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THE TREATMENT OF FIELD STABILIZATION IN DIELECTRICALLY STEMMED HELICAL LOADED WAVEGUIDES BY A TRANSMISSION LINE MODEL

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An approximate transmission line theory of the shielded helix is presented that allows the effect of certain discontinuities of the waveguide to be estimated. From this model a method of field stabilization of the TALIX-type sections is derived. The results are experimentally verified.

1. INTRODUCTION

Helical delay lines have become interesting as structures for heavy ion accelerators.[1,2] Their theoretical treatment is usually done by aid of the so called sheath model.[3–5] This model allows for easy and fairly accurate calculation of the line properties if the line is uniform and may be considered to be of infinite length. In case of nonuniformities, however, the treatment by this model becomes rather complicated. Some attempts in this direction are in progress at Frankfurt University, however.

It is the intention of this paper to present an approximate transmission line model by which some cases of nonuniformities may be treated in a simple and straightforward way and to apply it to the treatment of field stabilization in accelerator sections of the TALIX type.[1] This model has been developed for the treatment of closely wound helices several years ago by Dänzer and the author.[6] It has proven to be very useful in a number of applications (e.g. Refs. 7, 8), so its publication may be of interest. In case of not closely wound helices the idea is to present a simple possibility to estimate certain effects rather than to calculate them precisely.

2. THE TRANSMISSION LINE MODEL

Let us assume the following suppositions to be valid:

1) The phase velocity along the guide axis is small compared with c, so retardation may be neglected.
2) The guide wavelength is sufficiently large compared to the pitch of the helix that, at least in a rough approximation, quotients of field differences taken along the pitch may be replaced by differential quotients.
3) The distance between two windings is sufficiently small so that the electric flux between other than neighbouring windings may be neglected.
4) The pitch angle is small compared with unity.

If Φ is the magnetic flux going through the helix in axial direction and s the pitch of the helix, then the special dependence of the voltage U between helix and shield is given by the law of induction:

$$\frac{\partial U}{\partial x} = -\frac{1}{s}\frac{\partial \Phi}{\partial t} \tag{1}$$

Considering the dielectric fluxes from the surface of a winding one gets the relations

$$\Delta I_1 = K\frac{\partial}{\partial t}[U(x) - U(x-s)]$$

$$\Delta I_2 = K\frac{\partial}{\partial t}[U(x) - U(x+s)]$$

$$\Delta_3 I = C\frac{\partial}{\partial t}U(x)$$

where the notations of Fig. 1a are used. K is the capacity between two neighbouring windings, C the capacity between winding and shield. With the above assumptions the sum of these equations may be written

$$\frac{\partial I}{\partial x} = -\frac{1}{s}\frac{\partial}{\partial t}\left[CU - Ks^2\frac{\partial^2 U}{\partial x^2}\right] \tag{2}$$

† This work was done on leave of absence from Mainz at the University of Frankfurt (M).

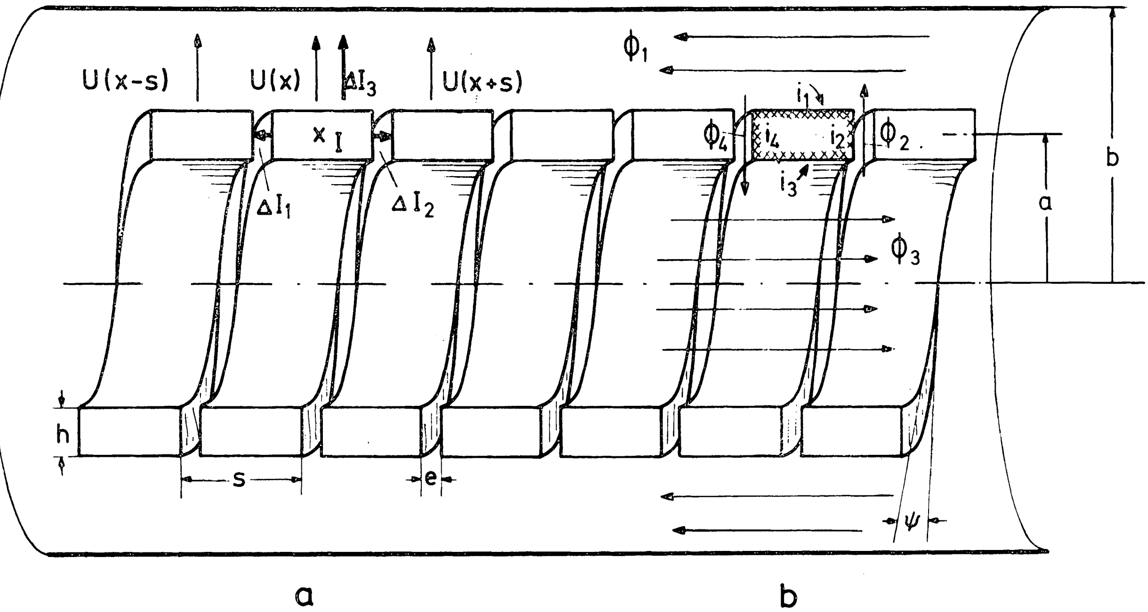

FIG. 1. Sectional view of a shielded helix. Current vectors pointing out of the drawing plane are denoted by crosses. (a) mutual dependence of voltages and displacement currents; (b) mutual dependence of eddy currents and magnetic flux.

with I being the current flowing along the helix wire.

Obviously a relation between I and Φ is still missing. It is obtained in the following way: Due to the skin effect, surface current density i and magnetic flux density B above the surface are linked by $B = \mu_0 i$. Applying this to the circumference of the helix wire which may be assumed for the moment to be of rectangular cross section, yields (see Fig. 1b):

$$I_1 = i_1 s = \frac{s}{\mu_0} \phi_1 \frac{1}{\pi(b^2 - a^2)}$$

$$I_2 = i_2 h = \frac{h}{\mu_0} \phi_2 \frac{1}{\mathrm{e} \cdot 2\pi a}$$

$$I_3 = i_3 s = \frac{s}{\mu_0} \phi_3 \frac{1}{\pi a^2}$$

$$I_4 = i_4 h = \frac{h}{\mu_0} \phi_4 \frac{1}{\mathrm{e} \cdot 2\pi a}$$

Here the assumption is used that the magnetic flux densities between the windings, inside the helix and between helix and shield are approximately constant with respect to r. Especially the latter assumption is very rough but a calculation by aid of more sophisticated fields (e.g. from the sheath model) does not change the results drastically, so for estimation purposes the expressions given above may be used. The restriction with respect to the field between the windings will be dropped later.

Taking into account that

$$\phi_1 = \phi_3 \equiv \phi$$
$$\phi_4 = \phi(x) - \phi(x-s)$$
$$\phi_2 = \phi(x) - \phi(x+s)$$

the sum of the current contributions $I_1 - I_4$ may be written

$$I = \sum_1^4 I_\nu = \frac{1}{\mu_0 \varepsilon_0 (2\pi a)^2} \left[\frac{4\pi\varepsilon_0 s}{1-(a/b)^2} \phi - \varepsilon_0 \frac{2\pi a h}{e} s^2 \frac{\partial^2 \phi}{\partial x^2} \right]$$

Here the coefficient $\varepsilon_0 2\pi a h/e$ is just the capacity K between two windings. The coefficient of Φ which is abbreviated by D in the following is a capacitance of the order of C. (In general $D > C$, D approaching C with $b \to a$). Thus the above equation may be written

$$I = \frac{c^2}{(2\pi a)^2} \left[D\Phi - Ks^2 \frac{\partial^2 \phi}{\partial x^2} \right] \tag{3}$$

It should be mentioned here that the identity of the coefficients K in (2) and (3) is in fact valid with any cross section of the helix wire, provided electrical and magnetical field lines may be considered to be orthogonal to each other. Between two windings of a closely wound helix this is always true to a good approximation.

U, I and Φ may now be calculated from Eqs. (1), (2) and (3). First, however, we write down the solution of Eq. (3) with respect to Φ:

$$\phi(x) = \frac{(2\pi a)^2}{2sc^2\sqrt{KD}} \int_0^l e^{-\gamma(|x-\xi|)} I(\xi) d\xi + b_1 e^{-\gamma x} + b_2 e^{+\gamma x} \tag{4}$$

where $\gamma = 1/s\sqrt{D/K}$, l being the length of the helix.

Obviously this is a representation of the magnetic flux by its sources, the coefficient $(2\pi a)^2/2c^2\sqrt{KD}$ being the inductance of a single winding of the helix. Equation (4) further states that the linked flux between two windings decays exponentially with the mutual distance of these windings. This has been found to be in good agreement with experimental results on a closely wound helix.

For further calculation an antiderivative Ψ will be introduced by the equivalent definitions

$$U \equiv \frac{1}{s}\frac{\partial \Psi}{\partial t};\ \phi \equiv -\frac{\partial \Psi}{\partial x} \tag{5}$$

Then Eqs. (1), (2), (3) and (5) lead to a differential equation of 4th order in Ψ:

$$\frac{\partial^2}{\partial t^2}\left(C\Psi - Ks^2\frac{\partial^2\Psi}{\partial x^2}\right) = \frac{s^2c^2}{(2\pi a)^2}\frac{\partial^2}{\partial x^2}\left(D\Psi - Ks^2\frac{\partial^2\Psi}{\partial x^2}\right) \tag{6}$$

For sinusoidal time dependence the complete solution of (6) is:

$$\Psi = (a_1 e^{-\mu x} + a_2 e^{-jkx} + a_3 e^{+\mu x} + a_4 e^{+jkx}) \cdot e^{j\omega t}, \tag{7}$$

where the a_ν are arbitrary constants and k and μ are given by

$$k_0 = k\frac{s}{2\pi a}\sqrt{\frac{D+K(ks)^2}{C+K(ks)^2}};\quad k_0 = \frac{\omega}{c} \tag{8}$$

$$\mu^2 - \gamma^2 = k^2 - k_0{}^2\left(\frac{2\pi a}{s}\right)^2;\quad \gamma \equiv \frac{1}{s}\sqrt{\frac{D}{K}} \tag{9}$$

The terms with real exponents represent cutoff modes which will in general be present at any kind of nonuniformity of the waveguide and will be necessary to fulfil the boundary conditions for U, I and Φ.

The characteristic impedance, defined as the quotient U/I with travelling waves is easily calculated to be

$$Z = \frac{(2\pi a)^2}{Ks^3c}\cdot\frac{k_0}{k(k^2+\gamma^2)} \tag{10}$$

The formulae (8) and (10) have been found in fairly good agreement with experimental results using closely wound helices[6] if C was calculated as the capacitance of a cylindric condenser of length s, K the capacitance between two parallel wires of length $2\pi a$ and diameter d and D as mentioned above, i.e.:

$$C = \frac{2\pi\varepsilon_0 s}{\ln(b/a)}$$

$$K = (2\pi^2\varepsilon_0 a)/\ln[(s/d) - \sqrt{(s/d)^2 - 1}]$$

$$D = \frac{4\pi\varepsilon_0 s}{1 - a^2/b^2}$$

3. THE TREATMENT OF INHOMOGENEITIES

From this model the effect of an inhomogeneity of the helix may be calculated, represented, e.g., by its coefficients of reflexion r and transmission d. Denoting the amplitudes in (7) left of the inhomogeneity by a second index 1, the amplitudes of the right-hand side by 2, these coefficients are defined by

$$r \equiv \frac{a_{41}}{a_{21}};\ d \equiv \frac{a_{22}}{a_{21}},$$

with $a_{11} \equiv a_{32} \equiv a_{42} \equiv 0$. The inhomogeneity is assumed to be located at $x = 0$.

In the case of a short-circuited winding of the helix, for instance, the boundary conditions are:

$$U(0-0) = U(0+0)$$

$$I(0-0) = I(0+0)$$

$$\Phi(0-0) = \Phi(0+0) = 0$$

The last condition takes into account that eddy currents will not allow the magnetic flux to pass through the shorted winding. Thus r and d are determined by the equations

$$a_{21}+a_{41}+a_{31}=a_{12}+{}_{22}$$

$$jk(\gamma^2+k^2)(a_{21}-a_{41})-\mu(\gamma^2-\mu^2)a_{31}=jk(\gamma^2+k^2)a_{22}+\mu(\gamma^2-\mu^2)a_{12}$$

$$jk(a_{21}-a_{41})-\mu a_{31}=jka_{22}+\mu a_{12}=0$$

and its evaluation yields:

$$d=\frac{1}{1-(jk/\mu)}; \quad r=1-d$$

Analogously d and r may be calculated in case of a series capacitance on a ordinary TEM transmission line. It will be seen then, that the shorted winding has the same effect on a helical waveguide as a series capacitance C_s of value $C_s=\mu/2\omega Zk$ has on an ordinary TEM transmission line of the same characteristic impedance Z (note that the low pass characteristic is conserved because of $C_s\to\infty^2$ with $\omega\to 0$).

As another example the effect of a lumped shunt capacitance C between helix and shield may be calculated in a similar way, and it turns out that the capacitance acts as a shunt capacitance C_{eff} of a somewhat different value would do in case of an ordinary transmission line of same characteristic impedance. The reduction of C to C_{eff} is given by

$$\frac{\omega C_{\text{eff}}Z}{2}=\left(\frac{\omega CZ}{2}\right)\bigg/\left(\frac{k}{\mu}\cdot\frac{\omega CZ}{2}+\frac{\mu^2+k^2}{\gamma^2+k^2}\right)$$

Small capacitances are seemingly increased, large ones are reduced and in the limit $C\to\infty$—giving the case of a short circuit—a finite value of C_{eff} is obtained.[7]

4. FIELD STABILIZATION OF HELICAL LOADED WAVEGUIDES

Helically loaded accelerator waveguides are usually proposed to operate in the standing wave mode, the helix being supported, if necessary, at the nodes of the voltage between helix and outer conductor. We will confine ourselves in the following to dielectrical stems.

The equivalent circuit of such a structure is a transmission line, periodically shunted by lumped capacitances. Its dispersion is given by

$$\cos k'l=\cos kl-\frac{\omega CZ}{2}\sin kl \tag{11}$$

where each k' solving (11) is a propagation constant of a possible partial wave and l the distance between stems. The resulting Brillouin diagram is shown in Fig. 2a. Operating the structure with the stems at the voltage nodes means that the upper cutoff frequency of the first stopband is used which gives the same phase velocity as the unloaded line. For this operation mode the transfer of voltage and current across a single cell is given by the matrix

$$T=\begin{pmatrix}-1 & j\omega C_pZ^2\\ 0 & -1\end{pmatrix}$$

where C_p is the capacity of a stem. The not vanishing matrix element t_{12} indicates that a change of impedance at the input of a cell will change the voltage at the input of the next. In π-mode this is equivalent to the possibility of unflat field distribution. In practice it is desirable to shield the dielectrical material of the stems from the axial electrical field, e.g. by means of metal strips as shown in Figs. 2 and 3. Thereby C_p might become high enough to cause serious flatness problems.

If, however, in the midst between two neighbouring stems a series capacitance C_s is inserted into the transmission line, the transfer matrix becomes

$$T=\begin{pmatrix}-1 & Z(g+\rho)\\ 0 & -1\end{pmatrix}$$

where $g=j\omega C_pZ$ and $\rho=1/j\omega C_sZ$. The Brillouin diagram may be calculated to be given by

$$\cos k'l=\frac{g\rho}{4}+\left(1+\frac{g\rho}{4}\right)\cos kl+j\left(\frac{g+\rho}{2}\right)\sin kl$$

As is readily seen, with $g=-\rho$ the transfer matrix of this biperiodic structure becomes equal to the negative unit matrix indicating unconditional flatness of the field distribution. In the Brillouin diagram the extremum at $kl=\pi$ is replaced by a finite slope and it turns out that in the application considered here the group velocity in the π mode

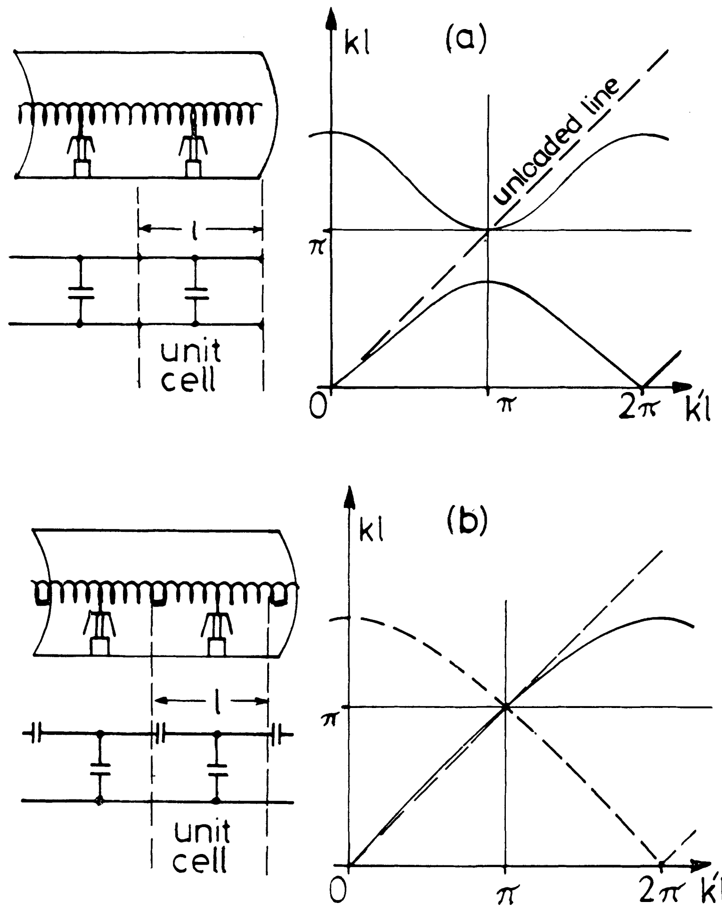

FIG. 2. Sectional view, equivalent circuit and Brillouin diagram (a) uncompensated; (b) compensated.

is not significantly lower than in case of an undisturbed uniform helix (see Fig. 2b). It is an important feature of this kind of stabilization that it still leaves field distribution and phase velocity unchanged with respect to the unloaded line.

As shown above the series capacitances may be represented simply by shorted windings. Since a shorted winding represents a capacitance of fixed value it is necessary to trim the capacitances of the stems to satisfy the stabilization condition. This may be done by adjusting the metal strips mentioned above. Fortunately, the values needed are of a suitable order of magnitude of some picofarad.

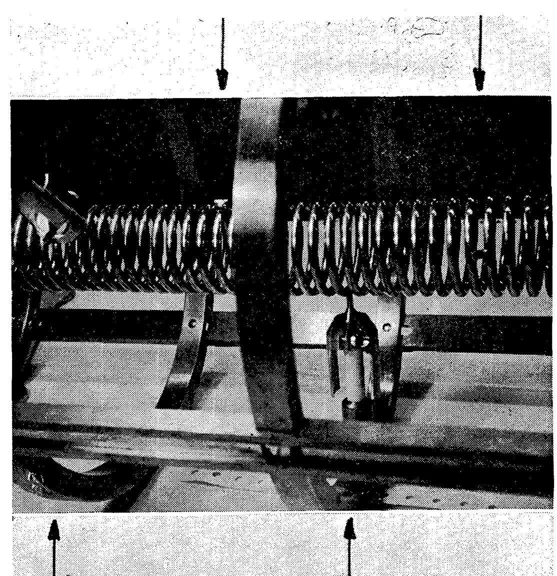

FIG. 3. Photograph of a compensated helix accelerator waveguide. Arrows from above point to shorted windings, arrows from below point to dielectrical stems.

To prove these considerations a helix accelerator section of a length of 1 m has been equipped with this kind of field stabilization, as shown in the photograph (Fig. 3). It has successfully been tested in CW operation to a field strength of 2.6 MV/m on the axis, limited by cooling capability. A reduction of unflatness effects due to tuning errors by a factor of five could easily be achieved by trimming the capacity of the stems to 1.32 pF each. From the transmission line model a value of 1.81 pF had been calculated.

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