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THE EFFECT OF SIMULTANEOUS PHASE ERROR AND PHASE JITTER ON THE ENERGY RESOLUTION OF MULTISECTION ELECTRON LINACS

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The output energy fluctuations of multisection electron linacs due to klystron voltage fluctuations are calculated assuming phase misalignments. It is shown that under realistic conditions the influence of klystron phase jitter may give a larger contribution than amplitude jitter.

1. INTRODUCTION

Concerning the energy spectrum of multisection electron linacs the effect of phasing errors has been treated in literature as well as the effect of phase jitter due to klystron voltage instabilities.[1–4] But as far as is known to the author each of these effects has been treated assuming the absence of the other. In the following it will be shown, however, that there exists a product term of both that might give a considerable additional contribution to spectrum width.

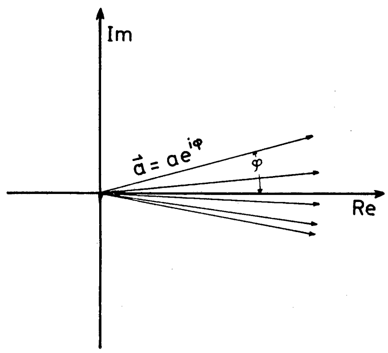

FIG. 1. Distribution of the complex rf amplitudes of the different sections due to phasing errors.

2. CALCULATION OF AMPLITUDE AND TIME JITTER OF THE ACCELERATING FIELD

Consider the distribution of the rf phases in the different sections all over a multisection linac as seen by a relativistic particle passing the linac.

Let the position of the reference particle be chosen in such a way that it is placed on the crest of the superposition of all waves in the sections ('phase closure'). In this case the individual phases are distributed due to misalignments around the abscissa of the complex plane with their sum coinciding with the abscissa (Fig. 1), and the complex amplitude of the total accelerating field is given by

$$\mathbf{A} = Ae^{i\phi} = \sum_\nu \mathbf{a}_\nu = \sum_\nu a_\nu e^{i\varphi_\nu}; \quad \phi = 0. \tag{1}$$

A and ϕ are amplitude and phase respectively of the total accelerating field (abbreviated as TAF in the following), a_ν and φ_ν refer to the waves on the individual sections. Each individual wave is submitted to coherent phase and amplitude jitter, $\mathrm{d}\varphi$ and $\mathrm{d}a$ respectively, due to fluctuations $\mathrm{d}U$ in the voltage U of the driving klystron:

$$\frac{\mathrm{d}a}{a} = \frac{5}{4}\frac{\mathrm{d}U}{U}; \quad \mathrm{d}\varphi = K\frac{\mathrm{d}U}{U}. \tag{2}$$

K depends on drift space and driving voltage of the klystron, e.g., $K = 9$ to 14 rad for the CSF type F2042, within the driving voltage of 280 to 160 kV. The fluctuations of the waves in each section will cause fluctuations in amplitude and phase of the TAF. These will be calculated in the following, taking into consideration pulse-to-pulse jitter only, which, as will be shown, gives rise to a severe limitation of phasing by the energy maximization method. The total differential of Eq. (1) with respect to all klystron voltages U_ν is

$$\mathrm{d}\mathbf{A} = \mathrm{d}A + iA\,\mathrm{d}\phi = \sum_\nu \left(\frac{\partial a_\nu}{\partial U_\nu} + ia_\nu \frac{\partial \varphi_\nu}{\partial U_\nu}\right) e^{i\varphi_\nu}\,\mathrm{d}U_\nu. \tag{3}$$

We assume that all klystrons are under the same working conditions, so that $\partial a_\nu/\partial U_\nu \equiv a'$ and

$\partial\varphi_\nu/\partial U_\nu \equiv \varphi'$ are independent of ν. Then Eq. (3) may be written

$$\left.\begin{aligned} \mathrm{d}A &= B\sum_\nu \sin(\varphi_c - \varphi_\nu)\,\mathrm{d}U_\nu; \\ A\,\mathrm{d}\phi &= B\sum_\nu \cos(\varphi_c - \varphi_\nu)\,\mathrm{d}U_\nu, \\ &\text{where} \\ B^2 &= (a\varphi')^2 + (a')^2;\quad \tan\varphi_c = \frac{a'}{a\varphi'} = \frac{5}{4K} \approx \varphi_c \end{aligned}\right\}\quad (4)$$

From Eq. (2) φ_c is calculated to be 0,09 to 0,14 rad. To get the variance δA^2 of the fluctuations of A with time we take a large number m of samples $\mathrm{d}A_\mu$ at different times t_μ. The variance is then given by

$$\delta A^2 = \sum_\mu \frac{\mathrm{d}A_\mu^2}{m} = \frac{B^2}{m}\sum_\mu\left[\sum_\nu \sin(\varphi_c - \varphi_\nu)\,\mathrm{d}U_{\nu\mu}\right]^2.$$

With the assumption that the fluctuations $\mathrm{d}U_\nu(t)$ of different klystrons are statistically independent† but of equal strength and that the mean value of $\mathrm{d}U_\nu(t)$ is zero this may be transformed to give

$$\delta A^2 = B^2\delta U^2 \sum_\nu \sin^2(\varphi_c - \varphi_\nu)$$

where δU^2 is the variance of the klystron voltage fluctuations. If the $(\varphi_c - \varphi_\nu)$ are not too large the $\sin^2$ may be replaced by its first order approximation. Furthermore, as may be seen from Eq. (1) the mean value of the φ_ν is very close to zero (it disappears rigorously in the limit of small φ_ν or if the distribution of the φ_ν is symmetrical). Therewith we may introduce the variance $\delta\varphi^2$ of the φ_ν, $\delta\varphi$ being a measure of the accuracy in the adjustment of rf phases on the different sections. This gives

$$\frac{\delta A}{A} = \frac{1}{\sqrt{n}}\frac{\delta U}{U}\sqrt{\left(\frac{5}{4}\right)^2 + K^2\delta\varphi^2}. \qquad (5)$$

Here Eqs. (2) and (4) are used and terms of less importance are neglected, n is the number of linac sections.

A similar calculation yields from the 2nd of Eq. (4)

$$\delta\phi = \frac{K}{\sqrt{n}}\frac{\delta U}{U}, \qquad (6)$$

$\delta\phi^2$ being the variance of the phase jitter of the TAF.

† Coherent phase jitter is not discussed here since its effect is not coupled with phase spread. It may easily be calculated by moving $\mathrm{d}U_\nu$ before the sum sign in Eq. (4) and leads to the simple results: $\mathrm{d}A/A = (5/4)\,\mathrm{d}U/U$; $\mathrm{d}\phi_{\mathrm{rel}} \approx 0$.

3. THE INFLUENCE OF EQS. (5) AND (6) ON THE ENERGY SPECTRUM

Let us consider the energy spectra after the transients due to beam loading are passed, and let the bunch cover the crest of the total accelerating field. Then a 'snapshot' of the spectrum shows a very sharp upper edge, the energy width of its ascent being given essentially by the energy resolution of the beam at the end of the injector, and a smooth slope to lower energies, the width of which is given by the bunch length. Figure 2

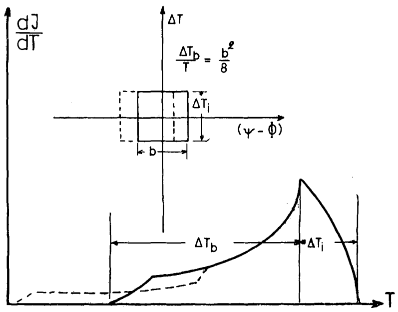

FIG. 2. The output energy spectrum as calculated from the injector phase space area shown. ψ is the angular coordinate along the bunch, $\psi = 0$ denoting the bunch midst. b is the bunch length. Dotted lines show the influence of small phase jitter.

shows the spectrum as it is obtained under the idealized assumption of a rectangular, uniformly filled phase space area at the end of the injector. The upper edge and the following peak consist of electrons on the crest of the TAF, i.e., they exist as long as the bunch still covers the crest, and their position is determined by the amplitude A of the TAF. A small shift between the bunch midst and the crest of the TAF mainly affects the lower end of the spectrum insofar as it is 'smeared out' if the position of the bunch ends is not symmetrical with respect to the crest of the TAF (dotted lines in Fig. 2). To affect the peak seriously a shift of about half a bunch length or more is needed.

We may assume that the phase jitter of the injector has the same amplitude as the phase jitter in any linac section. By adding its variance to $\delta\phi^2$, the variance $\delta\phi_{\mathrm{rel}}^2$ of the phase jitter between bunch and TAF is found to be

$$\delta\phi_{\mathrm{rel}} = \sqrt{\frac{n+1}{n}}\,K\frac{\delta U}{U}. \qquad (7)$$

As is seen from Eq. (7), $\delta\phi_{\rm rel}$ is almost independent of the number of sections but essentially given by the klystron working conditions and modulator performance. A bad estimate using $K = 14$ rad and $\delta U/U = 3 \times 10^{-3}$ gives $\delta\phi_{\rm rel} = 2.4°$. If one assumes a normal distribution for ∂U and a bunch length of 10° this means that the probability of affecting the peak is about 5 per cent.

Equation (5) gives the amplitude of energy fluctuations of the spectrum as a whole. It can readily be seen that the influence of the klystron phase jitter becomes predominant over the influence of klystron amplitude jitter if $K\delta\varphi > 4/5$, i.e., if $\delta\varphi > \varphi_c$ [see Eq. (4)]. In the limit $\delta_\varphi^2 \gg \varphi_c^2$ Eq. (5) becomes $\delta A/A = (K/n^{1/2})\ (\delta U/U)\delta\varphi$ showing direct proportionality between beam energy jitter and phase spread. As will be discussed later there is some evidence that many linacs are normally run in this region due to poor phase monitoring. For this discussion we will return to Eq. (5) and confine ourselves to the phasing by beam energy maximization which seems to be the most widely used method. The most serious handicap in this procedure is the fact that its precision is determined by the amount of fluctuations in the position of the upper edge of the spectrum, these, in turn, being given by the accuracy of phasing already obtained [Eq. (5)]. In the following it will be shown that this mutual dependence leads to an upper limit of accuracy in phasing.

The signal indicating the position of the phase in the ν-th section is the shift of the TAF amplitude when shifting this phase:

$$\frac{\mathrm{d}A}{A} = \frac{1}{n}(1 - \cos\varphi_\nu) \approx \frac{\varphi_\nu^2}{2n}. \tag{8}$$

Let $\mathrm{d}A/A$ be the interval of error within which a true shift of the TAF amplitude cannot be identified any more, then $(\pm\varphi_\nu)^2$ in Eq. (8) indicates the interval of error in phasing the ν-th section. We may numerically identify the interval of error in reading the TAF amplitude with the standard deviation $\delta A/A$ of its fluctuations and the phasing error given by Eq. (8) with the variance $\delta\varphi^2$ of the φ_ν. Then Eq. (8) denotes a state in which further improvement of phasing is inhibited by the reading error of the TAF amplitude. Introducing this into Eq. (5) gives

$$2nK^2\frac{\delta A_m}{A} = 2\sqrt{n}K^2\frac{\delta U}{U}\sqrt{\left(\frac{5}{4}\right)^2 + 2nK^2\frac{\delta A_m}{A}} \tag{9}$$

where the index m denotes the minimum value achievable by the beam energy maximization method. This relation is shown in Fig. 3. It is readily seen that in the limit $K \to 0$, corresponding to hypothetical klystrons being free from phase distortion, $n^{1/2}\delta A_m/A$ will be larger than $\delta U/U$ merely by a factor of 5/4, due to the amplitude jitter of the klystron outputs [see Eq. (2)]. But, if K and $\delta U/U$ are both finite, then $\delta A_m/A$ will

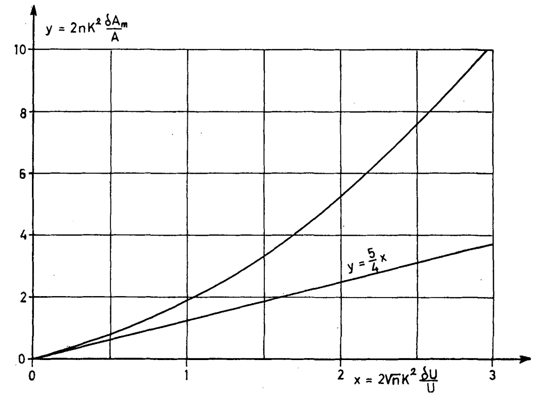

FIG. 3. The minimum achievable output energy jitter as a function of the klystron working conditions, phasing being done by beam energy maximization.

become larger due to the phase jitter of the klystron outputs. A numerical example for MUELL† may be given: With 8 sections, a K value of 12 rad (corresponding to about 200 kV klystron voltage), and a standard deviation of the voltage jitter of 0.25 per cent an x-value of 2.04 is obtained, giving $\delta A_m/A = 2.3 \times 10^{-3}$. This is more than twice of what one would get with $K = 0\ (\delta A_m/A = 1.1 \times 10^{-3})$. The additional spread in output energy of 0.23 per cent might be considered to be small, but it should be emphasized that these are fluctuations in time, causing severe fluctuations in intensity, if an 0.1 per cent interval near the peak of the spectrum is cut out by an energy defining system. The standard deviation of the phase spread may be calculated from Eq. (8) giving $\delta\varphi = 0.19$ rad. This compared with $\varphi_c = 5/4\,\mathrm{K} = 0.104$ assures that the beam energy fluctuations are predominantly caused by klystron phase jitter. It should be mentioned that other phasing methods may well lead to phase spreads of similar magnitude. Even the comparison of the phases of the impressed rf wave and the beam induced wave at the end of the sections by aid of a phase bridge, which up to now

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is considered to be the most precise method, may give severe errors if the sections are not exactly tuned or if their loads consist of low-Q cavities. At MUELL, for example, the beam energy maximization method gives even better results than the latter, and phasing by watching the beam loaded rf. amplitude at the ends of the sections, as was done earlier at MUELL and is still being done in some other places, leads to even worse results.

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