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RESEARCH ON THE ELECTRON RING ACCELERATOR†‡

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The desirable properties for a source of relativistic electrons suitable for ring formation are discussed. The properties of the ring in the so-called 'compressor' equipment used to shrink the ring are examined and compared with experiment. Problems with betatron resonance effects are treated, including possible strategies for circumventing the problems. Finally there is a brief description of the design of a high energy proton accelerator using current technological methods.

1. INTRODUCTION

The basic concept of a collective-effect accelerator involves the use of dense charge-clusters to produce fields larger than those normally available externally and thereby to accelerate beams of protons or other ions. This is in contrast to the conventional 'single-particle' accelerator in which the self-fields of the circulating particles usually constitute a minor perturbation in comparison with the applied electric and magnetic fields. Discussion of the possibility and advantage of using the self-fields of intense bunches of electrons dates back almost two decades,(1–4) and experiments to form and transport such clusters were performed by Alfvén and Wernholm(2) as early as 1952. If the self-fields are large enough to trap the ions (assumed small in number), the cluster can be accelerated and its gain in velocity is determined by the electron mass, thus the ions will gain energy at a rate higher by the ratio of (ion mass/electron mass). The central problem is to form a cluster with a high electron-density that is also stable. Harvie(1) referred briefly to the possibility that a toroidal configuration might be desirable, but the major impetus to concentrate on the electron ring as a suitable vehicle was provided by the extensive theoretical and experimental studies carried out by Veksler *et al.*(5) One of their suggestions was that by using relativistic electrons with, say, $\gamma = 40$, the repulsive space charge force could be reduced by a large factor, $(1-\beta^2) = 1/\gamma^2 \approx 1/1600$, and that the residual effect could be overcome by the addition of only a small fraction of ions, $N_\mathrm{i} \geqslant N_\mathrm{e}/\gamma^2$, thus creating a self-stable ion-focussed ring. In this case, the energy-gain factor for the ions contains the transverse mass, $m\gamma$, of the electrons; thus, for protons this factor is $M/m\gamma \approx 46$ for $\gamma = 40$. The choice of a suitable γ for the electrons in the ring will therefore involve a trade-off between the ease of stabilizing the ring and the lesser energy gain due to the use of collective fields.

As an example of parameters in the range of interest, consider an electron-ring with uniform charge density in its minor cross-section. If the number of electrons is N_e, the major radius R, and the minor semi-axes a (radially) and b (axially), then the peak electric holding field is

$$E_\mathrm{peak} = \frac{2e\,N_\mathrm{e}}{\pi R(a+b)} \tag{1}$$

For $N_\mathrm{e} = 10^{13}$, $R = 3.5$ cm, $a = b = 1$ mm we have $E_\mathrm{peak} = 131$ MV/m. In order to have a non-zero phase-space volume or bucket for the protons, acceleration should utilize a smaller field, $E_a < E_\mathrm{peak}$. If E_a is taken to be 100 MV/m, then the corresponding effective field that need be applied externally to the ring is only 2.2 MV/m.

Axial acceleration of the ring is effected in general by electric and magnetic fields. If we use z for the axial coordinate, the axial component of force on the electrons is $F_z = -e(E_z - B_r/c)$, where v is the transverse velocity of the electrons and is close to c. There are two cases that are easily understood and achievable geometrically. First, there is the case of electric field acceleration in a uniform field; here $B_r = 0$ and $F_z = -eE_z$. In the example given, if $E_z = 2.2$ MV/m the protons gain energy at a rate of 100 MV/m. Second is the case of magnetic field acceleration where† $E_z = 0$ and $B_r > 0$. If the field is supplied by a simple tapered solenoid the ring will expand in radius as it undergoes axial acceleration. A more general case discussed by

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† This article is intended primarily to cover ERA research in this country; developments by the Dubna Group will be treated elsewhere in this journal.

‡ Work done under the auspices of the U.S. Atomic Energy Commission.

† We use the sense of rotation of the electrons and the z-axis to define a right-handed coordinate system.

Veksler *et al.*(5) involves the situation where B_r alternately changes sign depending on whether the ring is in an electric-field-gap or in a drift space. When $|E_z| > 0$, $B_r < 0$ and when $E_z = 0$, $B_r > 0$. Thus some acceleration is taking place at all times and the process can be thereby smoothed. It is difficult, however, to arrive at a simple and economic design of magnet coils that will allow this to be exactly accomplished.

One attractive feature of the electron ring as a vehicle for proton acceleration is the high degree of order it possesses; this allows one to envisage and calculate with some confidence the type of accelerating structure needed to carry it to very high energies. Ion-acceleration from very intense electron beams (10^5 amps) travelling through gases has recently been observed,(6) but how these phenomena can be utilized in a controlled way to lead to a high-energy accelerator with reasonable properties is very difficult to see at this time within the range of present technology. It seems possible on the other hand, to design an electron ring accelerator with reasonable repetition rate (10–1000 pps) and intensity (10^{12}–10^{14} protons/sec). In addition there are other obvious applications, for example, the acceleration of heavy ions for nuclear chemistry ($\approx$ 10 MeV/nucleon) and biomedicine ($\approx$ 500 MeV/nucleon).

Following the report by Veksler *et al.*(5) presented at the Cambridge Accelerator Conference in September 1967, a major theoretical and experimental effort was begun by the Accelerator Study Group at LRL, Berkeley. A useful source-book of the early work in this country is the Proceedings of the ERA Symposium held at Berkeley in February 1968.(7) With the cooperation of members of the Astron CTR Group at LRL, Livermore, who have made time available at their unique 4MeV high-current linear induction accelerator, it has been possible for the Berkeley Accelerator Study Group to carry out a number of significant experiments oriented towards the development of an electron ring accelerator for high energy physics.(8,9) The first sequence on ring-forming and ion-loading was performed in late 1968 with the so-called Compressor II equipment, and the second sequence on ion-focussing and extraction with Compressor III is in progress (late 1969). Other groups in this country have mainly contributed in the region of theory, e.g. a study of static-field compression by workers at Maryland, and static-field compression and acceleration by Christofilos at LRL, Livermore. In addition, two groups in Western Europe, at Karlsruhe and Munich, have experimental programs on electron ring accelerator research.

2. FORMATION OF THE ELECTRON RING

The range of parameters most desirable for suitable rings has been the subject of much discussion (cf. Ref. 7, *ibid.*) but generally they are of the order of those given as an example in the last section. In what follows we will continue to use those numbers as representative, and later return to values that should be achieved in the future.

(a) *The Electron Source*

Using the typical ring parameters mentioned in the last section it can be seen that in principle the ring could be formed by direct injection into a guide field with a 2000 ampere one-nanosecond pulse from a 20 MeV injector. While this would pose a difficult inflection problem, in practice it is not relevant since an injector with such properties and suitable repetition rate does not exist at present. Instead, the approach used is to inject electrons at a few MeV into a time-varying field established by pulsed air-core coils whereupon the electron energy is raised by induction acceleration (analogous to a betatron). Unlike a betatron, the 2:1 condition is not adhered to and by suitable choice of field-shape versus time, both the major and minor dimensions of the ring contract leading to an enhancement of holding-power (cf. Eq. (1)). This function has led to the terminology 'compressor', to describe the ring-forming apparatus, in this country. This is not altogether apt, as the situation described corresponds to a *combined*-function induction accelerator and compressor. Another situation will be discussed later where it is desired to *separate* these functions into a high-energy induction accelerator (either circular or linear) followed by a static-field compressor whose sole purpose is to reduce dimensions without energy change.

An essential starting point is the high-intensity electron accelerator in the few-MeV range. There are several commercial machines with currents in the range 10^3–10^5 amperes, that are used to produce 'flash' X-rays, generally using cold cathodes. Although the German experimental groups have begun to use such machines in initial experiments, their properties are far from ideal for accelerator applications or research, notably in repetition rate and frequency of maintenance. For accelerator research one needs a repetition rate in the neighborhood of 1 Hz and a pulse number of the order of

10^5 between shutdowns for component replacement. The most promising approach is to use the linear induction technology for achieving high currents that has been pioneered by Christofilos and his co-workers.(10) The basic accelerating unit consists of a toroid of magnetic material surrounding the beam. A current pulse applied to a loop around the toroid accelerates the beam which acts as the secondary of a transformer. Energy is stored in high-voltage cables and is switched to the core-primaries by thyratrons. Following the construction of the first version of the Astron electron accelerator, a rather similar model was constructed in the USSR for the exclusive purpose of electron-ring research. Because of great pressures of the CTR and other programs on the Astron accelerator, the electron-ring experiments have necessarily had to be brief and infrequent, and the detailed systematic studies of various features of ring formation, stability, ion-loading, etc., must await the completion of the electron induction-accelerator at present under construction at Berkeley. This accelerator employs the essential features of induction acceleration but is scaled down in repetition rate and pulse-length to values that are more appropriate for electron-ring research. Typical operating parameters for the Livermore and Dubna accelerators, and design parameters for the Berkeley accelerator are compared in Table I.

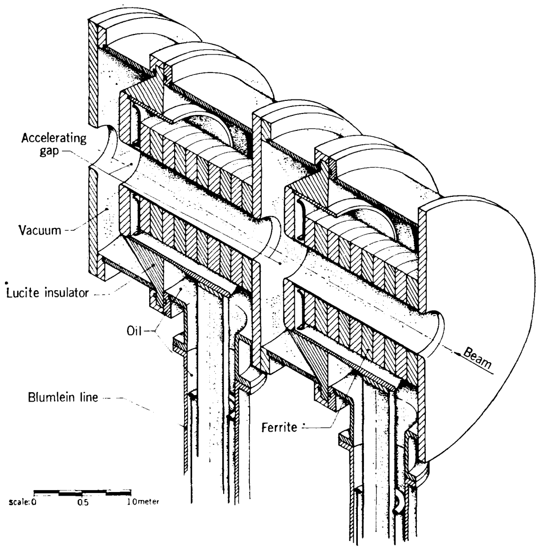
The much shorter pulse-length of the new injector allows several simplifications. Ferrite is used in place of iron tape for the core material, the voltage rating of the gap can be greatly increased, and the modulator is small enough to be mounted directly below the gap. The modulator is an oil-filled Blumlein line, with an electrical length of 35 ns and physical length of just over 12 ft. The high-voltage switch is a spark gap mounted as an integral part of the Blumlein line.

Figure 1 shows, in cutaway form, the way in which two accelerating units can be stacked in series with each other. Focussing along the length of the accelerator is provided by periodically-placed solenoids. The present plan calls for a gun section of 1.25 MeV obtained by stacking five cavities closely in series to excite a single high voltage terminal. Further increments in energy are provided by individual or paired cavities with solenoids in between. Initially, efforts will be made to develop a field-emission cold-cathode source of suitable brightness, but if this proves to be unsatisfactory the design incorporates provisions for later substitution of a barium oxide or press-sintered lanthanum boride hot cathode. Installation and testing of the accelerator is scheduled to take place during spring and early summer 1970, following which a sequence of detailed experiments with electron rings will begin.

TABLE I

Parameters for electron linear induction accelerators

	Astron Accelerator (typical operation) 1969	Dubna accelerator (typical operation) 1969	Berkeley (design)
Energy (MeV)	4.1	1.5	2.25 (4 by late '70)
Repetition rate (Hz)	5–30	1	1–10
Current (amps)	850	200	$\gtrsim 500$
Emittance at full energy in cm-rad	33×10^{-2}	$(1\text{–}3) \times 10^{-2}$	$\leqslant 20 \times 10^{-2}$
Pulse length (ns)	≈ 250	≈ 200	35
Cathode	Barium Oxide (Hot)	Barium Oxide (Hot)	Field Emission (Cold)
Cold material	Nickel-Iron Tape	Permalloy Tape	Ferrite
Voltage per core (kV)	10–12	10	250
Number of cores	≈ 450	≈ 100	9 (later 16)



Electron Induction-gun Ferrite Cavity

FIG. 1. This cut-away drawing shows two pulsed accelerating cavities stacked in series. Each cavity will have a voltage of approximately 250 kV.

(b) *Ring Compression*

Prior to compression a ring of large radius and relatively large betatron amplitude must be formed by proper injection and trapping in the compressor. An injection pipe, or 'snout', is used to conduct the beam from the linear injector into the compressor on a path tangential to the initial closed orbit. Shielding of the linear path from the pulsed field could be accomplished either by magnetic material or by an eddy-current shield of copper. The effect of such materials on the field inside the compressor would be a reduction in the first case and an increase in the second. In the compressor designs at Berkeley a combination has been chosen of copper-plated soft iron to eliminate the field from within the injection snout and to minimize the perturbations outside at the moment of injection. The Dubna group use current-windings on the injection snout to approximate the same conditions. Once the beam is led tangentially into the compressor an inflector, or orbit-contractor, is activated to reduce the closed-orbit radius by a few centimeters in a time of the order of 10^{-8} seconds. The Dubna group inject a single turn at a field condition where the field index, $n = -(R/B_z)(\delta B_z/\delta r)$, is about 0.5. In the Berkeley experiments success has been achieved with radial three-turn injection at an n-value slightly above 0.5, i.e., a radial tune, $\nu_R \approx 0.7$. In this instance the orbit-contractor applied an additional field over an annular region covering about two-thirds of the azimuth so that it corresponded somewhat to a 'beam-bump' effect. The closed-orbit appropriate to the injection energy was allowed to come about two centimeters within the snout radius before the contractor was activated by a high-voltage pulse that rose in 20 ns and fell in the same time. Thus the orbit position was deflected out to the mid-point of the snout, injection begun at that time and as three or four revolution times elapsed, the orbit collapsed back to its initial value. While this sequence of events can be planned on the basis of single-particle theory it rapidly became evident, from experimental evidence, that self-field effects played a large role in the injection process.(8) In fact, rather stable conditions of capture of 4×10^{12} electrons in Compressor II occurred when the voltage of the inflector, or orbit-contractor, was set at zero. Self-inflection was also observed in Compressor III although more intensity could be achieved by using the inflector. Several effects of approximately the right order of magnitude may be at work, e.g. deceleration of the electrons as the magnetic field of the ring becomes established, time-varying electric and magnetic images in the inflector structure, wake-fields from an early uncaptured fraction of the beam, etc. The mechanism of self-trapping is not understood and is an important subject for future experimental study.

Following injection the magnetic field rises as the sets of nested field coils are pulsed sequentially. To be specific, consider the geometry of Compressor III which is the equipment now being operated by the Berkeley group. Figure 2 shows the arrangement of the coils and Fig. 3 the nominal design circuitry. In the course of recent experiments several modifications have been made by adding time-dependent current corrections in certain of the coils but the gross features have remained substantially as shown in Fig. 3. The variation of the major radius of the ring, the energy of the electrons, the magnetic field and field index (at the ring location) with time is shown in Fig. 4. In broad terms, the way in which these quantities vary is not too different from that in the experiments with Compressor II. A photograph of the equipment is shown in Fig. 5.

The change in ring-parameters during the compression phase has been discussed by several authors in Ref. 7 and has been treated in detail by Ivanov *et al.*(11) As pointed out by Laslett(12) some results

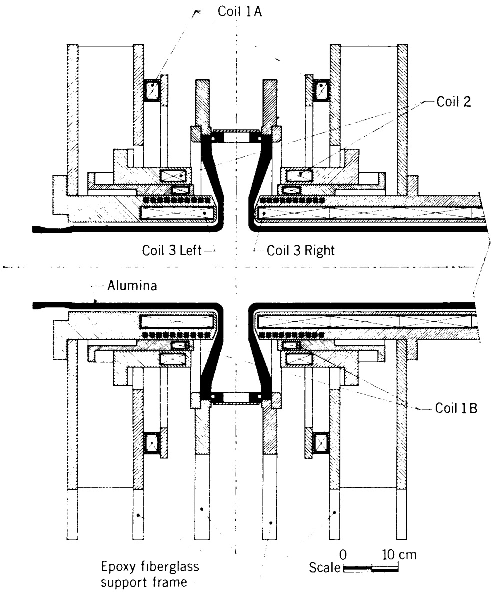


FIG. 2. The coil system and vacuum envelope of Compressor III.

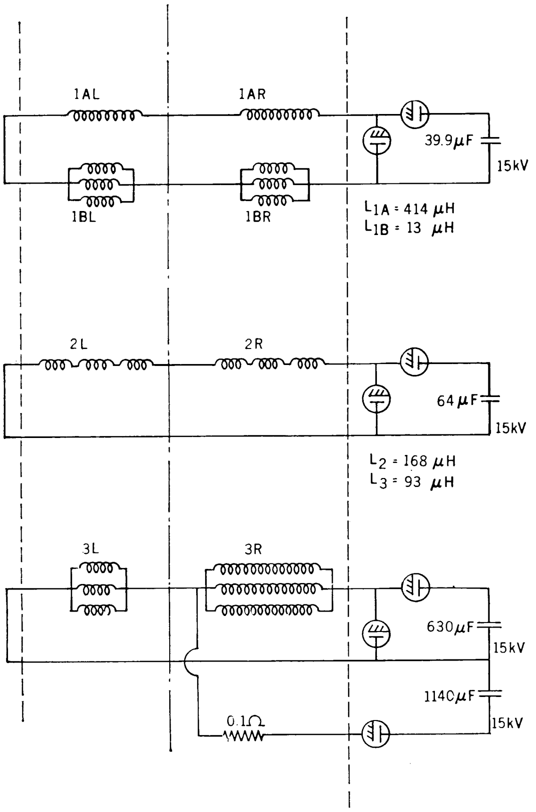


FIG. 3. Schematic of power supplies for pulsing the coils.

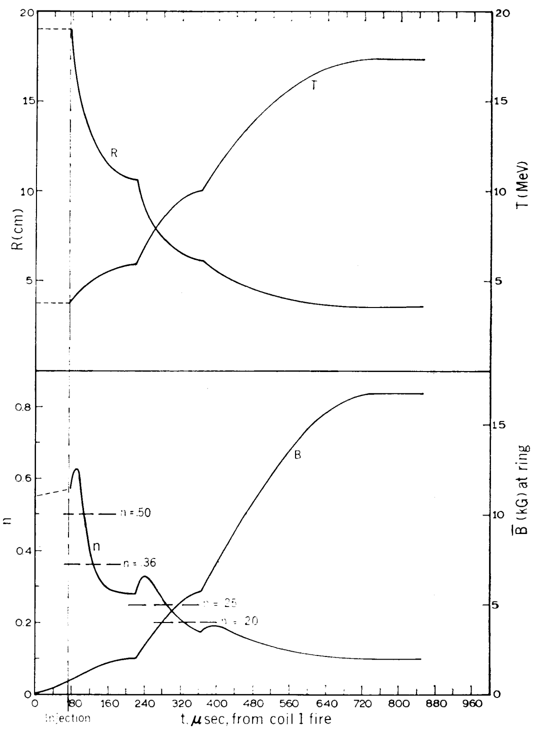


FIG. 4. The calculated variation of major radius, kinetic energy, field and field index with time during compression.

follow very readily from the fact that in the axisymmetric system the canonical angular momentum $(pR - RA)$ is conserved. Here, $p = BR$ is the electron momentum in gauss-centimeter, and the field B and vector potential A are evaluated at the orbit radius at the time that the particle is describing that orbit. Using a and b, as before, to represent the radial and axial extent of the minor-cross section due to betatron oscillations, one finds

$$a \propto B^{-1/2}(1 - n)^{-1/4}, \quad b \propto B^{-1/2}\, n^{-1/4}, \qquad (2)$$

and the variation of the radial spread, ΔR, due to energy spread in the electron beam is given by

$$\Delta R \propto (BR)^{-1}(1 - n)^{-1}. \qquad (3)$$

At least in the experiments with Compressor II it was found that the energy spread was small ($\approx 0.5\%$) and the radial width was determined almost entirely by betatron oscillations. The variation of the major radius, R, and electron momentum, p, depend on the details of the field configuration, but in practice it is found that for placement of the coils in a geometry like that in Fig. 2, and for the case of a reasonable compression

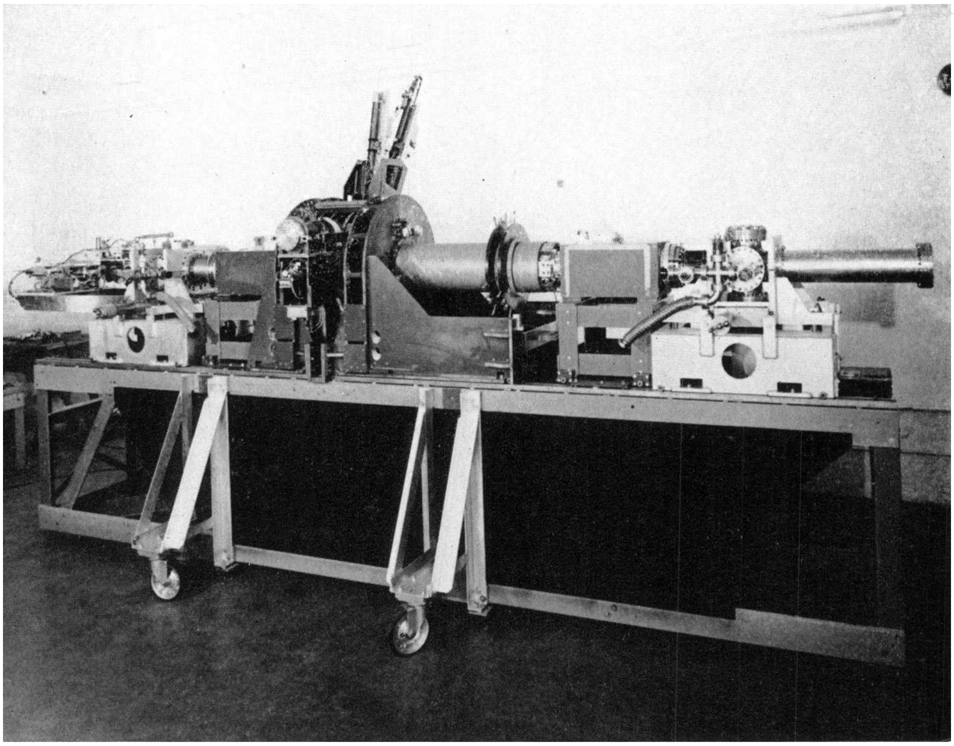


FIG. 5. A photograph of Compressor III in August 1969 before being moved to the Livermore site for experiments at the Astron facility. The coil support frame for the compression coils is clearly distinguishable as well as the actuators for certain of the radial probes. The accelerating solenoid and trim-coils are visible to the right of the compression chamber.

sequence, the following relations are approximately obeyed:

$$p \propto B^{1/2}, \quad R \propto B^{-1/2}, \tag{4}$$

corresponding to conservation of the magnetic moment BR^2.

The equations of motion of particles in the ring under the action of the external guide-field and the space-charge forces have been treated in detail by Ivanov *et al.*(11) and Laslett.(13) Some of their main conclusions can be summarized as follows. The space-charge effects are of two kinds: first, the well-known type where neighboring particles in a segment of the beam have a net repulsive force although the electrostatic term is largely cancelled by the magnetic term, and second, where one segment of the beam is acted upon by all other segments in the ring in which case both electric and magnetic terms are of the same sign. The first effect is present in a beam that is linear, the second is a consequence of its being a closed curve. The outcome is a bias field which produces a correction to the usual relation between momentum, field, and orbit radius, viz.,

$$B = -\frac{p}{eR}(1+\mu P), \tag{5}$$

where (to follow the terminology of Ref. 11):

$$\mu = \left(\frac{e^2}{mc^2\gamma}\right)\frac{N_e}{2\pi R} = \frac{1}{\gamma}. \text{ (Budker Parameter, } \nu\text{)},$$

and

$$P = 2\ln\left(\frac{16R}{a+b}\right).$$

The product μP is usually small compared with unity, and the change in orbit radius is therefore small. However, this 'toroidal' space charge term enters significantly into the expressions for the radial and axial betatron frequencies, ν_R and ν_z. Thus

$$\nu_R^2 = (1-n)(1+\mu P) - \frac{\mu P}{2} - \frac{4\mu}{\beta^2\gamma^2}\frac{R^2}{a(a+b)} \quad \text{(a)}$$

$$\nu_z^2 = n(1+\mu P) - \frac{\mu P}{2} - \frac{4\mu}{\beta^2\gamma^2}\frac{R^2}{b(a+b)} \quad (b) \tag{6}$$

The last term in each instance is the well-known 'linear' space-charge term and leads to *both* radial and axial defocussing. Note however, that in the regime of interest for the compressed ring where n becomes small, the 'toroidal' term leads to radial focussing and axial defocussing.

Experimentally it is of great importance in determining the quality of the rings formed to check on the ring dimensions. A simple but powerful tool used in the experiments with Compressors II and III was a tantalum target or 'flag', that could be placed at a wide variety of radial, azimuthal and axial positions, and which could intercept all or part of the beam. When electrons strike the flag, a burst of X-rays is produced that can easily be picked up by a scintillator and photo-multiplier external to the apparatus. The starting and ending times of the pulse indicate the times of arrival of the largest and smallest betatron amplitudes at the flag, while the pulse magnitude and shape give information about intensity and phase-space distribution. For example, if the flag lies in the median plane so that the ring as it is compressed is totally destroyed, then the plot of pulse end-times versus radius should correspond to the path of R, the closed-orbit radius, with time as shown for instance in Fig. 4. If the experimental points lie to the left of the calculated curve it is a strong indication that the injection closed-orbit is at too small a radius, that no small radial betatron amplitudes are present, or in other words, the beam is 'hollow' in the (x, x') phase-plane,—an undesirable situation leading to reduced holding power. Information about a combination of radial and axial oscillation amplitudes came from axial probing with the tip of the flag to produce partial loss. Another powerful diagnostic technique for studying ring dimensions was the use of a gated image-intensifier camera to take a 'snapshot' of the minor cross-section of the ring, by recording the synchrotron light. Photo-densitometric measurements allow the electron density distribution function in the spatial coordinates x, y, to be deduced.

The observations with Compressor II were all found to be consistent with a rather simple distribution in four-dimensional phase-space, $\exp(-\rho^2/2)$, where

$$\rho^2 = (x^2/a^2 + x'^2/a'^2 + y^2/b^2 + y'^2/b'^2).$$

In particular, the projection of this distribution on the xy, plane is

$$\frac{d^2N}{dx\,dy} = \frac{1}{2\pi ab}\, e^{-(x^2/a^2 + y^2/b^2)/2} \tag{7}$$

in agreement with the synchrotron light data which gave: $a = 2.4$ mm, $b = 1.6$ mm, at the end of compression, where $R = 3.5$ cm.

The most tedious and time-consuming task in arriving at efficient ring-formation is the empirical trimming of the magnetic field in order to cross certain single-particle betatron resonances. To achieve three-turn injection radially it is desirable to inject near $n = 0.5$ where the radial tune $\nu_R \approx 2/3$. At the end of compression it is desirable to have ν_z small, and the n-value close to zero, since after extraction the ring must be launched into a field with $n = 0$; hence it is necessary to cross several resonances between $n = 0.5$ and 0. Also, the amplitude of the circulating beam after injection is a few centimeters, so that even quite small driving terms may cause growth. As can be seen from Fig. 4, the radius of the closed orbit dwells for comparatively long times at certain radii corresponding to the peaks of current in the various coil sets, and it is important that the n-values at these radii be not too close to strong resonances. Of particular concern in the Compressor II experiments were resonances corresponding to $n = 0.5$ ($\nu_R = \nu_z$, coupling type) $n = 0.36$ ($2\nu - \nu_z = 1$, $\nu_R + 2\nu_z = 2$, $3\nu_R + \nu_z = 3$, sum and difference types) and $n = 0.25$ ($\nu_z = 1/2$, axial type).

By using a low current, small-emittance beam to avoid masking phenomena by high-current collective effects it is possible to diagnose which of the single-particle resonances are troublesome. The technique for eliminating their deleterious effects was to introduce small time-dependent current programs in certain coils with the aim either of postponing the resonance-crossing until as late as possible when the betatron amplitude has been substantially damped, or of crossing the resonance quickly. This was successful in Compressor II. The coil configuration for Compressor III was carefully designed to avoid the known troublesome n-values and to allow the addition of current-correction circuits. Nevertheless beam loss occurred at $n = 0.36$ and $n = 0.25$, and the loss at the former was difficult to eliminate completely.

3. ION-LOADING, EXTRACTION, AND ACCELERATION OF THE ELECTRON RING

The ring-acceleration process can be considered to begin when the ring has been loaded with ions to provide self-focussing, and positioned in a magnetic solenoid, which itself provides no axial focussing. A small positive radial field component then causes axial acceleration of the ring. Between its formation and launching, however, comes the important stage of loading the ring with ions and reducing the field gradient to zero at the appropriate place for launching, whereupon $n = 0$ and the resonances $\nu_R = 1$ and $\nu_z = 0$ are encountered. While the accelerating solenoid does not provide focussing it is possible that at the launching point and beyond, other sources of focussing may be present e.g. dielectrics or metal screens to provide images, an axial wire carrying current, ions, etc.$^{(14,15)}$

Extraction of the ring in the Compressor III experiments is accomplished as follows. At the end of compression when the fields in the coils have stopped rising, the ring has a major radius of 3.6 cm and its median plane (defined by $B_r = 0$, $(\partial B_r/\partial z) < 0$), is almost exactly centered in the left-right direction in the vacuum chamber. An unbalancing circuit is triggered to increase the current in Coil 3L and decrease the current in Coil 3R (see Fig. 6). At low current this moves the closed orbit ($B_r = 0$) continuously to the right and later, when the unbalancing current has decayed, it will move back to the center again. However, above a critical current level a stage is reached when, at a value of $z_0 \approx 12$ cm, both B_r and $\partial B_r/\partial z$ simultaneously vanish. At this time $n = -(R/B)(\partial B_z/\partial r) = -(R/B)(\partial B_r/\partial z) = 0$, the magnetic axial focussing is removed and later when $B_r > 0$ for $z > z_0$ the ring is accelerated to the right. While this condition for removal of the right-hand mirror is easy to achieve it becomes a matter of very careful adjustment of the residual currents in all the coils to ensure that beyond the release point B_r is kept at a small, rather constant value of the order of 10 gauss. If $|B_r|$ is too large the acceleration is so abrupt that ions will not be retained in the ring.

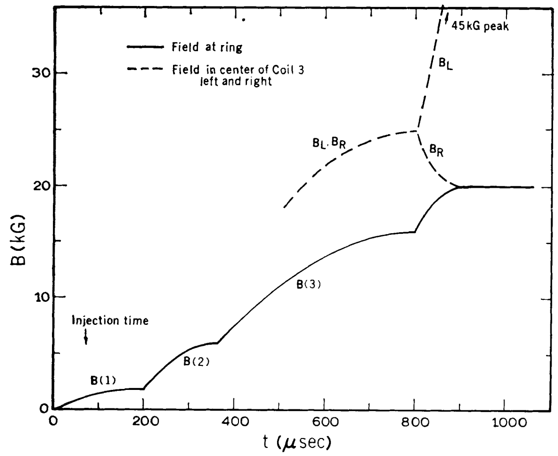


FIG. 6. The variation of magnetic field at the ring radius versus time during compression and extraction. The fields in the left-hand and right-hand solenoids are also shown to indicate the large changes needed to effect extraction.

Ions are added to the ring at or near the end of compression by actuating a gas valve that can provide a burst of gas molecules near the ring in a pulse some few hundred microseconds wide. The number of molecules ionized and trapped in the ring can be varied by adjusting the firing-time of the valve and the pressure of gas in the plenum. In the absence of other focussing methods after the release-point, the ion-loading must provide focussing sufficient to overcome the axial defocussing described by Eq. (6b) with $n = 0$.

In practice to date, the experiments addressed to extraction studies have used image focussing in addition and this must be included, along with the ions, in extending Eq. (6). The Dubna group used a set of copper strips to form a 'squirrel-cage' outside the ring as it was extracted axially. In Compressor III a central dielectric image cylinder consisting of titanium dioxide in a thin quartz envelope is mounted inside the ring. In addition, the outside surface of the quartz is plated with longitudinal copper stripes. The purpose of both of these devices is similar—to provide neighboring material in which the electric image of the ring is fully developed, but the magnetic current image is suppressed. The slotted metal surface cannot support azimuthal currents and acts in this case like a material with high dielectric constant. Laslett$^{(13)}$ has written the expressions for the radial and axial tunes for the case with ions and images as follows:

$$\begin{aligned}
\nu_R{}^2 = 1 - n - \mu \Bigg\{ & \frac{4R^2}{a(a+b)} \left(\frac{1}{\gamma^2} - f\right) - (1-f)\frac{P}{2} + \\
& + 4\left[\frac{(1-f)\,\epsilon_{1E}}{(S_E-1)^2} - \beta^2 \frac{\epsilon_{1M}}{(S_M-1)^2}\right] \\
& + n[(1-f/2)P + (1-f)K - \beta^2 L] \Bigg\} \\
\nu_z{}^2 = n + \Bigg\{ & -\frac{4R^2}{b(a+b)} \left(\frac{1}{\gamma^2} - f\right) - (1-f)\frac{P}{2} + \\
& + 4\left[\frac{(1-f)\,\epsilon_{1M}}{(S_E-1)^2} - \beta^2 \frac{\epsilon_{1M}}{(S_M-1)^2}\right] \\
& + n[(1-f/2)P + (1-f)K - \beta^2 L] \Bigg\} \qquad (8)
\end{aligned}$$

where f = fractional ion-charge $= N_i Z_i/N_e$ ($= N_P/N_e$ for protons); and S_E, S_M measure the distance to the electric or magnetic surface in units of R.

The image coefficients ϵ_{1M}, ϵ_{1E}, and the terms K and L are discussed by Laslett(15); in the present instance where the magnetic images are absent:

$$\epsilon_{1M} = 0, L = 0, \epsilon_{1E} \approx 1/8, K \approx \frac{1}{S_E - 1}.$$

The effect of the presence of ions is to provide both axial and radial focussing, the effect of the electric images is to provide axial focussing and radial defocussing. Table II summarizes the sign of the contribution to the betatron frequencies when n is small.

TABLE II

Origin of Effect	Contribution to betatron tune ν_z	ν_R
Magnetic field gradient	+	−
'Linear' space charge	−	−
Trapped ions	+	+
'Toroidal' space charge	−	+
Electric images in cylinder	+	−

It is clear that enough ions must be added to ensure axial stability, i.e. to overcome both space-charge terms, after the ring has begun to accelerate and is away from the region of the electric images. How many ions may be added beyond this lower limit and how the balance among the remaining contributions should be adjusted is much less clear, and hinges on the dangers to be encountered near the radial integral resonance, $\nu_R = 1$. As n approaches zero and the ring nears the release point, generally the incoherent (ion and space charge terms) resonance, $\nu_R^{\text{inc}} = 1$, is reached first and later the coherent integral resonance. Theoretical reasons suggest that one can cross $\nu_R = 1$ (incoherent) unless the spread in betatron frequencies is large.(16,17) One strategy for crossing $\nu_R^{\text{inc}} = 1$ would be to omit the image cylinder and rapidly to pulse a subsidiary magnetic field coil so that the resonance is traversed in nanosecond times. The alternative under consideration here is to remain below $\nu_R^{\text{inc}} = 1$ because of images,(18) (see Fig. 7); later, when acceleration has been begun and the ring is moving axially with relativistic speed the image cylinder can be abruptly terminated.

It was stated earlier that at the point where the ring is released for acceleration the conditions $B_r = 0$, $\partial B_r/\partial z = 0$ must be satisfied. There are further conditions required as pointed out by Pellegrini and Sessler, in particular on other derivatives, $\partial B_r/\partial r$ and $\partial^2 B_r/\partial z^2$, near the release point.(19)

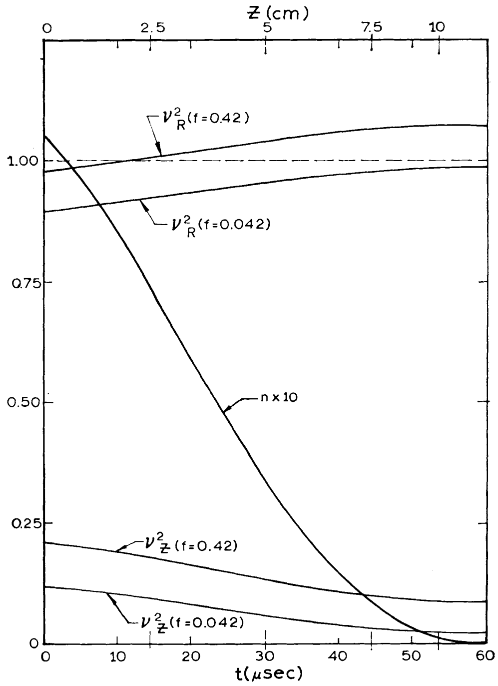


FIG. 7. The variation of the radial and axial betatron tunes (squared) as a function of time after activation of the extraction circuit. For moderate ion-loading (4.2%), in the presence of image focussing both the resonances $\nu_R = 1$ and $\nu_z = 0$ can be avoided.

Unless the self-focussing forces are strong enough the radial variation of B_r, acting differently on electrons with different energies, and the axial variation of $\partial B_r/\partial z$, acting differently on electrons of different amplitudes, can tear the ring apart. In this case the compact ring will not survive, the individual electrons, beginning with those of largest amplitudes, will be successively 'peeled off' on a slow time scale, and accelerated axially. Under conditions of field adjustment known to be poor, slow spilling of the ring consistent with these effects has been observed in Compressor III.(9)

Axial acceleration of the electron ring after release is designed to take place inside a one-meter long solenoid in the Compressor III equipment. By adjusting the residual currents in the compression coils and the currents in trim coils near the end of the solenoid a 'good-field' acceleration length of about 75 cm can be achieved. In a lightly-loaded ring with a holding field, E_{peak} (MV/m), the transverse magnetic field, B_r (gauss), must satisfy

$$B_r < \gamma E_{\text{peak}}/55, \tag{9}$$

otherwise no protons will be retained. For rings with a holding power 12 MV/m, created in Compressor II, one finds that B_r should be less than 10 gauss, which is quite demanding on the precision of coil fabrication and current adjustment. With a significant amount of ion-loading a higher transverse field is needed to achieve the same acceleration.

In the Dubna experiment reported recently by Sarantsev(20) extraction (i.e. cancellation of the mirror on one side) and acceleration were achieved by judicious placement of a sequence of unpowered coils which were shorted before the main field had risen to its maximum value. By observation of the reaction $N_7 + Ce_{58} \to Tb_{65}$ the Soviet group was able to establish the acceleration of nitrogen ions to an energy of $\gtrsim$ 4 MeV/nucleon, and inferred a peak holding power for their rings of 80 MV/m.

For applications in nuclear chemistry where ion-energies of the order of 10 MeV/nucleon are required, acceleration by a radial component of magnetic field in a solenoid about a meter in length seems quite adequate. This application for heavy ions has been discussed in more detail by Lewis(21) and Iovnovich *et al.*(22) The configuration appropriate for a high-energy proton accelerator will be discussed later and in that case it is probable that the initial magnetic acceleration will be used mainly as an injection stage into an electric-field acceleration column.

A variation of the scheme of magnetic acceleration referred to above is discussed by Furth and Rosenbluth,(23) and has many attractive features. They have pointed out that a solenoid placed inside the ring allows one to control independently the flux within the ring and the field at the orbit, freeing one of the constraint of conservation of magnetic moment. Thus, the radius of the ring can be held constant—or even decreased—while acceleration proceeds. If electron rings with high transverse energy ($\approx$ 500 MeV) could be created and high-field nested solenoids constructed with the ring travelling along in the intervening space—then proton energies in the range 10–100 GeV could be envisaged by magnetic acceleration alone.

4. STATIC FIELD COMPRESSORS

In Section 2 attention was drawn to the fact that the 'compressor' system common to the Dubna and Berkeley experiments performed a dual role, viz. acceleration of the electrons azimuthally, and compression of the major and minor dimensions leading to a higher holding power (Eq. (1)). There are considerable advantages to be gained by separation of these functions, and the concept of a device to compress the dimensions of the ring without adding azimuthal energy has been independently discussed by many authors.(24–30) Several proposals(24,26,28,29) involve injection into a mirror-type field, i.e. one in which B_z increases with z. If injection is at an angle to the axis and without a radial component of velocity the orbit is a conical helix of decreasing pitch and accumulate in a dense ring-configuration with small radius at a suitably high field-point along the solenoid. Having reached its maximum compression the ring would tend to expand again because of the absence of stabilizing longitudinal forces, unless a local trapping field is switched on at that point. Variants of this technique have been described by Berg *et al.*(26,29) and Hultschig.(22) The requirement that the ring be compact axially imposes very stringent requirements for small emittance and energy-spread of the injector. The Maryland group(26,29,30) have suggested several ingenious variations and improvements in this concept. They point out that if ions are created at a suitable time and density the axial spreading of the beam due to emittance and energy spread will be inhibited by Bennett pinching and as the pitch of the helix becomes small successive turns will be attracted to each other by mutual magnetic forces. In order to achieve high current it would be desirable to inject at several azimuths and a novel way of achieving this without a proliferation of linear injectors has been suggested by Nelson and Kim.(30) The electron beam is produced from an annular cathode of large radius to produce a cylindrical hollow beam. Sets of solenoid coils are placed both inside and outside the cathode, the inner one being rather short axially. As the electron stream leaves the region of the inner coils a portion of the axial velocity is converted to rotational velocity at a constant radius. Then the cylindrically rotating stream can be compressed into a ring by entering an increasing solenoid field, as before.

Christofilos(25) has proposed a system in which the axial stability is ensured by maintaining an axial focussing field, i.e. $\partial B_z/\partial r < 0$, over a single compression stage. By employing an inner cantilevered solenoid to avoid conservation of magnetic moment, the ring can be accelerated at constant radius during which the field at the orbit is reduced. Following the acceleration at constant radius the ring is then allowed to undergo adiabatic compression in an increasing field until it stops. In the overall process the major radius is compressed in the ratio of (initial field/final field) and the minor radius by the square root of this ratio. Several successive stages each of this type are envisaged to be placed in series, to obtain compression to very small dimensions. The transition between stages may be non-adiabatic and the behavior there has not yet been completely studied.

In the static field compressor discussed by Laslett and Sessler(27) no axial acceleration or deceleration is envisaged and hence there is no great sensitivity to initial energy spread. They have shown that with a suitable initial energy configuration of tapered solenoids nested one within the other, an axial magnetic field of zero gradient and varying inversely with radius, can be produced. Focussing is provided by an external winding to ensure the axial stability of the ring. This weak magnetic well can be made to propagate axially(31) thus causing the ring to move continually into the stronger field region and compress in radius. They quote an example in which the ring is compressed from $R = 57$ cm to $R = 7.3$ cm and demonstrate that the coil-currents required and the properties of the helical winding to supply the moving well are quite reasonable.

Implicit in the design of all static-field compressors is the availability of a high energy electron source ($\approx 20-30$ MeV). It is sometimes stated that the static field compressor eliminates the problem of providing pulsed magnetic fields. The problem is not eliminated but transferred into the injection accelerator where, however, more efficient solutions may be provided. Such an injector could be a linear induction accelerator of the type discussed earlier (See 2) extended in length by a factor of five or more, or it could be a high current betatron. The pulsed compressors at present used for electron ring experiments are, in essence, variable-orbit betatrons, but it is likely that the smaller volume of pulsed field needed for a constant-radius betatron injector would give greater efficiency and a higher repetition rate. Certainly a high energy electron source based upon an extension in energy of the type under construction at Berkeley imposes no new technological demands at least up to a repetition rate of 1000 Hz.

It should be noticed that the static-field compressors mentioned above fall into two classes: those where the electrons are considered to be already formed into a ring to begin with and those where the ring formation and compression take place together. The latter category of mirror-field compressors, if they were to work, might offer the additional advantage of relieving the injector of the requirement of very high output currents, since they should be capable of accepting a pulse-length very much greater than the few tens of nanoseconds required for the first category.

5. FUTURE INVESTIGATIONS

Of major importance to the future utility of electron ring accelerators is a continuing program to strive for rings of higher holding-power. At present, rings with holding fields in the range of tens of megavolts-per-meter have been achieved, enough to establish this type of collective accelerator as an extremely attractive device. However, a careful experimental program of studying injection and understanding collective instabilities and their relation to the geometrical and electrical environment near injection, is needed to find ways of achieving higher ring-currents. Discussions of many of the electron collective effects and ion-electron interactions are to be found in Ref. 7. A brief summary by Sessler(32) of the major limiting effects indicates that the number of electrons per ring should be capable of increase by one to two orders of magnitude. An electron source of high brilliance is needed to achieve compact rings in the region of $N_e \approx 10^{14}$. In this way the holding power should certainly be increased to several hundred MV/m and perhaps to 1–2 GV/m.

Parallel with these studies on stacking more intense rings is the need to study the behavior of electric-field acceleration. Magnetic field acceleration alone is capable of giving proton energies up to some tens of GeV and is certainly adequate for producing heavy ions for the needs of nuclear chemistry and biomedicine. But, for a high energy proton accelerator of 100 GeV or greater, electric acceleration is necessary. Both of these topics will be actively pursued at Berkeley upon completion of the electron accelerator.

At the time of the Berkeley Symposium(7) several conceptual designs for a high-energy proton accelerator using electron-rings were developed, and a variety of operating parameters were discussed. Further experiments, particularly on the topics just mentioned, are needed to give guidance on the most desirable and realistic parameters. There has been some further evolution in thought on the mechanical and electrical aspects of the accelerator design. In Ref. 7 an efficient method of induction acceleration of the ring was proposed (Pulsed-Line Acceleration(33)), but was recognized to involve a development program to learn how to fire a very large number of spark-gaps in sequence with jitter-times of the order of a few nanoseconds. This may indeed prove appealing in the future depending on the outcome of experiments with laser-triggered gaps. In the meantime development of ferrite-loaded cavities for the electron source (cf. Sec. 2) has suggested that a similar approach could be used for an accelerating column for the ring. The introduction of ferrite means that the space-factor is worsened leading to a significantly reduced average longitudinal electric field. It has however, several advantages: relaxation of the tolerance for jitter to a comfortable point, reduction in the number of spark gaps per cavity from four or five down to one, use of a technique upon which the development work is essentially complete, and a laterally more compact structure. Brief examination of mechanical and electrical requirements indicate that a column with an average field of several MV/m is quite feasible.(34)

Attention has also been paid to the practicality of using both electric and magnetic accelerating sections in series. For example, to obtain protons of energy E, one could accelerate electrically to energy $E/2$ and then enter a solenoid with decreasing axial field allowing the ring radius to expand and the longitudinal energy to increase. If the axial velocity of the ring entering the magnetic accelerating section is $\beta_L = (1-1/\gamma_L^2)^{1/2}$, the initial transverse electron energy described by γ, as before, and the final values on emergence denoted by β_L', γ_L', γ' after the ring has expanded, we have from conservation of energy in the static field:

$$N_P M_P \gamma_L + N_e m_e \gamma \gamma_L = N_P M_P \gamma_L' + N_e m_e \gamma' \gamma_L',$$

or

$$\frac{\gamma_L'}{\gamma_L} = \frac{\gamma + N_P M_P / N_e m_e}{\gamma' + N_P M_P / N_e m_e} \tag{10}$$

$$= \frac{\gamma + 18}{\gamma' + 18} \text{ for } 1\% \text{ loading with protons}$$

If the ring were allowed to expand by a factor of three in radius, say from $\gamma = 40$ to $\gamma' = 13$, then the protons will gain a factor of two in energy. This method of combining electric and magnetic field acceleration may be most suitable when rings of relatively modest intensity are used, for then γ must be chosen rather large to ensure that the holding power (proportional approximately to γ^2 for ring formation in the pulsed compressor) is adequate to retain the protons during electric acceleration. If very intense rings were available it would be preferable to place a magnetic accelerating solenoid upstream rather than downstream of the electric accelerator.

Apart from the lower cost per unit proton energy, it is desirable to exploit magnetic acceleration to the maximum to avoid the substantial energy loss by 'cavity-radiation' that will occur when the ring passes through the periodic structure of an electric column. The magnitude of this effect and its dependence on γ_L have been the subject of a remarkable variety of studies and have led to an almost equal variety of results. A detailed discussion and comparison of the methods of handling the theoretical calculation has been given by Sessler. The original formula derived by Kolpakov and Kotov(36) for the energy loss of a charge Q in passing through a hole, of radius R, in a cavity of length G, was

$$\Delta E = -\frac{Q^2 G}{2R^2} \tag{11}$$

While there is still lack of agreement among those working on this subject, a current estimate is that the loss shown in Eq. (11), should be multiplied by a factor $(1+\frac{4}{3}G/R)$ which typically is about 2 or 3 for geometries under consideration. At this time the cavity radiation loss is believed to be an annoyance but far from limiting for the electron ring accelerator.

6. CONCLUSION

As regards the immediate future of the Berkeley program, experiments will continue with Compressor III to study and define the complicated interaction of variables needed to achieve efficient injection and successful extraction of a compact ring. It is expected that the new electron accelerator will be ready by late spring 1970 to carry out further experiments.

The Berkeley program on ERA, under the overall direction of E. J. Lofgren, is an effort in which many people are deeply involved. Among the principal members of the accelerator study group responsible for the work summarized here are:

Physics Aspects: W. W. Chupp, A. Garren, G. R. Lambertson, L. J. Laslett, A. U. Luccio, A. Nakach, C. Pellegrini, W. A. Perkins, J. M. Peterson, J. B. Rechen, A. M. Sessler.

Electrical Aspects: A. Faltens, E. C. Hartwig, C. D. Pike.

Mechanical Aspects: R. T. Avery, H. P. Hernandez, J. R. Meneghetti, W. W. Salsig.

The experimental observations with Compressors

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