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EFFECT OF RANDOM FIELD ERRORS WITH APPLICATION TO SLOW EXTRACTION†

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The effects due to magnetic field perturbations caused by random errors in the coil positions in the magnets of a high-field superconducting accelerator are investigated. An error in the position of a particular current block of a bending magnet with a cosine current distribution will introduce nonlinear sextupole and higher-pole terms in the median plane magnetic field. These nonlinear terms may produce serious effects by exciting nearby nonlinear resonances, and, in particular, they may seriously affect a slow extraction system that uses sextupoles to excite a nearby one-third resonance. The nonlinear fields due to random errors of the current blocks are estimated and the resulting nonlinear harmonics around the accelerator are found to be comparable to the nonlinear harmonics introduced by the slow extraction sextupoles. A method for correcting the field effects due to the random errors in the current blocks is investigated. This correcting method uses additional correcting sextupoles and octupoles, and was found to be able to correct for the effects of the random field errors in the slow extraction system.

1. INTRODUCTION

This paper investigates the effects due to field perturbations caused by random errors in the magnet coil positions in the dipole magnets of a high-energy superconducting accelerator. A possible high-field superconducting bending magnet is shown in Fig. 1. The current distribution is a circular cosine distribution which is approximated by blocks of current arranged around a circle. Within each block, the current density is constant and proportional to the cosine of the azimuthal angle. The current distribution is surrounded by a circular iron shield. It has been found(1) that the magnetic field in this kind of dipole remains remarkably uniform up to field levels of the order of 60 kG.

The cosine dipole differs from conventional bending magnets in that the field shape is determined primarily by the current distribution rather than by the iron shape. An error in the position of a particular current block will therefore introduce a second harmonic and higher harmonics in the current distribution, which will produce a nonlinear sextupole as well as higher-pole terms in the median plane magnetic field. These nonlinear terms may produce serious effects by exciting nearby nonlinear resonances. In particular, a slow extraction procedure that uses sextupoles to excite a nearby one-third resonance may be seriously affected by the sextupole field produced by random errors in the coil positions.

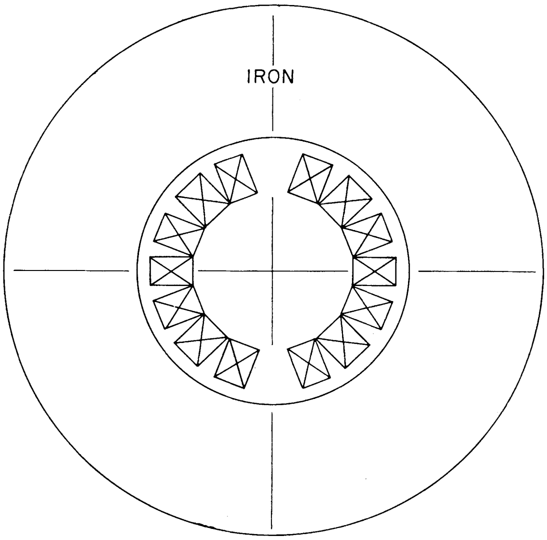

FIG. 1. Geometry of a superconducting dipole magnet with a cosine current distribution.

This paper estimates analytically the nonlinear fields due to random errors of the current blocks and the resulting harmonics introduced into the magnetic field around the accelerator. It appears

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that the nonlinear harmonics introduced by the random errors in the current blocks may be comparable to the nonlinear harmonics introduced by the extraction sextupoles.

One method for correcting the field effects due to the random errors in the current blocks is investigated. In addition to the two sets of extraction sextupoles, one 90° out of phase with the other, a set of octupoles is included in the lattice. The choice of the number and position of the octupoles does not appear to be too important from the orbit viewpoint and can be chosen on other considerations.

This correcting method may be roughly understood as follows: Let us suppose that the extraction sextupoles are used to generate a kth harmonic (k not divisible by three) in order to excite the $\nu = k/3$ resonance. It would appear then that the most important effect of the random errors in the current blocks is to introduce a disturbing kth harmonic, which can be corrected out by the two sets of sextupoles. However, the other harmonics due to the random errors in the current blocks and the sextupoles also produce an undesirable effect. This effect may be thought of as an octupole-like detuning effect, which moves the particles away from the $k/3$ resonance at large enough oscillation amplitudes, with the result that the growth in the oscillation amplitude due to the $k/3$ resonance is limited by the detuning effect, and the particles cannot be extracted. This detuning effect of the other harmonics can be compensated by the zeroth harmonic of the set of octupoles.[2]

2. FIELD PERTURBATIONS DUE TO RANDOM ERRORS IN THE CURRENT BLOCKS

In this section, the magnetic field perturbations introduced by random errors in the current blocks will be estimated analytically. The first task is to compute the sextupole field produced by a displacement of one of the current blocks shown in Fig. 1. The calculation will first be done assuming there is no iron shield present. The corrections to the results due to the iron shield will be indicated later in the section where it will be seen that the presence of iron does not affect the results greatly.

The vector potential due to an arbitrary two-dimensional current distribution can be written as

$$A(r,\theta) = -\frac{\mu}{2\pi}\int \mathrm{d}S' \ln R\, j(r',\theta'), \tag{2.1a}$$

$$R = \{r'^2 - 2rr'\cos(\theta-\theta') + r^2\}^{\frac{1}{2}}, \tag{2.1b}$$

where $\mu = 4\pi/10$ and $\mathrm{d}S' = r'\,\mathrm{d}r'\,\mathrm{d}\theta'$.

In order to see the sextupole term in the magnetic field, we use the expansion[3]

$$\ln\sqrt{1-2x\cos\theta + x^2} = -\sum_{K=1}^{\infty}\frac{x^K}{K}\cos K\theta. \tag{2.2}$$

Then for small r near the center of the dipole magnet we find, using Eqs. (2.1) and (2.2),

$$A(r,\theta) = \sum_{K=1}^{\infty}(A_K\cos K\theta + C_K\sin K\theta)r^K, \tag{2.3a}$$

where

$$\begin{bmatrix} A_K \\ C_K \end{bmatrix} = \frac{\mu}{2\pi K}\int \mathrm{d}S' \frac{1}{r'^K} j(r',\theta')\begin{bmatrix}\cos K\theta' \\ \sin K\theta'\end{bmatrix}. \tag{2.3b}$$

The median plane magnetic field can be found from Eq. (2.3) using

$$B_\theta = -\frac{\partial A}{\partial r} \tag{2.4a}$$

$$B_r = \frac{1}{r}\frac{\partial A}{\partial \theta}, \tag{2.4b}$$

which gives

$$B_\theta = \sum_{l=0}^{\infty}(l+1)A_{l+1}\, r^l \tag{2.5a}$$

$$B_r = \sum_{l=0}^{\infty}(l+1)C_{l+1}\, r^l. \tag{2.5b}$$

In an error-free magnet, $C_K = 0$ as there is no B_r field in the median plane. A_1 and A_3 give rise to the dipole and sextupole fields.

Dipole Field

Equations (2.3) will first be used to find the field of a pure dipole magnet, and this result will be used later to compare with the perturbed fields that develop when positioning errors in the coils are present. A current distribution is assumed which corresponds to a current sheet at the radius R and is given by

$$j(r,\theta) = \frac{I_t}{2R}\cos\theta\,\delta(r-R), \tag{2.6a}$$

where I_t is the total current which flows in half of the circular distribution.

The field is then given by Eq. (2.3) as

$$A = \frac{\mu I_t}{4R} r, \tag{2.6b}$$

$$B_\theta = \frac{\mu I_t}{4R}. \tag{2.6c}$$

In the same way one can treat a K-pole current distribution given by

$$j(r, \theta) = \frac{KI_t}{2R} \cos K\theta \, \delta(r-R), \tag{2.7a}$$

which then produces the field

$$A = \frac{\mu}{4} \frac{I_t}{R^K} r^K, \tag{2.7b}$$

$$B_\theta = \frac{\mu K I_t}{4R^K} r^{K-1}. \tag{2.7c}$$

Here I_t is the total current in $1/K$ of the circle, and B_θ is the median plane field.

Radial Displacement of a Current Block

It will be assumed that the current blocks are centered at $\theta = \theta_n$, and the current density in each block is given by

$$j(r, \theta) = \frac{I_b}{R\,\delta\theta} f(\theta - \theta_n)\, \delta(r-R), \tag{2.8}$$

where $f(\theta-\theta_n) = 1$ over the $\delta\theta$ width of the block, and is zero outside the block. I_b is the current carried by the block.

A radial displacement of the block by ΔR will produce the current perturbation

$$\Delta j(r, \theta) = \frac{I_b}{R\,\delta\theta} f(\theta-\theta_n)\, \delta'(r-R)\, \Delta R. \tag{2.9a}$$

The corresponding perturbing field will have all poles present, and from Eq. (2.3) this field is given by

$$\begin{bmatrix} \Delta A_K \\ \Delta C_K \end{bmatrix} = \frac{\mu}{2\pi} \frac{K-1}{K} \frac{I_b}{R^{K+1}} \Delta R \begin{bmatrix} \cos(K\theta_n) \\ \sin(K\theta_n) \end{bmatrix}, \tag{2.9b}$$

$$\begin{bmatrix} \Delta B_{\theta,l}/B_0 \\ \Delta B_{r,l}/B_0 \end{bmatrix} = \frac{4}{2\pi} \frac{I_b}{I_t} \frac{l}{R^{l+1}} \Delta R \begin{bmatrix} \cos(l+1)\theta_n \\ \sin(l+1)\theta_n \end{bmatrix} r^l. \tag{2.9c}$$

It has been assumed that $\cos K\theta$ does not vary much over the $\delta\theta$ extent of the block. B_0 is the dipole central field.

Azimuthal Displacement of a Current Block

An azimuthal displacement of a block by $\Delta\theta$ will produce the current perturbation

$$\Delta j(r, \theta) = \frac{I_b}{R\,\delta\theta} f'(\theta-\theta_n)\, \delta(r-R)\, \Delta\theta. \tag{2.10a}$$

One may note that $f'(\theta-\theta_n)$ may be approximated by $f'(\theta-\theta_n) = \delta\theta\, \delta'(\theta-\theta_n)$.

The corresponding perturbing field is given by Eq. (2.3) as

$$\begin{bmatrix} \Delta A_K \\ \Delta C_K \end{bmatrix} = \frac{\mu}{2\pi} \frac{I_b}{R^{K+1}} R\,\Delta\theta \begin{bmatrix} \sin(K\theta_n) \\ \cos(K\theta_n) \end{bmatrix}, \tag{2.10b}$$

$$\begin{bmatrix} \Delta B_{\theta,l}/B_0 \\ \Delta B_{r,l}/B_0 \end{bmatrix} = \frac{4}{2\pi} \frac{I_b}{I_t} \frac{l+1}{R^{l+1}} R\,\Delta\theta \begin{bmatrix} \sin(l+1)\theta_n \\ \cos(l+1)\theta_n \end{bmatrix} r^l. \tag{2.10c}$$

RMS Field Error in Each Magnet

Having found the field error produced by a radial displacement, ΔR, and an azimuthal displacement, $\Delta\theta$, of a current block, one is now ready to find the rms field error in each magnet due to the random errors in all the current blocks, and then the rms field harmonic around the accelerator produced by the random field errors in each magnet.

The rms field error in each magnet is given by the square root of the sum of squares of the rms field errors due to ΔR and $\Delta\theta$ displacements of each of the current blocks in the magnet. If it is assumed that the rms radial displacement error ΔR and the rms azimuthal displacement error $R\Delta\theta$ of the current blocks in a magnet are equal and denoted by ε, then one may write for the rms field error in each magnet,

$$\left(\frac{\Delta B}{B_0}\right)_{\text{rms}} = \frac{4}{2\pi} \frac{l+1}{R^{l+1}} \frac{1}{I_t} \varepsilon \sqrt{\sum_n I_b^2(n)} \cdot r^l. \tag{2.11}$$

In Eq. (2.11) the sum over n is over all the current blocks in a magnet which carry the current $I_b(n)$. In using Eqs. (2.9) and (2.10) to obtain (2.11) the difference between the factors l and $l+1$ has been neglected to simplify the result, which already has comparable errors in it due to the block thickness and shape, and other similar parameters. Equation (2.11) is valid for either the error in the B_r field or the B_θ field in the median plane.

The $I_b(n)$ may be approximated by

$$I_b(n) = \frac{\pi I_t}{N_b} \cos\theta_n, \tag{2.12}$$

where N_b is the total number of current blocks in the magnet. One finds then from Eq. (2.11),

$$\left\{\sum_n I_b^2(n)\right\}^{\frac{1}{2}} = \frac{\pi}{(2N_b)^{\frac{1}{2}}} I_t, \tag{2.13}$$

$$\left(\frac{\Delta B_l}{B_0}\right)_{\text{rms}} = \sqrt{\frac{2}{N_b}\frac{l+1}{R^{l+1}}}\, \varepsilon r^l. \tag{2.14}$$

RMS Field Harmonics

The magnetic field errors in each dipole given by Eq. (2.14) will give rise to field harmonics. It is assumed that there are M magnetic dipoles around the ring. The random field errors will generate all harmonics having harmonic numbers smaller than M roughly equally. The rms harmonic is given by $1/\sqrt{M}$ times the field error in each dipole if we neglect the presence of straight sections in the ring. Thus the contribution of each field multipole to the nth harmonic is given by

$$\frac{\Delta B_{l,n}}{B_0} = \frac{1}{\sqrt{M}} \sqrt{\frac{2}{N_b}\frac{l+1}{R^{l+1}}}\, \varepsilon r^l. \tag{2.15}$$

Equation (2.15) is valid for $n < M$ and for either B_r field or B_θ field in the median plane.

Comparison of Extraction Harmonics with the Error Harmonics

The error harmonics introduced by the random errors in the coil positions may be estimated from Eq. (2.15). Consider as an example the 120-GeV superconducting synchrotron described in Ref. 4. With the values $l = 2$, $N_b = 16$, $M = 192$, $R =$ 1.31 in., one finds for the sextupole contribution to the harmonics

$$\frac{\Delta B_{2,n}}{B_0} = 0.3 \times 10^{-4} x^2, \tag{2.16}$$

where it was assumed that $\varepsilon = 0.001$ in., and x is the radial displacement in inches.

The above result is to be compared with the harmonics introduced by the slow extraction sextupoles. In the study for a possible 120-GeV synchrotron, it has been assumed that there are four extraction sextupoles around the ring which introduce the 22nd harmonic needed to excite the $\nu_y = 22/3$ resonance. Each sextupole has a strength so that it changes the particle direction by $\Delta x' = gx^2$, where $g = 0.27 \times 10^{-4}/\text{cm}^2$. In terms of the magnetic field, the magnitude of g is given by

$$g = \frac{1}{2}\frac{1}{B_0 \rho} \int B''\, \mathrm{d}s. \tag{2.17a}$$

The sextupole harmonics are then given by

$$\frac{S_n}{B_0} = \frac{1}{2}\frac{1}{LB_0} \int B''\, \mathrm{e}^{in\theta}\, \mathrm{d}s \cdot x^2 \tag{2.17b}$$

for $n = 2, 6, 10 \ldots$, and θ is the azimuthal angle around the synchrotron. This gives, for four sextupole magnets (equally separated around the accelerator, with equal strength and alternating polarity),

$$\frac{S_n}{B_0} = \frac{2}{\pi}\frac{\rho}{R} g x^2, \tag{2.17c}$$

where R is the geometrical radius of the accelerator. For $R/\rho = 1.3$, we get

$$\frac{S_n}{B_0} = 0.85 \times 10^{-4} x^2. \tag{2.18}$$

One sees that an rms error of 0.001 in. in the coil positioning already introduces error harmonics which are about a third of the extraction harmonics. It therefore appears necessary to have a correction procedure to eliminate the error harmonics as it does not appear likely that one can satisfy the tolerances required to make them negligible.

Effect of the Iron Shield

The previous calculation of the effect of random errors in the coil positions was carried out assuming no iron shield was present. It is possible to repeat the calculation if one assumes the iron has infinite permeability. In this case, Eq. (2.1) for the vector potential due to an arbitrary current distribution is replaced by

$$A(r,\theta) = -\frac{\mu}{2\pi} \int \mathrm{d}S' G(r,\theta,r',\theta') j(r',\theta'), \tag{2.19a}$$

where $G(r, \theta, r', \theta')$ is the Green's function in the presence of a circular iron shield of infinite permeability and with the radius a. This Green's function can be found analytically[5] and is given by

$$G = \log\left\{1+\frac{2r}{r'}\cos(\theta-\theta')+\left(\frac{r}{r'}\right)^2\right\}^{\frac{1}{2}} +\log\left\{1+\frac{2rr'}{a^2}\cos(\theta-\theta')+\left(\frac{rr'}{a^2}\right)^2\right\}^{\frac{1}{2}}. \quad (2.19b)$$

And this Green's function has the expansion

$$G = -\sum_{K=1}^{\infty}\frac{1}{K}\cos K(\theta-\theta')r^K\left(\frac{1}{r'^K}+\left(\frac{r'}{a^2}\right)^K\right). \quad (2.19c)$$

This expansion is valid for $r < r'$. For $r > r'$, one simply interchanges r and r'.

The effect of the presence of iron on the multipole is to replace Eq. (2.3b) by

$$\begin{bmatrix}A_K\\ C_K\end{bmatrix} = \frac{1}{2\pi}\frac{1}{K}\int dS'\frac{1}{r'^K}\left(1+\left(\frac{r'}{a}\right)^{2K}\right) \cdot j(r',\theta')\begin{bmatrix}\cos K\theta'\\ \sin K\theta'\end{bmatrix}. \quad (2.20)$$

The net effect of the iron is to introduce an additional factor of $1+(r'/a)^{2K}$ in the integral. This would increase both the unperturbed field due to the unperturbed current distribution and the field errors by very roughly a factor of two. The relative field errors $\Delta B_l/B_0$ would remain approximately unchanged.

3. CORRECTION OF THE EFFECTS OF RANDOM FIELD ERRORS ON SLOW EXTRACTION

In this section, one method of correcting for the effects of the random field errors on the slow extraction system is investigated. This correction procedure employs two sets of sextupoles (four in each set) and four octupoles which are excited in a way which is discussed below.

We use an idealized lattice comprised of 48 cells, each having one focusing and one defocusing point quadrupole, as well as four bending magnets. Such a FODO type lattice is illustrated in Fig. 2.

When there are no sextupole errors in the bending magnet field, vertical resonant extraction can be excited at $\nu_y = 7\frac{1}{3}$ by a 22nd harmonic of an introduced azimuthal sextupole distribution.[4,6] This can be done with sextupole magnets placed immediately following vertically focusing quadrupoles, to take advantage of the vertical β-function maximum at these points of the azimuth. Four sextupoles with alternating polarity are placed symmetrically around the synchrotron ring, as illustrated in Fig. 3. Each of the four has an integrated strength of magnitude g_s, that is

$$\Delta y_i' = \pm g_s y_i^2, \quad (3.1)$$

where $\Delta y_i'$ is the change in particle slope at the ith sextupole, and y_i is the particle vertical amplitude at this azimuth. This distribution produces a pure sine component of the 22nd harmonic at the thin septum azimuth. Another set of four sextupoles of alternating polarity, but with integrated strength g_c, is included in the lattice to provide a cosine component.

These eight sextupole magnets have two purposes:

(1) They provide a 22nd harmonic appropriate for slow extraction at the azimuthal position of the

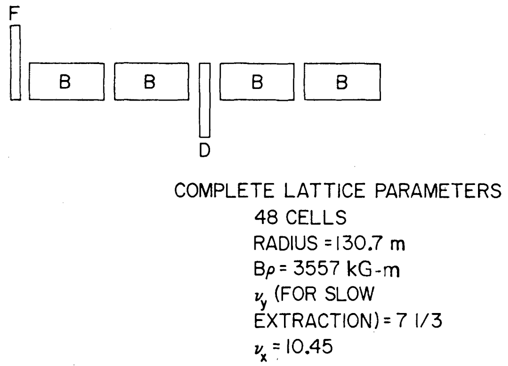


FIG. 2. Cell of the separated function lattice. *F*—focusing quadrupole; *D*—defocusing quadrupole; *B*—bending magnet.

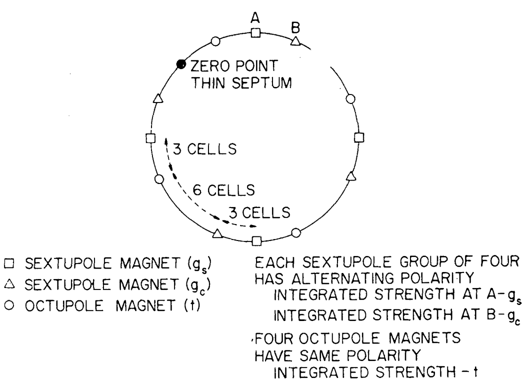
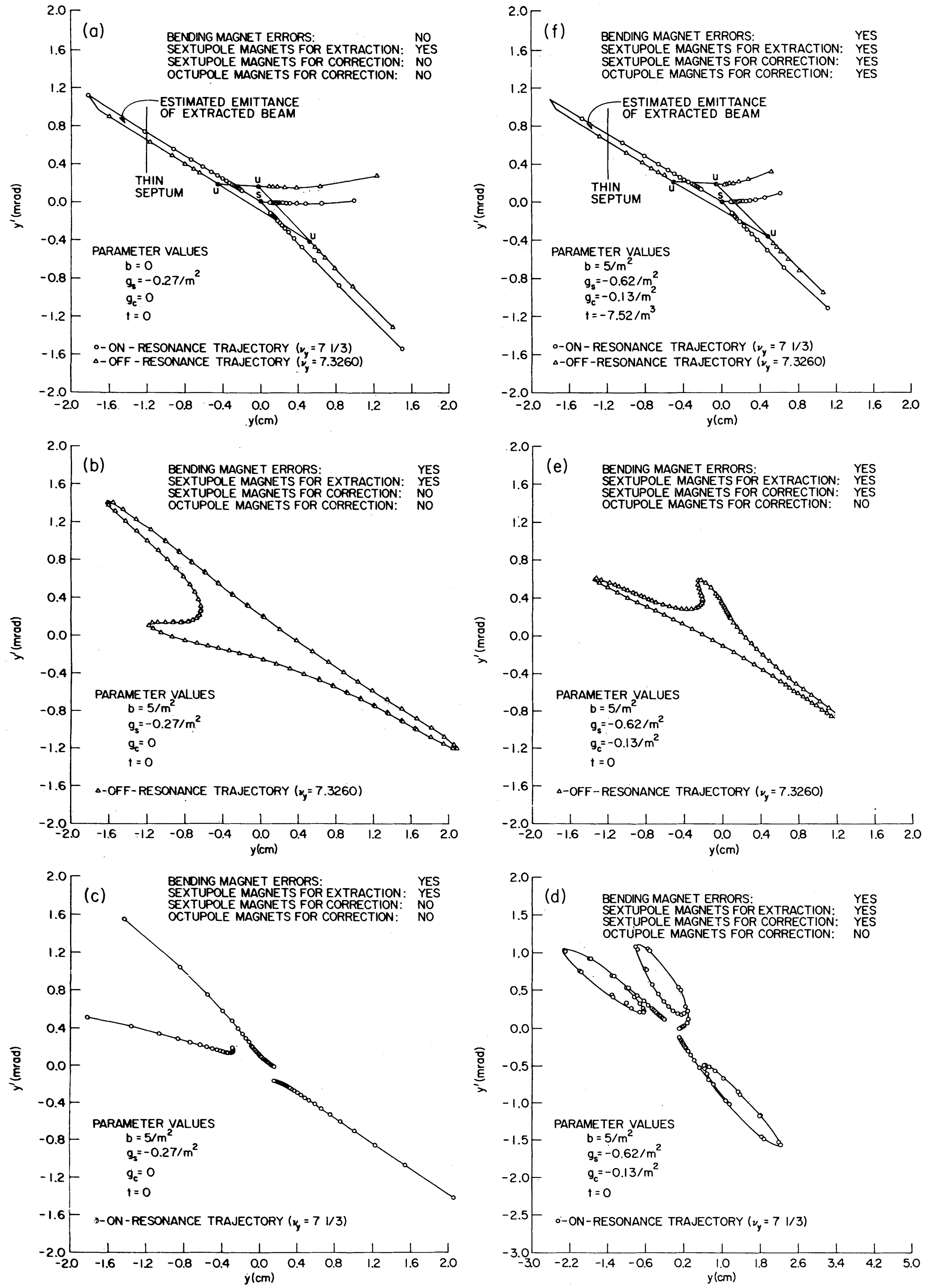


FIG. 3. Extraction sextupoles and correcting elements.


(a)
BENDING MAGNET ERRORS: NO
SEXTUPOLE MAGNETS FOR EXTRACTION: YES
SEXTUPOLE MAGNETS FOR CORRECTION: NO
OCTUPOLE MAGNETS FOR CORRECTION: NO
ESTIMATED EMITTANCE OF EXTRACTED BEAM
THIN SEPTUM
PARAMETER VALUES
b = 0
gs = -0.27/m2
gc = 0
t = 0
o-ON-RESONANCE TRAJECTORY (νy = 7 1/3)
Δ-OFF-RESONANCE TRAJECTORY (νy = 7.3260)
y'(mrad)
y(cm)
(f)
BENDING MAGNET ERRORS: YES
SEXTUPOLE MAGNETS FOR EXTRACTION: YES
SEXTUPOLE MAGNETS FOR CORRECTION: YES
OCTUPOLE MAGNETS FOR CORRECTION: YES
ESTIMATED EMITTANCE OF EXTRACTED BEAM
THIN SEPTUM
PARAMETER VALUES
b = 5/m2
gs = -0.62/m2
gc = -0.13/m2
t = -7.52/m3
o-ON-RESONANCE TRAJECTORY (νy = 7 1/3)
Δ-OFF-RESONANCE TRAJECTORY (νy = 7.3260)
(b)
BENDING MAGNET ERRORS: YES
SEXTUPOLE MAGNETS FOR EXTRACTION: YES
SEXTUPOLE MAGNETS FOR CORRECTION: NO
OCTUPOLE MAGNETS FOR CORRECTION: NO
PARAMETER VALUES
b = 5/m2
gs = -0.27/m2
gc = 0
t = 0
Δ-OFF-RESONANCE TRAJECTORY (νy = 7.3260)
(e)
BENDING MAGNET ERRORS: YES
SEXTUPOLE MAGNETS FOR EXTRACTION: YES
SEXTUPOLE MAGNETS FOR CORRECTION: YES
OCTUPOLE MAGNETS FOR CORRECTION: NO
PARAMETER VALUES
b = 5/m2
gs = -0.62/m2
gc = -0.13/m2
t = 0
Δ-OFF-RESONANCE TRAJECTORY (νy = 7.3260)
(c)
BENDING MAGNET ERRORS: YES
SEXTUPOLE MAGNETS FOR EXTRACTION: YES
SEXTUPOLE MAGNETS FOR CORRECTION: NO
OCTUPOLE MAGNETS FOR CORRECTION: NO
PARAMETER VALUES
b = 5/m2
gs = -0.27/m2
gc = 0
t = 0
o-ON-RESONANCE TRAJECTORY (νy = 7 1/3)
(d)
BENDING MAGNET ERRORS: YES
SEXTUPOLE MAGNETS FOR EXTRACTION: YES
SEXTUPOLE MAGNETS FOR CORRECTION: YES
OCTUPOLE MAGNETS FOR CORRECTION: NO
PARAMETER VALUES
b = 5/m2
gs = -0.62/m2
gc = -0.13/m2
t = 0
o-ON-RESONANCE TRAJECTORY (νy = 7 1/3)

thin septum. With the two independent sets of sextupoles, both spiral pitch and separatrix orientation can be independently chosen.

(2) They correct, in both magnitude and phase, the 22nd harmonic produced by the random sextupole errors in the lattice bending magnets.

Although the 22nd harmonic introduced by the random errors can be effectively controlled by the sine and cosine groups of sextupole magnets, we find that this is insufficient to ensure proper extraction conditions. The fact that only the 22nd harmonic is responsible for resonant extraction is a first order concept. With increasing nonresonant harmonics, the first order picture becomes distorted. It has been shown[(2)] that the dominant distortion of the ideal resonant extraction picture, that due to the nonresonant harmonics, can be removed with a 0th harmonic azimuthal octupole distribution. We therefore introduce into the lattice four octupole magnets, all of equal integrated strength, t; that is,

$$\Delta y_i' = t y_i^3. \tag{3.2}$$

Their positions in the lattice are indicated in Fig. 3. Thus, the correcting procedure that is used here is the following: given a particular random distribution of sextupole errors in the lattice bending magnets, the values of g_s, g_c and t are varied so as to optimize the extraction conditions. In fact, the goal is to achieve an extraction picture which is essentially unchanged from the ideal one (that is, the extraction scheme without errors and without correcting devices).

The errors in coil positioning introduce errors in all the harmonics of the magnetic field of the bending magnets. In particular, a sextupole error is produced. Using Eq. (2.14) we obtain for the rms sextupole term introduced into the radial magnetic field (if x is assumed zero),

$$\left(\frac{\Delta B}{B}\right)_{\text{rms}} = \langle b \rangle_{\text{rms}} \, y^2, \tag{3.3}$$

where B is the ideal bending field, y is the vertical distance from the center of the magnet, ΔB is the rms radial field introduced by the coil positioning errors, and b is the resulting sextupole coefficient, given by

$$\langle b \rangle_{\text{rms}} = \sqrt{\frac{2}{N_b}} \frac{3}{R^3} \langle \varepsilon \rangle_{\text{rms}}. \tag{3.4}$$

Here, N_b is the number of coil blocks, R is the magnet radius, and ε is the positioning error for the block placements. Using the values $N_b = 16$, $R = 1.31$ in., $\langle \varepsilon \rangle_{\text{rms}} = 0.004$ in., we have for the rms sextupole coefficient, $\langle b \rangle_{\text{rms}} = 0.0019/\text{in.}^2 = 2.9/\text{m}^2$.

Consider a random distribution of sextupole coefficients, $\{b_i\}$, within a fixed range, limited by the value, b; that is,

$$-b \leqslant b_i \leqslant b. \tag{3.5}$$

We choose for the purpose of doing a numerical study, $b = 5/\text{m}^2$. The $\langle b \rangle_{\text{rms}}$ which corresponds to this choice of b is $\langle b \rangle_{\text{rms}} = 2.9/\text{m}^2$, which would be generated by a coil position error $\langle \varepsilon \rangle_{\text{rms}} = 0.004$ in., as indicated above.

We show, in Figs. 4(a) to 4(f), the procedure in detail for a specific case. The case we have chosen to demonstrate has a particularly large azimuthal 22nd harmonic sextupole distribution, and thus requires greater use of the correcting elements than was generally required, and in this sense is a difficult error distribution to correct. The actual 22nd harmonic of the sextupole errors is $\langle b \rangle_{\text{22nd harmonic}} \approx 0.44/\text{m}^2$, whereas the rms 22nd harmonic is roughly given by

$$\langle b \rangle_{\text{rms, 22nd harmonic}} \approx \frac{\langle b \rangle_{\text{rms}}}{\sqrt{M}} \approx 0.21/\text{m}^2. \tag{3.6}$$

FIG. 4. Vertical resonant extraction at a one-third-integer ν-value. The bending magnet sextupole errors are indicated by the parameter b. The two groups of sextupole magnets that ensure the proper azimuthal 22nd harmonic have integrated strengths g_s and g_c. The octupole magnets for the correction of the nonresonant distortions have integrated strength t.

The off-resonance ν-value corresponds to a separatrix (triangular region) size which is equivalent to a beam radius of 1/3 cm.

u—Unstable fixed points; s—Stable fixed point (equilibrium orbit).

In (a), no bending magnet errors are present, $b = 0$. In (b), (c), (d), (e), and (f), a random distribution of sextupole errors in the bending magnets is present. This distribution has a limiting value, $b = 5/\text{m}^2$, and

$\langle b \rangle_{\text{22nd harmonic}} \approx 0.44/\text{m}^2 = 2.84 \times 10^{-4}/\text{in.}^2$,

while

$\langle b \rangle_{\text{rms, 22nd harmonic}} \approx 0.21/\text{m}^2 \approx 1.26 \times 10^{-4}/\text{in.}^2$.

The values of g_s, g_c and t were numerically chosen to optimize the extraction, and in the relevant region of phase space, the correction procedure was successful. Various other random distributions were tried and were successfully corrected.

The random field errors due to coil position errors generate both a B_z and B_r field in the median plane. In the computer results presented above, only the B_r component was assumed to be present as the B_r field drives the vertical resonance used in the extraction procedure. Some computer runs were done with the B_z sextupole field also present. They showed that the B_z component does not affect the results appreciably. Computer runs were also done with nonzero horizontal betatron oscillations, and the coupling found between the vertical and horizontal oscillations was somewhat less than 10 per cent, which means that starting with a very small horizontal betatron amplitude, the horizontal oscillations grow to somewhat less than 10 per cent of the vertical oscillations.

In conclusion, the computer studies reported here show that the correction procedure employing additional sextupoles and octupoles can correct for the effects of random field errors on the slow extraction orbits when the rms error in the coil positions is $\langle \varepsilon \rangle_{\mathrm{rms}} = 0.004$ in. Further computer studies indicate that this correction procedure will correct for the effects of random field errors due to errors in the coil positions as large as $\langle \varepsilon \rangle_{\mathrm{rms}} = 0.008$ in.

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