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ENERGY AND ANGULAR RESOLUTION IN PION-PION SCATTERING AND OTHER COLLIDING BEAM EXPERIMENTS

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In certain types of colliding beam experiments, the interaction region may contain particles within a wide momentum range. The paper describes how the energy and angle dependence of the cross section can be measured under these circumstances. Pion–pion scattering in a miniring is given as an example.

1. GENERAL DISCUSSION OF COLLIDING BEAM EXPERIMENTS

Most of our information concerning elementary particle physics is derived from experiments in which two elementary particles are permitted to scatter on each other and the particles produced in the scattering are observed. Let us refer to one of the two incoming particles as 'target' particle, and to the other as the 'projectile' particle. Unfortunately, of all the numerous conceivable scattering events of this type, only a small fraction has ever been observed directly, namely those in which the target particle is either a proton, or a neutron, or an electron. Let us refer to these as experiments of the 'usual kind'. In experiments of the usual kind, detailed measurements of energy and angular dependence of the differential cross section are achieved in the following way. First, one restricts the absolute value and angle of the momenta of both colliding particles to lie within well defined narrow limits. This is done by choosing beams with momentum vector resolution of typically a few per cent and if solid material targets are used, then choosing those to be practically at rest. Second, one induces a reaction between the two particles of well defined momenta, and measures the cross sections of interest for that particular reaction center of mass energy, and at various angles relative to the well defined incoming momenta. Third, one repeats this measurement for various values of the incoming momenta, and plots the measured cross sections as a function of energy and angle.

If we try to perform experiments other than those of the usual kind, then we encounter serious difficulties. The reason is that in such experiments the target particle is either (a) shortlived and decays before enough scattering events take place, or (b) neutral and cannot simply be prevented from leaving the interaction region too soon. The experimental technology in common use today, is unable to overcome these difficulties. On the other hand, in recent years four important new developments have taken place: the design or construction of new accelerators, storage rings, more powerful magnets, and better detectors. The question arises whether the appropriate combination of these new tools could enable us to perform at least some of the heretofore unfeasible experiments. This question was investigated several years ago, at the time when the planning and construction of the new generation of accelerators and the CERN-ISR began in earnest. The results of this investigation concerning some colliding beam experiments of type (a) were reported in references 1–5 and concerning experiments of type (b) in reference 6. It was found that the new developments in technology could indeed enable us to perform some of these experiments.

All of the experimental arrangements discussed in these references make use of one or more of the following four methods to reach reasonable counting rates: (1) Use more powerful sources of primary particles. (2) Reduce the duty factor of the primary particles, e.g. by using storage rings for this purpose. The counting rate in these experiments is proportional to the crossing factor of the primary beam, defined as $C_b \equiv \bar{I}^2/D \cdot d \cdot c$, where $\bar{I}$ is the average intensity, D and d are the macro and micro duty factors respectively of the primary beam, and c is the velocity of light in vacuum. The first two methods, therefore, clearly have the effect of increasing C_b for the primary beam. (3) Increase the number of secondaries reaching the interaction region, per primary particle. This is done by collecting into the interaction

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region all secondary particles whose momentum, p, has the absolute value $p_0 - \Delta p/2 \leqslant p \leqslant p_0 + \Delta p/2$ and lies within a solid angle $\Delta\Omega$. The $\Delta p/p$ and $\Delta\Omega/4\pi$ are *not* small. In fact, for certain suggested experimental arrangements $\Delta p/p$ and $\Delta\Omega/4\pi$ are both of the order unity. (4) Shape the interaction region in such a manner that the secondary particles have as much chance to interact as possible.

It is clear from number (3) above, that in these experimental arrangements the interaction region contains not only incoming particles with well-defined momenta, but instead, it contains particles with a wide range of kinetic energy, and momentum vectors pointing anywhere within a large solid angle. Consequently, it may appear at first sight that in such experiments one cannot determine the energy dependence of cross sections, and that only some average (over energy and angle) value can be measured. It is the purpose of this paper to clear up this misunderstanding. In fact, although the methods used in experiments of the usual kind, can clearly not be employed now, nevertheless, the energy and angle dependence of cross sections can be measured even in these experiments. To be specific, we will illustrate this point on one particular example, namely the suggested pion-pion scattering experiment in a miniring.[1,2,5] However, our considerations will have general validity, and, with the appropriate changes, can be applied to other colliding beam experiments as well.

2. ENERGY AND ANGULAR RESOLUTION IN A MINIRING

The miniring[1] is a device which can contain low energy stable or unstable particles. Its purpose is to provide a target of low energy particles, so that the scattering of the particles in the miniring on each other or on other particles, can be observed. It consists of a relatively small volume (hence the name) filled by a magnetic field shaped in such a manner that low energy particles produced inside the ring within a wide energy and angular range, are guided by the field, and most of them are prevented from leaving the miniring for an average time T. One particular design for a miniring is outlined in Fig. 1. The primary protons of about 1 BeV kinetic energy, hit the target located at one end of the ring. The pions are then gathered and guided by the narrow channel section of the miniring into the wider interaction region. Both the channel section and the interaction region are filled with a predominantly longitudinal magnetic field along which the pions with transverse (to the magnetic field) momentum up to say 300 MeV spiral. The field at both ends of the interaction region is such that most pions are reflected by it, and are trapped. The T is chosen to be approximately equal to the lifetime, τ, of the average pion. The miniring is surrounded by a spark chamber. The (possibly superconducting) magnet surrounding the interaction region may be the first plate of the chamber.†

Both the miniring and the conventional storage rings make use of magnetic fields in order to guide particles along approximately closed orbits.†† The

† I am gratefull for R. W. Kennedy for pointing this out to me.

‡ Recently the pion orbits were calculated in detail for one particular magnetic field distribution.[7] (These authors refer to the particular miniring they investigated as 'precetron'.)

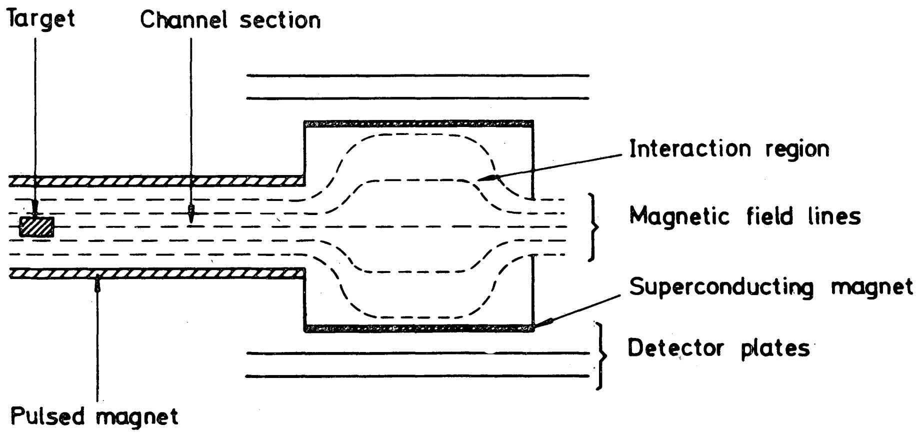

FIG. 1. One possible arrangement of the target, the miniring consisting of two sections: the channel section and the interaction region, and the detector. Perturbing fields are not indicated.

minining, however, differs from conventional storage rings in the following ways. First, it is smaller; a miniring may have a volume between 10^3 cm^3 and 10^6 cm^3. Second, in a miniring the particles in the magnetic field interact usefully along most of their orbits, while in storage rings useful interactions can occur only along that relatively small fraction of the orbit which lies inside the interaction regions. Third the miniring is capable of containing particles with $\Delta p/p$ and $\Delta\Omega/4\pi$ both of the order of unity, while in conventional storage rings both of these parameters are usually less than or of the order of a few per cent.

The fact that the interaction region contains particles with momenta lying within a wide range of Δp and $\Delta\Omega$, implies that in the miniring, reactions with various values of center-of-mass energy and various incoming momenta take place simultaneously.

For the purposes of the following discussion we will consider the reactions

$$\pi^+ + \pi^- \rightarrow \pi^0 + \pi^0 \qquad (1)$$

$$\pi^+ + \pi^- \rightarrow \pi^+ + \pi^- \qquad (2)$$

Energy resolution

(A) Neither the two final π^0 in reaction (1), nor the two pairs of γ rays into which they decay, are affected by the magnetic field. From the showers produced by the four γ-rays, it should be possible to locate the origin of the γ-rays within a cylinder of about 5 cm radius. Fourfold γ coincidence can not be produced by the background in the miniring, therefore, a coincidence of four γ-rays, all of which originate at the same point (within errors) indicates an event of reaction (1). Measurement of the showers produced by the γ-rays should make reconstruction of the γ-ray energy possible within 20–30 per cent.

The paths of the final particles in reaction (2) depend on the magnetic field. Those which result from elastic scattering with little change of momentum, will continue to be trapped by the magnetic field. On the other hand, those which result from elastic scattering near the backward direction in the reaction center-of-mass frame, have a good chance (depending on exactly where the scattering took place) to leave the interaction region. These particles can be tracked by the surrounding spark chamber. To establish coincidence between the origins of a π^+ and a π^-, one has to reconstruct their orbits inside the interaction region. For this purpose, an accurate knowledge of the field in the miniring is necessary. The accuracy of this extrapolation will determine the background which can be tolerated in the measurement of reaction (2). On the other hand, the curvature of the pion tracks in the detector, is determined by the magnetic field between the detector plates, and does not depend on the field in the miniring. This should enable one to make a good measurement of the pion energies.

We see that for both reactions it is possible to measure the number of scattering events whose total momentum lies in the interval $\bar{P} \pm \Delta\bar{P}$, center of mass energy is within $E_{\text{cms}} \pm \Delta E_{\text{cms}}$ and in which, as seen from the reaction center of mass, the positive pion momentum forms an angle within $\chi \pm \Delta\chi$ with the axis of the interaction region, and the reaction takes place within the volume $\bar{r} \pm \Delta\bar{r}$ in the miniring, at time t. Let us denote this quantity by M_k ($\bar{P}$, $\Delta\bar{P}$ E_{cms}, ΔE_{cms}, χ, $\Delta\chi$, $\bar{r}$, $\Delta\bar{r}$, t), where k can have the value 1 or 2, depending on which of the two reactions we are discussing.†

Before we can determine the energy dependence of the cross section, we have to know the density of $\pi^\pm$ with momentum within $\bar{p} \pm \Delta\bar{p}$, contained in the volume $\bar{r} \pm \Delta\bar{r}$, at time t. Let us denote this quantity by $N^\pm(\bar{p}, \Delta\bar{p}, \bar{r}, \Delta\bar{r}, t)$. Furthermore, let us denote by $n^\pm(\bar{p}, \Delta\bar{p}, E_0, X)$, the number of $\pi^\pm$ with momentum within $\bar{p} \pm \Delta\bar{p}$, produced by an incoming proton beam of kinetic energy E_0, on one atom of the target material X. Let us denote by $\alpha^\pm(\bar{p}, \Delta\bar{p}, X, d_t)$ that fraction of the $\pi^\pm$ with momentum within $\bar{p} \pm \Delta\bar{p}$, which is not absorbed by the target made out of material X, and which has target dimensions symbolized by d_t. Note that α may be larger than one for certain momenta, because pions of high momentum may be slowed down by the target and transformed into low momentum pions. We will denote the target volume by V_t. Finally, let us denote by $\beta^\pm(\bar{p}, \Delta\bar{p}, \bar{r}, \Delta\bar{r}, t, \bar{H})$, that fraction of the $\pi^\pm$ leaving the target with momentum within $\bar{p} \pm \Delta\bar{p}$, which can be found within the volume $\bar{r} \pm \Delta\bar{r}$, at time t, when the field intensity is $\bar{H}$. The $\bar{H}$ is a function of $\bar{r}$, but its value is assumed to be constant over the whole time interval of interest. This time interval is of the order of T, and therefore this assumption is justified. Again, β need not be less

† In the degenerate case when $\bar{P}$ is parallel to the axis of the interaction region and $\chi = 0$ or 180°, then two different scattering events may have the same $\bar{P}$, E_{cms}, and χ. These two events can be distinguished, if one also specified, e.g., E^+, the laboratory energy of the π^+. However, this variable is not essential for our discussions here, and we will not write it out.

than one. From the above discussion it follows that

$$N^{\pm}(\bar{p}, \Delta\bar{p}, \bar{r}, \Delta\bar{r}, t) = n^{\pm}(\bar{p}, \Delta\bar{p}, E_0, X)\, V_t\, \alpha^{\pm}(\bar{p}, \Delta\bar{p}, X, d_t) \cdot \beta^{\pm}(\bar{p}, \Delta\bar{p}, \bar{r}, \Delta\bar{r}, t, \bar{H}). \quad (3)$$

We denote the density of pion pairs with total momentum within $\bar{P} \pm \Delta\bar{P}$, relative center-of-mass energy within the interval $E_{\mathrm{cms}} \pm \Delta E_{\mathrm{cms}}$, and relative momentum in the reaction center of mass forming an angle within $\theta \pm \Delta\theta$ with the axis of the cylindrical interaction region, in the volume $\bar{r} \pm \Delta\bar{r}$ at time t, by $Q(\bar{P}, \Delta\bar{P}, E_{\mathrm{cms}}, \Delta E_{\mathrm{cms}}, \theta, \Delta\theta, \bar{r}, \Delta\bar{r}, t)$. It is clear that Q is some function of $N^{\pm}$ only:

$$Q(\bar{P}, \Delta\bar{P}, E_{\mathrm{cms}}, \Delta E_{\mathrm{cms}}, \theta, \Delta\theta, \bar{r}, \Delta\bar{r}, t) = f[N^{\pm}(\bar{p}, \Delta\bar{p}, \bar{r}, \Delta\bar{r}, t)], \quad (4)$$

and that the total (i.e. integrated over all angles) cross section of reaction $k(k = 1, 2)$ for center-of-mass energy within $E_{\mathrm{cms}} \pm \Delta E_{\mathrm{cms}}$, is

$$\sigma_k(E_{\mathrm{cms}}) = \frac{\int d\chi\, M_k(\bar{P}, \Delta\bar{P}, E_{\mathrm{cms}}, \Delta E_{\mathrm{cms}}, \chi, \Delta\chi, \bar{r}, \Delta\bar{r}, t)}{\int d\theta\, Q(\bar{P}, \Delta\bar{P}, E_{\mathrm{cms}}, \Delta E_{\mathrm{cms}}, \theta, \Delta\theta, \bar{r}, \Delta\bar{r}, t)(v \pm \Delta v)}, \quad (5)$$

where v is the relative velocity of the two pions. For all relativistic pions $v \approx 1$, and $\Delta v \approx 0$.

Note that Eq. (5) is valid for all $\bar{P} \pm \Delta\bar{P}$, $\bar{r} \pm \Delta\bar{r}$, and t, and can be so applied if sufficient number of events are available. Alternatively, Eq. (5) can be integrated over some or all of $\bar{P}$, $\bar{r}$ and t. By integrating over $\bar{r}$, one can determine $\sigma_k(E_{\mathrm{cms}})$ without determining the approximate location of each observed pion-pion scattering event, except for purposes of establishing coincidence between the outgoing particles.

(B) One can measure the energy dependence of the cross section even without measuring the energies of the outgoing particles in reactions (1) or (2). This method will be described next. This method and method (A) described above, are not mutually exclusive. They can be used simultaneously to improve resolution.

To begin with, rearrange Eq. (5), integrate over $\bar{P}$ and E_{cms}, and, to avoid unessential complications, assume that $v \approx 1$, $\Delta v \approx 0$. (These assumptions are well satisfied in most cases of practical interest). We obtain

$$\int d\bar{P} \int dE_{\mathrm{cms}}\, \sigma_k(E_{\mathrm{cms}}) \cdot \int d\theta\, Q(\bar{P}, \Delta\bar{P}, E_{\mathrm{cms}}, \Delta E_{\mathrm{cms}}, \theta, \Delta\theta, \bar{r}, \Delta\bar{r}, t) = \int d\bar{P} \int dE_{\mathrm{cms}} \int d\chi\, M_k(\bar{P}, \Delta\bar{P}, E_{\mathrm{cms}}, \Delta E_{\mathrm{cms}}, \chi, \Delta\chi, \bar{r}, \Delta\bar{r}, t). \quad (6)$$

Next, observe that Q will change if $N_k^{\pm}$ is changed. On the other hand, $N_k^{\pm}$ can be varied in several ways: One may change $n_k^{\pm}$ for a given $\bar{p}$, $\Delta\bar{p}$ by varying the primary beam energy E_0, or target material X, one may change V_t and $\alpha^{\pm}$ by changing the target dimensions d_t, and $\beta^{\pm}$ can be modified by changing $\bar{H}$ or d_t. In practice, a change of target material is difficult, since few materials can fulfill the requirements which have to be satisfied when large number of pions are produced in short bursts. It is also difficult to induce effective changes by varying the target dimensions. On the other hand, E_0 and $\bar{H}$ can be changed easily, and, therefore, in the following we will vary only these two parameters. Furthermore, for simplicity, we assume that the variation of $\bar{H}$ consists of multiplying it everywhere by a constant (independent of $\bar{r}$), which can be achieved by simply increasing or decreasing the current in the coils of the miniring. When E_0 is varied, then the spectrum of the produced $\pi^{\pm}$ will change; when $\bar{H}$ is, e.g., increased, then the miniring will contain pions with higher transverse (to $\bar{H}$) momenta. Under small such variations of E_0 and H, the variation of Eq. (6) will be

$$\int d\bar{P} \int dE_{\mathrm{cms}}\, \sigma_k(E_{\mathrm{cms}}) \cdot \delta \int d\theta\, Q(\bar{P}, \Delta\bar{P}, E_{\mathrm{cms}}, \Delta E_{\mathrm{cms}}, \theta, \Delta\theta, \bar{r}, \Delta\bar{r}, t) = \int d\bar{P} \int dE_{\mathrm{cms}}\, \delta \int d\chi\, M_k(\bar{P}, \Delta\bar{P}, E_{\mathrm{cms}}, \Delta E_{\mathrm{cms}}, \chi, \Delta\chi, \bar{r}, \Delta\bar{r}, t). \quad (7)$$

One can obtain a whole set of equations of this type, by choosing various variations δ_l for E_0 and $\bar{H}$. Let us denote by S the number of such equations which we wish to use. Each of these equations can be approximated by writing summation (over $\bar{P}$ and E_{cms}) instead of integration.

$$\sum_i \sigma_k(E^i_{\mathrm{cms}})\delta_l\, \bar{Q}(E^i_{\mathrm{cms}}, \bar{r}, \Delta\bar{r}, t) = \sum_i \delta_l\, \bar{M}_k(E^i_{\mathrm{cms}}, \bar{r}, \Delta\bar{r}, t),$$

where

$$\bar{Q}(E_{\mathrm{cms}}, \ldots) = \int d\bar{P} \int d\theta\, Q(\bar{P}, E_{\mathrm{cms}}, \theta, \ldots) \quad (8)$$

and

$$\bar{M}_k(E_{\mathrm{cms}}, \ldots) = \int d\bar{P} \int d\chi\, M_k(\bar{P}, E_{\mathrm{cms}}, \chi, \ldots).$$

Let us denote by S' the total number of $\sigma_k(E^i_{\mathrm{cms}})$ which appear in this set of equations. The $\delta_l M_k$ can be measured, the $\delta_l \bar{Q}$ can be calculated, and the set of numbers $\sigma_k(E^i_{\mathrm{cms}})$ can then, in general, be obtained by solving Eqs. (8), provided that $S' \leqslant S$. Just as for Eq. (5), it is true that Eqs. (8) are valid for all $\bar{r}$, $\Delta\bar{r}$ and t, but may be integrated over r or t or both.

Angular resolution

The miniring is so constructed that at the two ends of the interaction region, most pions are reflected many times by the magnetic barrier, before they are able to leave the region. Each time a pulse of pions reaches the magnetic barrier, the number of pions is reduced only by a small fraction. Therefore (see Fig. 1), the number of pions moving from left to right in the interaction region, is approximately the same as the number of pions moving from right to left (except during the short time interval which immediately follows the entrance of the first pions into the region). Consequently, the function Q satisfies, to a good approximation,

$$Q(\bar{P}, \Delta\bar{P}, E_{\text{cms}}, \Delta E_{\text{cms}}, \theta, \Delta\theta, \bar{r}, \Delta\bar{r}, t) = Q(-\bar{P}, -\Delta\bar{P}, E_{\text{cms}}, \Delta E_{\text{cms}}, \theta+180°, \Delta\theta, \bar{r}, \Delta\bar{r}, t), \tag{9}$$

which expresses symmetry of Q under rotation by 180° around an axis perpendicular to the axis of the cylindrical interaction region.

Keeping in mind Eq. (9), we will at first turn to the discussion of reaction (1), and later we will discuss reaction (2). Isotopic spin conservation will be assumed.

Reaction (1) can proceed only if the isotopic spin of the final (also initial) two pions is even, and, therefore, if their relative angular momentum, L, is even, so that the scattering amplitude will be an even function of ξ, the center of mass scattering angle:

$$A_1(E_{\text{cms}}, \xi) = A_1(E_{\text{cms}}, \xi+180°) \tag{10}$$

From Eq. (9) it follows, that M_1 has the approximate symmetry

$$M_1(\bar{P}, \Delta\bar{P}, E_{\text{cms}}, \Delta E_{\text{cms}}, \chi, \Delta\chi, \ldots) \approx M_1(-\bar{P}, -\Delta\bar{P}, E_{\text{cms}}, \Delta E_{\text{cms}}, \chi+180°, \Delta\chi, \ldots) \tag{11}$$

The measured quantity M_1 is given by

$$M_1(\bar{P}, \Delta\bar{P}, E_{\text{cms}}, \Delta E_{\text{cms}}, \chi, \Delta\chi, \bar{r}, \Delta\bar{r}, t) = \int d\xi \int d\theta\, \sigma_1(E_{\text{cms}}, \xi) K(\bar{P}, E_{\text{cms}}, \chi, \xi, \theta) \cdot Q(\bar{P}, \Delta\bar{P}, F_{\text{cms}}, \Delta E_{\text{cms}}, \theta, \Delta\theta, \bar{r}, \Delta\bar{r}, t), \tag{12}$$

where the function K is a kinematical factor, and determines the probability with which a π^+ is scattered in a direction which forms an angle χ with the axis of the interaction region. In particular, for a given set of $\bar{P}$, E_{cms}, θ and χ, the K is nonzero at most for one particular value of ξ (except in degenerate cases where two values of ξ may contribute to M_1 with the same θ). Therefore, the double integral can be replaced by a single integral over ξ, and that can be approximated by a sum. One can write down Eq. (12) for S different values of ξ (all for the same value of E_{cms}), and obtain in this manner a set of S equations. Let us denote by S' the total number of $\sigma(E_{\text{cms}}, \xi^i)$ which appear in this set of equations. One can, in general, find $\sigma(E_{\text{cms}}, \xi^i)$ for S' different values of ξ^i from these equations, provided that S is chosen so that $S \geqslant S'$. However, even if $S \geqslant S'$, the determinant of the set of equations may, in special cases, be equal to zero. In these special cases the dependence of σ on ξ can not be determined by this method. Such would be the case if Q were a constant as a function of $\bar{P}$ and θ, but this can be avoided according to Eqs. (3) and (4) by varying, if necessary, X, d_t, E_0 or $\bar{H}$. When E_{cms} is so low that only the state with $L = 0$ contributes significantly to the scattering, then, of course, σ is independent of ξ, M is independent of χ, and the angular dependence measurement is trivial. Again we note that Eqs. (18) are valid for any $\bar{P}$, $\bar{r}$, and t, but may be integrated over some or all of these three variables.

Reaction (2) can take place both when the isotopic spin of the two final (also initial) pions is even and when it is odd. Consequently, this elastic scattering can occur both when L, the relative orbital momentum of the scattered pions, is even and when it is odd. Therefore, one expects in general, that the scattering amplitudes $A_1(E_{\text{cms}}, \xi)$ and $A_2(E_{\text{cms}}, \xi+180°)$ are not equal. Nevertheless, it follows from Eq. (9) that Eq. (11) holds for M_2 also, not only for M_1. Furthermore, M_2 and σ_2 satisfy an equation like Eq. (12), and this equation can be obtained by simply substituting subscript 2 for subscript 1 everywhere in Eq. (12).

One may now think that $\sigma_2(E_{\text{cms}}, \xi)$ can be measured by writing down Eq. (12) for various values of χ, and then solving this set of equations as described earlier. However, this is not so. In fact, the set of equations so obtained for the unknown σ_2 will have a determinant equal to zero to the same approximation as Eq. (9) is true. The reason for this is easy to see: the backward forward asymmetry in θ will not be measurable in this way, because the measured M_2 will always satisfy the 'backwards-forwards symmetry' (11) to the same approximation as Q satisfied Eq. (9). Using this method one can measure the quantity

$$\bar{\sigma}_2(E_{\text{cms}}, \xi) \equiv \tfrac{1}{2}[\sigma_2(E_{\text{cms}}, \xi) + \sigma_2(E_{\text{cms}}, \xi+180°)] \tag{13}$$

but not $\sigma_2(E_{\text{cms}}, \xi)$ and $\sigma_2(E_{\text{cms}}, \xi+180°)$ separately.

On the other hand, if additional information is available, then one may proceed further, and determine $\sigma_2(E_{\rm cms}, \xi)$. For example, let us assume, that we already know the pion-pion scattering amplitude for isotopic spin 0 and 2, e.g. as a result of previous measurements of reaction (1) and elastic $\pi^+ - \pi^+$ scattering. Let us write the amplitude for reaction (2) as a sum of two terms

$$A_2 = A_2{}^s + A_2{}^a, \tag{14}$$

where the symmetric and antisymmetric parts satisfy

$$A_2{}^{\overset{s}{a}}(E_{\rm cms}, \xi) = \pm A_2{}^{\overset{s}{a}}(E_{\rm cms}, \xi + 180°). \tag{15}$$

In terms of these amplitudes, the measured quantity $\bar{\sigma}$ can be written (we suppress the argument $E_{\rm cms}$) as

$$\bar{\sigma}_2(\xi) \sim |A_2{}^s(\xi) + A_2{}^a(\xi)|^2 + |A_2{}^s(\xi) - A_2{}^a(\xi)|^2. \tag{16}$$

Using isotopic spin invariance, we can calculate $A_2{}^s$, and measuring $\sigma_2(\xi)$ for a large enough number of ξ values, one can determine, in general, from the resulting set of Eqs. (16), any finite set of phase shifts which contribute significantly to $A_2{}^a$. Of course, for certain special Q functions (such as when Q is independent of $\bar{P}$ and θ), the determinant of this set of equations vanishes, and then $\sigma(\xi)$ can not be determined. Equations (16) are valid for any $\bar{P}$, r and t, and may be integrated over some or all of these variables.

We conclude with some remarks concerning experimental errors. The quantities M_k or their integrals over any or all of $\bar{P}$, r and t are measured directly. Their relative statistical error decreases as (number of events)$^{-1/2}$, as usual. On the other hand, according to some of the methods described above, the differential cross section $\sigma(E_{\rm cms}, \xi)$ is obtained from these quantities by solving sets of equations. The solution involves sometimes the taking of differences between measured quantities. Whenever a difference is taken, the relative error of the resultant quantity is, generally speaking, larger than that of the directly measured quantities. Therefore, when solving systems of equations, one should, as a general rule, try to minimize the number of times differences are taken. How often the taking of differences is unavoidable, depends on the structure of the set of equations under discussion. In particular, it may happen that the equations are such that by using colliding beams of large Δp and $\Delta\Omega$, not only the directly measured quantities, but also the differential quantities, can be determined faster and with greater statistical relative accuracy than when beams with the customary small Δp and $\Delta\Omega$ are used. On the other hand, in other cases, the structure of the equations may be such that the statistical relative accuracy of the differential quantities is not smaller than would be obtained by the usual method. It is clear that the use of small Δp and $\Delta\Omega$ in colliding beam experiments need not be the best method to perform measurements with good resolution. On the other hand, using beams with large Δp and $\Delta\Omega$, as in experiments in a miniring, we may be able not only to measure heretofore unobservable total cross sections, but also differential cross sections.

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REFERENCES

1. Paul L. Csonka, Report CERN-TH 836 (1967).
2. Paul L. Csonka, Yellow report CERN 67–30 (1967).
3. Paul L. Csonka and Andrew M. Sessler, Report CERN-ISR-TH/67–29 (1967).
4. Paul L. Csonka, Phys. Letters **25B,** 423 (1967).
5. Paul L. Csonka, Nuclear Instruments and Methods (to be published).
6. Paul L. Csonka, Physics Letters **24B,** 625 (1967) and Yellow report CERN 67–15 (1967).
7. R. Macek and B. Maglic, Particle Accelerators, **1,** 121 (1970).

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