Particle Accelerators
1978, Vol. 8, pp. 107–113

HEAVY ION TOROIDAL COLLECTIVE ACCELERATOR†

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(*Received January 18, 1977; in final form April 14, 1977*)

Experiments on HIPAC at Maxwell Laboratories have shown that almost all of the confined electrons are trapped and do not go around the torus. A toroidal electric field produces a negligible toroidal electron current. We consider an ion accelerator where electrons are magnetically contained and their space charge contains ions. A toroidal electric field of suitable magnitude can be applied so that it accelerates all of the ions but does not accelerate most of the electrons. This is possible if the magnetic moment of electrons $\mu_e > \mu_i/Z$, where μ_i is the ion magnetic moment and Z is the charge of the ion. Ions would be contained by the electron space-charge electric field E, for energies up to $ZeER/2 \sim 100$ GeV where $Z = 60$, $E = 10^7$ V/cm and the major radius of the torus is $R = 3.3$ meters.

I INTRODUCTION

The motivation for this work is the Berkeley[1] summer study of heavy ion accelerators for pellet fusion. The requirements of the accelerator are that it produce a few megajoules of heavy ions such as uranium at about 100 GeV with a power of about 100 terawatts. The beam quality must be such that it can be focussed to a spot size of about 5 mm at a distance of 10 meters. This involves an emittance $\varepsilon/\pi \lesssim 12$ mrad·cm. These requirements can possibly be met with conventional linear accelerators or storage rings. However, the dimensions will be of the order of kilometers and the price of the order of 10^9 $.

We consider a collective accelerator based on the principles of HIPAC[2] which has been studied experimentally and theoretically at AVCO Corporation and at Maxwell Laboratories, Inc. The idea is to use magnetic confinement to contain electrons in a torus. Ions would be contained and focussed by the electric fields interior to the toroidal electron cloud. Collective effects are only employed to focus ions. The acceleration itself would be conventional and accomplished with an externally produced toroidal electric field. The essential point of this paper is that it is possible to accelerate only ions and not electrons around the torus by making use of the fact that the magnetic moment of the electrons μ_e is much larger than the magnetic moment of the ions divided by the ion nuclear charge μ_i/Z. Then with a mildly "bumpy torus" the magnetic mirror effect would prevent electrons from being accelerated around the torus by the external electric field, $|\mu_e\, \partial B/\partial z| > eE_z$, while ions would be accelerated, $ZeE_z > |\mu_i\, \partial B/\partial z|$.

HIPAC experiments[3] with the Mark II device showed that a toroidal electron ring of density $n_0 = 4 \times 10^9$ cm^{-3} could be confined for 10 msec. The three types of instabilities that were identified and studied were the diocotron instability,[4] the magnetron instability,[5] and the ion resonance instability.[6] The diocotron instability can be absent with proper selection of the electron cloud configuration. The magnetron instability is unimportant if the stability parameter $q = (\omega_p/\Omega_e)^2 < 0.05$, where $\omega_p = \sqrt{4\pi n_0 e^2/m}$ is the electron plasma frequency and $\Omega_e = eB_z/mc$ is the electron gyrofrequency. The ion resonance instability involves a resonance between ions oscillating in the potential well of electrons at the same frequency as electron guiding centers precess. It depends on q, the ratio M/Z of the ions and a/b, where a is the electron cloud radius and b is the minor radius of the torus. For ion charge up to 10% of the electron charge, it is easy to choose parameters so that this instability can be avoided, and the theory is supported by the HIPAC–Mark II experiments. For our present considerations, this is sufficient since the ion density should not be large enough to significantly depress the potential well of the electrons.

† Supported by the National Science Foundation.

‡ This paper is based in part on a thesis submitted to the University of California, Irvine, in partial fulfillment of the requirements for the Ph.D. degree.
UCI Technical Report #77-1.

In recent experiments at Maxwell Laboratories,[7] electron densities of 10^{10} cm^{-3} were confined. The most important result for present purposes was the discovery that almost all of the electrons were in trapped orbits and did not circulate around the torus. When a toroidal electric field of 0.5 V/cm was applied, the resultant current was less than 50 amperes; according to potential measurements the confined charge was 100 μ Coulombs, which should have led to a toroidal current of 10 kA if the electrons were not trapped.

In this paper we consider the conceptual design of an accelerator to produce 2 MJ of U^{60+} ions at 100 GeV. It appears that it is in principle possible with a torus of major radius 3.3 meters. In order to switch out the ions, we propose the use of a plasma gun to locally neutralize the electron cloud, in which case the ions should move tangentially out of confinement. If this can be done at five locations around the torus and the resultant beams brought to focus on a pellet, a beam power of 100 terawatts can be achieved. By considering the oscillations of the ions within the electron cloud we estimate an upper bound to the beam emittance of $\varepsilon/\pi = 27.5$ mrad $\cdot$ cm.

II PARTICLE ORBITS

Electron and ion orbits are illustrated schematically in Figure 1. The electron motion consists of a rapid gyration at frequency $\Omega_e = eB_z/\text{mc}$ and a slower precession around the axis at frequency $\omega_D = cE_r/rB_z$. The magnetic moment of the electron is

$$\mu_e = \frac{1}{2}\frac{mv_\perp^2}{B}. \tag{1}$$

If B changes slowly and periodically around the torus, the mirror force would be $-\mu_e\,\partial B/\partial z$. If a toroidal electric field is applied,

$$m\frac{\mathrm{d}v_z}{\mathrm{d}t} = -\mu_e\frac{\partial B}{\partial z} - eE_z. \tag{2}$$

If $|E_z| > |(\frac{1}{2}mv_\perp^2/eB)\Delta B/\Delta z|$, the electron would be accelerated around the torus. For example, if $\Delta B/B = 10^{-2}$ over a distance of 17 cm and $\frac{1}{2}mv_\perp^2 = 10$ kV, E_z must exceed 6 V/cm. These data are typical of the experiment at Maxwell Laboratories[7] where a negligible toroidal current was observed with $E_z = 0.5$ V/cm.

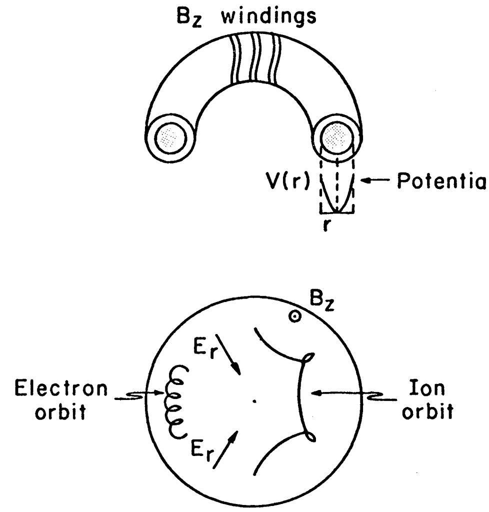

FIGURE 1 Particle orbits in a toroidal collective accelerator.

The ion orbit is illustrated in Figure 1. The ion gyroradius is not small so that the usual theory of adiabatic motion is not applicable. We consider the ion motion in cylindrical geometry which should be sufficient for a very small b/R torus. b is the minor radius and R is the major radius. The equations of motion are

$$\begin{aligned} M\ddot{x} &= Ze\left(-2\pi n_0 ex + \frac{1}{c}\dot{y}B_z\right), \\ M\ddot{y} &= Ze\left(-2\pi n_0 ey - \frac{1}{c}\dot{x}B_z\right). \end{aligned} \tag{3}$$

The electric field $E_x = -2\pi n_0 ex$, $E_y = -2\pi n_0 ey$ is due to a uniform cylinder of electrons of density n_0. Introduce $\Omega_i = ZeB_z/Mc$, $\omega_i^2 = 2\pi n_0 Ze^2/M$, and $\zeta = x + iy$. Equations (3) simplify to

$$\ddot{\zeta} + i\Omega_i\dot{\zeta} + \omega_i^2\zeta = 0. \tag{4}$$

With the initial condition that at $t = 0$, $\dot{\zeta} = 0$, $\zeta = \zeta_0$, the solution is

$$\zeta = \frac{\zeta_0}{(\omega_- - \omega_+)}\left[\omega_- e^{i\omega_+ t} - \omega_+ e^{i\omega_- t}\right]. \tag{5}$$

The angular momentum of an ion is $L_z = M(x\dot{y} - y\dot{x})$, and from Eqs. (3)

$$L_z + \frac{Ze}{2c}r^2B_z = \text{constant} = \frac{Ze}{2c}r_0^2B_z, \tag{6}$$

where $r^2 = |\zeta|^2$ and $r_0^2 = |\zeta_0|^2$. The ion magnetic moment is

$$\mu_i = \frac{Ze}{2Mc} L_z$$

$$= \frac{(Ze)^2}{4Mc^2} B_z r_0^2 \left[1 - \frac{\omega_+^2 + \omega_-^2}{(\omega_+ - \omega_-)^2} - \frac{2\omega_+ \omega_-}{(\omega_+ - \omega_-)^2} \cos(\omega_+ - \omega_-)t \right]. \tag{7}$$

We have made use of Eq. (5) to calculate μ_i and

$$\omega_{\pm} = \frac{-\Omega_i \pm \sqrt{\Omega_i^2 + 4\omega_i^2}}{2}. \tag{8}$$

Averaging out the fast oscillation, the ion magnetic moment becomes

$$\mu_i = -\frac{(Ze)^2}{2Mc^2} B_z \frac{\omega_+ \omega_-}{(\omega_+ - \omega_-)^2} r_0^2. \tag{9}$$

Consider the following numerical example:

beam radius $a = 1$ cm
electron density $n_0 = 10^{13}\ \mathrm{cm}^{-3}$
magnetic field $B_z = 50$ kG
uranium ions $M \cong 238$ proton masses
ionization state $Z = 60$
gyrofrequency $\Omega_i = 1.2 \times 10^8\ \mathrm{sec}^{-1}$
$\omega_i = 1.6 \times 10^9\ \mathrm{sec}^{-1}$

The ion magnetic moment for $\omega_i \gg \Omega_i$ is

$$\mu_i \cong \frac{1}{8} \frac{M(\Omega_i r_0)^2}{B} = \frac{W_\perp}{B} \tag{10}$$

and

$$\frac{\mu_i}{Z\mu_e} = \frac{W_\perp}{\frac{1}{2}mv_\perp^2 Z} = 1, \qquad \text{if } r_0 = a$$

and $\frac{1}{2}mv_\perp^2 = 7.5$ keV. The electron energy depends on the nature of the injector and the degree of magnetic compression of the electron cloud. We assume an electron energy after compression of 100 keV. This satisfies the requirement that $B_z^2/8\pi \gg \frac{1}{2}n_0 mv_\perp^2$. For this electron energy $\mu_i/Z\mu_e = 0.075$. If $\Delta B/B = 10^{-2}$ over a distance of 20 cm and $\langle mv_\perp^2/2 \rangle = 100$ keV, ions would be accelerated around the torus for $E_z > 3.75$ V/cm and for electrons $E_z > 50$ V/cm would be required.

III EQUILIBRIUM TOROIDAL EFFECTS

Consider a solenoid with n turns/unit length carrying a current I wound on a torus of major radius R

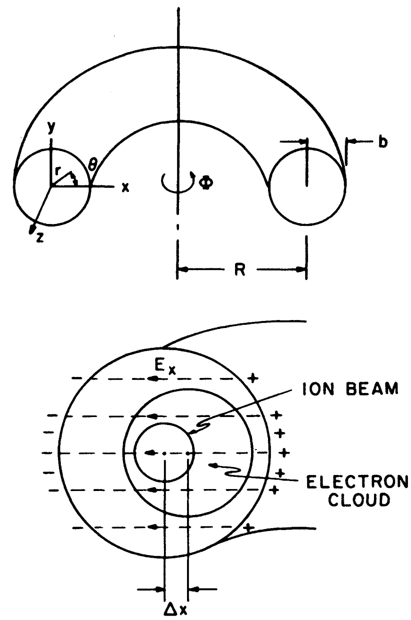

FIGURE 2 Equilibrium positions of electrons and ions.

and minor radius b. The toroidal magnetic field is

$$B_z = \frac{4\pi n I}{c} \frac{1}{\left(1 - \frac{r}{R}\cos\theta\right)} \cong B_0 \left[1 + \frac{r}{R}\cos\theta \right], \tag{11}$$

where the coordinates are illustrated in Figure 2. It has been shown for an electron beam in a torus[8] that the only explicit toroidal effect for $b/R \ll 1$ is to shift the axis of the beam towards the inside wall of the torus as indicated in Figure 2 by the amount

$$\Delta x_e = \frac{2\gamma_0^2 V_z^2}{\omega_p^2 R}. \tag{12}$$

V_z is the electron velocity around the torus which in the present study is negligible because electrons are trapped and do not go around the torus.

Consider a toroidal electron cloud in which ions move with the equations of motion

$$M\left(\frac{\mathrm{d}v_x}{\mathrm{d}t} + \frac{V_z^2}{R}\right) = Ze\left[E_r \frac{x}{r} + \frac{1}{c} v_y B_z\right]$$

$$M\frac{\mathrm{d}v_y}{\mathrm{d}t} = Ze\left[E_r \frac{y}{r} - \frac{1}{c} v_x B_z\right] \tag{13}$$

$$v_z = V_z\left[1 + \frac{r}{R}\cos\theta\right].$$

These equations are expressed in a local Cartesian coordinate system such as is shown in Figure 2. It has been assumed that $r/R \ll 1$ and only the lowest order corrections to cylindrical geometry are included. The origin is taken to be on the axis of the electron beam. Since $ZeE_r x/r = -M\omega_i^2 x$, the only explicitly toroidal effect disappears if we make the transformation

$$x' = x + \frac{V_z^2}{R\omega_i^2}. \tag{14}$$

For 100 GeV uranium ions, $V_z = 2.2 \times 10^{10}$ cm/sec. For $Z = 60$ and $R = 3.3$ m, $\Delta x_i = 0.6$ cm. The ion orbit shifts outward so that the collective electric field of the electron cloud can provide the centripetal force. Thus we would expect the center of the ion beam to be shifted outward from the center of the electron cloud.

The centripetal force for all of the ions must be provided by the electron cloud and hence a reaction force must act back on the electron cloud. What force keeps the electron cloud within the torus? The electron cloud will shift toward the inner wall of the torus. The charge density on the torus will be of the form $\sigma = \sigma_0 \cos\theta$ so that there will be a uniform electric field $E_x = -2\pi\sigma_0$ within the torus and the resultant force on the electron cloud will be $F_x = -eN_e E_x$, where F_x is the force per unit length and $N_e = n_0 \pi a^2$ is the number of electrons per unit length. If the center of the electron cloud is shifted by Δx_e

$$\Delta E_n = \frac{2N_e e}{b^2}\Delta x_e \cos\theta = 4\pi\sigma. \tag{15}$$

ΔE_n is the incremental normal electric field due to the shift of the electron cloud. Thus $\sigma_0 = N_e e\Delta x_e/2\pi b^2$ and $E_x = -N_e e\Delta x_e/b^2$. The electron cloud will shift inward until force balance is achieved.

$$F_x = \frac{(N_e e)^2}{b^2}\Delta x_e = N_i \frac{MV_z^2}{R}$$

$$\frac{\Delta x_e}{a} = \frac{4N_i}{N_e}\frac{M}{m}\left(\frac{b}{a}\right)^2\left(\frac{c}{\omega_p a}\right)^2 \beta_i^2 \frac{a}{R} = 0.3. \tag{16}$$

$\beta_i = V_z/c$, $\omega_p^2 = 4\pi n_0 e^2/m$, and we have assumed 100 GeV uranium ions with $N_i = 0.6 \times 10^{11}$ corresponding to 1.9 MJ of ions in a torus of major radius $R = 3.3$ m. We have also assumed that $a = 1$ cm, $b = 1.5$ cm. The polarization electric field E_x is too small to provide the centripetal force for the ions: $E_x \cong 1$ MV/cm, and of course it is in the wrong direction. However, it is sufficient to hold the electrons in place which in turn provide the centripetal force for ions.

IV ACCELERATOR CHARACTERISTICS

In Table I a set of possible parameters is shown for a heavy ion toroidal accelerator.

It is necessary to inject, trap and compress 10 milli-Coulombs of electrons. Injection must take place during the rise time of the toroidal field to 50 kG. Assume that $B_z = B_0 + (B_1 - B_0)t/\tau$. Electrons are injected beginning at $t = 0$ at a rate $\dot{N}$. Each element can be labelled by $\mu = \dot{N}t$. Due to the rising magnetic field each element moves inward according to

$$\left(\frac{\partial r}{\partial t}\right)_\mu = -\frac{1}{2}\frac{r}{B_z}\frac{\partial B_z}{\partial t}. \tag{17}$$

At $t = \mu/\dot{N}$, $r = b$. For $t > \mu/\dot{N}$

$$r(\mu, t) = b\frac{[1 + (B_1 - B_0)(\mu/\tau B_0 \dot{N})]^{1/2}}{[1 + (B_1 - B_0)(t/\tau B_0)]^{1/2}} \tag{18}$$

and the density is

$$\begin{aligned} n(\mu, t) &= \frac{1}{2\pi r(\mu, t)\left(\frac{\partial r}{\partial \mu}\right)_t} \\ &= n_0[1 + (B_1 - B_0)(t/\tau B_0)], \end{aligned} \tag{19}$$

where $n_0 = \tau B_0 \dot{N}/\pi b^2(B_1 - B_0)$. The density depends only on time. At $t = \tau$, the element $\mu = N_e$ should be at radius a. Thus,

$$\dot{N} \cong \left(\frac{b}{a}\right)^2 \frac{N_e}{\tau}. \tag{20}$$

The injection period is $\tau_i = (a/b)^2\tau = 450$ μsec if the magnetic field rise-time is 1 msec. During this time the injection current should be 22.5 A. The thermionic injectors employed at AVCO produced such a current, but injection only took place for rather small toroidal fields, less than 1 kG. A different thermionic injector developed by Amnon Fisher[9] has been employed in the Maxwell torus experiment[7] where injection took place at several kilogauss. 100 μ Coulombs were injected with a single injector. However, it has not been demonstrated that such injectors will work in an environment where the electric field is of the order of 10 MV/cm and the magnetic field is 50 kG. Probably an electron accelerator will be required but, since

TABLE I

Design parameters for heavy ion toroidal accelerator

TORUS PARAMETERS			
Minor radius	b = 1.5 cm	Magnetic Field	B = 50 kG
Major radius	R = 3.3 m	Magnetic Energy	$\frac{B^2}{8\pi}V$ = 150 kJ
Volume	V = 1.5 × 10^4 cm^3	Base Vacuum	p_0 = 5 × 10^{-9} Torr
Surface area	S = 2 × 10^4 cm^2		
ELECTRON PARAMETERS			
Beam radius	a = 1 cm		
Electron density	$n_0 = 1.1 \times 10^{13}$ cm^{-3}		
Total number of electrons	$7.2 \times 10^{16} \equiv 10^{-2}$ Coulombs		
Peak electric field	$2\pi n_0 ea = 10^7$ V/cm		
Electric field energy	37.5 kJ		
Stability parameter	$q = (\omega_p/\Omega_e)^2 = 0.04$		
Thermal velocity	$V_{\perp e} = 1.9 \times 10^{10}$ cm/sec		
ION PARAMETERS U^{60+} – 100 GeV			
Total number of ions	$1.2 \times 10^{14} \equiv 1.15$ milli-Coulombs		
Ionization time	36 msec		
Toroidal electric field	10 V/cm		
Acceleration time	9 msec		
Pulse length	94 nsec		
Ion current	12.5 kA		
Total ion energy	1.9 MJ		
Power (5 switches)	100 terawatts		
Maximum emittance	27.5π mrad · cm		

the current requirement is modest, this appears to be a soluble problem.

Uranium would be injected in the form of neutral atoms which would be ionized by the electron cloud and trapped in it. The ionization times have previously been studied in great detail at AVCO.[10] The value of $n_0 t_i$ for the production of $Z = 60$ is 4×10^{11} cm^{-3} sec. For $n_0 = 1.1 \times 10^{13}$ cm^{-3}, the time is 36 msec. The acceleration time to 100 GeV or a velocity of 2.2×10^{10} cm/sec is $t_a = MV_z/ZeE_z = 9$ msec assuming $Z = 60$ and a toroidal electric field $E_z = 10$ V/cm. Of course there would be a distribution of charge states of the ions and the beam would not be mono-energetic. However, the ions can change their charge state without being lost as in a conventional storage ring.

During the process of ionization, it is assumed that the potential well produced by the electron cloud does not change. Let E_i be the initial total energy of the ion prior to an ionization when the particle is at the top of the well. Let E_m be the energy immediately after ionization and E_f be the final energy at the top of the well after the ionization. Initially $E_i = Z_i e\varphi_i$ and $T_i = 0$. Immediately after ionization $E_m = Z_f e\varphi_m + T_m$ and $T_m = Z_i e(\varphi_i - \varphi_m)$. The final state is $E_f = Z_f e\varphi_f$ and $T_f = 0$. Noting that $E_m = E_f$ we conclude that $\varphi_f = (Z_i\varphi_i + (Z_f - Z_i)\varphi_m)/Z_f$. Since $\varphi_m < \varphi_i$, it follows that $\varphi_f < \varphi_i$ so that ionization makes the particle oscillate deeper in the potential well. The kinetic energy at the bottom of the potential well is

$$\begin{aligned} T_0 &= Z_i e(\varphi_i - \varphi_m) + Z_f e\varphi_m \\ &= Z_i e\varphi_i + (Z_f - Z_i)e\varphi_m, \end{aligned} \tag{21}$$

so that it is increased.

The largest kinetic energy the ion can have at the bottom of the well is $T_{max} = Z_f e\varphi_i$ which we use to estimate an upper bound to the emittance

$$\frac{\varepsilon}{\pi} = \theta a_i, \tag{22}$$

where $\theta \cong V_\perp/V_z = \sqrt{2Z_f e\varphi_i/MV_z^2} = 55$ mrad. Since ions would be shifted to the outside of the electron cloud as illustrated in Figure 2, because of the centripetal force, the ion radius a_i should be less than half the electron cloud radius. Assuming $a_i = 0.5$ cm, we obtain an upper limit to the emittance of $\varepsilon/\pi = 27.5$ mrad · cm. This is about a factor of 2 greater than required, but it is of course only an upper bound estimate.

The instabilities that have previously been considered for HIPAC are diocotron, magnetron, and ion resonance. They can be avoided and this has been documented in previous HIPAC experiments. In the present concept there are many other possible instabilities. A complete study is beyond the scope of the present paper, but we shall consider some of the most important instabilities that might be expected and show that they can be controlled. The electrons, being trapped between mirrors, should have a loss-cone distribution and it is well known that such a distribution leads to a loss-cone instability. However, there is a density threshold given by $\omega_p \gtrsim \Omega_e$ or $q \sim 1$ so that this instability would not be expected in this device where $q = 0.04$. There is some experimental support for this in experiments with non-neutral plasmas confined in mirrors.[11] There are some well-known instabilities that arise from ions streaming past electrons. We consider the most dangerous instabilities when $\mathbf{k} = (0, 0, k_z)$ is parallel to the streaming velocity and the magnetic field. For electrons we assume a distribution function of the form

$$f_e(\mathbf{v}) = \frac{n_0 v_\perp^2}{2(2\pi)^{3/2} V_{\parallel e} V_{\perp e}^4} \exp - \left[\frac{v_\perp^2}{2V_{\perp e}^2} + \frac{v_z^2}{2V_{\parallel e}^2}\right], \tag{23}$$

and for ions

$$f_i(\mathbf{v}) = \frac{n_i}{(2\pi)^{3/2} V_{\parallel i} V_{\perp i}^2} \exp - \left[\frac{(v_z - V_z)^2}{2V_{\parallel i}^2} + \frac{v_\perp^2}{2V_{\perp i}^2}\right]. \tag{24}$$

The dispersion relation for electrostatic waves is

$$D(k, s) = 1 - \frac{\omega_{pe}^2}{2(k_z V_{\parallel e})^2} Z'\left(\frac{\omega}{\sqrt{2}|k_z| V_{\parallel e}}\right) - \frac{\omega_{pi}^2}{2(k_z V_{\parallel i})^2} Z'\left(\frac{\omega - k_z V_z}{\sqrt{2}|k_z| V_{\parallel i}}\right) = 0, \tag{25}$$

where

$$Z(z) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \mathrm{d}\zeta \frac{e^{-\zeta^2}}{\zeta - z} \qquad \text{for } \operatorname{Im} z > 0$$

and is defined by analytic continuation for $\operatorname{Im} z < 0$; $Z'(z) = \mathrm{d}Z/\mathrm{d}z$. From Eq. (25) there is a possible high frequency instability obtained by using the asymptotic form for $Z'(z) \cong 1/z^2 + 3/2z^4 \ldots$ when $|z| \gg 1$. Equation (25) simplifies to

$$\frac{\omega_{pe}^2}{\omega^2 - k_z^2 C_e^2} + \frac{\omega_{pi}^2}{(\omega - k_z V_z)^2 - k_z^2 C_i^2} = 1, \tag{26}$$

where $C_e = \sqrt{3}\, V_{\parallel e}$, $C_i = \sqrt{3}\, V_{\parallel i}$, $\omega_{pe}^2 = 4\pi n_0 e^2/m$ and $\omega_{pi}^2 = 4\pi n_i (Ze)^2/M$. There will be an instability if $V_z > C_e + C_i$ which is essentially a plasma-wave instability of frequency $\omega_{pe} \gg \omega_{pi}$. There is also a possible low-frequency instability obtained by using the small argument asymptotic form of $Z'(z)$ for the second term in Eq. (25) and the large argument form for the third term. This leads to an ion-wave instability of frequency

$$\omega = \frac{\omega_{pi}(k_z L_e)}{\sqrt{1 + (k_z L_e)^2}} - k_z V_z \tag{27}$$

and growth rate

$$\gamma = \frac{1}{2}\sqrt{\frac{\pi}{2}} \frac{\omega}{1 + (k_z L_e)^2} \left\{\frac{V_z}{V_{\parallel e}} - \frac{1}{\sqrt{1 + (k_z L_e)^2}} \left(\sqrt{\frac{m}{M}} + \left(\frac{T_{\parallel e}}{T_{\parallel i}}\right)^{3/2} \exp - \left[\frac{1}{2}\left(\frac{T_{\parallel e}}{T_{\parallel i}}\right) \frac{1}{1 + (k_z L_e)^2}\right]\right)\right\}, \tag{28}$$

where $L_e = V_{\parallel e}/\omega_{pe}$, $T_{\parallel e} = mV_{\parallel e}^2$, $T_{\parallel i} = MV_{\parallel i}^2$. This instability can be avoided if $T_{\parallel e} \lesssim T_{\parallel i}$ in which case the Landau damping term dominates and $\gamma < 0$.

These instabilities must be avoided during the acceleration of ions to a final velocity of $V_z = 2.2 \times 10^{10}$ cm/sec. During the stripping and acceleration time the electron and ion temperatures $T_{\parallel e}$, $T_{\parallel i}$ will change. Assuming only classical collisions, the electron distribution will become isotropic on a time scale of $\tau_{ee} = 10^9 W_e^{3/2}/n_0 = 100$ msec assuming $W_e = 100$ keV and $n_0 = 10^{13}$ cm^{-3}. After 45 msec, $T_{\parallel e} \sim 45$ keV and $\sqrt{3}\, V_{\parallel e} \sim 2 \times 10^{10}$ cm/sec $\cong V_z$ so that the criterion for high-frequency instability would not be satisfied. Initially the ion temperatures would be $T_{\perp i} = MV_{\perp i}^2/2 \cong 5$ MeV and $T_{\parallel i} \cong 0$. The ion distribution would become isotropic on a time scale $\tau_{ii} = 6.6 \times 10^{11}$ $(W_i^{3/2}/n_i Z^4) = 500$ msec assuming $W_i = 5$ MeV, $n_i = 0.7 \times 10^{11}$ cm^{-3} and $Z = 60$. If initially $T_{\parallel i} = 0$, after 10 msec $T_{\parallel i} \cong 100$ keV $> T_{\parallel e}$ so that the low-frequency instability would be Landau damped. It is thus clear that with the parameters selected for the present accelerator concept, the streaming instabilities can be avoided during the acceleration of ions.

In order to extract the ions after they have been accelerated to the required energy, we consider the use of a plasma gun to fire a plasma stream into the electron cloud. This would neutralize the space charge of the electron cloud at a particular location. The uranium ions would then move tangentially to their previous circular path into a suitably placed drift tube. The pulse length of the ions extracted would be $t = 2\pi R/V_z = 94$ nsec. Since the total ion energy is 1.9 MJ, this would give a beam power of 20 terawatts. To obtain a beam power of 100 terawatts, it would be necessary to switch out the beam simultaneously at 5 locations and guide 5 beams to the pellet target. It would be necessary to fire 5 plasma guns simultaneously with a few nanoseconds jitter, which is a non-trivial technical problem. It may be easier to use a single plasma gun and propagate the plasma beam in 5 separate channels.

Conventional accelerators for heavy ion pellet fusion[1] are attractive because they are based on a developed mature technology which, with some extrapolation, can possibly meet the requirements discussed in the introduction. They also have an established repetition rate capability. In contrast, the present collective scheme is yet to be developed on a one shot basis. However, in view of the fact that it offers the possibility of reducing the scale size by three orders of magnitude with a similar reduction in cost, it is of interest for pellet fusion, where economics will be a decisive consideration.

ACKNOWLEDGEMENT

We were greatly assisted by discussions with John Dawson of the University of California, Los Angeles, and Alfred Mondelli of Maxwell Laboratories.

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