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ON FREQUENCY SHIFT OF AXIAL OSCILLATIONS UNDER THE EFFECT OF THE BEAM SPACE CHARGE

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Experimental studies of the frequency shift of incoherent axial oscillations under the effect of the beam space charge in the electron model of the circular cyclotron are described. The results of the studies are in satisfactory agreement with the existing presentation on the mechanism of this shift.[1]

1. INTRODUCTION

The assembly of particles accelerated in the isochronous cyclotron can be presented as a bunch of finite azimuthal width with continuous distribution of charge density in the vicinity of the median plane from the initial radius to the full one. Since the radial Δr and azimuthal $\Delta\varphi$ sizes of such a bunch are usually much larger than the axial size Δz, it can be approximated by a sufficiently thin charged layer. Under the assumption of a uniform distribution of charge density the electric field generated by it inside this layer will be directed along the vertical axis z, the equation of axial oscillations being

$$\frac{d^2 z}{d\varphi^2}+\left(Q_{z0}^2-\frac{4\pi x r_\infty^2 r_e}{\gamma^3}\right)z=0 \qquad (1)$$

where Q_{z0} is axial frequency determined by the parameters of the magnetic system, x is particle density in the lab system, $r_\infty = c/\omega_0$, $\omega_0 = 2\pi f_0$ is particle revolution frequency in the isochronous magnetic field, $r_e = e^2/4\pi\varepsilon_0 m_0 c^2$ is the electron radius, $\gamma = E/E_0$ is the relativistic factor. It follows from Eq. (1) that the self-field of the beam reduces the focusing force affecting each particle, which results in decreasing axial oscillation frequency. The reduction of the Q_z frequency down to the nearest parametric resonance can cause beam intensity loss. Taking Eq. (1) into account the frequency shift of axial oscillations with the known current value[2] i, one obtains

$$\Delta Q_z = \frac{i}{2Q_{z0}\,\varepsilon_0 f_0\, V\Delta z\,\Delta\varphi} \qquad (2)$$

The tolerable frequency shift of axial oscillations is determined by the condition of noninterception of the parametric resonance band. From expression (2) by substituting the known values of Q_{z0}, f_0, V, $\Delta\varphi$ and Δz the limit current of the electron model was found which turned out to be equal to 1 mA. This value of the limit current was obtained experimentally on the operating accelerator.[3]

It is necessary to emphasize that the space charge affects the motion of a separate particle but not that of the accelerated bunch as a whole. Therefore the axial frequency of the bunch centre-of-gravity (coherent frequency) remains unchanged while the oscillation frequency of separate particles (incoherent frequency) is reduced under the effect of the space charge. Hence, in order to discover the axial frequency shift under the effect of the space charge it is necessary to excite the parametric resonance. Indeed, the measurements of the coherent axial oscillation frequency by exciting the exterior resonance[4] have shown that it does not depend on the value of the space charge for beam currents from 10 to 600 μA.

2. PARAMETRIC RESONANCE EXCITATION

One of the possible ways of exciting the parametric resonance is the use for this purpose of an electric quadrupole lens,[5] whose field changes with coordinates.

Another way of exciting the parametric resonance applied for the first time in the present investigation is the modulation of the longitudinal momentum of accelerated particles with a frequency equal to the doubled frequency of axial oscillations.† The equation of vertical oscillations has the following form:

$$\frac{d^2z}{dt^2}+\omega_z^2\left(1-\frac{\widetilde{\Delta p}}{p_0}\cos\omega t\right)z=0 \tag{3}$$

where $\widetilde{\Delta p}$ is the amplitude of the momentum part oscillating at the frequency ω.

For this purpose one can use a special 'modulating' electrode with small azimuthal and radial dimensions. If the amplitude of voltage applied to the electrode is U and varies slightly for the time lT/L, where l is the electrode length, L is the orbit length, and T is the particle revolution period in the accelerator magnetic field, then the energy modulation of the required resonance harmonic is $E \sim U(l/L)$, and the 'resonance force' h is

$$h=\frac{\widetilde{\Delta p}}{p_0}=\frac{\gamma}{\gamma^2-1}\cdot\frac{Ul}{E_0 L} \tag{4}$$

where E_0 is electron rest energy. Our evaluations show that the h force attained practically by this method of excitation is about two orders of magnitude smaller compared to resonance excitation by the quadrupole. The advantage of the momentum modulation method is that there are no special requirements for the dependence of the exciting field upon the transversal coordinate. Obviously this advantage for the cyclotron is essential since unlike in synchrotrons (fixed space orbit accelerators) it is impossible to use the quadrupole in the 'pure' form in cyclotrons. Indeed, in order to lead the beam up to the quadrupole axis it is necessary to leave in one of its walls (electrodes) a slit for beam entrance. The slit deforms the field considerably. The field calculation by the method of a rectangular grid for some practically possible electrode configurations has shown that the linear part of the field at the excitation system axis is diminished comparing to the ideal quadrupole, at least, by an order of magnitude. Besides, in the beam slit area the fringing field of the opposite sign appears whose effect on particles is difficult to take into account by means of calculations.

In order to excite the parametric resonance both by the first and second methods a sufficiently large number of particle revolutions is necessary (50–100) within the range of the exciting field. This condition is impractical for the cyclotron particle acceleration mode.

If, however, beam acceleration is stopped at the radius of exciting electrode position and the beam is left circulating at this radius,[7] a number of revolutions limited only by the lifetime of the circulating beam can be obtained. The evaluations made for the electron cyclotron model show that particle loss in the circulating beam due to scattering on gas limits the beam lifetime to some hundred microseconds. Since the particle revolution period in the electron model is ~ 25 nsec, particles make $(10 \text{ to } 20)\times 10^3$ revolutions in the circulating mode.

Practically, the circulating beam can be obtained by modulating the accelerating voltage.[8] The information on the circulating beam current can be obtained by watching the signal on the oscilloscope screen taken from the pickup electrode. It should be stressed that the signal induced on the pickup electrode is proportional to the variable component of the amplitude of the circulating beam current and thus depends not only on the number of particles in the bunch but on its azimuthal width. Therefore, the reduction of the induced signal can be due not only to particle loss in residual gas scattering but due to the increase of the beam azimuthal width. Since particle density depends on both the effects, an oscillogram can be used for the determination of particle density in the circulating beam, if the particle density of the accelerated beam is known.

The experiments on the parametric resonance excitation have shown that the minimal effective time for the excitation of the parametric resonance necessary for noticeable reduction of the circulating beam was about 10 μsec, which corresponds to 400 particle revolutions in the field region. For this time the signal from the pickup electrodes is reduced on the average down to 0.8 compared to that from the accelerated beam.

3. EXPERIMENT

The measurements of the frequency of incoherent axial oscillations versus the accelerated beam current were made for three modes of the accelerator

† A similar method was applied earlier for exciting the exterior resonance of radial oscillations in the proton synchrotron.[6]

magnetic field tuning. With the first mode the exterior frequency of axial oscillations in the area of the exciting electrode location is $Q'_{z0} = 1.290 \pm 0.002$, with the second one $Q''_{z0} = 1.460 \pm 0.002$ and the third one $Q'''_{z0} = 1.244 \pm 0.002$.

In order to obtain more accurate results the frequency shift from the value of the beam current was observed by means of multiple frequencies $Q_z = \frac{1}{2}|f_g/f_0 \pm K|$ with $K = 0 \div 5$, $1 \div 4$ which were excited from one generator tuned within $f_g = 45$–105 MHz.[4]

Figure 1 is the shift of axial oscillations frequencies versus the circulating beam for two modes of the magnetic field. The dee accelerating voltage was not varied and was taken to be $U_d = 1.3$ kV.

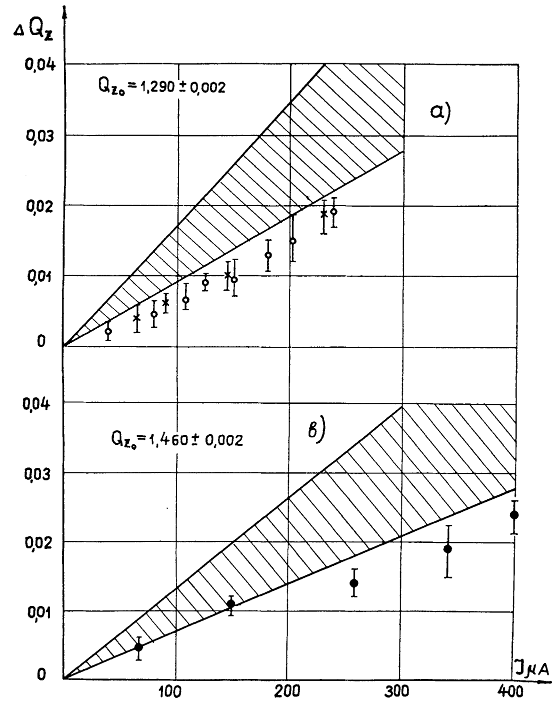

FIG. 1 Frequency shift ΔQ_z versus beam current for two modes of the magnetic field.

Figure 1a is a comparison of results obtained by various methods of the parametric resonance excitation. Circles are frequency shifts measured by exciting the resonance with a modulating electrode and crosses are frequency shifts measured by exciting the resonance with a quadrupole element. It follows from the diagram of Fig. 1a, that the frequency shift within the above accuracy does not depend upon the method of parametric resonance excitation.

To compare the obtained results with theoretical predictions it is necessary to determine more precisely some parameters of the accelerated beam and investigate their behaviour in varying current. Within the area of the exciting electrode and the pickup electrode location having less effective aperture compared to that of the dee, the current transmission coefficient was determined. One of such radial dependences is shown in Fig. 2a. For all the curves $i = f(R)$ having currents of 340–400 μA the transmission coefficient was about 80 per cent. Figure 2b is the dependence of the beam FWHM versus radius for three values of the beam $i = 40$,

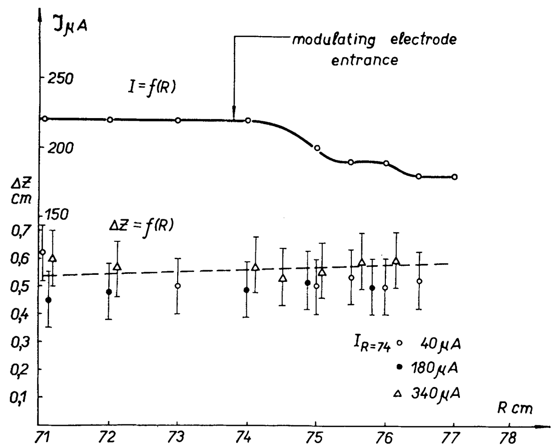

FIG. 2.(a) Beam current versus radius. (b) Beam FWHM versus radius for three values of beam current.

180, 340 μA. As is seen from the diagrams, the beam FWHM does not depend on current within ± 10 per cent. It is known from oscilloscope measurements that the total duration of the current pulse[8] is about 3 nsec, which corresponds to the bunch width of $\Delta\varphi = 0.75$ rad. The bunch width remains constant within 10–15 per cent with varying currents within a wide range. Besides, the quantities entering into formula (2) Q_{z0}, f_0 and V are known to accuracies of 2×10^{-3}, 2.5×10^{-5} and 5 per cent, respectively. The above-mentioned parameters measured experimentally to the indicated accuracies make it possible to determine the theoretical shift of the frequencies of incoherent axial oscillations under the effect of the beam space charge. Taking into account the errors in the determination of the beam parameters one obtains a band of possible shifts of oscillation frequencies relative to beam current. Such bands are shown in Figs. 1 and 3.

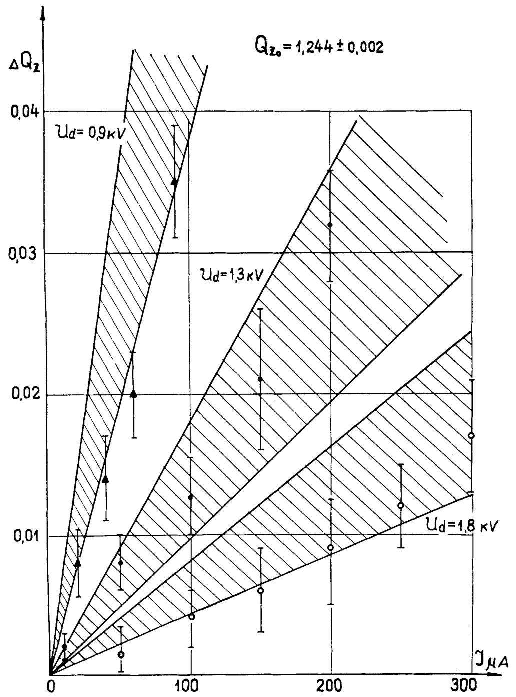

FIG. 3. Frequency shift ΔQ_z versus beam current for three values of the accelerating voltage.

As follows from Eq. (2), the observed frequency shift with the fixed current must be inversely proportional to the energy gain eV, the coherent oscillation frequency Q_{z0}, the azimuthal bunch width $\Delta\varphi$ and its height Δz. It is of obvious interest to check experimentally these dependences. As has been mentioned above, the Coulomb frequency shift was determined in three magnetic field modes corresponding to various values of Q_{z0}. The analysis shows that the obtained results in retuning Q_{z0} are in good agreement with the theoretical evaluations made by formula (2).

Figure 3 shows diagrams obtained with various accelerating voltages. It is clearly seen from the figure that, when the accelerating voltage is reduced, the frequency shift is considerably increased. The nonlinear dependence Q_z upon eV is observed due to the fact that the bunch width is changed immediately when V is retuned. If one takes into account the change of $\Delta\varphi$ and eV entering into (2), one obtains satisfactory agreement of the performed experiment with the theory of the Coulomb shift mechanism developed by the present time.

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