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ELECTRON ORBITS IN THE ERA (ELECTRON RING ACCELERATOR) WITH AN ELECTROSTATIC-TYPE INFLECTOR SYSTEM

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Electron orbits in the ERA with an inflector system are calculated. The operational condition for effective capture of the electron beam is presented.

1. INTRODUCTION

The ERA (Electron Ring Accelerator) in the Institute of Plasma Physics is now under construction. The assembly of the ERA system has as components a field emission high intensity electron gun, a post accelerator and a compression system in a large vacuum chamber. The beams are injected into the chamber through the snout and are captured with the assistance of the inflectors. The inflector system is an electrostatic one, which can produce easily a rectangular pulse with very short rise and fall times. The electrodes are set along two concentric circles in the vacuum chamber as shown in Figure 1. They consist of copper plates divided azimuthally into four equal sections. A snout is made of ferromagnetic material with current-carrying conductors to shield the static magnetic field and the pulsed electric field.[1] It extends close to the outer electrode and the electron beam is injected tangentially into the region between the plates. In normal operation, a pulsed radial electric field is applied uniformly along the total circumference.

In this paper we have calculated electron orbits to get operational conditions for effective capture of electrons.

2. CALCULATION

The equation of motion of a particle of charge e and mass m in a combined electric and magnetic field is

$$\frac{d}{dt}(m\mathbf{v}) = e(\mathbf{E}+\mathbf{v}\times\mathbf{B}). \qquad (1)$$

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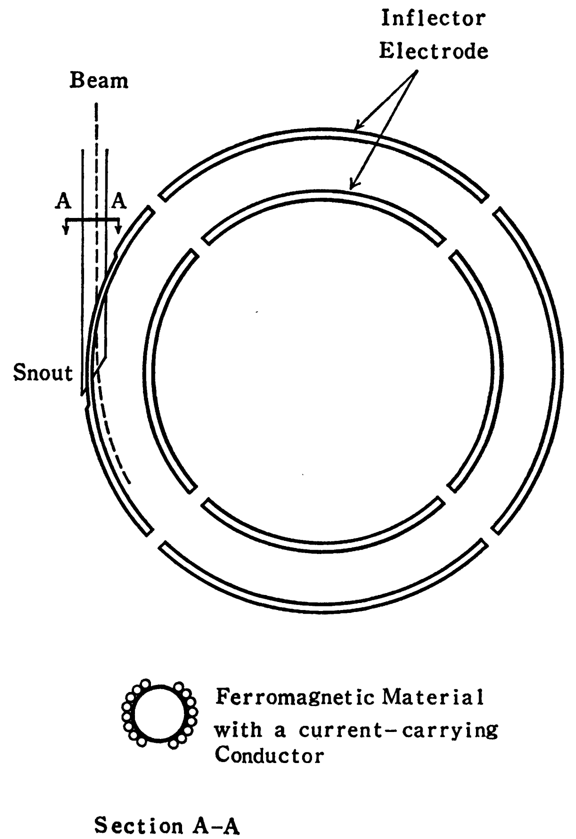

FIGURE 1. Schematic diagram of the inflector system.

In cylindrical coordinates Eq. (1) becomes

$$\frac{d}{dt}(m\dot{r}) = mr\dot{\theta}^2 + eE_r + e(r\dot{\theta}B_z - \dot{z}B_\theta), \qquad (2)$$

$$\frac{1}{r}\frac{d}{dt}(mr^2\dot{\theta}) = eE_\theta + e(\dot{z}B_r - \dot{r}B_z), \quad (3)$$

$$\frac{d}{dt}(m\dot{z}) = eE_z + e(\dot{r}B_\theta - r\dot{\theta}B_r), \quad (4)$$

where the dots indicate differentiation with respect to time. For relativistic motion $m = m_0\gamma$, where m_0 is the rest mass of the particle and

$$\gamma = (1 - v^2/c^2)^{-1/2}. \quad (5)$$

Introducing nondimensional variables

$$x = t/T,$$
$$y_1 = r/R,$$
$$y_2 = \theta,$$
$$y_3 = z/R,$$
$$y_4 = (T/R)\gamma\dot{r},$$
$$y_5 = (T/R^2)\gamma r^2\dot{\theta},$$
$$y_6 = (T/R)\gamma\dot{z},$$

we can write Eq. (2) in the form

$$\frac{dy_i}{dx} = f_i(x, y_1, y_2, \cdots, y_6), i = 1, 2, \cdots, 6, \quad (6)$$

$$f_1 = y_4/\gamma, \quad (7)$$

$$f_2 = y_5/(\gamma y_1{}^2), \quad (8)$$

$$f_3 = y_6/\gamma, \quad (9)$$

$$f_4 = \frac{y_5{}^2}{\gamma y_1{}^3} + Ae_r + B\left(\frac{y_5}{\gamma y_1}b_z - \frac{y_6}{\gamma}b_\theta\right), \quad (10)$$

$$f_5 = y_1\left[Ae_\theta + B\left(\frac{y_6}{\gamma}b_r - \frac{y_4}{\gamma}b_z\right)\right], \quad (11)$$

$$f_6 = Ae_z + B\left(\frac{y_4}{\gamma}b_\theta - \frac{y_5}{\gamma y_1}b_r\right), \quad (12)$$

$$\gamma = \left[1 + \left(\frac{R}{cT}\right)^2\left(y_4{}^2 + \frac{y_5{}^2}{y_1{}^2} + y_6{}^2\right)\right]^{1/2}, \quad (13)$$

$$e_\alpha = E_\alpha/E_0,$$
$$b_\alpha = B_\alpha/B_0 \quad (\alpha = r, \theta, z),$$
$$A = \frac{eE_0 R}{m_0 c^2}\left(\frac{cT}{R}\right)^2$$

and

$$B = \frac{eB_0}{m_0}T,$$

where c is the velocity of light in free space, and T, R, E_0, and B_0 are scaling factors.

In the inflector system of the ERA, electric and magnetic fields are as follows:

$$E_\theta = E_z = B_r = B_\theta = 0,$$

$$E_r = \begin{cases} E_0 & 0 \leqq t \leqq \tau \\ 0 & \text{otherwise} \end{cases}$$

$$B_z = B_0\left[1 + \frac{R}{B_0}\frac{\partial B_z}{\partial r}\left(\frac{r}{R} - 1\right)\right] = B_0\left[1 - n\left(\frac{r}{R} - 1\right)\right],$$

where

$$n = -\frac{R}{B_0}\frac{\partial B_z}{\partial r}.$$

Numerical calculations of the electron trajectory in these fields were performed on a HITAC-8500 digital computer by using Eqs. (6–13) with the following initial conditions and parameters:

Initial conditions:

$r = 27\,\text{cm}$, $\theta = 0$, $z = 0$, $v_r = 0$, $v_z = 0$,
electron energy = 600 keV.

Parameters:

radius of equilibrium orbit $r_0 = 22$ cm,
$E_0 = -0.2, -0.4, -0.6, -0.8, -1.0, -1.2\,\text{kV/cm}$,
$\tau = 1, 3, 5, 7, 9$ nsec,
$n = 0.7$.

The gyration period on the equilibrium orbit is 5.2 nsec. Typical examples of the orbits are shown in Figures 2, 3, 4 using FACOM GRAFIC DISPLAY System.[2] A dotted circle E is the equilibrium orbit.

When the electric field E_0 is zero (corresponding to Figure 2), the electron does not clear the snout at the 2nd revolution ($\sqrt{1-n} = 0.55$). In the case of $E_0 = -0.4$ kV/cm and $\tau = 7$ nsec, the electron clears the snout as shown in Figure 3, but the amplitude of the betatron oscillation is rather large.

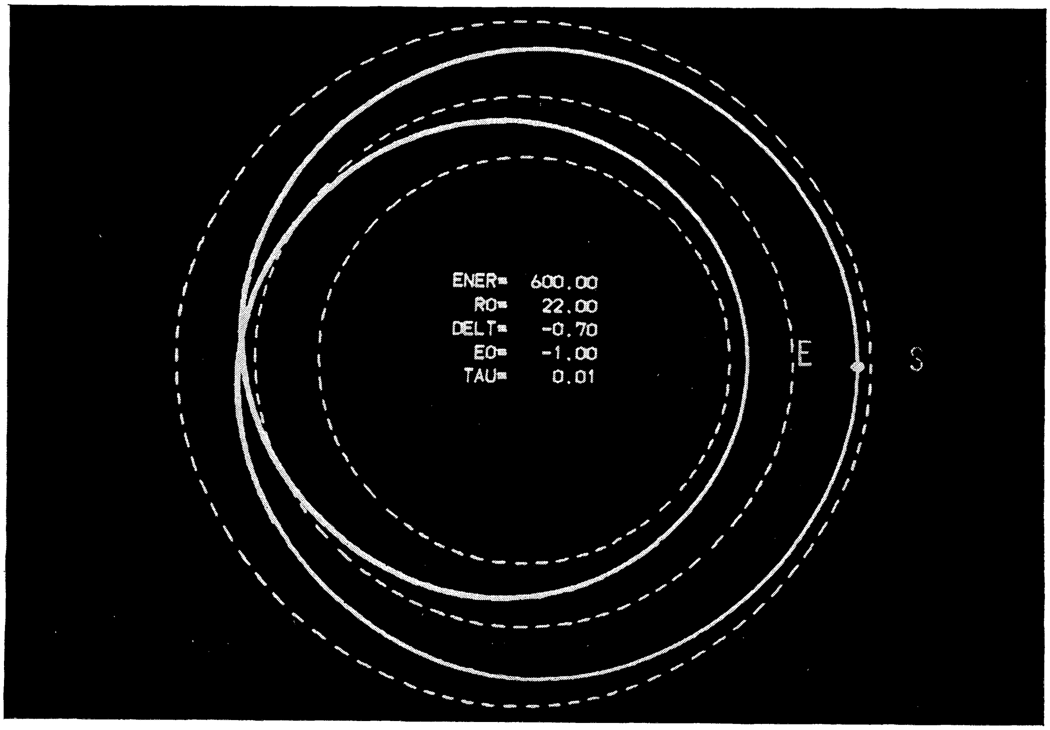

FIGURE 2. Electron orbit. $E_0 = 0$. S: Start. A dotted circle E: Equilibrium orbit.

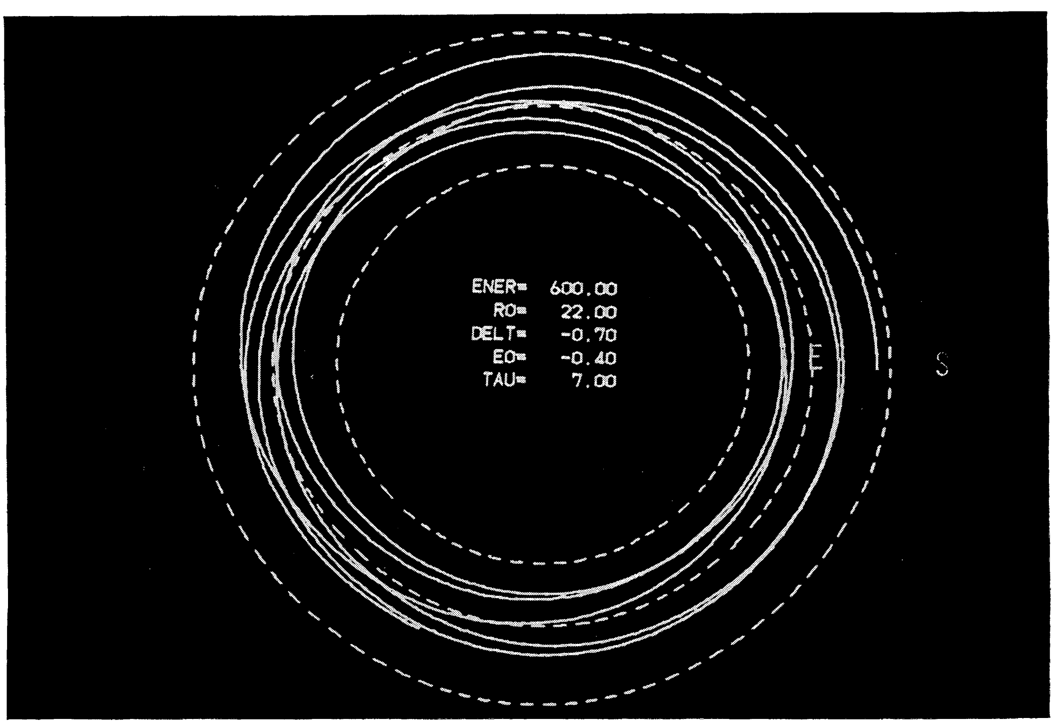

FIGURE 3. Electron orbit. $E_0 = -0.4$ kV/cm and $\tau = 7$ nsec.

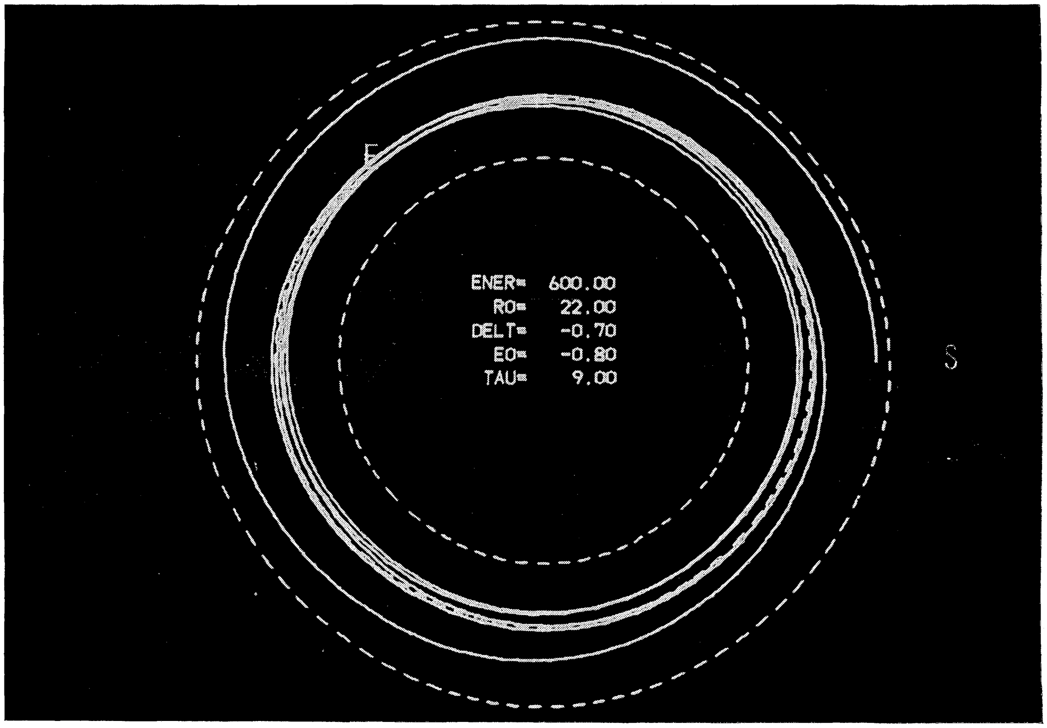

FIGURE 4. Electron orbit. $E_0 = -0.8$ kV/cm and $\tau = 9$ nsec.

It is desirable that the electron is moving nearly on an equilibrium orbit $r = r_0$ at the instant the electric field is turned off. In the system under consideration, the right side of Eq. (2) becomes as follows:

$$m_0 \gamma \frac{v^2}{r} + eE_0 + evB$$

$$= \frac{m_0 \gamma v^2}{r_0} \frac{1}{1+(x/r_0)} + eE_0 + evB_0\left(1 - n\frac{x}{r_0}\right)$$

$$= e\left[E_0 + vB_0 x\left(\frac{1}{r_0+x} - \frac{n}{r_0}\right)\right],$$

where $v = r\dot{\theta}$ and $x = r - r_0$. Thus, the electric field required to balance the force is

$$E_0 = -1.03 \text{ kV/cm},$$

in the case of the electron energy = 600 keV, $r_0 = 22$ cm, and $x = 5$ cm. When an electric field close to but less than 1k V/cm is applied to the inflectors for a long enough time, the electron orbit shrinks gradually such as shown in Figure 4 ($E_0 = -0.8$ kV/cm and $\tau = 9$ n sec). This operational condition is also beneficial for the capture of an electron beam of finite length.

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