=\gamma_b-\gamma_0. \quad (3)$$

Here γ_0 is the initial relative electron energy, γ_b is the relative energy of the electron at the point where the velocity component directed along the decelerating field becomes zero.

The conversion efficiency Π is equal to the ratio between the energy lost by the electron during its deceleration in the annular traveling wave resonator and the initial kinetic energy. Thus

$$\Pi=\frac{\gamma_0-\gamma_b}{\gamma_0-1}\,. \quad (4)$$

For ultrarelativistic electrons ($\gamma_0 \gg 1$) $\Pi \rightarrow \nu/1+\nu$ and at $\nu = 1.84$ $\Pi \rightarrow 0.65$. In the Gyrocon, this reduction of the conversion efficiency is eliminated with the help of the static field of a compensating electromagnet (9) (Fig. 1).

In Appendix I it is shown that the magnetic induction, averaged over the flight gap of the output resonator (b in Fig. 3), required to compensate the traveling-wave magnetic-field effect is

$$B_k=\frac{U_0}{b\nu c}\,, \quad (5)$$

where U_0 is the accelerating voltage of the electron source. For $U_0 = 1500$ kV, $b/\lambda = 0.4$, $\lambda = 0.7$ m, $\nu = 1.84$, the required induction $B_k = 0.019$ T.

Another method of improving the electron deceleration efficiency in the Gyrocon is the use of a static magnetic field in front of the resonator entrance. This field directs the electron into the output resonator at some angle ψ (Fig. 3) with a value defined by the condition $\gamma_b = 1$ (4), equivalent to the complete deceleration of the electron at the resonator output. It is shown in Appendix I that this angle is equal to

$$\psi=\text{arc sin}\left[\frac{1}{\nu}\sqrt{\frac{\gamma_0-1}{\gamma_0+1}}\right]. \quad (6)$$

For $\nu = 1.84$, and $\gamma_0 \gg 1$, $\psi \rightarrow 33°$. Any additional component of the initial velocity that is imparted to the electron in this case must be directed parallel to the motion of the traveling wave. The fulfillment of conditions (5) and (6) ensures the possibility of obtaining electron conversion efficiencies of the Gyrocon close to 100%.

The capability of making use of either some or all of the methods used to increase the electron deceleration efficiency is one of the features intrinsic in such a device.

II. ESTIMATE OF ATTAINABLE GYROCON PARAMETERS

In order to understand the possibilities of the Gyrocon, let us estimate its main parameters that are attainable in principle. For each parameter we shall make the assumptions required to simplify the analysis, list the limitations and conditions to achieve the optimum and finally give the consequent formulae and the table of estimates.

a) Maximum power of the Gyrocon

Let us assume that the transverse beam dimensions do not exceed those due to divergence caused by the action of the space-charge forces, without external fields.[10] Moreover, let us assume that the electron beam is optimally focused at the accelerator output[11] and that it has sharp boundaries. Under these conditions, for nonrelativistic energies, the beam current is equal to

$$I_{0\,\max}=39\cdot10^{-6}\left(\frac{D}{L}\right)^2U_0^{3/2}\;[\text{A}],$$

whereas in the relativistic case it is higher by a factor of $[(\gamma_0+1)/2]^{3/2}$. Here D and L are the initial (and final) diameter of the beam and its length.

Assuming that the dimensions of holes and slots of the resonators of the Gyrocon are such that $D \approx \lambda/20$, and their locations are such that $L = 2\lambda$, where λ is the Gyrocon operating wavelength, one can then estimate the maximum power of beam which is limited by the carrying capacity of an electron-optical channel of the Gyrocon. At an energy of $eU_0 = 500$ keV, this power is 8 MW, and 270 MW for $eU_0 = 1500$ keV. For high-power accelerators, operating in the continuous regime, the first-mentioned energy level can be attained for the beam powers of a few megawatts,[16] whereas in the regime of microsecond pulses, the second energy level can be achieved for beam powers of the order of hundreds of megawatts.[18] These levels correspond to the main boundaries of the rated power of the Gyrocon. It seems possible to considerably improve this power in the future.

b) Electron conversion efficiency of the Gyrocon

To estimate the electron conversion efficiency, let us consider only the azimuthal dimension of the beam cross section in the output resonator, and assume that the other transverse dimension can be reduced substantially by means of focusing after scanning.

In the scanning resonator, the beam cross section is assumed to be circular. The factors in principle limiting the electron conversion efficiency of the Gyrocon are the following:

1. The final transverse beam dimension in the output resonator and consequent unequal electron-deceleration conditions.
2. Beam energy spread caused by annular scanning.
3. The partial electron deceleration required for extracting the beam to the collector.

The effect of the azimuthal beam dimension ($\Delta\varphi$) on the electron conversion efficiency is estimated with the assumption of a uniform distribution of electrons over the beam cross section. If the decelerating voltage across the resonator gap is equal to $U_1 \cos \varphi$, where φ is the azimuthal coordinate (with reference to the beam center), then the energy carried by the electron to the collector is $e(U_0 - U_1 \cos \varphi)$. Here eU_0 is the initial energy. In this case the total lost energy is proportional to

$$e(U_0 \Delta\varphi - U_1 \int_{-\Delta\varphi/2}^{\Delta\varphi/2} \cos \varphi d\gamma).$$

The deceleration of the central electron will be complete when $U_1 = U_0$. The electron conversion efficiency is

$$\eta_{el} = \frac{\sin \dfrac{\Delta\varphi}{2}}{\dfrac{\Delta\varphi}{2}} .$$

Assuming that a dimension of $\Delta\varphi \leq 30°$ is achieved, the deviation from unity of the electron conversion efficiency does not exceed 1.5%.

The effect on the electron conversion efficiency of the electron energy spread caused by the circular scanning is analyzed in Appendix II. The results of this analysis show that for a deflection angle $\alpha = 5.7°$, a beam diameter $D \approx \lambda/20$, and an electron energy $eU_0 = 500$ keV the conversion efficiency is not decreased more than 3%. Since during scanning the energy spread is substantial, the third effect mentioned above has not been taken into account.

There are a number of additional factors which limit the electron conversion efficiency, such as the initial energy spread, beam-current instabilities, inaccuracy of level of the load for the output resonator, and so on. Nevertheless, their action can be eliminated or minimized by reasonable selection of the parameters of the stabilizing systems and of the tolerances on the design components. There is only one factor that can be hardly removed, namely the difference between the real electromagnetic field in the output resonator and the traveling wave. This is characterized by the ratio of the maximum to the minimum value of the voltage amplitude across the gap of the output resonator along the cicular slot (VSWR). Assuming that one can maintain this ratio at a 1.03 level, we tolerate a decrease of the electron conversion efficiency of the Gyrocon by an additional 1.5%.

c) Efficiency of the output resonator

Let us examine the losses in the output resonator walls when the amplitude of the electric wave field excited by the beam in the output resonator is lowest. In order to find these conditions, it is necessary to integrate the relativistic equations of motion of an electron in a given field (1). The result, given in Appendix I, permits us to find a velocity component directed along the decelerating field. At the output point from the resonator ($y = b$, see Fig. 3), this velocity component vanishes for complete deceleration of the electron [i.e., when condition (6) is satisfied]. Its vanishing occurs for different electron output and input phases, which depend on detuning of the output resonator, its coupling with the load and on the flight-gap dimension. In this case, the magnitude of the decelerating field will be also varied. A minimum of the field strength will occur if the input electron phase on the resonator is equal to $-\pi/2$, and the output phase to $+\pi/2$ (the reference point being the maximum of the decelerating voltage). Let us call such a regime critical, and the flight gap at which the latter takes place as optimum.

After having integrated a set of relativistic equations of motion, given in Appendix I, one finds the value of the optimum flight gap with the critical regime conditions taken into account.

With this, let us calculate the power of the losses in the output resonator, approximated by a section of waveguide of rectangular cross section. This leads to a relation which defines the conversion efficiency of the output resonator. In the critical regime, the field in the output resonator is minimum and so consequently are the specific losses in its walls. Actually, the total losses are somewhat higher, but this difference is not essential for an approximate analysis, and therefore the conversion efficiency found will be close to the peak value.

d) The overall conversion efficiency and the conversion efficiency of the system

The overall conversion efficiency of the Gyrocon (η) is lower than the electron conversion efficiency because of the beam current losses, which are taken not to exceed 1%, and also because of the losses in the output resonator walls, which constitute 2 to 4% of the maximum beam power, i.e., $\eta \geq 90\%$ for copper resonators of meter and decimeter wavelength range.

The efficiency of a system defined as the ratio of the output VHF power to the power consumed, is required because of the losses in the circuits of the vacuum and other equipment of the Gyrocon ($\approx 3\%$). Moreover, the energy losses in the electron HV accelerator stabilization system can amount to 5%. These estimates apply to a Gyrocon of continuous generation ($U_0 = 500$ kV). Increasing the acceleration voltage ($U_0 \geq 1000$ kV), the maximum beam power grows approximately as U_0^4 and the losses both in the output resonator and in the circuits of auxiliary equipment of the Gyrocon can be neglected to estimate the conversion efficiency.

Thus, the estimates of the top electron conversion efficiency $\eta_{el} = 90$ to 95% and the conversion efficiency of the system $\eta_{syst} = 85$ to 90% give some idea about the energetic possibilities of the Gyrocon.

e) Amplification factor

For the sake of simplicity, we shall assume that the deflection angle of the electron beam is small during scanning ($\alpha \leq 20°$), and the flight gap of this resonator is chosen in such a way that the losses in the walls are approximately equal to those due to acceleration of the electrons in the scanned beam, which corresponds to the minimum of excitation power.

In Appendix II an analysis is given of the electron motion in the field of the scanning resonator formed by two modes of oscillations of type E_{110} (Fig. 2) shifted by $\pi/2$ in space and in phase. The estimation of the maximum amplification factor of the Gyrocon is also made there.

The analysis shows that the losses in the walls of scanning resonator and those due to the electron beam acceleration are proportional to $\tan^2\alpha$. For $\tan\alpha = 0.3$, at energies of 500 to 3000 keV and for extreme beam currents in a wavelength range of 1.6 m $\geq \lambda \geq$ 0.1 m, the maximum amplification power factor does not exceed $K_{max} =$ 10 to 25 db for a scanning resonator made of copper.

One can increase the amplification factor by reducing the deflection angle during scanning. If, for example, $\tan\alpha = 0.1$, $K_{max} = 20$ to 35 db. However, in the simplest design (Fig. 1) operation with a small deflection angle is possible only by elongation of the beam, namely by decreasing the maximum power of the Gyrocon.

Without reducing the power, we can increase the amplification factor by additional deflection of the beam with a stationary field not requiring HF power. The design version of the Gyrocon, where this method is realized at the shortest beam length, is shown in Fig. 4. This is a radial Gyrocon.[12] Here an additional deflection system (5) carries a slightly scanned beam into the plane perpendicular to the Gyrocon axis. In this version an output resonator (6) is a rectangular waveguide with a traveling wave H_{10} closed in a ring in the E plane, rather than in the H plane, as in Fig. 1. A system (5) (Fig. 4), deflecting the relativistic beam almost by 90°, is made as a bending-magnet system (for example, in the form of a conic coil), since an electrostatic system does not possess the required electric strength.

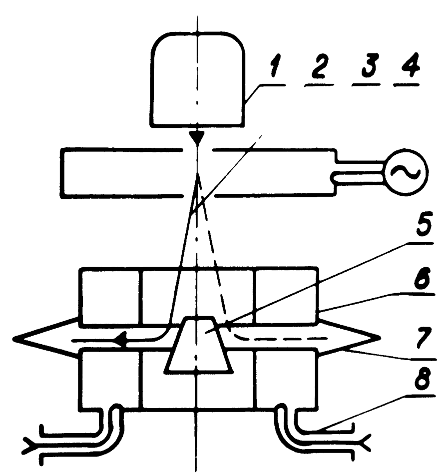

FIGURE 4 The design of the radial Gyrocon. 1) HV accelerator; 2) electron beam; 3) scanning resonator; 4) HF exciter; 5) bending magnet system; 6) output resonator; 7) collector; 8) energy outputs.

The magnetic field near the top of a conical coil (5) (Fig. 4) is similar to the field of a "magnetic charge." Moving in such a radial field, the electron describes a trajectory of double curvature[13]: it does not just pass along a plane perpendicular to the Gyrocon axis, but it gains the azimuthal velocity component necessary for its entrance in the output resonator at a given angle ψ.

The results of the calculation of the electron trajectories in the radial Gyrocon given in Ref. 12 show that the azimuthal beam dimension in the output resonator does not exceed $\Delta\varphi = 30°$.

The magnetic fields of the electron trajectories are comparatively small, and for a Gyrocon of decimeter wave range with a beam energy $eU_0 = 3$ MeV, do not exceed 0.3 to 0.5 T. One should, however, bear in mind that inside the deflection system (near the top of the cone) the field is several times higher, which limits the usefulness of ferromagnetic materials in systems of similar type.

The additional deflection system allows operation with a small deflection angle during scanning. Hence an amplification factor of 20 to 35 db can be obtained in the radial Gyrocon at a beam length $L \approx 2\lambda$, i.e., without decreasing the maximum power and the electron conversion efficiency.

This method for increasing the amplification factor can also be applied for the design of the so-called axial Gyrocon which is shown in Fig. 1. An additional bending system located behind the scanning resonator can increase the initial beam deflection so that the amplification factor reaches a level of 20 to 35 db, but the total beam length in this case somewhat increases in comparison to that of the radial Gyrocon design. An advantage of the axial version is the lower value of the magnetic field of the deflecting system and an advantage of the radial version is the enlarged area of the collector which dissipates the residual beam energy.

A further increase of the amplification factor can be achieved with the use of the scanning device with a passive resonator (Fig. 5).[14] In this device, a passive resonator (5), identical to the resonator (3) but not having the power inputs, is located behind the scanning resonator. The electron beam scanned by the field of the resonator (3) at a small angle excites a wave traveling along the circle in a resonator (5) and with a magnetic field deflecting the electrons at a larger angle. In this case, some fraction of the beam power is lost because of the HF heating of the passive resonator walls.

The increase over the amplification for such a design is $[(\tan\alpha)/(\tan\alpha_0)]^2$, where α_0 and α are the deflection angles of the beam in the active and passive resonators. The estimate given in Appendix II gives a value of the ratio $(\tan\alpha/\tan\alpha_0) \approx 10$ for the distance between the resonators which is equal to $\lambda/2$ and for ultrarelativistic energies of electrons. Thus there is a possibility of increasing the amplification factor of the Gyrocon one hundred times more, though this will cause some decrease of the maximum power.

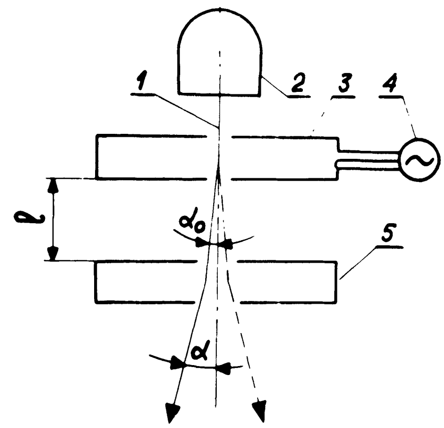

FIGURE 5 The design of the scanning device with a passive resonator. 1) electron beam; 2) HV accelerator; 3) scanning resonator; 4) HF exciter; 5) passive resonator; α_0, α are the deflecting angles in the active and passive resonators.

f) Estimation of the minimum operating wavelength of the Gyrocon

The limitation due to insufficient electric strength at the output resonator gap determines a minimum operating wavelength.

In Appendix I, we derive a relation which relates a wavelength to the electric field strength of the Gyrocon output resonator. This relation is derived for the critical regime and for the optimum gap, at which the electric field strength is the lowest.

If a field strength $E_{tol} \leq 10$ MV/m is assumed to be permissible in the continuous regime, and $E_{tol} \leq 50$ MV/m in the regime of pulses of microsecond duration, then for these conditions we obtain the following values of the minimum operating wavelength λ_{min} at the electron beam energy $eU_0 = 0.5$ to 3 MeV.

For the continuous regime,

$$0.3\text{m} \leq \lambda_{min} \leq 1.1\text{m},$$

and for the pulsed regime,

$$0.06\ \text{m} \leq \lambda_{min} \leq 0.22\ \text{m}.$$

It should be noted that for waves no longer than 10 cm, two of the basic assumptions, the achievement of small diameter of the beam ($D \approx \lambda/20$) at powers of tens and hundreds of megawatts, and the achievement of the current losses on the walls [$(\Delta I)/I_0$] which do not exceed 1%, may turn out to be unrealizable. This can impose additional limitations on the conversion efficiency and power of the Gyrocon.

Table I lists the limiting parameters of the Gyrocon at various wavelengths.

g) The results of the analysis and the estimation of attainable parameters of the Gyrocon

Under the specified assumptions, the approximate analysis of the Gyrocon operation leads to the following relations for estimating its parameters.

Output power

$$P_{max} = 4 \cdot 10^{-5} \left(\frac{D}{L}\right)^2 U_0^{5/2} \left(\frac{e}{mc^2} \cdot \frac{U_0}{2} + 1\right)^{3/2} \cdot \eta. \tag{7}$$

Electron conversion efficiency

$$\eta_{el} = \frac{\sin\frac{\Delta\varphi}{2}}{\frac{\Delta\varphi}{2}} \left(1 - \frac{\pi D}{\lambda}\sqrt{\frac{\gamma_0 + 1}{\gamma_0 - 1}} \cdot \tan\alpha\right) \cdot \frac{1 + \frac{1}{\rho}}{2}. \tag{8}$$

Conversion efficiency of the output resonator

$$\eta_c = 1 - \left(\frac{\delta}{\lambda}\right)\left(\frac{U}{u_0}\right)^2 \frac{u_0^2}{30\,P_{max}} \cdot \frac{\lambda\nu}{4b_0}\left[\left(1 - \frac{1}{\nu^2}\right) + \frac{\lambda}{4b_0\sqrt{\frac{1}{\nu^2} - 1}} \cdot\right] \tag{9}$$

Overall conversion efficiency

$$\eta = \eta_{el} \cdot \eta_c \cdot \left(1 - \frac{\Delta I}{I_0}\right). \tag{10}$$

Amplification factor

$$K_{max} = \frac{5 \cdot 10^{-6}}{\tan^2\alpha}\sqrt{\left(\frac{\lambda}{\delta}\right)\frac{P_{max}}{\gamma_0(\gamma_0 + 1)}}\ . \tag{11}$$

Shortest wavelength

$$\lambda_{min} = \frac{1.6 \cdot 10^6 \sqrt{\gamma_0^2 - 1}}{E_{tol}} \tag{12}$$

In these relations P_{max} is the maximum output power of the Gyrocon, η_{el} is the ratio of the power transformed into VHF oscillations to the beam power, $1 - \eta_c$ is the ratio of the loss power in the output resonator to the beam power, K_{max} is the ratio of the output VHF power to the excitation power in the scanning resonator, η is the ratio of the output VHF power to the beam power, and λ_{min} is the minimum operating wavelength of the Gyrocon.

Further, D, L are the initial diameter and the beam length, U_0 is the accelerating voltage of the electron source, e, m are the charge and the mass of the electron, c is the velocity of light [$(mc^2)/e = 5.11 \times 10^5$ V], $\Delta\varphi$ is the azimuthal beam dimension in the output resonator, [$\gamma_0 = (eU_0/mc) + 1$] is the initial relative energy of electrons, α is the deflection angle of the scanned beam, ρ is the VSWR in the output resonator, and δ is the skin-depth of the resonator walls,

$$\frac{U}{U_0} = \frac{\pi b_0}{\lambda}\sqrt{\frac{\gamma_0 + 1}{\gamma_0 - 1}} \cdot \cos\psi. \tag{13}$$

U is the HF voltage amplitude on the output resonator flight gap,

$$\sin\psi = \frac{1}{\nu}\sqrt{\frac{\gamma_0 - 1}{\gamma_0 + 1}} \qquad \text{see (6)}$$

b_0 is the optimum flight gap of the output resonator (see Appendix I), $\nu = (V/c)$ is a parameter of the output resonator, and V is the phase velocity of the wave traveling in it, see (2). $\Delta I/I_0$ is the relative amount of the beam current losses, and E_{tol} is the permissible electric field strength in the output resonator.

The parameters and numerical coefficients in relations (7)–(12) are given in the units of the International System (SI). The data of Table I are obtained for the diameter of the beam $D = 0.05\,\lambda$, for beam length $L = 2\lambda$, for azimuthal dimension of the beam in the output resonator $\Delta\varphi = 30°$, for scanning angle $a = 5.7°$, for current beam losses $\Delta I/I = 1\%$, for the difference between the wave in

TABLE I

Attainable Parameters of the Gyrocon

U_0 kV	P_{max} MW	Conditions for complete deceleration of the central elelctron		η_{el}	$\lambda = 1.65$ m				$\lambda = 0.7$ m			
		Ψ^0	$\frac{b_0}{\lambda}$		E $\frac{MV}{m}$	K^*_{max} db	$1 - \eta_c$	η	E $\frac{MV}{m}$	K^*_{max} db	$1 - \eta_c$	
500	8	18	0.29	0.94	1.65	26	0.020	0.91	2.9	25	0.030	0.
1000	70	23	0.35	0.95	2.7	29	0.006	0.93	6.3	28	0.010	0.
1500	270	25	0.39	0.955	3.7	31	0.003	0.94	8.4	30	0.004	0.
2000	720	26	0.41	0.96	4.7	32	0.002	0.95	11	31	0.003	0.
3000	3000	28	0.44	0.96	6.6	34	0.001	0.95	16	33	0.002	0.

*The amplification factor is given with no taking into account the possibilities of the scheme with a passive resona the output resonator from the traveling one $\rho = 1.03$, and for copper resonators.

In this Table are also given the values of the angle ψ at which the central electron of the beam is directed to the output resonator for providing complete deceleration, as well as the values of a relative flight gap (b_0/λ) of the output resonator and of its electric field (E) for the critical regime.

Table I illustrates the properties of the Gyrocon as a relativistic generator: a high power and a high conversion efficiency can be attained only at sufficiently high accelerating voltage U_0.†

h) About the estimation accuracy of the conversion efficiency and the power of the Gyrocon

The Gyrocon theory, on which basis the formulae of the maximum power and the conversion efficiency have been derived, takes into account relativistic energies and space-charge effect, and is suitable for large signals in the output resonator. The simplifying assumptions reduce mostly to idealization of the beam and output resonator geometries. This leads to underestimation of the power (7) which can be transferred through the electron optical channel of the Gyrocon, and to overestimation of the conversion efficiency (8), (9), (10).

As a matter of fact, in an idealized (unscanned) freely divergent beam the transverse space charge forces are not smaller than in a real, scanned beam with components located along the helical line upon the conical surface. Therefore, formula (7) can be treated as the lowest estimate. In the scanned beam, however, there arise longitudinal space-charge forces (which are cancelled in the unscanned beam) and this increases the azimuthal dimension of the scanned beam and leads to some decreasing of the electron conversion efficiency (not more than 1% at $\nu = 1.84$).

As for the additional energy losses in the walls of a real annular traveling wave resonator, also due to finite beam dimensions, their account would cause a decreasing of the overall conversion efficiency of the Gyrocon in comparison to that calculated with formula (10) not more than 1 to 2% at $\eta \geq 90\%$ and $\nu = 1.84$. These refinements follow from the more detailed approximate analysis and from the comparison with the results of numerical calculations of energy losses in real designs of the Gyrocon of meter and decimeter wavelength range.

† By decreasing U_0 (under the conditions of Table I), for example, down to 200 kV or 100 kV, the Gyrocon power falls to $p_{max} = 0.6$ MW and $p_{max} = 0.1$ MW, and the overall conversion efficiency to $\eta = 0.75$ to 0.85 and $\eta = 0.45$ to 0.7, respectively.

III. DEVELOPMENT OF GYROCONS AT NOVOSIBIRSK

The development of Gyrocons was started at the Nuclear Physics Institute of the Siberian Division of the USSR Academy of Sciences on the initiative of G. I. Budker.

The first operating model of the gyrocon was built according to the scheme shown in Fig. 1. In 1971, a load power of over 600 kW was produced at an electron energy of 320 KeV with a pulse of 20 μsec long. Thus an electron conversion efficiency of over 90% at 430 MHz was proven possible.

Then two Gyrocons were built with the following design characteristics:

1. A Gyrocon for continuous generation, for HF supply of resonators of the electron-positron storage ring VEPP-4.

Output power	5000 kW
Accelerating voltage	500 kV
Operating wavelength	1.65 m ($f = 181$ MHz)
Amplification factor	23 db
Overall conversion efficiency	80%

2. A pulsed Gyrocon, for HF supply of the linear accelerator supplying positrons to VEPP-4.

Pulsed power	200 MW
Accelerating voltage	2000 kV
Pulse duration	10 μsec
Repetition rate	1 Hz
Operating wavelength	0.7 m ($f = 430$ MHz)
Amplification factor	25 db

Below the data of the design and of the tests of these Gyrocons are given.

a) Gyrocon of continuous generation

The scheme of this radial Gyrocon and its general view are given in Figs. 6 and 7.[15]

The high-voltage accelerator, consisting of a transformer-rectifier ESU-2, similar to that earlier described as ESU-1,[16] and of an accelerating gap of the Gyrocon, is the source of electrons. A 500-kV diode gun is designed for a maximum current of 12 A. The electrons are focused by a magnetic lens so that the location of the beam waist is near the bending magnet system. This enables one to obtain the minimum azimuthal beam dimension in the output resonator.

For annular scanning of the beam at an angle $\alpha = 7°$, at its maximum power, the scanning resonator will be supplied with a HF power of 25

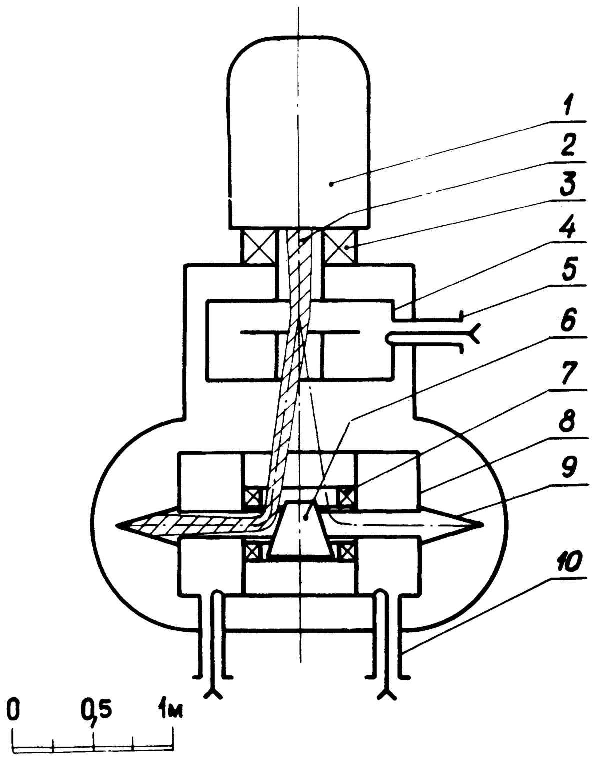

FIGURE 6 The radial Gyrocon for continuous generation of meter wavelength range. 1) source of relativistic electrons; 2) electron beam; 3) magnetic lens; 4) scanning resonator; 5) power input; 6) bending magnet system; 7) cylindrical lens; 8) output resonator; 9) collector; 10) energy output.

kW. In order to reduce the sizes and to cut down the losses, the periphery section of the cylindrical scanning resonator with E_{110} mode of oscillations is deformed so that the field distribution in the flight beam region is conserved. A flight gap is chosen so that about 20 kW is lost in the resonator walls, and about 5 kW is spent for the acceleration of the electron beam. The bending-magnet system is made in the form of conical coil with a uniform winding. In addition to the major rotation of the trajectories by 90°, this system provides compensation of the relativistic effect decreasing the conversion efficiency, by introducing the electrons in the output resonator at an angle of $\psi \approx 18°$. Without such a precaution, the conversion efficiency would decrease by 12%.

The beam, scanned in a plane perpendicular to the Gyrocon axis, is focused by a cylindrical lens so that the vertical size of the beam cross section becomes much smaller than the azimuthal one.

To estimate the overall conversion efficiency of this Gyrocon, one takes into account the losses in the output resonator walls, equal to 200 kW, and the losses in the circuit of the electron voltage stabilization of the ESU-2, which will reach 250 kW for a 5000 kW beam power.

By now a number of tests for the separate systems and the Gyrocon as a whole have been carried out. The HV gap of the electron gun has been tested without the beam at 270 kV constant voltage. Near-breakdown phenomena have not been observed. For beam powers over 500 kW and voltage to 250 kV, breakdowns have been observed but not more often than every two hours. The maximum electric strength of the gap is increased during the training process.

One of the most complex problems was the transfer of the high-power continuous beam through the electron-optical system of the Gyrocon. The maximum power of the beam, scanned at 181 MHz which has been transferred to the collector through the detuned output resonator has exceeded 1000 kW at 220-keV electron energy. The beam boundaries have been found by measuring the current losses on the sectionalized water-cooled electrodes and shields around the beam. We succeeded in tuning the electron-optical channel in such a way that the total current losses did not exceed 1%.

The voltage rise from the initial level of 35 kV to the given value has been performed in steps of 1 kV by the control computer ODRA-1325, with automatic change of the currents in the lenses and the deflection system, and of the input signal level as well. The results of the measurements of the supply regimes, the annular scanning parameters and the current losses on the electrodes have been extracted from the display.

During this test, one of the Gyrocon energy outputs was loaded to a resistance dissipating 200 kW, the second to the accelerating resonator of the storage ring VEPP-4, which was tested on the stand prior to its installation in the storage ring, and the remaining two were removed. The peak HF power measured at 181 MHz at the voltage of the Gyrocon resonator attained 500 kW at the beam power of about 600 kW (electron conversion efficiency > 80%). The Gyrocon load (which in this test limited the power level) was fed by 400 kW.

Performance of the tests became possible after tuning of the HF beam scanning system with its small (0.5%) deviation from the circular system, of the voltage stabilization system ESU-2 which decreases the instabilities down to 0.5% and also of the fast protection which limited an energy release of

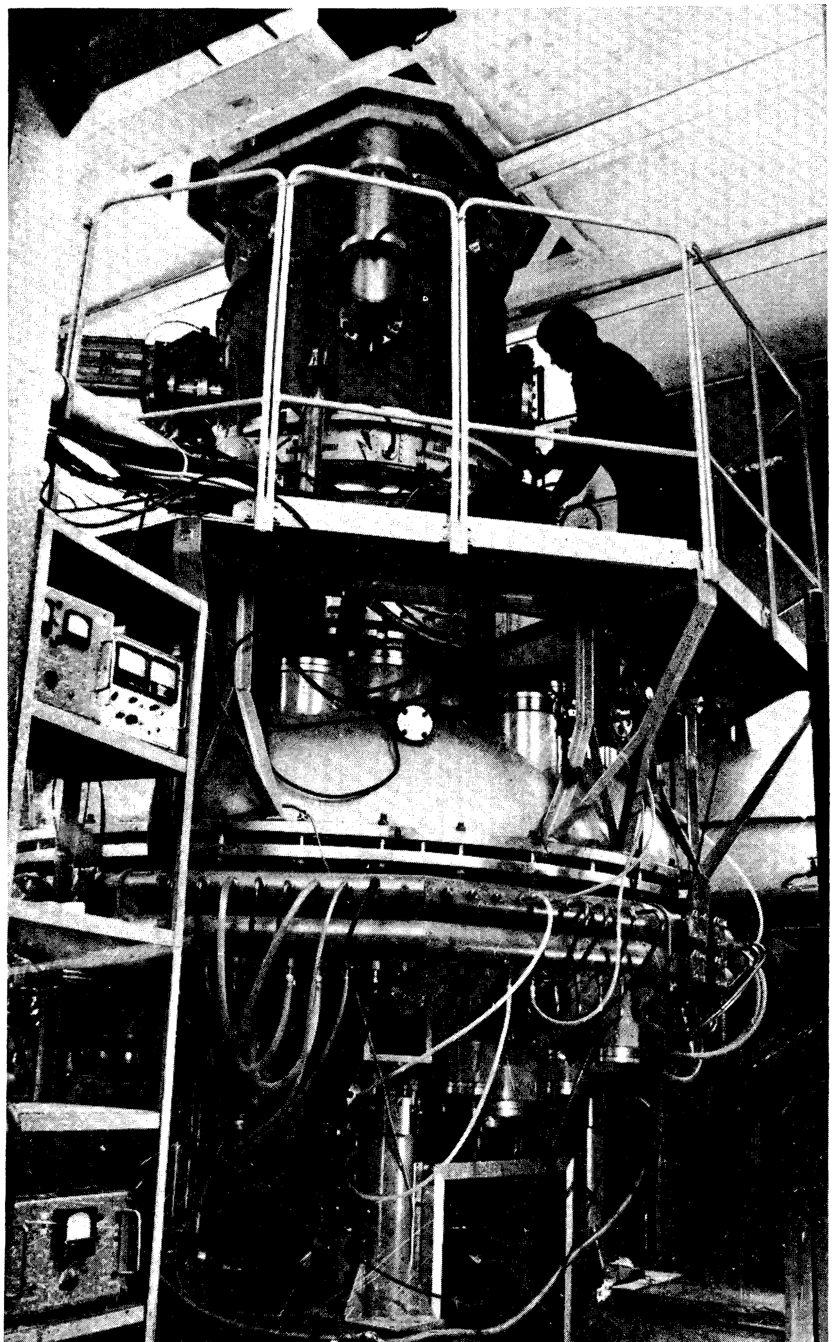

FIGURE 7 The Gyrocon of continuous generation with a design power of 5000 kW at a 181 MHz frequency.

1 kJ at the moment that the focused beam hits the Gyrocon elements.

The results obtained allow us to consider the developed Gyrocon as a HF power source for VEPP-4.

c) A pulsed Gyrocon

The design of the pulsed Gyrocon built according to the axial scheme is shown in Fig. 8 and its general view in Fig. 9.[17] The electron source is of the ELIT-type high-voltage accelerator rated at 2 MeV electron energy, beam current of 100 A, 10 μsec pulse duration, $\pm 1\%$ spread of electron energy, and 1-Hz repetition rate (ELIT-3A).

In the cylindrical scanning resonator, the polarization of a rotating field differs from that of the circular one by less than 1%. The scanning angle $\alpha = 5°$. Approximately half the output power is spent on the losses in the walls, and another half transferred to the beam.

The scanned beam is carried into the first deflecting system, where it is deflected by the constant magnetic field by an angle close to 45°, and then into the second directing the electrons perpendicular to the side walls of the output resonator. The first deflecting system is made as a solenoid of conical shape, and the second with the shape of a coaxial conductor with a current producing the azimuthal magnetic field. The side walls of the second deflecting system are made of wires and each has a 97% transparency to the particles. The magnetic field on the electron trajectory is 0.04 to 0.06 T.

In the output resonator, a constant magnetic field of the order of 10^{-2} T is produced to compensate the relativistic effect that reduces the conversion efficiency. The HF power is transmitted from the output resonator to the vacuum waveguides through two identical energy outputs. These waveguides connect the Gyrocon with the sections of the linear accelerator of the positron injector VEPP-4.

The first tests were done with the lower paramaters of ELIT-3A: an electron energy of 1.3 to 1.4 MeV and a beam current of 45 A. At 430 MHz, 40 MW were produced in a pulse 6 μsec long between the points of half power, at a repetition rate of 0.5 Hz. The electron conversion efficiency was about 80%, the amplification factor 24 db, the current transferred 90%. The purpose of these tests was to work out a way for generating the positrons by means of the linear accelerator fed by the Gyrocon.

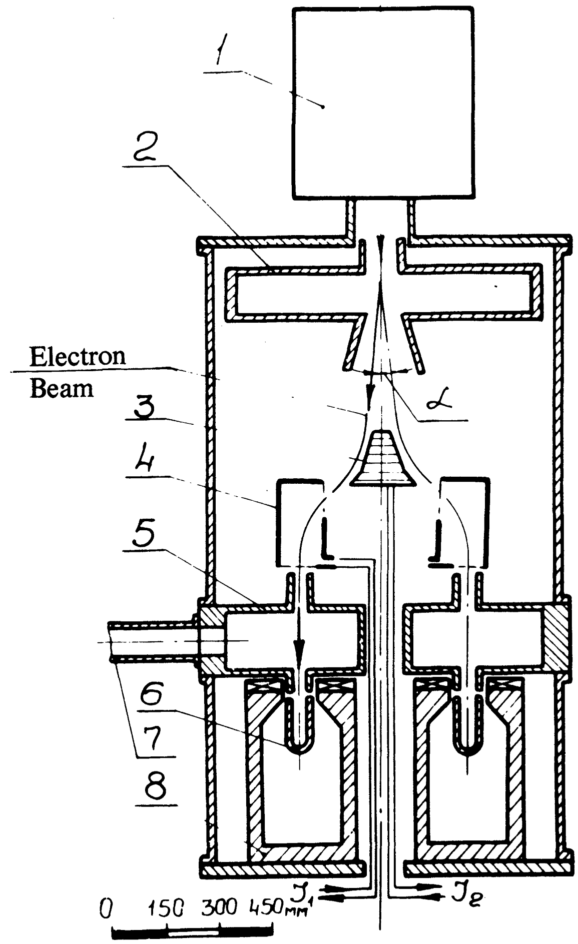

FIGURE 8 The pulsed axial Gyrocon of decimeter wave range. 1) source of relativistic electrons; 2) scanning resonator; 3) first deflecting system; 4) second deflecting system; 5) output resonator; 6) collector; 7) energy output; 8) compensating electromagnet.

After raising the ELIT-3A parameters up to the values close to the design ones, a pulsed Gyrocon will operate as a component of the positron source VEPP-4.

IV. CONCLUSION

Many VHF deflection-modulated devices described in the literature are close enough to the Gyrocon but do not have its capabilities. The new elements required to achieve the high levels of power, efficiency and amplification factor that dis-

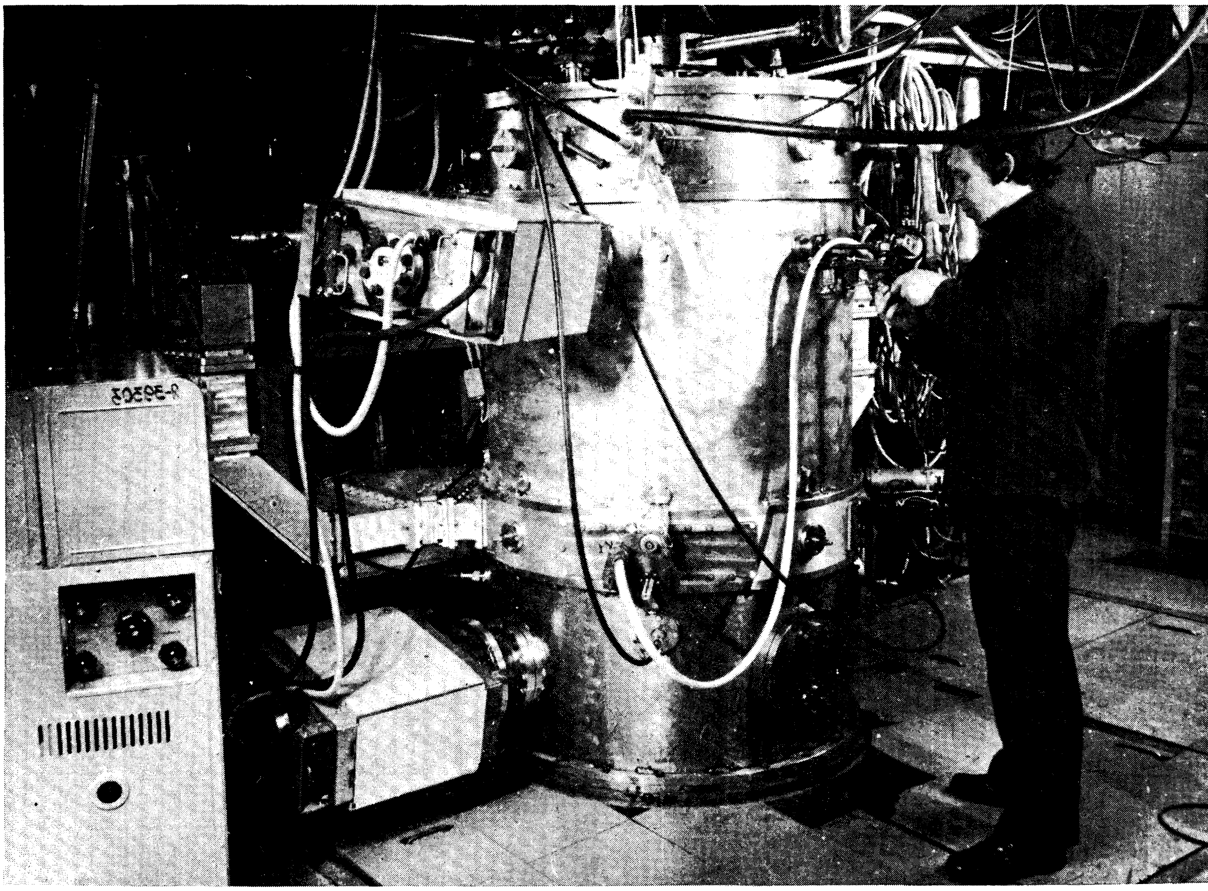

FIGURE 9 The pulsed Gyrocon with a designed power of 200 MW at a 430-MHz frequency.

tinguish the Gyrocon from the other devices are the following:

a) a source of relativistic electrons,

b) a compensating electromagnet or a magnetic system designed to direct electrons into the output resonator at a given angle; with these systems the relativistic electrons are completely decelerated in the output resonator;

c) a bending magnet that enables one to work with a small scanning angle, keeping the minimum beam length which is required to achieve high amplification at high power.

The estimates of the main parameters presented here show that in a range of meter and decimeter waves, the Gyrocon can in principle attain a rf power substantially exceeding the powers of klystrons and control-grid tubes with an amplification factor of over 40 db and with 90–95% electron conversion efficiency, unusually high for VHF devices.

Whether these parameters can be realized is connected to the solution of a series of complex technical problems. Some arise for the development of superpower VHF devices of any type, the others only for the development of the Gyrocon. In the latter case one can refer to the problem of inventing an electron-optical system such that the current losses on the walls are not more than 1% under the conditions when constant magnetic guidance of the scanned electron flux encounters difficulties. The planning of the HF system keeping a high accuracy of the magnetic-field circular polarization in the scanning resonator, and also of the energy-stabilization system of the electrons from the HV accelerator are connected with this problem. In order to operate with a high conversion efficiency, the system is required to maintain accurately the traveling-wave regime in the output resonator.

Despite these difficulties, the possibility of achieving HF power and the conversion efficiency unattainable by means of the devices of other types, justify the efforts to produce such a complex device as the Gyrocon.

Application of the Gyrocon in the systems for information transfer will apparently be limited because of the narrow-band scanning resonator.

The first experimental results confirm the correctness of our understanding of the Gyrocon operation and its advantages. They show that the new source of VHF oscillations will find application in the accelerating technique and VHF energies, especially in those fields where a high power per unit equipment at a high conversion efficiency is required.

ACKNOWLEDGEMENTS

The authors are very pleased to thank the coworkers of the Novosibirsk Nuclear Physics Institute participating in the design of Gyrocons and also in developing the systems of supply, control and electronics necessary for operation of these devices.

The authors express their gratitude to the designers of the HV electron accelerators, which have been used for the Gyrocons as the sources of the relativistic beam of particles.

APPENDIX I

INTEGRATION OF THE EQUATIONS OF MOTION OF A RELATIVISTIC ELECTRON IN ELECTROMAGNETIC WAVE FIELD. CALCULATION OF AN OPTIMUM GAP IN THE GYROCON OUTPUT RESONATOR

1. For convenience of integration, let us write a set of equations of electron motion in a field of the wave (1) traveling in the waveguide (Fig. 3) with a phase velocity V (2) in the four-dimensional form:

$$\left.\begin{aligned} \ddot{x} &= -k\dot{y}\cos\beta q + k_0(y)\cdot\dot{y} \\ \ddot{y} &= -k\dot{q}\cos\beta q - k_0(y)\cdot\dot{x} \\ \ddot{z} &= 0 \\ c\ddot{t} &= -kv\dot{y}\cos\beta q \end{aligned}\right\} \quad \text{(I.5)}$$

Here

$$k = \frac{eB}{mc}, \quad k_0(y) = \frac{eB_0(y)}{mc} \quad \text{(I.6)}$$

B is the amplitude of the magnetic induction of the traveling wave field (1), $B_0(y)$ is the compensating-field magnetic induction, $[v = (V/c) = (E/cB)]$ is a parameter of the waveguide (2), E is the amplitude of the electric field strength of the traveling wave (1),

$$q = v\cdot ct - x, \quad \omega t - \frac{\omega}{V}x = \beta q, \quad \beta = \frac{\omega}{V}.$$

Dots denote differentiation with respect to s, where $ds = cdt\ [1 - (|v|^2/c^2)]$ is the four-dimensional element of length. The initial conditions in this notation are of the form at $s = 0$

$$\left.\begin{aligned} c\dot{t} &= \gamma_0, & t &= t_0, \\ \dot{x}_0 &= \sqrt{\gamma_0^2 - 1}\cdot\sin\psi, & x &= x_0, \\ \dot{y}_0 &= \sqrt{\gamma_0^2 - 1}\cdot\cos\psi, & y &= 0, \\ \dot{z}_0 &= 0. & z &= 0, \end{aligned}\right\} \quad \text{(I.7)}$$

where ψ is the so-called input angle (Fig. 3). After elimination of the term $-k\dot{y}\cos\beta q$ from the first and fourth equations of Eq. (I.5) and a single integration, taking account of the initial conditions (I.7), we get a first integral of motion

$$v\dot{x} - c\dot{t} - \int_0^y k_0(y)dy = v\sqrt{\gamma_0^2 - 1}\cdot\sin\psi - \gamma_0. \quad \text{(I.8)}$$

If the compensating field is absent $k_0(y) = 0$ and the initial electron velocity is transverse ($\psi = 0$), then the first integral (I.8) takes the form

$$v\dot{x} - c\dot{t} = -\gamma_0. \quad \text{(I.9)}$$

Taking into account that $\dot{x} = \gamma\beta_x$, $c\dot{t} = \gamma$, where $\beta_x = v_x/c)$ is the relative longitudinal velocity of the electron, and also the fact that at the point where the electron deceleration is completed ($\gamma = \gamma_b$, $\beta_x = \beta_{bx}$), its transverse velocity becomes equal to zero, the expression (I.9) can be written as

$$\gamma_b(v\beta_{bx} - 1) = -\gamma_0. \quad \text{(I.9a)}$$

Then

$$\gamma_b = \frac{1}{\sqrt{1 - \beta_{bx}^2}}, \quad \text{(I.9b)}$$

since at the indicated point the total electron energy is connected only to its longitudinal velocity. Eliminating the quantities β_{bx} and γ_b from (I.9a), (I.9b), and (4), we obtain an expression for the electron conversion efficiency of a device with annular traveling wave resonator

$$\Pi = \frac{1-\sqrt{1-\beta_0^2\left(1-\frac{1}{\nu^2}\right)}}{\left(1-\sqrt{1-\beta_0^2}\right)\left(1-\frac{1}{\nu^2}\right)},$$

$$\beta_0 = \frac{|v_0|}{c}, \nu = \frac{V}{c} \qquad \text{(I.10)}$$

2. If the compensating field and the input angle are not equal to zero ($k_0(y) \neq 0$ and $\psi \neq 0$), then at the point where the electron is completely decelerated, not only the transverse but also the longitudinal velocity can be equal to zero. The condition of such a complete deceleration of the electron at the point $y = b$, where b is the flight gap dimension, follows from Eq. (I.8) at $c\dot{t}_b = \gamma_b = 1$ and $\dot{x}_b = 0$. Thus

$$\frac{1}{\sqrt{\gamma_0^2-1}} \int_0^b k_0(y)dy + \sin\psi = \frac{1}{\nu} \sqrt{\frac{\gamma_0-1}{\gamma_0+1}} \qquad \text{(I.11)}$$

At $k_0(y) = 0$, for the required input angle the expression (6) follows from (I.11), and at $\psi = 0$ the expression (5).

3. In the subsequent integration of the set (I.5) it will be assumed that $k_0(y) = 0$, and $\psi \neq 0$.

From the second equation of this set we get another integral of motion

$$\dot{y} = \dot{y}_0 + \frac{k}{\beta} (\sin\beta q_0 - \sin\beta q). \qquad \text{(I.12)}$$

If the electron completes its deceleration during the transfer ($q = q_b$), then both the transverse and longitudinal velocities are converted into zero ($\dot{y}_b = 0$). In order to ensure this, it is required to impose conditions on the input phase and on the value of the output resonator flight gap so that the relation (I.13) is fulfilled. That is,

$$\dot{y}_0 = \frac{k}{\beta} (\sin\beta q_b - \sin\beta q_0). \qquad \text{(I.13)}$$

The relation (I.13a) follows from the previous one

$$\sqrt{\gamma_0^2-1} \cdot \cos\psi = (\gamma_0 - 1) \left(\frac{\lambda}{2\pi b}\right)\left(\frac{U}{U_0}\right) (\sin\beta q_b - \sin\beta q_0). \qquad \text{(I.13a)}$$

Here βq_0 and βq_b are the input and output phases of the electron. The condition (I.13a) can be fulfilled at various values of the difference ($\sin\beta q_b - \sin\beta q_0$). In this case, quantities $(U/b) = E$ are also different. A minimum of E will occur at $\beta q_0 = -\pi/2$ and $\beta q_b = +\pi/2$. This is the critical regime at which $b = b_0$ is the optimum flight gap.

The minimum electric field strength in the gap is here equal to

$$E = E_{\min} = \frac{U_0\pi}{\lambda} \sqrt{\frac{\gamma_0+1}{\gamma_0-1}} \cdot \cos\psi, \qquad \text{(I.13b)}$$

whence the minimum operating wavelength of the Gyrocon is connected with the permissible electric field strength as

$$\lambda_{\min} = \frac{1.6 \cdot 10^6 \sqrt{\gamma_0^2-1}}{E_{\mathrm{tol}}} \cos\psi.$$

At $\cos\psi = 1$, this yields the relations (12). The expression (13) for the overvoltage coefficient (U/U_0) in the critical regime also follows from (I.13a).

4. Solving the first and fourth equations of the set (I.5) and the Eq. (I.12), one obtains a differential equation with separate variables

$$\ddot{q} = -k(\nu^2-1)\left(\dot{y}_0 + \frac{k}{\beta}\sin\beta q_0\right)\cos\beta q + k(\nu^2-1)\frac{k}{\beta}\sin\beta q \cdot \cos\beta q. \qquad \text{(I.14)}$$

The integration of this equation gives

$$\dot{q} = \frac{dq}{ds} = \frac{A}{b}\sqrt{(\sin\beta q - \sin\beta q_b)^2 + \left[\frac{b^2}{A^2}\dot{q}_0^2 - (\sin\beta q_0 - \sin\beta q_b)^2\right]}. \qquad \text{(I.15)}$$

Here $A = k\sqrt{\nu^2 - 1}$, b is the flight-gap size ensuring the relation (I.13). When this relation is fulfilled, the first integral (I.12) is also simplified to

$$\dot{y} = \frac{dy}{ds} = \frac{k}{\beta}(\sin\beta q_b - \sin\beta q). \quad \text{(I.16)}$$

After introducing a new variable,

$$\theta = \sin\beta q_b - \sin\beta q, \quad \text{(I.17)}$$

Eq. (I.18) follows from Eqs. (I.15) and (I.16).

$$\frac{dy}{d\theta} = -\frac{k}{A\beta} \cdot \frac{\theta}{\sqrt{G(\theta)}} . \quad \text{(I.18)}$$

The solution of these equations makes it possible to find the coordinate of an output point of the electron from the resonator ($y = b$) in the case of zero transverse velocity, i.e., a value of the flight gap. In Eq. (I.18) we introduce

$$\left.\begin{aligned} &G(\theta) = \sqrt{(\alpha_1 - \theta)(\theta - \alpha_2)(\theta^2 + n^2)},\\ &\alpha_1 = 1 + \sin\beta q_b,\ \alpha_2 = -1 + \sin\beta q_b,\\ &n^2 = \frac{\beta^2}{A^2}\dot{q}_0^2 - (\sin\beta q_0 - \sin\beta q_b)^2. \end{aligned}\right\} \text{(I.19)}$$

The value of the flight gap b is equal to

$$b = -\frac{k}{A\beta}\int_{\theta_0}^{\theta_b}\frac{\theta d\theta}{\sqrt{G(\theta)}} . \quad \text{(I.20)}$$

For the regime where $\beta q_0 = -\pi/2$ and $\beta q_b = +(\pi/2)$ [i.e., $\theta_0 = 2$, and from (I.17), $\theta_b = 0$] with the above notation taken into account, we have

$$\frac{2\pi b_0}{\lambda} = \frac{\nu}{\sqrt{\nu^2 - 1}}\int_0^2\frac{\theta d\theta}{\sqrt{G(\theta)}} . \quad \text{(I.21)}$$

This integral is reduced to the complete elliptic integrals and the expression for the relatively optimum flight gap takes the form

$$\frac{2\pi b_0}{\lambda} = \frac{\nu}{\sqrt{\nu^2 - 1}} \cdot \frac{1}{\sqrt{\Gamma}}\sqrt{\frac{\Gamma + 1}{\Gamma - 1}}\left[\pi\sqrt{\Gamma}\sqrt{\frac{\Gamma - 1}{\Gamma + 1}}\right.$$

$$\left. + (\Gamma - 1)K(k^2) - (\Gamma + 1)\Pi(m,k^2)\right], \quad \text{(I.22)}$$

where

$$K(k^2) = \int_0^{\pi/2}\frac{d\varphi}{\sqrt{1 - k^2\sin^2\varphi}} ,$$

$$\Pi(m,k^2) = \int_0^{\pi/2}\frac{d\varphi}{(1 + m\sin^2\varphi)\sqrt{1 - k^2\sin^2\varphi}} ,$$

$$k^2 = \frac{\Gamma - 1}{2\Gamma} ,\ m = \frac{2}{\Gamma - 1} ,\ \Gamma = \frac{1}{\nu^2} + \left(1 - \frac{1}{\nu^2}\right)\gamma_0.$$

APPENDIX II

ANALYSIS OF ELECTRON MOTION IN THE RESONATOR OF BEAM CIRCULAR SCANNING. ESTIMATION OF THE AMPLIFICATION FACTOR OF GYROCON

1. For the electromagnetic field **E**, **B** in the cylindrical resonator with an E_{110} mode of oscillation, which forms a circular traveling wave (see Fig. 2), the following relation is fulfilled between the field components E_z and B_r at any point.

$$\frac{E_z}{B_r} = \omega r, \quad \text{(II.1)}$$

where E_z is the electric field strength, B_r is the radial component of the magnetic flux density, ω is the circular frequency of oscillations, and r is the radial coordinate of the point.

Cylindrical coordinates (r, φ, z) are chosen so that the z axis coincides with the resonator axis. The equations of motion of the electron are

$$\frac{d\mathbf{p}}{dt} = -e\mathbf{E} - e[\mathbf{vB}],$$

$$\frac{d\mathcal{E}}{dt} = -e\mathbf{E}\cdot\mathbf{v},$$

where $\mathbf{p} = \gamma mc\boldsymbol{\beta}$, $\boldsymbol{\beta} = (\mathbf{v}/c)$, $\gamma = 1/[\sqrt{1 - (|v|^2/c^2)}]$, $\mathcal{E} = \gamma mc^2$, c is the velocity of light, e, m, $\mathbf{v}$ are the charge, mass and velocity of the electron. In the field (II.1), the equations have a first integral of the following form

$$\gamma\left(\frac{\omega r^2}{c^2} \cdot \frac{d\varphi}{dt} - 1\right) = \gamma_0\left[\frac{\omega r_0^2}{c^2}\left(\frac{d\varphi}{dt}\right)_0 - 1\right] . \quad \text{(II.2)}$$

The coordinates, relative energy and velocity of the electron at the input to the resonator are denoted by zero.

The electron entering such a field along the resonator axis will exit off the axis and will change its energy. The expression (II.2) for the electron with $(d\varphi/dt)_0 = 0$ takes the form

$$\frac{\gamma_0}{\gamma} = 1 - \frac{\omega r^2}{c^2}\left(\frac{d\varphi}{dt}\right) = 1 - \frac{\omega}{c^2}\left(x\frac{dy}{dt} - y\frac{dx}{dt}\right). \tag{II.3}$$

Here $x = r\cos\varphi$, $y = r\sin\varphi$ are Cartesian coordinates, wherein the equation of motion may be integrated completely if it is assumed that the rotating magnetic field in the region of electron motion is uniform and that the longitudinal velocity (along the z-axis) of the electron also does not change. The change in the electron energy γ_0/γ is estimated by (II.3) from the coordinates and velocities of the electron at its output from the resonator, which are found in these approximate solutions. The deflecting angle during the scanning α is estimated from the same solutions. By definition,

$$\tan\alpha = \frac{\sqrt{(dx)^2 + (dy)^2}}{dz}, \tag{II.4}$$

and this can be calculated by formula (II.5) with the above assumptions

$$\tan\alpha = \frac{1}{2J_1(t'_{11})}\sqrt{\frac{\gamma_0 - 1}{\gamma_0 + 1}}\cdot\left(\frac{U_m}{U_0}\right)\cdot\frac{\sin\kappa}{\kappa}. \tag{II.5}$$

Here $J_1(t'_{11})$ is the Bessel function of the first order in the first maximum, U_m is the amplitude of rf voltage in the maximum of the resonator electric field, U_0 is the accelerating voltage of an electron source, $\kappa = [(\pi h)/\beta_0\lambda]$ is half the transit time angle, h is the flight gap, λ is the resonant wavelength ($\lambda = 1.64a$), and a is the resonator radius. The error of the formula (II.5) does not exceed 10% at $\alpha \leq 20°$ and $\kappa \leq 0.6$.

2. The HF power required to produce the field deflecting the electron by an angle α during circular scanning is defined in terms of the resonator voltage (U_m) and the shunt impedance R_{sh} as

$$P_1 = \frac{U_m^2}{2R_{sh}}\,;\; R_{sh} = \frac{h}{\delta}\sqrt{\frac{\mu_0}{\varepsilon_0}}\cdot\frac{0.175}{1 + \frac{a}{h}}.$$

The voltage U_m is calculated from (II.5).

The value of P_1 obtained is the power lost in the resonator walls.

$$P_1 = \frac{U_0^2}{\sqrt{\frac{\mu_0}{\varepsilon_0}}}\cdot 12.3\left(\frac{\delta}{\lambda}\right)\left(\frac{\gamma_0}{\gamma_0 - 1}\right)^2\cdot\frac{\beta_0\kappa + 1.9}{\sin^2\kappa}\tan^2\alpha \tag{II.6}$$

The change in energy of the electron γ/γ_0 (II.3) which travels through the scanning resonator enables one to calculate the interaction power of the beam with the resonator field

$$P_n = P_0\frac{\gamma_0}{\gamma_0 - 1}\left(\frac{\gamma}{\gamma_0} - 1\right), \tag{II.7}$$

where P_0 is the beam power.

This power is spent on electron acceleration in the beam scanning

$$P_n = P_2 = P_0\left(1 + \frac{1}{\gamma_0}\right)(1 - \kappa\cot\kappa)\cdot\tan^2\alpha \tag{II.8}$$

3. By definition the amplification factor of the Gyrocon is equal to

$$K_p = \frac{P_{out}}{P_1 + P_2}, \tag{II.9}$$

where $P_{out} = P_0\eta$, and $P_1 + P_2 = P_{inp}$ is the excitation power of the Gyrocon. Depending upon the transit time angle 2κ, the minimum of the P_{inp} occurs under the condition that

$$\kappa = \kappa_0 \approx \sqrt[4]{M}, \tag{II.10}$$

where

$$M = \frac{5\cdot 10^{10}(\delta/\lambda)\gamma_0^3}{P_0(\gamma_0 + 1)} \tag{II.11}$$

is the parameter taking into account the ratio between the power of losses in the walls and the beam power P_0.

Under the condition (II.10), the amplification factor is maximum and equal to

$$K_p = \frac{3}{2\sqrt{M}}\cdot\frac{\eta}{\left(1 + \frac{1}{\gamma_0}\right)}\cdot\frac{1}{\tan^2\alpha} \tag{II.12}$$

At $\kappa \leq 0.6$ and $M \leq 0.15$, the error of this formula is not much more than 20%.

The maximum amplification factor K_{max} (11) is calculated from (II.12) and (II.11) at $P_{out} = P_{max}$ and $\eta = 0.9$.

4. In the scanning device with a passive resonator (Fig. 5) the power P_n (II.7) is taken from the electrons and lost in the passive resonator walls. From the energy-balance conditions one can determine the relation between the angle α_0, at which

the beam was deflected in the active resonator, the latter being located at the distance l from the passive one, and the deflecting angle α in the passive resonator. We find

$$\frac{\tan\alpha}{\tan\alpha_0} = \frac{[(\pi l/\beta_0\lambda) + \kappa_0]\cos\kappa_0 - \frac{1}{2}\sin\kappa_0}{1 - \kappa_0\cot\kappa_0} \qquad \text{(II.13)}$$

The relation (II.13) is obtained for the flight gap in the passive resonator, which is defined from the condition (II.10). In this case, the gain in the amplification factor is close to the maximum.

5. The energy spread of the electrons during the beam scanning is estimated from the change in energy of the electron being at the edge of the beam.

For the electron with initial coordinates x_0, y_0 we get the following estimate from the results of approximate determination of the coordinates and velocities at the resonator output by the relation (II.3):

$$\frac{\Delta U}{U_0} = \frac{2\pi r_0}{\lambda}\sqrt{\frac{\gamma_0 + 1}{\gamma_0 - 1}} \cdot \tan\alpha \qquad \text{(II.14)}$$

Here $\pm e\Delta U$ is the maximum energy spread and $r_0 = D/2 = (x_0^2 + y_0^2)^{1/2}$ is the beam radius in the scanning resonator. The electron conversion efficiency is reduced by an amount $\Delta U/U_0$, that is taken into account by the second factor in parentheses in Eq. (8).

REFERENCES

1. G.I. Budker et al., U.S. Patent 3,885,193 (1975).
2. M.S. Neyman, Electron-Beam Generator, USSR Patent (Author's Certificate) No. 67,770 (1940).
3. P.V. Hartley, U.S. Patent No. 2,381,539 (1945).
4. J.W. McRae, U.S. Patent No. 2,408,437 (1946).
5. A.V. Whale and J.R. Pickin, British Patent No. 954,840 (1964).
6. I. Kaufman, U.S. Patent No. 3,219,873 (1965).
7. G.T. Conrad, *IEEE Trans. Nucl. Sci.*, **NS-22,** 1249 (1975).
8. L.L. Clampitt, *Proc. IEEE* **61**, 279 (1973). (Powerful Electrovacuum Instruments of the SVCh "Mir", 1974, p. 50, 126.)
9. M.M. Karliner et al., Facility for the Circular Generation of a Beam of Charged Particles, USSR Patent (Author's Certificate) No. 471,847 (1975).
10. I.N. Meshkov and B.V. Chirikov, *ZhTF* **35**, 12, 2202 (1965).
11. W.I. Cleen, *Electronics of Microwave Tubes* (Academic Press Inc., New York, 1958) (Introduction to UHF Electronics. Moscow, Sov. Radio, 1963, p. 443.)
12. G.I. Budker et al., U.S. Patent No. 4,019,088 (1977).
13. S.A. Boguslavsky, *Selected Papers on Physics* (Moscow, 1961), p. 162.
14. M.M. Karliner et al., Gyrocon, USSR Patent (Author's Certificate) No. 503,444 (1975).
15. G.I. Budker et al., High-Frequency Electron-Positron Generating System VEPP-4 Based on the Gyrocon—A Powerful Ultrashort-Wave Generator of Non-Group Relativistic Beam, *Proc. V USSR National Conf. on Particle Accelerators,* Moscow, 1976, p. 284.
16. V.A. Gaponov et al., Development and Practical Application of Electron Accelerators, *Proc. USSR National Conf.* Tomsk National University Press, 1975, p. 131.
17. G.I. Budker et al., Source of Positrons for the VEPP-4 Generators, *Proc. V USSR National Conf. on Particle Accelerators,* Moscow, 1976, p. 280.
18. S.B. Vasserman et al., Pulsed High-Voltage Electron Accelerators, *Proc. IV USSR National Conf. on Particle Accelerators,* Moscow, 1974, p. 285.