Particle Accelerators
1973, Vol. 5, pp. 99–103

PROSPECTS FOR THE DEVELOPMENT OF NEW COLLECTIVE METHODS OF ACCELERATION †

M. S. RABINOVICH and V. N. TSYTOVICH
Lebedev Physical Institute, Moscow, USSR

The future of new collective methods of acceleration is discussed. For beam coherent acceleration the possibility is emphasized of longitudinal ring stabilization by using the transitional radiation. For impact acceleration a transient compressor is proposed. Acceleration by nonlinear waves and solitons‡ in relativistic electron beams is considered in more detail.

During the past few days we have listened to reviews of various acceleration methods under different stages of development, from models and designs of particular installations to hypotheses and sketchy ideas.

To clarify the situation we propose a classification. In general, new methods of acceleration involve methods of creating superhigh fields; intentionally we do not specify the kind of field, whether it be magnetic or electric. Using methods of circular acceleration we can always equal the efficiency of one field in terms of the other. In the Gaussian system the condition of equivalence can be written as follows:

$$H = 2\pi E \tag{1}$$

or

$$H\ (\mathrm{kOe}) = (E/5)\ (\mathrm{MV/m})$$

Thus the following classification is obvious

1) Traditional methods:
   electric field $E = 10$–100 MV/m
   magnetic field $H = 2$–20 kOe

2) New methods (collective methods and superconducting accelerators)
   $E = 100$–1000 MV/m
   $H = 20$–200 kOe

3) Future methods
   $E = 1000$–$10{,}000$ MV/m
   $H = 20$–200 kOe and higher

It should be noted that in all three divisions the accelerators can have different energies ranging from low to superhigh. This is why a classification according to attainable energy is not appropriate.

At present we witness the completion of the evolution of new methods in the 100–1000 MV/m range (having in mind the relativistic ring accelerator) and in the 20–200 kOe range (having in mind superconducting accelerators). Perhaps the time has come to analyze in greater detail future methods; these involve superhigh electric fields in collective methods of acceleration and in laser beams, and also superhigh magnetic fields. Leaving aside the discussion of possibilities in the megagauss range of magnetic fields (this problem would require special consideration) we shall pay our attention to future collective methods.

Of course, our considerations will be far from being as rigorous as is characteristic of traditional methods and from what is required for the relativistic electron ring method.

Although we cannot suggest a fully worked out scheme for future collective methods, we shall try to make our considerations concrete.

Let us assume, as has already been done in the papers of Ya. B. Feinberg and A. A. Kolomensky, that plasma and stabilized relativistic electron rings are used as the media for collective acceleration. Actually, it is well known that it is relatively easy to produce fields of 10,000 MV/m and higher in plasmas and also in laser beams. In particular, such fields retard electron beams very effectively.

† Paper presented at the Symposium on Collective Methods of Acceleration, Dubna, USSR, September 1972.

‡ EDITORS' NOTE. Although a familiar term to plasma physicists, the term 'soliton' may not be familiar to all of our readers. It is an abbreviation for 'solitary-wave pulses propagating in non-linear dispersive media'; it has come to refer to any propagating pulse-like excitation (in a plasma) which resists dispersion or break-up.

It is much more difficult, if at all possible, to utilize such fields in plasma, as well as in lasers, for acceleration.

But, if it is possible, high pulse powers will inevitably be required. For example, when 10^{11} ions are accelerated in 10 GV/m fields, the ions absorb power at a rate of 5×10^{12} watts; consequently the total power required is at least one or two orders of magnitude higher. Thus the problems of collective acceleration must be solved in parallel with the problem of creating a pulsed power source, taking into account the efficiency problem. The utilization of all kinds of superhigh frequency pulse power becomes involved with the difficult problems of superconductivity. Apparently electron and laser beams are good pulsed power sources.

Apart from external souces of pulsed power, it is rather attractive to use accumulated beam energies. This is the case, for instance, in the well-known method of utilizing the accumulated transverse energy of relativistic rings by transforming it into longitudinal energy. The problem of pulsed power does not arise here, of course, but there remains the problem of the duration of the action. Usually it is not possible to make it long. In summary, these methods are rather convenient, but not for superhigh energies (that is, in the 1-GeV range).

Let us return to the first problem of the use of plasma fields. The problem is closely connected with the physics of collective processes in plasma and will be discussed further especially from this point of view.

To be specific, these problems will be considered for the cases of beam and impact collective acceleration mechanisms and also relativistic solitons and nonlinear waves.

V. I. Veksler assumed that high electric fields could be produced by an impact mechanism. In this case the forces arise during short time intervals and only at the position of the accelerated particles; this greatly reduces the possibility of instabilities.

Let us first consider a simpler problem—an electron ring immersed in an electron beam. As usual we shall consider the behavior of the ring in the system where the beam is at rest and the ring is moving. Coherent Cerenkov acceleration will play an important role. To determine the parameters, it is necessary to write down the condition of coherence and simultaneous electron and ion acceleration.

Actually, if the electron flow accelerates the ring, the maximum electric field is determined, not by the electron flow, but by the number of particles in the ring, as in the electron ring accelerator, and the electron beam acts as a power source. With such an interpretation it is clear that this method is suitable (if it is valid) for the 1–100 GeV range, and apparently it is not suitable for the 1000-GeV range. For this reason primary attention should be given to the effective use of electron beam power.

The maximum coefficient for utilization of power is

$$K_{\max} = 4\pi^2 (\nu^R)^2 \gamma_\perp^2 \left| \ln \frac{\pi}{\nu^C \gamma_\parallel} \right| \tag{2}$$

where ν^R and ν^C are Budker factors for the ring and for the beam.

$$\nu^R \gamma_\perp \cong \frac{10^{-13} N_e}{2R} \tag{3}$$

Although the formula is valid only for $\nu^R \ll 1$, it is evident that, for $N_e \lesssim 10^{14}$, the power efficiency is sufficiently high; but this cannot be said about the energy efficiency.

In the previous considerations it was assumed that the problem of bunch stability has been solved. But it is far from being solved and it is necessary to consider possible methods of longitudinal bunch size stabilization. This has not been done rigorously as yet.

Other mechanisms than Cerenkov radiation can be used in collective flow acceleration. The use of transitional radiation in a modulated flow is rather an interesting possibility. In contrast to Cerenkov emission, transitional radiation gives successive longitudinal focusing and defocusing effects (like strong focusing in ordinary accelerators).

If the modulation period is chosen to be $(R/2)\sqrt{\gamma_\parallel/\nu^C}$, then the transitional emission exceeds the Cerenkov emission when $\nu^C \gamma_\parallel \ll 1$.

Another forgotten acceleration process proposed by V. I. Veksler is known as 'impact acceleration'. Veksler has always associated this method with the attainment of superhigh particle energies although with low beam intensity. In this connection we should recall Lawrence's well-known but yet unproved prediction made at the first Conference on Accelerators in 1951, that with increasing particle energy the intensity of accelerated beams will decrease and, at the high energy limit, will tend to zero.

Let us consider the impact acceleration method using as an example the collision between two electron rings. One light ring (L) is at rest and contains a small number of ions, which should be accelerated in the collision process.

It is known that in the process of direct elastic collision

$$(\gamma^L)_{\max} = 2(\gamma_{\parallel}{}^H)^2 \tag{4}$$

This is rather an attractive formula; using it in a straightforward fashion we obtain very high energy values. But a number of difficult conditions must be fulfilled for an elastic collision, first, the condition for simultaneous motion of ions and electrons and, second, the condition for elastic reflection.

The collision distance can be written formally in the following form

$$d = \frac{fR}{\gamma_{\parallel}{}^C} \tag{5}$$

where $\gamma_{\parallel}{}^C$ is the factor corresponding to the transfer from the center-of-mass system to the laboratory system, and f is the parameter determined by the collision dynamics; f actually depends on all beam parameters. If the collision occurs over a distance d, then the effective acceleration field is

$$E_{\text{eff}} = \frac{2m_e c^2}{eR} \frac{\gamma_{\perp}{}^L(\gamma_{\parallel}{}^H)^3}{f} \tag{6}$$

$$\nu^L > \frac{m_i}{m_e} \frac{2\pi a}{Rf} (\gamma_{\parallel}{}^H)^3 \tag{7}$$

$$M_H \gg 2M_L \gamma_{\parallel}{}^H \tag{8}$$

These formulae also are attractive, but, if one inserts realistic figures, their straightforward use leads to values that are too high.

If it is assumed that $N_e{}^L = 5 \times 10^{15}$, $N_e{}^H = 3 \times 10^{16}$, $\gamma_{\perp}{}^L = 15$, $\gamma_{\perp}{}^H = 50$, $R = 5$ cm, $a/R = 10^{-3}$, $\gamma_{\parallel} = 10$, then the relations above lead to the conclusion that the collision length should be 50 m and the energy achieved should be 200 GeV. Even if acceleration with $f > 10^5$ can possibly be attained, it cannot be assumed that the collision is a rapid one. Then radiation losses and radiation instability may prevent the functioning of the coherent impact mechanism of acceleration.

But radiation effects can be neglected if

$$\frac{M_H}{M_L} > \begin{cases} (\gamma_{\parallel}{}^H)^2 \\ (\gamma_{\parallel}{}^H)^3/[20\nu^L \ln(8R/a)] \end{cases} \tag{9}$$

The figures mentioned above have been chosen so as to satisfy these conditions. However, we can speak of the impact mechanism as an established one only when the collision mechanism has been studied in detail and when the actual value of f has been precisely determined. The result seems to be discouraging at first sight but we should take into consideration the following items.

In the calculations we used static values of the bunch parameters corresponding to the values before the collision, assuming that there is little change during the collision. At this moment, the arriving heavy bunch can produce an increasing field in the neighborhood of the light one, and the latter will be compressed. However, as yet the self-consistent problem of collisions has not even been defined. It is not necessary to decrease the minor radius of the heavy bunch. Perhaps the heavy bunch could be less like a ring and more like the E-layer in Christofilos' 'Astron'. Finally, we abandoned Veksler's principles in the collision mechanism. We required the bunch form to be preserved during the collision and the deformation to be small. Thus the acceleration of a large number of particles was achieved (on paper). But the collision mechanism should be thought of as a large acceleration of a relatively small group of particles.

The main drawback of the collective methods of acceleration just described is the necessity for the creation of charge bunches by some external forces. Of course, the procedure is reasonable if the electric fields used to produce bunches are considerably weaker than those in the bunch, or if magnetic fields are used for this purpose.

This approach has been consistently maintained in the density wave accelerator, proposed by Feinberg early in 1956. Perhaps the plasma accelerator will be the main collective accelerator in the future. A great deal of work remains to be done in this direction; progress during the past sixteen years has been very slow, but always positive.

The modern state of the problem has been sufficiently discussed in Feinberg's paper. In this paper we shall concentrate on future possibilities of this method using relativistic solitons and nonlinear waves of large amplitude.

The creation of bunches in a plasma is connected with the development of several types of instabilities. Stimulated emission is one such instability. It is this emission that amplifies the initially weak effects. The beam particles in the beam–plasma interaction can generate oscillations and waves and, since the presence of the beam is equivalent to an inverse population of energy levels, a maser effect is created. As was mentioned, we are interested in density waves. Such waves can be excited by the bunch if slow waves can exist in the system with phase velocities equal to or close to the bunch velocity. But the usual case is one in which the density oscillations are excited with a wide spectrum of wavelengths. Then superposition of these waves creates a pattern of rapidly changing oscillating fields which vary randomly from point to point. Enormous electric fields exist in such oscillations but it is impossible to use them for acceleration.

Ya. B. Feinberg has proposed an effective method of synchronization of oscillations by forced superposition of oscillations having frequencies corresponding to the maximum growth rate. In this case the induced emission is effectively stimulated.

We shall consider the conditions under which the bunch excites a single wave in the plasma. External weak fields also can be used, if necessary, to suppress the effects of spectrum broadening.

A necessary, but not sufficient, condition for the excitation of a single wave is a small energy spread in the beam:

$$\frac{\Delta E}{E} < (n_1/n_0)^{1/3}\gamma \tag{10}$$

Here n_1 represents beam density and n_0 is plasma density.

The conditions for optimal excitation are achieved when

$$n_1/n_0 \gg 1/\gamma^3 \tag{11}$$

and the density of excited energy is approximately $n_1 m_e c^2\gamma$; for a current density of 5×10^5 A/cm^2 and a γ of 100, the field strength produced is of the order of tens of gigavolts/meter. Apparently similar fields can be produced when the beam interacts with a retarding wave structure (of the type of corrugated or spiral waveguide).

The amplitude of a nonlinear wave can be described conveniently by the relative amplitude of the potential difference in the system where the wave profile is stationary.

$$\alpha = \frac{2e\phi_{\max}}{m_e v_\phi^2} \tag{12}$$

where v_ϕ is the phase velocity of the wave. The maximum value of α for which unstable oscillation and multi-velocity flow do not set in is $4\gamma_\phi^2$. This result is rather important; we note that in the nonrelativistic case $\alpha_{\max} \cong 1$.

A nonlinear wave can be either a soliton or a periodic configuration. Solitons are not produced in a cold plasma in the absence of magnetic field, or in propagation of waves along a magnetic field.

For $\alpha \gg 1$, the wave splits into a number of quasi-independent bunches. As α increases, the space between the bunches in the system with the front at rest also increases.

$$x_0 = \frac{2c\sqrt{2\gamma_\phi\alpha}}{\omega_{pe}} \tag{13}$$

where $\omega_{pe}^2 = 4\pi n_0 e^2/m_e$ and n_0 is the plasma density. In our system the distance between bunches is x_0/γ_θ and the electric field is

$$E_{\max} \cong \phi_{\max}/x_0 \tag{14}$$

The full average wave energy is $W = (\overline{E^2}/8\pi)(1+4)$.

In this formula the fact has been taken into account that the kinetic energy density of particles taking part in the wave build-up is higher by a factor of four than the average field energy. Numerical calculations indicate that the beam gives two thirds of its energy to the wave. This means that the amplitude

$$\alpha = (3/5)\gamma^2 n_1/n_0 \tag{15}$$

cannot reach its maximum value of $4\gamma_\phi^2$ for $n_1/n_0 \ll 1$. It is true that this estimate must be used with care since it has been derived using only qualitative energy conservation arguments.

Evidently the nonlinear wave is rather strong. However, before it is used for acceleration a number of problems should be completely solved.

We have mentioned the fact that it seems quite possible to satisfy the condition for excitation of a single wave. However there is no straightforward evidence supporting this conclusion. The stability of such waves has not been studied completely. But it has been shown that, as γ is increased, the time interval during which local perturbations of the wave appear also increases. Also, if multi-velocity flows do occur, the difference in γ-factors is rather small. This leads to insignificant redistribution of particle velocities and to insignificant changes in the structure and shape of the relativistic wave. Parametric instabilities also cannot affect the shape of the wave, and electron-ion instabilities do not develop if

$$n_1/n_0 < (1/\gamma^3)(m_i/m_e)^{2/3} \tag{16}$$

The possibility of using nonlinear waves depends also on the ability to control their velocities. It is possible, in principle, to regulate these velocities by profiling plasma density and by use of varying magnetic fields. It is also necessary to evolve methods of ion injection into a nonlinear wave.

Of course the situation would be simplified if it were possible to use stochastic acceleration, but this is effective only for multicharged ions (the rate of acceleration is proportional to Z^2).

Thus, if we are to use nonlinear waves in a plasma a great deal of work must be done. Nevertheless, it may be that this will be the trend of the future and, in the future, plasma accelerators will compete with laser accelerators and with megagauss magnetic field accelerators.

Our main conclusions are

1) The development of new collective methods requires further intensive work.
2) The time has come to pay more attention to the study of nonlinear waves in plasma, using high energy relativistic electron beams.

Received 9 May 1973