RESISTIVITY INTERPRETATION

IN

GEOPHYSICAL PROSPECTING

by

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S. B. , Massachusetts Institute of Technology

(1955)

SUBMITTED IN PARTIAL FULFILLMENT

OF THE REQUIREMENTS OF THE

DEGREE OF DOCTOR OF

PHILOSOPHY

FOR

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

June, 1959

Signature of Author . . . .
Department of Geology and Geophysics, May 18, 1959.

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Submitted to the Department of Geology and Geophysics on May 18, 1959, in partial fulfillment of the requirements for the degree of Doctor of Philosophy.

ABSTRACT

The direct interpretation of electrical resistivity measurements in applied geophysics requires the solution of an inverse boundary value problem. Although an exact solution for the vertical variation of specific resistivity with depth has been known for some time, no exact method exists for the cases of two and three dimensional variation of resistivity.

The earth can be considered as subdivided into a number of small homogeneous regions. A first approximation to the exact forward solution allows the compositing of a number of these regions and the superposition of their effects. The direct interpretation of resistivity data is then accomplished by a least squares fitting of the effects of the regions to the observed field data. By a proper choice of these sub - surface regions, two and three dimensional variations of resistivity can be represented and the field data interpreted on this basis.

It has been necessary to modify the first approximation of Stevenson (1934) in order that symmetry of source and receiver be maintained. The modified Eltran array, essentially a dipole-dipole electrode configuration, forms the basis for an application of the interpretation scheme developed. However, there is no fundamental reason why any other array cannot be used in conjunction with this approach.

A number of practical direct interpretation operators have been developed and tested on model, theoretical and field results of apparent resistivity surveys. The method is quite successful in the majority of the cases. It is capable of resolution of resistivity data on approximately the same scale that the measurements represent.

ACKNOWLEDGEMENTS

The author acknowledges the continued interest and counsel of Prof. T.R. Madden during the period when this research was conducted. Several fruitful discussions have been held with Prof. George Backus of the MIT Mathematics Department on the inverse boundary value problems in resistivity interpretation.

During the school years 1956-1957, 1957-1958 the IBM Corporation supported the author at the MIT Computation Center. The numerical work reported on in this thesis has been done at the MIT Computation Center. The author has been a Gulf Oil Fellow during this last year 1958-1959. He expresses his appreciation to these two industrial organizations for their sponsorship. In addition the staff of the MIT Computation Center has provided valuable assistance in expediting the completion of this work.

Finally he gratefully acknowledges the efforts of his wife Amelia who has performed all of the secretarial work involved in this thesis.

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1.0 Introduction and Objectives

Geophysical prospecting represents the attempt of man to make intelligent decisions about the physical character of his environment. The environment is subjected to certain excitations, its response measured and predictions made on the basis of how it has responded.

In quantizing this process man uses his knowledge of many different physical forces and fields and interprets the response in terms of the physical laws which his environment must obey. This thesis is concerned with the apparent resistivity method of prospecting which measures the response of the earth to a source of electrical current.

Other investigators have considered this problem but have interpreted the response on the basis that the only variation of electrical parameters influencing the current flow occurs in the vertical direction. The work discussed represents the extension to interpret electrical responses in terms of two and three dimensional variations of parameters.

The method of Stevenson in which the potential is expanded in a series is the basis for the research carried out in this thesis. The work has only been possible because of the existing electronic computers now available for scientific calculation. In application the interpretation scheme developed is most efficient when used in conjunction with such computers although it is not necessary in order to achieve an application of the method developed.

1.1 Definition of Apparent Resistivity Prospecting

The resistivity method of geophysical exploration is based upon the measurement of electrical fields which are conductively generated within the earth by means of grounded electrodes. By assuming the earth to be homogeneous and isotropic in its electrical characteristics it is possible to determine the resistivity necessary to produce the observed fields from the known sources. It is this effective physical parameter which is referred to as the apparent resistivity of the earth. Passing current through two electrodes into the earth and observing the voltage between two others leads to the calculation of the mutual resistance of these two circuits (source and receiver). The geometrical value which is required to transform this resistance to the apparent resistivity of the earth can be computed from a knowledge of the relative distances of the four electrodes.

There is no unique arrangement of electrodes in prospecting applications and indeed the current may be introduced into the earth by means of long cables grounded their entire length rather than with point electrodes. It is also possible to qualitatively investigate the resistivity by observing the equipotential surfaces associated with the current flow. In each modification of the method a systematic distribution of source and receiver circuits within an area can lead to an evaluation of the subsurface variation of electrical resistivity and the possible geological structure and material inferred. It is only through a combined use of geological and geophysical data that an intelligent prediction about the details of the subsurface can be made.

1.2 Geological Considerations

The specific resistance or resistivity of matter on a microscopic scale is defined from the equation $R = \rho L/A$ where R is the resistance measured across a sample of cross-sectional area A, length L and ρ is the resistivity in units of ohms-length. Many samples of minerals and rocks show anisotropic properties in their electrical parameters and in some of the more refined methods of electrical well surveying in regions with parallel boundaries between formations it is necessary to consider this possibility. In general the small scale of the inhomogeneities and the random orientations of anisotropic minerals combine to yield an approximate isotropic resistivity.

The range of resistivities measured in geological materials is tremendous, being over many orders of magnitude------ samples of minerals with metallic luster such as the sulphide of lead will have a value as low as 10^{-6} ohm-meters while some igneous and sedimentary rocks may present values as high as 10^{8} ohm-meters. However, for material in situ the influence of the electrolytic solutions and fluids filling the always present pore spaces, fractures and shears tends to dominate the resistivity pattern and it may be stated that for very low frequency electrical current flow the main transport of current is by ionic solutions. The range of resistivities observed in the field is thus much less than might be anticipated from laboratory measurements but still is over a wide enough range, several orders of magnitude -- 1 to 10^{4} ohm-meters, to be useful in prospecting applications.

It is possible to infer the structural relationships existing in the subsurface from the variations in the apparent resistivity measured in an areal survey, on the assumption that the material causing the variations of electrical fields is directly associated with the structure. In the majority of cases this will certainly be true but it must not always be deduced that electrical variations are only associated with structural variations. Since the measurements are quantitative at each geometrical configuration not only the structure may be inferred from the variations but also the possible material from the absolute values of the apparent resistivity and any deduced specific resistivities. This last remark points out the basic aim of all geophysical prospecting systems: the detection of regions of anomalous physical parameters and the interpretation of the field data in terms of location, size and possible composition of such regions. Since current flow is strongly influenced by ionic solutions the resistivity of different geological materials depdnds largely upon the relative amount of void space present in each sample. With particular reference to mining geophysical applications, mineralization often occurs in regions where the fracturing or shear is much greater than in the surrounding material and thus the detection of the zone in these cases is direct in electrical parameters but indirect with regards the mineralization.

1.3 Field Procedures for Resistivity Measurements

As previously indicated, the apparent resistivity method requires the measurement of a voltage existing across two electrodes which are in contact with the earth. Because of the electrochemical reactions which take place between an ionic solution and an electronic conductor a resulting potential difference is measured between them. Therefore if two metallic electrodes are inserted into the ground they will not correctly measure the electrical fields within the earth. This difficulty may be overcome by the use of non-polarizable electrodes made of porous clay pots into which is placed a saturated solution of a metallic salt in equilibrium with a metallic electrode of the same element, commonly copper culphate and a copper rod. This reduces most of the ectraneous contact potential although there may be a small potential difference between the solution within the pot and the electrolyte within the ground. Another source of error in resistivity measurements is due to naturally occuring earth currents which arise from chemical potential gradients within the earth. There currents are at times used as indications of the electrical and geological character of the subsurface in the self-potential or spontaneous potential method of prospecting.

In any event, these influences must either be determined by measurement and their effects subtracted from the artificially induced fields observing the correct polarity or eliminated from the actual measurement. This can be done by inserting an opposing voltage of the same magnitude in series with the measured voltage. In addition the impedance of the voltage sensing device must be high enough so as not to alter the current flow within the earth or the characteristics of the device be modified by the contact resistance of the porous pots.

Methods have also been developed to utilize the naturally occuring fields of alternating currents in the telluric methods and the recently applied magnetotelluric techniques.

These require respectively the simultaneous observation of the horizontal electric field at two different locations or the horizontal magnetic and electrical fields at one station. These fields may well be due to extraterrestrial causes but their origin is only now being investigated.

It is not necessary to take similar precautions with the source electrodes although it may be necessary to add more ground contacts in the immediate vicinity of the source points and to 'salt' the area with a solution of NaCl or other similarly highly dissociated electrolyte. This is required in order that sufficiently low contact resistance for the power source allow adequate current flow to be established. The calculation of apparent resistivity, ρ_A, will not be seriously in error if the distances between the electrodes is much larger than the distances between the multiple ground contacts at each source position.

1.4 Historical Summary of Resistivity Measurements

Historically, the electrical methods of prospecting in one form or another are amongst the oldest of applied geophysical techniques. However, a lack of adequate quantitative treatment of the basic phenomena involved in the early work and even up to the present has hindered the proper development of some of these methods and may in some cases have had negative effects on their acceptance and utility in the profession.

In the electrical resistivity methods the first definitive work was performed by Frederick Wenner in 1915 in which he explicitly showed that the apparent resistivity of the subsoil could be measured by a four electrode system placed on the surface. His analytical treatment derived the necessary geometrical factor to transform the observed mutual resistance to ρ_A and he indicated the scale of the sample of earth material measured by such a system. The electrode arrangement proposed used equally spaced intervals between the four co-linear electrodes with the current being applied to the extreme ground contacts while the voltage was observed between the interior two. By varying the spacing interval the size of the sampled earth region varied and it is this electrode configuration, the Wenner array, which has been widely used in its original form since then. There have been attempts made to utilize and introduce other arrangements or to permute the electrode connections but no other system has received as much consideration nor achieved as similar a success until quite recently.

It was not until 1923 that actual field tests were begun to test the theory and system proposed by Wenner and in 1925 these results were published by Gish and Rooney in connection with the growing interest of the Carnegie Institution's Department of Terrestrial Magnetism in earth current phenomena. These two investigators made a series of measurements at various localities throughout the world and compared the results with geological information to ascertain the value of the method in estimating the resistivity structure of large masses of the subsoil. The problems of polarization of receiver electrodes and naturally occurring earth currents were overcome by simultaneously reversing the voltage leads and the source leads with a hand operated double commutator which operated at approximately 30 cps. Thus the voltage observed was independent of the self-potentials within the earth and the polarization potentials were never allowed to build up due to the rapid reversal of current flow.

The field results of Gish and Rooney showed the usefulness of apparent resistivity as an exploration technique and since then many investigators have used this system of resistivity prospecting with a good degree of success. As might well be anticipated the evaluation of the method depends most heavily on the interpretation of the data obtained and its subsequent confirmation by geological examination. A great deal of effort on the part of many investigators during the 10 years following this original field work improved on the initial system and advances were made in the interpretational techniques used to predict the subsurface structure. Since 1935 articles have appeared from time to time regarding the resistivity method but the basic steps were well defined during this first 10 year period. Applications of this method have not been limited to geological exploration for economic minerals or petroleum and extensive work in civil engineering problems of building foundations and highway construction has been done. The objective there is essentially the same as that of geophysical exploration: the determination of the subsurface structure from surface measurements of the electrical field.

1.5 The Indirect Method of Interpretation

Existing methods of interpretation of apparent resistivity field data to yield subsurface variations have had one point in common: all assumed that the only variation of resistivity was vertical, and that the field data represented the response of such an earth to whatever array was used. Since this assumption was very limiting in its validity, modifications were made which allowed for horizontal variations by correlating the vertical interpretations at different locations within an area and forming a composite model of the subsurface. However this is an approximation useful only when vertical variations are rather similar at all the locations or in other words when the actual variation departs little from being truly vertical over the entire area.

The initial attempt at interpretation of resistivity data was made by Gish and Rooney in the series of field experiments already mentioned. Without analytically justifying their mathod it was applied to their field data with a great degree of success, although it is impossible to reconstruct the detailed steps involved in reaching their final results. Graphically plotting the apparent resistivity measured as a function of the spacing interval and qualitatively observing that interval at which the character of the resulting curve changed, the depth to a horizontal interface separating two different media was obtained. Thus an exact correlation of electrode spacing to depth variation was inferred and this idea has been widely used in field operations since then.

This 'break point' method of interpretation is strictly an empirically motivated and derived approach that is in reality a very qualitative approximation to an interpretive scheme since it places a great deal of responsibility on the part of the individual doing the interpretation. However, there have been recent extensions and modifications to this method by Moore (1945), whose use of 'cumulative resistivity' as an interpretational basis has been subject to much critical review. An integration of the curve is effected by summing the apparent resistivity of all the preceeding intervals as the electrode spacing is increased in equal intervals. Although Ruedy (1945) has attempted to analytically justify the interpretation by 'break points' of this integrated data when graphically presented it is not clear from his work that any improvement over the original Gish-Rooney concept is made. Published field results have not been capable of duplicate interpretations by other investigators.

In 1930 Tagg and Lancaster-Jones criticized the interpretation method of Gish-Rooney and Tagg made a contribution to a more quantitative approach in presenting theoretical curves of a Wenner array for the response of an earth of two horizontal layers, or two vertical layers. These theoretical curves were employed to deduce from field data the depth to an interface separating two media, whose specific resistivities were also determined. This was accomplished by graphically plotting the theoretical variation of depth versus electrical resistivity contrast factor ($k = \frac{\rho_1 - \rho_2}{\rho_1 + \rho_2}$) which each data value could represent, since any value for a given spacing interval could be due to an infinite number of different two layer earth models. The upper layer's resistivity ρ_1 was found from the limiting value of apparent resistivity for small electrode spacing. Combining the plots for the different field stations an intersection of all such curves at a certain depth and k value, hence ρ_2, led to the solution.

While this method is as accurate as the field data for a real two layer earth, any departures from a perfect single intersection of curves would yield a range of two layer models which would approximately satisfy the observed field data. It would remain for the interpretor to deduce whether the data was in error or the earth was not an ideal two layer configuration. However, this method is a rapid technique for determining a two layer equivalent to field data and represents the first analytically correct interpretation scheme within the limiting assumptions outlined. Tagg has since this original work modified it slightly in order to eliminate the need for knowing ρ_1 with the use of dimensionless parameters and has also attempted to apply it to more than two layers. Other authors have published articles essentially using some further modification of Tagg's method as well as the theoretical solution of problems involving more than two layers.

The next original approach to interpretation was made by Irwin Roman in 1931 with a technique for graphically comparing theoretically derived curves representing apparent resistivity versus the spacing interval with field curves. The plots for two horizontal layers were made on log-log scales and the theoretical curves were plotted in dimensionless variables so that by properly superposing the field curve with a theoretical curve the correct depth and contrast factor could be obtained directly. This approach is not only rapid but accurate, and required little training on the part of the interpretor. There is however, the problem that the real geometry may not be that assumed in the derivation of the theoretical curves, and in these cases the slight mis-match existing between the two curves might be attributable to data errors and/or variation in the horizontal formations rather than as the number of layers involved. Experience on the part of the interpretor in these situations would again be relied upon to resolve the difficulty. Since this work was done families of theoretical responses and curves for two, three or four layers have been added but the basic principle of coincidence of curves is still utilized.

In 1940 Rosenzweig proposed a method of parametric curve interpretation in which a master plot would be made for all solutions of a specified geometry. He used dimensionless parameters in the form $\rho_A(na)/\rho_A(a)$ evaluated from the field data at predetermined relative locations. Using two of these parameters for different values of n as coordinates, the master plot would allow a direct reading to be made of the corresponding depth and contrast values from the family of intersecting curves representing the theoretical solutions for these particular parameters. The success of this method, which is ideally simple in its applications, has not been great since the resolving power of the master plots is poor. Since the field data slways is in error, a range of solutions would be possible and for the difficulty alluded to above these would vary over a wide limit. Moreover unless this process were repeated for a number of values of the interpretor could never be certain that the real geometry was simply a two layer, and that the best solution had been obtained.

These represent the main methods of indirect interpretation in resistivity prospecting since they require the knowledge of theoretical solutions to assumed geometries. As indicated other workers have modified these basic approaches but the principles remained the same. The method of superposition introduced by Roman appears to be the most useful developed although it requires the possision of a set of theoretical curves for many geometries and contrasts, theoretically an infinite number. However, by judicious choice of these values it is possible to retain only a small number of solutions and interpolate between these when the field curve does not coincide exactly. Again the experience of the observer is required to decide upon the correct solution in such cases.

1.6 The Direct Methods of Interpretation

The direct methods of interpretation would not require the use of extensive tables or curves of theoretical solutions, instead by operating in some specific manner on the field data the exact solution to the resistivity variation with depth would be obtained. The use of direct methods however implies a knowledge of the theoretical solutions for the apparent resistivity as a function of all the parameters involved, and then an inversion of this solution to obtain these parameters as a function of the apparent resistivity measured. The functional obtained from this inverse operation however, is not simply related to the depth and resistivity parameters and some method must be utilized to abstract these values from the functional.

There are two methods to solve the physical problem of predicting apparent resistivity for a given vertical or horizontal variation of specific resistivity. One is limited to direct current flow in either horizontal or vertical homogeneous, isotropic layers and corresponds to the familiar image theory of electrostatics. The other is more general and is useful for either a discrete number of layers of a continuous variation of resistivity which may be anisotropic and is the integral formulation using eigen-functions convenient for the geometry. During the early work in resistivity prospecting much effort was devoted to obtaining numerical solutions by these two methods. It is possible to show that if an image theory solution exists it can always be obtained formally from the integral derivation and vice versa.

The potential measured on the surface of an infinite halfspace for a point source with only vertical variations of specific resistivity can be represented generally as:

$$\varphi(r) = c\int_0^\infty K(\lambda, x_i)\, J_0(\lambda r)\, d\lambda \qquad 1.6.1$$

where φ is the potential

K the kernal function

J_o the ordinary Bessel function of zeroth order

r distance between source and receiver

c constant

x_i the physical parameters

As already indicated the potential linearly determines the apparent resistivity. The kernal function K is related to the resistivity variation in different manners, dependent upon whether the variation is discrete or continuous. Utilizing this integral formulation and the requisite inversion formulae, King (1933) and Slichter (1933) independently presented a method for interpretation which was semi-direct since although it rigorously derived the kernal function from the field data the final scheme was comparison of known kernal functions for certain geometries. Langer (1933) treated the problem also and solved the Sturm-Liouville differential equation which related the resistivity variation to the kernal function and Slichter applied this development to solve the problem of direct interpretation.

King recognized the necessity of measuring the electrical field in all practical applications and hence used a forward solution which predicted the surface potential gradient due to a point source:

$$\frac{\partial \varphi}{\partial r} = -c \int_0^{\infty} \lambda K(\lambda, x_i) J_1(\lambda r) d\lambda \qquad 1.6.2$$

where J_1 is the Bessel function of order 1.

Use of the Hankel Fourier-Bessel inversion formula for this equation led to the kernal function which he referred to as the characteristic function of the system.

$$K(\lambda, x_i) = \frac{-1}{c} \int_0^{\infty} r \left(\frac{\partial \varphi}{\partial r}\right) J_1(\lambda r) dr \qquad 1.6.3$$

By assuming different variations of resistivity, he suggested obtaining a family of forward solutions with which comparison of field derived data could be made.

The work of Slichter was based upon the point potential receiver and Eq. 1.6.1 represents the forward solution which was also inverted by Hankel Inversion theory to yield the kernal function as:

$$K(\lambda, x_i) = \frac{1}{c} \int_0^{\infty} \lambda r \, \varphi(r) \, J_0(\lambda r) dr \qquad 1.6.4$$

Expansions of the logarithmic derivative of the conductivity in a Taylor's series and substitution in the differential equation following Langer resulted in an algebraic expression relating the variation of conductivity with depth as a function of the derived kernal. Slichter presented various forward solutions and their associated kernal functions for graphical comparison with field derived kernals. Until the very recent development of high speed electronic computers, the amount of numerical computations involved in applying this rigorously developed interpretation scheme precluded its frequent application. There are problems regarding the resolving power of the actual field data for certain geometries and conductivities but a more fundamental difficulty is that use of the Taylor series expansion is valid only for continuous variations of resistivity and hence discrete layering could only be approximated. Langer extended his original work so as to include one vertical discontinuity in the resistivity but the amount of computation involved was even greater.

Vozoff (1959) used a high speed computer to overcome the numerical hardships in applying Slichter's method of kernal comparison for discrete layering. He employed a trial solution which was obtained by comparing the derived kernal function with certain theoretical solutions and fixing the number of layers involved. Various measures of the fit of the derived kernal and the theoretical kernal were used in modifying the values of depths and resistivities until the variations in these parameters were less than the accuracy of the field data would indicate. It is to be noted that since field data is taken in a four electrode arrangement only for a discrete number of data points, approximations regarding the behavior of the apparent resistivity, and the derived point potential and point source must be made. The resolution problem in the case of thin conducting or resistive layers was demonstrated in his work and the analytical treatment showed clearly why this would be anticipated.

Shortly after Slichter and Langer's work Stevenson (1934) published a paper presenting a rather different approach to the problem of interpretation which was sufficiently general to consider both two and three dimensional variations of resistivity. It was based upon an expansion of the potential in a series of higher order terms whose physical significance was pointed out by Madden (1953). He indicated that it corresponded to secondary sources created by the primary source in regions of conductivity variation and the interactions of these secondary sources representing the higher order terms in the series. Stevenson presented an example of the interpretation possible with this method which used only the first term in the series expansion for a one dimensional variation of resistivity with depth and compared his results with those obtained by using the Slichter-Langer method. The example chosen was for discrete layering, and Stevenson's method gave much better results than the other. Recently Belluigi and Maaz (1956) have critically reviewed these two methods and employed a continuous variation of resistivity to illustrate the relative failure of Stevenson's method in this case. In addition they have pointed out that basic proofs of uniqueness and existence which are lacking in his original mathematical formulation, and which Stevenson himself readily acknowledged.

1.7 Induced Polarization Phenomena

A recent development closely associated with the measurement of apparent resistivity of the earth provides a more direct evaluation of the metallic content of the sub-surface rocks. This method of geophysical prospecting has been referred to as the over-voltage or induced-polarization method and is based upon the measurement of the artificial electrical field within the earth. It differs radically from the ordinary resistivity methods in that either the variation with frequency of the resistivity is measured for an alternating current source or the chargeability of the earth is determined by the decay of voltage from its steady value at the receiver electrodes after the primary source is turned off. Schlumberger in 1920 referred to this second form of the same phenomena as provoked polarization but his attempt to utilize it or to completely understand it were not successful. It is only recently (since 1949) that practical results have been obtained and these mainly in mining geophysical applications.

As mentioned previously, when an ionic solution is in contact with a metallic conductor a potential will arise because of the electrochemical reactions occuring at the interface of the two media. In addition to this will be added a voltage drop across the circuit thus formed when current is passed through it. This voltage, and in reality the impedance of the circuit, will vary dependent upon the frequency of the current source.

Another method of observing this electro-chemical phenomena and even perhaps a better manner in which to visualize this process is to pass a direct current thru such a system for a certain time interval. If the source is then removed the voltage across the circuit will not instantaneously decay to zero but rather will take a finite amount of time and it is this which is referred to as an over-voltage. This may be thought of as an internal storage of electrical charge at the interface of the two different media somewhat as a capacitor stores charge.

If the only process within the material commonly found in the earth that could store charge was always associated with an ionic solution in contact with a metallic conductor then truly a direct method of prospecting for such mineralization would exist. However, since the phenomena as outlined depends upon the difference in electrical current transport any other mineral or rock which is a good electronic conductor such as graphite will yield a similar electrical response. Even more critical than this is the very recently experimentally observed and theoretically investigated phenomenon of membrane polarization of current flow which arises in many geologic material. In this physical process the flow of ions is highly selective with regards to the sign of charge carried, yielding a result for electrical measurements that is identical with the electrode polarization. While there is some overlap in the magnitude of the membrane and ionic-electronic polarization effects, usually the latter is the most significant in hard-rock areas.

1.8 Frequency-Time Relationships in IP

The results of observing this polarization of geological material, regardless of its origin, are identical for electrical measurements and either the time or frequency methods may be employed with equal success in field operations. Measurements on the phenomenon indicate that it is linear for the current densities commonly applied to geologic materials and this allows the use of sophisticated mathematical transform theory to relate time and frequency behavior to each other.The usual method of observing overvoltage phenomena is to establish a current in the ground for a fixed time interval and then to observe the decay after the current is withdrawn. The value of the decay voltage immediately after the current is turned off divided by the normal voltage appearing when the source is active is taken to be a measure of the amount of charge and hence of the metallic mineralization within the earth. Modifications to this are measurement of decay voltages at other time intervals or the integration of the decay voltage for a certain time interval.

In the frequency domain, measurements are made upon the apparent resistivity for two different frequencies and the change in their reciprocals, the conductivity, is used to evaluate the subsurface geologic composition. Field measurements in an area proceed just as a normal resistivity survey, with variation of source-receiver position and distances involved with the objective to evaluate the variation of polarizability of subsurface material.If the time domain method uses the fractional decay voltage appearing as a quantitative estimate of the electrical character of the ground it can be shown from the limit theorems of Laplace Transform theory that this is identically equal to the fractional decrease in apparent resistivity from very low to very high frequencies.

Successful applications of this most recently developed geophysical prospecting method have obtained by investigators with the most significant results in areas of disseminated mineralization such as the porphyry copper deposits throughout the world which have not been amenable to any of the other geophysical methods of prospecting in general. The anomalies obtained from these areas are at times outstanding in the sharpness of the resolution of mineralized zones as compared with normal resistivity results in the same areas. However, as indicated the membrane phenomenon is important and as more field work is done in areas with less geological control greater experience will be required to interpret the data regarding the presence of metallic mineralization.

1.9 Electromagnetic Coupling

Attractive as this new method appears, there are certain complications which arise because of the necessity to observe either the response to an alternating current source or the decaying voltage from an interrupted steady current flow. Associated with every electrical current flow is a magnetic field calculable from Ampere's law. If the current flow varies with time however, and hence its magnetic field, then by Faraday's law of induction eddy currents will be set up. These will act in the same way as the original source currents and in order to properly describe the current flow it is necessary to consider the problem as one of electromagnetism.

That is, starting from Maxwell's equations describing the differential behavior of electric and magnetic fields, the complete description of the relative position of source and receiver circuits, the connections to them and the distribution of conducting material must be made in order to predict the correct response of the system to any excitation. There will be differences in the electrical fields observed as compared with direct current and two phenomenon are responsible for this: in addition to the normal resistive coupling of the two circuits there will be inductive coupling through the conductive media and the capacitive coupling between the wires forming the two circuits. Formally it is necessary to refer to the coupling as the mutual impedance rather than the mutual resistance in any of the existing resistivity methods.

The use of a commutator in the Gish-Rooney equipment actually creates a square wave source of current. There have been reports of variations of resistivity measured at a given location dependent upon the rate of turning of the commutator in the G-R systems and this is no doubt attributable to an induced polarization effect although the possibility of an EM coupling phenomenon must also be considered.

Since the distribution of conducting material in the subsurface is not known before a survey is made within an area the electromagnetic coupling effects can not be predicted and their influence eliminated from the measurement to yield the correct value of the frequency dependent resistivity. Theoretical investigations on a uniform region however can be used as indications of when EM coupling effects became important. As anticipated, the more rapid the change in current flow the larger will be the effect but the scale of the electrode configuration and the conductivity of the material must also be considered. Fortunately there are few areas where the EM coupling effects are large enough to be troublesome and the polarization effects occur well before this critical point is reached. The solution to the problem when it arises is relatively straight forward in either the time or frequency method: measure the decay voltage a short time after the current flow is interrupted rather than immediately or use two frequencies which are low enough not to yield EM effects but still sufficiently far apart to show polarization effects in the resistivities measured.

In the Eltran and Sawtran methods of geophysical prospecting introduced wome years ago, the basic phenomenon measured was the EM coupling of the two circuits through the ground, and the polarization effect was not known to be important. Observations of the decaying field were made for a four-electrode array similar to the Wenner but the connections to the electrodes were modified so that the current source was applied to the two electrodes at one end of the spread while the voltage was measured at the other two. The rate of decay or time constant of the voltage was taken as the parameter indicative of subsurface conditions but both instrumental and analytical problems caused the method to suffer adversely in its applications.

1.10 Modified Electrode Array

There have been two main groups working on the application of polarization phenomena as a geophysical prospecting mtehod and each has developed their respective system about either the time or frequency basis of measurement. The group using the time domain has utilized the Wenner array and prospected areas by profiling along grid lines often using several spacing intervals yielding both a lateral and vertical probing of the subsoil. Those using the frequency domain have utilized a modified Eltran array by fixing the spacing between the pair of electrodes forming the source circuit and similarly for the receiver circuit while varying the distanee between the two circuits in integral multiples of the electrode interval.

The use of the modified Eltran array arose from the necessity to minimize as much of the capacitive coupling effects as possible by geometrical arrangement of the source and receiver circuits. Experience has also indicated the advantages of separating the source and receiver circuits as contrasted to the Wenner array with respect to sensing the subsurface structural variations. Operationally this method has proven very successful in both efficient use of field personnel and equipment without sacrificing the quality of the results in the details of the resistivity variation.

Indeed there has been a substantial improvement in the method of obtaining data, and its presentation and interpretation regarding the lateral and vertical variations of resistivity by using the modified Eltran array. The normal field procedure has been to fix the position of the source circuit while successively moving the receiver circuit farther away, then moving the source along the line in the same direction and repeating the receiver movement but for a slightly different set of positions. The diagram below illustrates the details of this system along one grid line with the most noteworthy aspect being the two-dimensional character of the data measured. The data is plotted at a point midway between the two circuits with its distance below the surface line proportional to the spacing between the two circuits.

```
1       2       3       4       5       6       7
*-------*-------*-------*-------*-------*-------*

          S 1-2   S 2-3   S 3-4   S 4-5
          R 3-4   R 4-5   R 5-6   R 6-7
              S 1-2   S 2-3
              R 4-5   R 5-6
                            etc.
```

S= Source Electrode Positions   R= Voltage Measurement Electrodes

Because of the symmetry principle regarding current flow in any media, there is no difference as to which circuit is the source and hence there is no variation of results dependent upon the direction in which the profile was made. Very characteristic patterns have been obtained from two dimensional model experiments using this array as well as for certain theoretical subsurface geometries. The applied interpretation system which this thesis will develop is based upon this electrode array although its basic principles can be applied to any array.

2.1 The Forward Problem in Resistivity Prospecting

The forward problem in resistivity prospecting is defined as the prediction of the electrical potential within a given region for a specified distribution of sources and variation of conductivity. This can be accomplished by obtaining the solution to the differential equation governing the flow of current within the media which satisfies the conditions of continuity of potential and current flow at any interior point in addition to certain boundary conditions imposed by the type of source and geometry employed.

Representing the vector current density by $\vec{J}$ and the source distribution by q, application of Gauss's theorem equating the net out flow of charge from within a small element of volume to the enclosed source strength yields:

$$\nabla \cdot \vec{J} = q \qquad 2.1.1$$

where $\nabla\cdot$ is the divergence operator. This equation is actually a statement of the conservation of electrical charge at all points within the region. The generalized form of Ohm's law for continuous media states that the current density is linearly proportional to the electrical field strength $\vec{E}$, the constant being the conductivity; that is

$$\vec{J} = \sigma \vec{E} \qquad 2.1.2$$

This form of Ohm's law is not strictly general since it implies that the current flow parallels the electrical field in all cases. For anisotropic media this is not the case and either a modified form of Eq. 2.1.2 for each component must be stated or equivalent use made of the tensor concept and notation. However, this will not be done here and it is possible to solve certain physically important anisotropic problems by choosing the coordinate directions to coincide with the directions of anisotropy, which are assumed to be constant throughout the media.

For steady current flow the total work done on moving an electrical charge in a closed circuit must be zero since the conservation of energy must be upheld. This implies that the vector E field is conservative and thus derivable as the gradient of a scalar electrical potential φ . Thus it is possible to write

$$\vec{J} = \sigma \vec{E} = -\sigma \nabla \varphi \qquad 2.1.3$$

where is the gradient operator. Substitution of this equation into 2.1.1 yields the general equation relating the electrical potential to the source distribution and conductivity variation as:

$$\sigma \nabla^2 \varphi + \nabla \sigma \cdot \nabla \varphi = -q \qquad 2.1.4$$

There are two types of problems in resistivity prospecting for which solutions to this equation have been obtained. The first represents the physical system in which the conductivity is constant within certain subregions of the entire region of interest and in this case the differential equation reduces to the familiar Poisson's Equation $\nabla^2 \varphi = -q/\sigma$ if sources are present and Laplace's equation $\nabla^2 \varphi = 0$ if there are none. Both of these equations have been studied in other fields of physics and solutions for different coordinate systems are well known which also satisfy the necessary boundary conditions for the actual sources used. This forward problem of resistivity prospecting is generally referred to as a boundary value problem in mathematical physics and with a complete set of solutions for different coordinate systems many specific problems of interest can be solved by satisfy-ing the requisite boundary conditions.

For those problems with eintinuous variations of resistivity only a limited number have been capable of solution, and these in particular for a one-dimensional variation. Even for this latter group completely arbitrary variations of conductivity cannot be solved in closed analytic form because of formidable mathematical operations required.

A second method of solution utilizes the concept of an influence function which corresponds to that solution for the physical system in which the source is a mathematical point. By superposition of these influence functions, which satisfy the necessary boundary conditions, so as to form a source identical with the actual source the net response of the system will be obtained. This is the Green's function formulation which at present has been only fully developed for homogeneous regions in which the governing differential equation reduces to Poisson's equation. In the general forward problem of resistivity prospecting it will be necessary to develop a corresponding Green's function for equation 2.1.4 when q is a point source but this has yet to be done.

2.2 The Inverse Problem in Resistivity Prospecting

As contrasted with the problem of predicting the system response to a known excitation with a knowledge of the parameters is the inverse problem of determining the distribution of parameters from a known response. This is the inverse boundary value problem of mathematical physics which forms the core of this thesis investigation, and is essentially a formalism of the interpretation procedure associated with all geophysical prospecting methods. Certain advantages in the theoretical aspects of the general prospecting problem are associated with the use of artificial sources as is done here although it is not possible to measure every physical parameter of interest by their use. As indicated previously there are two approaches to interpretation: the direct methods use the forward solutions of Section 2.1 while the indirect method is exactly the inverse problem.

By surface measurements of apparent resistivity the inverse problem solution would predict the subsurface variation of electrical conductivity. This extremely difficult problem has been approached only for a one dimensional conductivity variation in the work of Slichter and Langer. A completely rigorous treatment necessitates the formulation of basic proofs of existence and uniqueness which for the one dimensional problem have applied only to the characteristic function derived from the field data by the use of the Fourier-Bessel inversion theorem. In addition there are additional requirements that the surface potential, which is derivable from the apparent resistivity, be known for all distances of the receiver from the source. Such a requirement for data is unrealistic in practical field operations and Vozoff was only able to apply the formal inversion techniques developed by assuming a form for the continuous interpolation of the field data between the measured points. This limits the accuracy which may be expected but by proper choice of spacing intervals for the number of discrete data points obtained the uncertainity for many cases may be minimized. Also of useful assistance in the interpolation are the theoretical results of known geometries so that the potential's interpolated behavior correspond somewhat to a real system.

2.3 Stevenson's Series Solution

In the work of Stevenson which has already been mentioned, a series expansion of the solution was made for the forward problem for an arbitrary 3 dimensional variation of conductivity. In addition he suggested that this approach be used in the solution of the inverse problem although certain points of mathematical rigor were not clarified. By transposing equation 2.1.4 into the following form:

$$\nabla^2 \varphi = -\frac{1}{\sigma}\left[q + \nabla\sigma \cdot \nabla\varphi\right] \qquad 2.3.1$$

and recognizing the close analogy between this equation and Poisson's equation he formally solved both the forward and inverse problems with a Green's function development as:

$$\varphi(\vec{r}) = \int \frac{1}{\sigma}\left[q + \nabla\sigma\cdot\nabla\varphi\right] G(\vec{r},\vec{\rho})\, dVol_{\rho} \qquad 2.3.2$$

It is not possible to actually solve this integral equation for either problem as it stands because of the lack of adequate analytical methods for generalized integral equations and Stevenson obtained a solution in his series expansion.

He assumed that the final potential could be expressed as the sum of an infinite sequence of potentials as:

$$\varphi = \varphi_0 + \sum_{i=1}^{\infty} \varphi_i \qquad 2.3.3$$

Here φ_0 was the solution to the differential equation

$$\nabla^2 \varphi_0 = -q/\sigma \qquad 2.3.4$$

obtained by the Green's function development as

$$\varphi_0(\vec{r}) = \int \frac{q}{\sigma}\left[G(\vec{r},\vec{\rho})\right] d\,Vol_{\rho} \qquad 2.3.5$$

The remaining φ_i were obtained by succesive solution of the differential form:

$$\nabla^2 \varphi_i = \frac{-1}{\sigma} \nabla\sigma \cdot \nabla\varphi \qquad 2.3.6$$

as before using the Green's function to yield

$$\varphi_i(\vec{r}) = \int \frac{1}{\sigma}\left[\nabla\sigma \cdot \nabla\varphi_{i-1}\right] G(\vec{r},\vec{\rho})\, dVol_p \qquad 2.3.7$$

Stevenson did not show that this series development would converge to the proper solution and the recent paper of Belluigi and Maaz have pointed up this lack of fundamental mathematical rigor. Assuming the convergence it can be shown that the series 2.3.3, whose terms satisfy 2.3.4 and 2.3.6, also satisfies the general differential equation 2.1.4.

2.4 Green's Function for Half Space

The geometry used in all theoretical work thus far has been that of an infinite half space whose upper surface corresponds with the earth's surface within the area of interest. While this assumption would not be valid for resistivity measurements on an extremely large scale, comparable with the earth's radius, it is valid for the problems of specific interest in geophysical prospecting. The maximum distances involved in these cases are less than a kilometer, which is negligible when compared with the 6370 km radius of the earth. Essentially the earth's surface may be considered as perfectly flat for all of the present day geophysical prospecting systems and this is exactly what the previous assumption does.

The Green's function solution to Poisson's equation for a point source in such a geometry must satisfy the following boundary conditions:

$$G(\vec{r},\vec{p}) \Rightarrow \frac{1}{|\vec{r}-\vec{p}|} \quad \begin{array}{l} \text{as } |\vec{r}-\vec{p}| \rightarrow 0 \\ \text{as } |\vec{r}-\vec{p}| \rightarrow \infty \end{array} \qquad 2.3.8$$

where $\vec{r}$ represents the position of the source point and $\vec{p}$ the point at which the potential is measured. Also there can be no flow of current across the upper surface so that:

$$\frac{\partial G}{\partial n} = 0 \qquad 2.3.9$$

where $\vec{n}$ represents the normal to the surface. There are two methods of constructing a proper Green's function for the region considered. The first is to consider the exact problem as stated: conducting material below the upper surface. In this case for a rectangular coordinate system with the origin on the surface and the 3 direction vertical the Green's function is given by:

$$G(\vec{r},\vec{p}) = \frac{1}{4\pi}\left[\frac{1}{[(r_1-p_1)^2+(r_2-p_2)^2+(r_3-p_3)^2]^{1/2}} + \frac{1}{[(r_1-p_1)^2+(r_2-p_2)^2+(r_3+p_3)^2]^{1/2}}\right] \qquad 2.3.10$$

The second method and the one which Stevenson used is to reflect the conducting media about the upper surface so that:

$$\sigma(p_1,p_2,p_3) = \sigma(p_1,p_2,-p_3)$$

$$G(\vec{r}, \vec{p}) = \frac{1}{4\pi}\left[\frac{1}{\left[(r_1-p_1)^2 + (r_2-p_2)^2 + (r_3-p_3)^2\right]^{1/2}}\right] \quad 2.3.11$$

but the integrations must now be performed over all space rather than as previously only the half space. Either method is analytically correct, but the second allows a physical interpretation which is helpful in qualitatively studying the responses of various geometries. It is to be noted that the Green's functions developed are symmetrical in the source and receiver points. That is, an interchange of them will not yield any change in the value of the Green's functions nor the boundary conditions which they satisfy.

This principle of symmetry allows the position of source and receiver to be reversed without yielding a different result for the apparent resistivity. In field operations this principle can lead to savings in expenses and time while maintaining the same quality and even increasing on the quantity of results obtained. A specific example occurs when using the modified eltran array if the source and receiver circuits are located respectively in high resistivity material and low resistivity material. The voltage developed at the receiver electrodes might then be too small for the instrument employed to accurately measure but by exchanging the two circuits the amount of current employed could be made larger and thus so also would be the voltage measured.

2.5 First Approximation Inversion of Stevenson

As already noted Stevenson suggested that the series expansion be used also in determining the solution to the inverse problem. He was not able to obtain such a solution except in a specific geometry and this included only the first term beyond the homogeneous solution. That is, Stevenson used the general form:

$$\varphi \approx \varphi_0 + \int \frac{1}{\sigma}\left[\nabla\sigma \cdot \nabla\varphi_0 \right] G \, dVol \qquad 2.5.1$$

He considered the case for only vertical variations of conductivity and performed the integrations in the horizontal directions without a knowledge of the conductivity variation. The one dimensional integral in the vertical direction was finally inverted by the use of the Hankel inversion theorem. Because of the finite series expansion used in this approach and the linearization in the conductivity which it effected, the results of this inversion yielded the conductivity directly. This is quite different than the exact procedure of Slichter- Langer which yielded the kernal function which was then related to the conductivity variation.

2.6 Physical Interpretation of Series Expansion and Inversion

It is useful to consider a physical interpretation of the series expansion developed by Stevenson and the significance of the inversion resulting when only including the first term. The Green's function development indicates that the term in Eq. 2.3.2 essentially represents another source term whose strength and distribution is dependent upon the gradients of the electrical potential and the conductivity and the cosine of their included angle. However, the potential referred to is the final potential and thus in both the forward and inverse problems the correct distribution is not known.

In the series development the potential which is used to create the additional higher order sources is dependent upon a known potential of lower order and by iteration of these sources the final potential is derived. It is possible to consider the creation of secondary sources yielding φ_1 as being an interaction between the field of the primary source and the conductivity variations to a first approximation. The potential φ_2 is then capable of interpretation as the result of the interaction of these secondary sources upon each other, again to a first approximation. Continuing this interaction process finally approaches the real system in which the existing sources, both primary and secondary, and their fields satisfy equation

2.1.4. For those problems in which the changes in conductivity are limited to discrete jumps between regions of uniform but different values the sources induced will be distributed over the surfaces of the boundaries separating them.

In either situation the interactions of the secondary sources will be governed by the Green's function which for a half-space is given by Eq. 2.3.10. Because of the two distances involved in this formulation it is seen that what is happening is not only an interaction of the sources with themselves within the half space but an interaction with image sources located above the plane boundary. By using Stevenson's form of the Green's function Eq. 2.3.11 and reflecting the half space about the plane boundary these added interactions are seen to correspond to the interaction of the actual conductivity variation with its image conductivity region above the plane. This method of interpretation of the series expansion as induced sources and their interactions was originally proposed by Madden (1953).

For plane surfaces of discontinuity the interactions of the sources on themselves are zero since the angle between the conductivity gradients and potential gradients of the secondary sources is 90°. Here the interactions of the secondary sources with their images produce all the higher order terms. For horizontal layering there are interactions of sources on plane discontinuities with one another but the first approximation solution corresponds to the induction of sources only by the primary source and no interactions with themselves or their images. The sources induced on these planes are equivalent to placing an image source at the point mirrored about the plane by the primary source. For three horizontal layers these images will be located at different depths in the media with strengths dependent upon the conductivity contrasts. The diagram below illustrates this interpretation with two planes of discontinuity and the equivalent image sources.

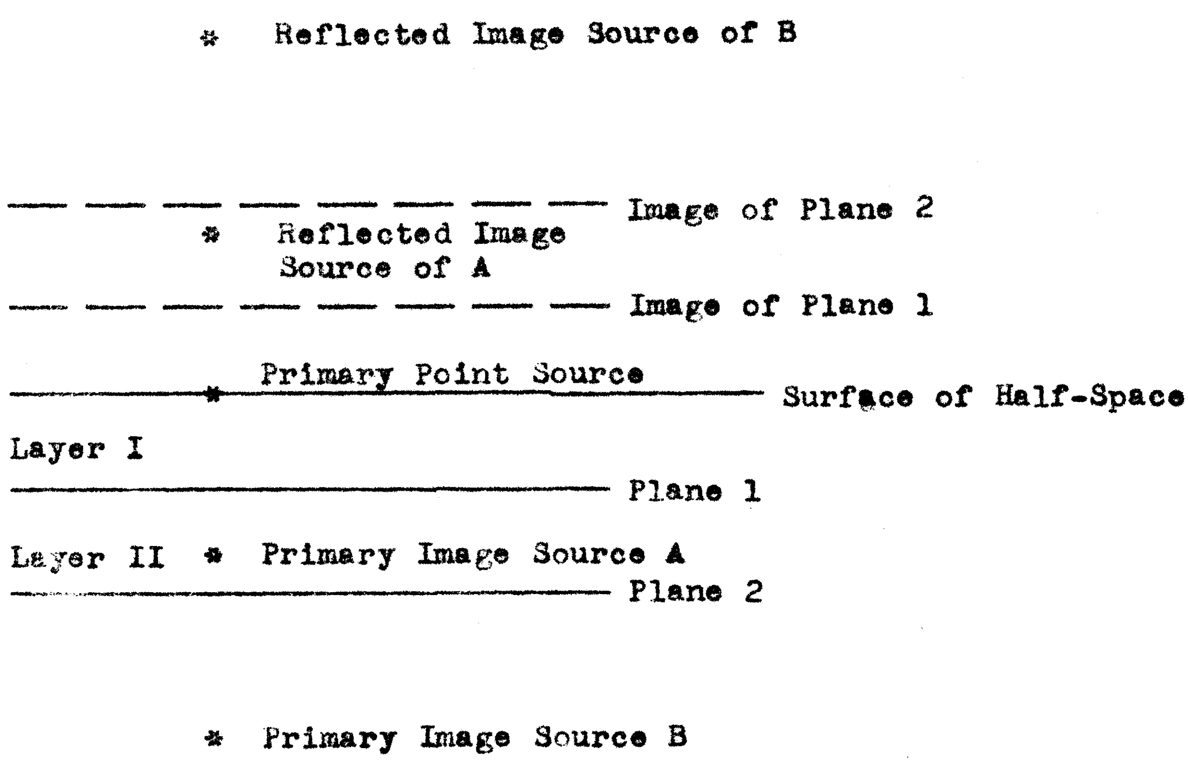


Image Source Distribution for three horizontal layers

The electrical potential ψ observed on the surface is then readily interpretated as being due to the total effect of the 5 sources; 1 primary, two secondary images and their reflected images. This interpretation can be extended to any number of plane discontinuities and is recognized as paralleling the image theory solutions of electrostatics. For the image theory solutions in resistivity problems with horizontal layers there is a decided difference between the manner in which the strength of the sources is calculated and the following section discusses the problems of the first approximation in this connection.

2.7 Source Strength and Saturation

From a consideration of the principles of energy conservation it is known that the secondary electrical fields associated with non-homogeneous regions are bounded in magnitude, regardless of the conductivity variations. The maximum values for these fields are reached asymptotically as the contrasts between different regions increase. This phenomenon when viewed from the concept of equivalent induces induces sources implies that the strength of such sources reaches a finite saturation value when the contrasts become infinite.

In the first approximation of Stevenson the strength of the sources induced in his series development depends upon $\nabla\sigma/\sigma$. This can be rewritten as $\nabla \ln \sigma$ so that across a discontinuity in conductivity (σ_1 to σ_2) the value is given as $\ln(\sigma_2/\sigma_1)$. This term however, does not exhibit any saturation behavior and although its singularity for both large and small contrasts is of low order, it does reach infinity for the limiting values of $\sigma_2/\sigma_1 =$ 0 or ∞. This is a serious failure of the first approximation and has not been capable of a direct explanation other than that there must be interactions of the induced sources on themselves in the neighborhood of each surface point, and this leads to saturation.

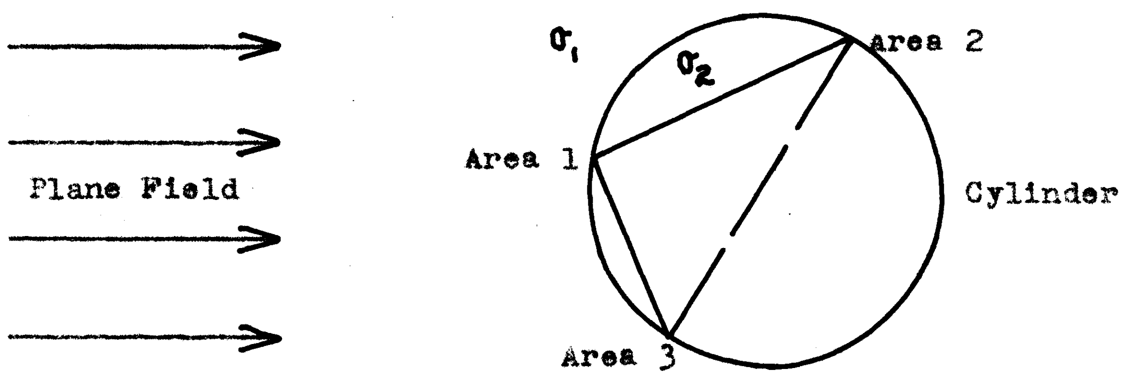
Stevenson's series expansion does represent interactions between induced sources but does not refer to any which may take place between one surface point and its immediate neighborhood. That interactions of this nature occur in real problems is evidenced in the exact solutions for two geometries in which the interactions among surfaces are zero. Thus any saturation which occurs must be due to an interaction of the surface on itself.

The first example is that of a point source in a whole space containing two different media separated by an infinite plane. In this case the strength of the sources induced on the surface of the discontinuity is dependent upon the two conductivities as:

$$2\left[\frac{\sigma_1 - \sigma_2}{\sigma_1 + \sigma_2}\right] \qquad 2.7.1$$

and this does show saturation. According to Stevenson's series however, there would never be any interactions since the gradient of the conductivity is perendicular to the gradient of the induced sources'potential. The first approximation does yield the correct current flow lines, but the wrong magnitude.

The second example is that of an infinite cylinder in a plane field with the axis of the cylinder perpendicular to the field. For a given infinitesimal surface area interacting with another there is always a third area diametrically opposite the second whose effect exactly cancels that of the second on the first. The diagram below illustrates this:



Hence the net effect of all the interactions will be zero but the exact solution predicts that the induced sources depend upon the conductivities as:

$$2\left[\frac{\sigma_1 - \sigma_2}{\sigma_1 + \sigma_2}\right] \qquad 2.7.2$$

and again this exhibits the phenomenon of saturation.

The conclusion reached is that saturation must occur in the first approximation even if the quantity $\nabla\sigma/\sigma$ does not appear to explicitly indicate this. Hallof (1957) has shown that for a continuous variation of conductivity across a thin region separating two different but homogeneous media the interactions of the surface on itself do not go to zero as the thickness approaches zero. Thus in the forward problems and also the interpretations based upon the use of the first approximation, consideration must be given to the strength of the induced sources depending upon the conductivity contrasts in some manner which will demonstrate saturation. That is a form such as 2.7.1 or 2.7.2 should very possibly be considered as the correct expression of source strength as a function of the conductivities.

2.8 Modified First Approximation

A more serious failure of the first approximation as it now stands than the lack of a saturation phenomenon is its non-symmetrical character. That is, because the real problem must show symmetry in the source and receiver points it would be expected that so should the 1st. approximation. This trouble does not arise in the application of the 1st. approximation to only depth variations of conductivity but does arise when 2 or 3 dimensional variations are considered. The following explicit expression of the first approximation solution of the potential for the general case illustrates this fact.

$$\varphi(\alpha|\varrho) \approx \frac{1}{|\alpha-\varrho|} + \int \frac{\nabla\sigma}{\sigma}\cdot\nabla\varphi_o(\alpha)\, G(\alpha-\varrho, \vec{p})\, d\mathrm{Vol}_p \qquad 2.8.1$$

Here α represents one position on the surface and ϱ the other, with the vertical line separating the source and the receiver positions respectively. This problem has been resolved by the introduction in this thesis of a modified first approximation solution, which will always demonstrate the required symmetry of source and receiver.

In order to preserve the linearity in the conductivity variation afforded by the first approximation a linear operation on the existing approximation of Stevenson has been made. The modified first approximation potential is defined as the average of the potentials observed when the original first approximation is used for both combinations of source and receiver point. That is, using the same notation as Eq. 2.8.1, the modified potential is defined as:

$$\bar{\varphi}(\alpha|\xi) = \frac{1}{2}\left[\varphi(\alpha|\xi) + \varphi(\xi|\alpha)\right] \qquad 2.8.2$$

It is this modified first approximation which has been utilized to solve both the forward problem and the inverse problem. The solutions obtained are only approximations but they represent the first quantitative attempt to predict and interpret apparent resistivity data for more than a one dimensional variation of conductivity.

3.1 Formulation in Discrete Regions

As previously indicated, Stevenson was unable to obtain an analytic inversion of his approximate solution for more than a one-dimensional variation of conductivity. The fact that point sources and receivers are utilized in the field operations leads to a singularity of the integrand for the forward problem which can only be treated properly in the one-dimensional problem. In this case it is possible to integrate out the other two dimensions in the forward solution without a knowledge of the conductivity and invert the remaining integral by the Fourier-Bessel transform. The author has attempted to obtain an analytic inversion of the modified first approximation when considering two or three dimensional conductivity variations without success. The main problem has been the singularity of the integrand associated with point sources and receivers.

Backus (1959) has eliminated the problem of the singular integrand by using a source and receiver which are distributed over the entire half-space plane boundary. The voltage source varies in magnitude sinusoidally in both surface dimensions and the current outflow is measured over the entire plane. Thus he is effectively considering the two-dimensional Fourier transform of the point response of the earth. It is possible to determine the response of the earth to such an extended source and receiver by superposing the responses for point sources and receivers which are distributed continuously over the plane surface. However the number of mathematical inversions necessary to yield the conductivity variation will be increased by two if this method of obtaining data is employed in this approach. It is doubtful whether this analytic modification of the basic resistivity problem of interpretation will be helpful in establishing a useful procedure because of the numerical computations involved and also because field operations may never yield the required data. The use of point sources and receivers dictates that the mutual resistance be known for all combinations of points on the surface and this leads to field operations which measure the response for a given point source over the entire surface. At present there appears good reason to restrict the collection of data to profile lines and this field procedure eliminates the opportunity to apply the approach of Backus.

Since there has not been effected an analytic solution to the interpretation of resistivity data another approach has been taken which is based upon the concept of finite sized regions in the subsurface. The geometry of these regions is pre-determined and then their effects calculated from the modified first approximation. Combining a number of these regions allows the representation of the subsurface geologic structure as well as the fitting of the measured field data to that predicted from the known effects of each region. The unknowns involved once the geometry has been determined are the conductivity contrasts for each region.

Vozoff (1956) originally suggested for the subsurface "...that the region be considered as being made of homogeneous blocks of given geometry but unknown conductivity..." and it is this concept which has been fundamental in the interpretation scheme developed. He proposed that the region be divided into a three dimensional array of cubes and that by applying the first approximation solution of Stevenson a completely linear problem would be formulated. The effect of each cube was linearly combined with all the others and if the number of observations equaled the number of cubes then a linear set of equations in the unknown conductivities would allow a solution to be obtained. He pointed out that the lateral resolving power of such an approach would approximately be equal to the minimum spacing interval which the set of observations represented.

However, Vozoff's efforts along this line of approach did not reach the actual numerical computation and certain problems have arisen in its application which have necessitated a modification of his approach. In addition certain considerations regarding the choice of the shape and distribution of the regions have led to a rather different subsurface geometry but the original suggestion of using finite regions is due to Vozoff's work.

The first approximation has been modified in order that it demonstrate symmetry in source and receiver point. However, the linearity of the solution in the conductivity contrasts has been preserved so that a compositing of a number of regions yields a net response identical with the entire region. This is not the case when a contrast factor such as 2.7.1 is used. Here the sum of induced sources at adjacent interfaces will not be equal to the source induced when the two regions are considered as being present simultaneously, unless the adjacent regions have the same conductivity value or one of them has a value equal to the background. Hence any modification to the strength factor $\nabla\sigma/\sigma$ to allow that saturation phenomenon be adequately treated must also consider the ramifications when a linear superposition is attempted with the finite sized regions.

3.2 General Problem of Data Fitting for the Inverse Problem

In order to determine the values of the conductivities of the different regions within the earth when applying the discrete region concept, it is necessary to fit the data to the effects of all the regions. By fitting the data is meant essentially that the results of the forward problem with the determined conductivities yield a response identical, or approximately so, to the response measured in the field.

There are many problems which arise in scientific research which require the fitting of data to an assumed physical system. If the data fitting is satisfactory, according to some previously defined criterion, then the system is considered to be a possible model of the phenomenon studied. However, simply because the data is well approximated by the model proposed is no guarantee that it is a true representation of the actual physical system. Often a model will explain phenomena other than the original one investigated and as the number of independent sets of data explained by the model increases so does the confidence of the investigators that they have the correct model. It may well be that they have not yet exhausted the number of experiments possible or that there are certain ones they cannot at present undertake. Whatever the situation, the basic fact that a model proposed and tested is just that and nothing more must always be considered. These remarks apply to the use of models in both economic and scientific research.

A set of data is associated with a certain number of variables and may be linearly or non-linearly dependent upon their values. Also there may be the same number of data points as variables, or more or less data than variables. In general, scientific disciplines have only been concerned with the first two cases but there are possibilities for considering the last case of less data than variables. Certain very recent developments in operations research concerned with the efficiency of business and military logistics as well as the optimization of 'return' have led to the analysis of such problems. They are referred to as linear 'programming' problems when the relations between the data and the variables are linear as well as the function which is to be optimized. Correspondingly when the dependence of the data and/or the function is not linear then it is called non-linear 'programming'. It is to be noted that this use of the word 'programming' has a very different connotation than when associated with the directions and commands used to program a computer.

When the number of variables is equal to the number of data points then unless there are non-linear relations to be considered the problem is usually capable of solution. There is no freedom in a completely linear problem and either a solution exists or it does not. That is, a formal solution may be stated although numerically it may require the use of an electronic computer and highly developed computer codes to solve the resulting equations. Few formal statements can be made for the non-linear case other than that in general the difficulties are much greater than the linear problems.

For those cases with more data than variables some technique to utilize some or all of the data must be employed which will fit the data in some 'best' sense. The most highly developed measure of 'best' is the familiar least squares approach which minimizes the aggregate squared error of the data fit. A great deal of computational effort is eliminated by the use of orthogonal functions and a better fit can be obtained by using additional variables without recalculating those already considered. But most important of the properties of a least squares analysis is that it may require only linear operations to yield a solution. This allows the use of many different linear techniques to obtain the 'best' fit and it is this method which has been widely applied since its original discovery independently by Gauss and Legendre in the early

1800's. No other method of dealing with the problem of surplus data has been as successful and modern statistical methods are based fundamentally upon its concept and utilization.

The 'programming' problems previously referred to have arisen to create an entirely new field of applied mathematics although the theoretical aspects of linear inequalities upon which they are based had been studied some time before. As a result of the desire of the military and business interests to make their operations more efficient by the use of decisions made on a quantitative and scientific basis, these methods of analysis have been very rapidly developed since 1946. In order to compensate for the additional variables, certain restraints are placed upon them, commonly that they be positive, or integers or be bounded in range. Finite algorithms for solving certain of these linear problems have been derived and prepared for computer use and a wide variety of problems have been solved. A typical example is that of supplying a number of different stores from a group of warehouses so that transportation costs are minimized while never allowing return shipments from the stores nor the capacity of any warehouse to be exceeded.

In geophysics there are possibilities of applying this method of analysis to data fitting in a variety of interpretation problems. In gravity prospecting it would be possible to assume a background density value sufficiently low so that all density contrasts would be positive and bounded in magnitude. The limits and background values would be based upon independent geological information about the area of interest and an assumption made regarding the geometry within the area. In any application adequate consideration must always be given to the basic physical principles underlying the phenomenon. The fact that the total mass of the disturbing body can be computed from the anomaly without knowledge of the geometry and the fundamental non-uniqueness of gravity interpretation must be realized. The decrease in the resolving power of gravity anomalies as the depth to the source body is increased must be included by increasing the scale for deep anomalies.

With regards to resistivity interpretation, the conductivity contrasts for a given geometry of subsurface regions could all be bounded in magnitude and a solution obtained. Bounding the contrasts would simulate the saturation phenomenon which must occur and which at present is not evidenced in either first approximations.

Both of these interpretation problems have been linear in their dependence upon the variables of density and conductivity contrast but there certainly are also non-linear problems. There is however a more basic consideration to be given geophysical data fitting by any 'programming' technique.

Usually there is a surplus of data relative to the amount of information that is desired. Seldom is there ever a lack of enough data, more often it is a lack of sufficiently good data that hinders the interpretation problems in most of geophysics. However, the fact that theoretical and numerical work on these 'programming' approaches has made them available as useful methods may lead to their utilization in certain problems of geophysics where the scarcity of data is the major difficulty.

3.3 Least Squares Formulation and Inclusion of Background

There has not been any numerical attempt in this thesis at using a 'programming' approach to the data fitting for the interpretation of apparent resistivity prospecting. The main effort has been to use a fitting by least squares to determine the conductivity contrasts and the background resistivity. It is necessary to determine the contrasts since the induced sources are dependent upon the relative conductivity of the regions and not their absolute value. The following presentation will be valid for a finite number of regions in the subsurface of any geometrical shape and for any electrode array. It will indicate explicitly the particular procedure used in this research to determine an effective resistivity interpretation scheme. Moreover, this formulation will also be valid for the case in which the number of data points is is exactly equal to the number of variables.

Vozoff suggested that in the finite-region forward problem the secondary potential due to the various regions be calculated and employed in the data fitting.

$$\varphi_s = \iint \left[\frac{\nabla\sigma}{\sigma} \cdot \nabla\varphi_0\right] G \, dVol_p \qquad 3.3.1$$

This form of the data variable requires that the field data as measured in apparent resistivities be transformed into potential data before initial computation begins. Any such operation on the initial field data should be avoided and in this particular case an integration of the data must be made twice to yield the desired form for the analysis. The amount of error involved in such a procedure may be great unless data observations are made at small spacing intervals and extended to large distances relative to the scale of the anomalous region.

Since the apparent resistivity measurements depend linearly upon the potentials, it is possible to utilize the resistivity data directly as a form for the data variable. Let $V_j^i(a)$ be the secondary potential at the j^{th} receiver position due to the i^{th} region when the source point is at a and let $V_j^i(b)$ be defined in a similar manner for the source point at b. Any four electrode measurement of apparent resistivity will depend upon the primary field plus the sum of the secondary fields due to the disturbing regions. Let

ΔV_o represent the voltage measured across the j, j+1[st] receiver electrodes due to the primary field and let ΔV_s be the secondary voltage difference across the same set so that the apparent resistivity is derived from:

$$\rho_A = g \left[\frac{\Delta V_o + \Delta V_s}{I}\right] \qquad 3.3.2$$

Here g is a geometrical factor necessary to transform the mutual resistance to the resistivity and I is the magnitude of the current. The ΔV_s is equal to the sum of the secondary potentials due to the different regions. Considering the source at (a) to be positive and that at (b) negative it can be written for N regions as:

$$\Delta V_s = \sum_{i=1}^{N} \left[V_j^i(b) + V_{j+1}^i(a) - V_j^i(a) - V_{j+1}^i(b)\right] \qquad 3.3.3$$

Now the V_j^i are linearly dependent upon the strength factors of the induced sources on the surfaces of the regions. Let K_i be this value and let A_{ij} represent the normalized secondary potential at j due to the i[th] region so that:

$$V_j^i(a) = K_i A_{ij} \qquad 3.3.4$$

If there were no conductivity contrasts then the apparent resistivity would be equal to the specific resistivity of the entire region. Let ρ_o be this value and hence ΔV_o must be given by the expression:

$$\Delta V_o = \frac{\rho_o I}{g} \qquad 3.3.5$$

Substituting Equations 3.3.5 and 3.3.4 into 3.3.2 yields:

$$\rho_A = \rho_o + \sum \frac{K_i}{I} \left[(A_{i,j+1}^{a} - A_{ij}^{a} - A_{i,j+1}^{b} + A_{ij}^{b}) g \right]$$

The term in brackets is dependent only upon the geometry of the regions and relative electrode positions. Hence it may be determined once a choice of these parameters is made.

Redefine A_{ij} by the following equation so that it now represents the normalized fractional effect on the apparent resistivity measurement at the j^{th} source-receiver combination due to the i^{th} region per unit current:

$$A_{ij} = \frac{g}{I \rho_o} \left(A_{i,j+1}^{a} - A_{ij}^{a} - A_{i,j+1}^{b} + A_{ij}^{b} \right) \qquad 3.3.7$$

Finally the explicit expression of the apparent resistivity as a linear function of the K_i is given as:

$$\rho_{Aj} = \rho_o \left[1 + \sum_{i=1}^{N} A_{ij} K_i \right] \qquad 3.3.8$$

The transformation of the data variable from potential to apparent resistivity eliminates the necessity of integrating the field data and the errors introduced by such as operation. This procedure is valid regardless of the particular K_i used as long as the potentials are linearly dependent upon them. The ρ_{Aj} are the predicted values of apparent resistivity and are to be fitted by a least squares analysis to the observed apparent resistivities ρ_j .

Assume that there are M data values and N regions and that $M \geq N+1$. The error in the prediction of the j^{th} apparent resistivity is:

$$\epsilon_j = \rho_{Aj} - \rho_j = \rho_o\left[1 + \sum_{i=1}^{N} A_{ij}K_i\right] - \rho_j$$

so that the aggregate squared error of the fitting is defined as:

$$\{\epsilon^2\} = \sum_{j=1}^{M} \epsilon_j^2 = \sum_{j=1}^{M}\left[\rho_j - \rho_o\left(\sum_{i=1}^{N} A_{ij}K_i + 1\right)\right]^2 \qquad 3.3.9$$

The variables are the K_i and the background resistivity ρ_o although it is preferable to use the background conductivity σ_o so that a completely linear set of equations will result from the least squares analysis. In order that a minimum of 3.3.9 be obtained it is necessary that:

$$\frac{\partial}{\partial K_k}\{\epsilon^2\} = 0 \qquad \text{for } k = 1 \text{ to } N$$

$$\frac{\partial}{\partial \sigma_o}\{\epsilon^2\} = 0 \qquad 3.3.10$$

The solution to these equations is best obtained by rewriting the form 3.3.9 as follows:

$$\epsilon_j = \rho_0 \left[\sigma_0 \rho_j - \left(1 + \sum_{i=1}^{N} A_{ij} K_i \right) \right]$$

$$\left\{ \epsilon^2 \right\} = \rho_0^2 \sum_{j=1}^{M} \left[\sigma_0 \rho_j - \left(1 + \sum_{i=1}^{N} A_{ij} K_i \right) \right]^2$$

Since whatever value of ρ_0 is determined will only scale the aggregate squared error, it is possible to cancel the term ρ_0^2 and minimize the following form:

$$\sum_{j=1}^{M} \left[\sigma_0 \rho_j - \left(1 + \sum_{i=1}^{N} A_{ij} K_i \right) \right]^2 \qquad 3.3.11$$

According to 3.3.10, a minimum occurs when:

$$\sum_{j=1}^{M} \left(\left[\sum_{i=1}^{N} A_{ij} K_i + 1 - \sigma_0 \rho_j \right] \left[A_{kj} \right] \right) = 0 \qquad \text{for } k = 1 \text{ to } N$$

$$\sum_{j=1}^{M} \left(\left[\sum_{i=1}^{N} A_{ij} K_i + 1 - \sigma_0 \rho_j \right] \left[\rho_j \right] \right) = 0 \qquad 3.3.12$$

These N + 1 equations in the N + 1 unknowns are best solved through the use of matrix notation and operations as follows:

Let $[A]$ be the M-row, N-column matrix of the A_{ji}

$[K]$ be the N-row, 1-column matrix of the K_i

$[\rho]$ be the M-row, 1-column matrix of the

$[S]$ be the M-row, 1-column matrix with all elements = 1.0

Also indicate the transpose of a matrix by the superscript T and an augmented matrix by a vertical or horizontal line separating the original two matrices. It will be necessary to consider $[\sigma_o]$ as a 1-row, 1-column matrix in the equations. Equations 3.3.12 can now be compactly rewritten as:

$$[A^T][A][K] - \sigma_o[A^T][\rho] = -[A^T][S]$$

$$-[\rho^T][A][K] + \sigma_o[\rho^T][\rho] = [\rho^T][S]$$

Combining these two matrix equations by the use of augmented matrices yields:

$$\left[\frac{A^T}{-\rho^T}\right]\left[A \mid -\rho\right]\left[\frac{K}{\sigma_o}\right] = -\left[\frac{A^T}{-\rho^T}\right]\left[S\right] \qquad 3.3.13$$

The solution is obtained by premultiplying both sides of this equation by the inverse of $\left[\frac{A^T}{-\rho^T}\right]\left[A \mid -\rho\right]$ as:

$$\left[\frac{K}{\sigma_o}\right] = -\left(\left[\frac{A^T}{-\rho^T}\right]\left[A \mid -\rho\right]\right)^{-1}\left[\frac{A^T}{-\rho^T}\right]\left[S\right] \qquad 3.3.14$$

Each time a new set of data is obtained this entire set of operations must be repeated, and this is only sensibly possible with the use of a high speed computer. However, many of the operations involved in the limited interpretation problem of assuming the background resistivity and only determining the N values of the K_i can be done without knowledge of the data.

Indeed, in this case it is possible to reduce the entire interpretation procedure to a single matrix multiplication. In this case it is necessary to introduce the parameter B_j defined as:

$$B_j = \left[\frac{\rho_j - \rho_0}{\rho_0}\right] \qquad 3.3.15$$

and also the M-row, 1-column matrix $[B]$ it forms. The analysis by least squares leads to a linear set of equations represented by:

$$[A^T][A][K] = [A^T][B] \qquad 3.3.16$$

The solution is obtained as:

$$[K] = \left([A^T][A]\right)^{-1}[A^T][B]$$

and here it is seen that the matrix $\left([A^T][A]\right)^{-1}[A^T]$ can be considered as an operator which when post-multiplied by the matrix $[B]$ derived from the field data will yield the K_i. Since this operator is independent of the field data it need be computed only once after the geometry and array are chosen and this can be done on a high speed machine.

This limited interpretation problem actually is very useful since an experienced geophysicist can often make a reasonable estimate of the background resistivity and a simple matrix multiplication will yield the entire interpretation of the K_i values. It is this limited problem approach which is to be used by the field personnel when doing preliminary interpretations. The complete problem of actually determining the background is only attempted when the final results of an area's survey are available and need to be interpreted in a more sophisticated manner.

3.4 Formulation of Rectangular Blocks and Resolving Power

The prospecting of an area in many geophysical methods is done with the aid of a grid-work of lines covering the area of interest. This grid is either rectangular or square in the shape of the individual smaller areas which it forms and is utilized so that a systematic investigation of the area may be made. The four-electrode resistivity methods often use such a grid and measurements are made along one of the series of parallel lines thus formed. That is, both the source and receiver circuits are moved along one line at a time, varying both their exact position within the grid and also their relative distances to one another. This procedure allows a profile of the subsurface conditions to be constructed from the interpretation of the measurements.

There are two properties of any geophysical prospecting method other than the physical parameter measured which are of prime importance in determining its specific applications. It is not really possible to completely separate these two properties but there is a difference in what each may accomplish. One is the ability to detect an anomalous subsurface region and the other is the ability to resolve the anomaly into a set of values describing its position, size and physical parameter. The success of much geophysical prospecting is often based only on its ability to detect anomalous regions. There are certain minimum geometries and contrasts which may be detected with any method and hence there is some overlap in these two properties. In general, present day prospecting techniques have no problem detecting anomolous regions since the instrumentation of basic physical concepts to measure different parameters is highly developed. The greatest failure is in the interpretation or resolution of these anomalies, and it is this resolution of resistivity measurements with which this thesis is concerned.

Profiling for the modified Eltran array is operationally quite rapid as the vertical variations of resistivity are measured as the distance between the two circuits is increased while the lateral variations parallel to the line are determined as the entire array is moved along the line. A similar procedure is used for the Wenner array , movement along the line for a given spacing interval sensing the lateral variations to a certain depth. Then by going back over the same line additional times for larger spacing intervals the vertical variations of resistivity are detected.

Although the profiling is in a sense only two-dimensional, it does detect changes in conductivity which occur perpendicular to the profile line. However, it is not possible for the measurements to determine on which side of the profile line these changes occur. The effect of changes either at depth or to one side or another cannot be separated and both possibilities are only capable of interpretation as being associated with a change in conductivity an approximate distance away. Nothing can be said about the distance being to the side or vertical from the single profile line. By profiling over a series of parallel lines however the possibility of the changes occuring laterally can actually be investigated and final interpretations about the subsurface based on this added information.

The ability of surface resistivity measurements to detect changes in subsurface resistivity is dependent upon the actual conductivity contrasts but most strongly upon the geometry of the anomalous region. As the distance of the region from the profile line increases so must the size in order that its effect for measurements along the profile line remain the same. If the scale of the measurements is not large enough they may never detect certain very large anomalous regions which are effectively too far away.

Rectangular blocks which are symmetric about the profile plane have been chosen to form the finite-sized subsurface regions. The choice of regions which are symmetric about the profile plane eliminates the problem associated with the failure of the line measurements to resolve certain geometries as indicated. It forces the variations to be essentially two dimensional for each profile line but by using a series of parallel profile lines the three-dimensional character of the subsurface resistivity may be determined. The ability to predict the vertical and lateral variations along the line of measurements in a quantitative method is an improvement of the empirical method of comparison with known responses for a few geometries and contrasts. While the approximation used to determine the A_{ij} introduced in Section 3.2 is limited in its validity, this approach does represent a direct method of interpretating resistivity data on the basis of two and three dimensional variations of the subsurface conductivity.

3.5 Final Blocks and their Resolution

Model results have indicated that the limits to which the modified Eltran array can detect anomalous regions is approximately 3 units, when using a maximum separation of 5 units between source and receiver. A highly conducting horizontal block (4 units parallel to the line, 1 unit in thickness and 4 units extent on either side of the line) centered at a depth of 3½ units was not detectable in an otherwise homogeneous half-space. That the body was not detected is due to the rapid decrease in the strength of the secondary sources created as the region recedes from the source, proportional to the inverse distance squared. The effect of the block at this depth is masked in the experimental error of approximately 10%. A similar error is anticipated in field operations and thus this block represents the limit of the detection power of the array.

Because of the fixed separation of each set of electrodes in the modified Eltran array the primary field is essentially dipolar and its strength falls off as the inverse distance squared. Also the receiver makes essentially a dipole measurement, hence measures the rate of change of the potential. The primary field is then measured as depending upon the inverse cube of the distance separating source and receiver. The very rapid decrease of response to a subsurface region which is observed in model experiments for finite regions is of great help in resistivity prospecting for near surface features such as usually occurs in mining geophysics. It indicates that only those regions immediately next to the profile line need to be considered as the cause of the observed secondary electrical fields.

Although explicit model results for bodies displaced to either side of the profile line have not been obtained it seems quite reasonable to assume that their maximum detection range is also the 3 units limit. Hence any regions which lie outside these limits need not be considered as being the cause of the secondary fields and in the application of the modified first approximation this will in general be true. It is important to realize that the presence of conducting regions within this limited volume around the profile line will yield a much larger field than similar or even larger regions outside. In a sense this is a shielding phenomenon in which the near surface variations completely dominate the creation and behavior of the secondary fields.

The choice of the dimensions for the final blocks to be used in a finite region interpretation scheme has been strongly influenced by the previous considerations and also various trial sizes. The modified first approximation forward problem response for blocks that extend 3 units on either side of the line are almost identical with blocks that extend 4 units on either side. Thus it appears that the optimum length of the blocks perpendicular to the profile plane is three units since any greater lengths will yield the same effect. Parallel to the line it has been found that the dimensions must not be less than a unit or the resolution of these endividual regions will fail. Finally the vertical dimensions must not be less than a unit unless the regions are close to the surface, in which case they may be approximately ½ unit and still be resolved individually.

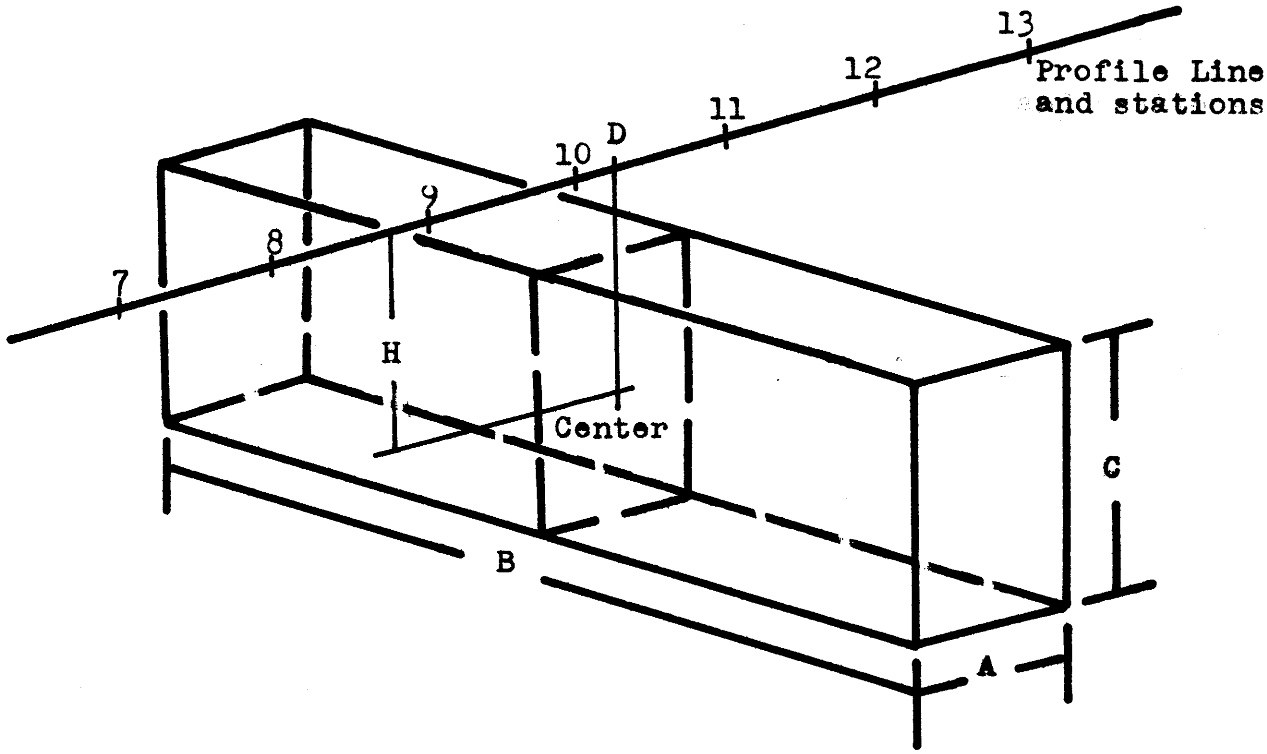
The dimensions of each region must be chosen so that the magnitude effects are approximately equal in order that a stable interpretation operator will be developed. That is, the regions must increase their lateral and/or vertical dimensions as their depth is increased. The type of stability referred to is that slight changes of the resistivity measurements, such as arise in the errors present in the field data will not greatly alter the resistivity interpretation.

There is a difference in the accuracy of the first approximation forward solution when compared to the model results for rectangular blocks of certain geometries. It appears that if the vertical surfaces which bound the block lie directly under a source and/or receiver position along the profile line, the comparisons are less valid for certain source-receiver positions. This no doubt is due to the fact that the interactions between surfaces are important in effecting the final distribution of induced sources in the real system and the first approximation yields zero induced sources on such surfaces for those positions directly above them. Thus, bodies with surfaces which when projected to the profile line intersect it at station positions should be avoided. Basically the idea is to use those blocks for which the first approximation presents a fairly accurate representation of the true response.

It has already been noted that the modified Eltran array presents a system that is symmetric with respect to an interchange of source and receiver circuits. This is extremely useful not only in field operations but also in the interpretation of resistivity measurements since it allows any existing subsurface symmetries to be accurately displayed. For those regions not symmetrical it permits good estimates of the existing orientation to be made since the symmetric measurements will not distort the relative geometry of these regions.

In order to preserve this very useful property of symmetry in the interpretation the blocks as well as the resistivity data measurements should be chosen symmetrically about the center of the composite configuration. Thus if any symmetries in the data actually or approximately occur the subsurface is capable of modelling this geometry satisfactorily. The forced symmetry of the modified first approximation as well as the symmetry in the blocks and data points lead to a consistent development in which the very useful and necessary property of symmetry is preserved.

The diagram on the following page illustrates the final choice of blocks used in developing the interpretation operators. It will be appropriate to consider the group of blocks, their responses and the least squares fitting as being an operator on the resistivity data which directly determines the conductivity variations once the geometry is chosen. Only three blocks have been utilised in forming the different operators and their dimensions and relative positions have been chosen so as to approximately model either mainly horizontal or vertical oriented structures. Each block extends three units on each side of the profile plane consistent with the resolution possible using the modified eltran array. The forward problem solutions for these blocks as determined by the modified first approximation are presented, for which the general format employed in the tabulation of the results is explained on page 75. The values given are the A_{ij} parameters multiplied by 100 so that they represent the % changes from the background resistivity.



Blocks Employed in Operator Formation

(1)	A= 1.0	B= 6.0	C= 0.6	D= 10.0	H= 0.7
(2)	A= 1.0	B= 6.0	C= 2.0	D= 10.0	H= 2.0
(3)	A= 2.0	B= 6.0	C= 1.0	D= 10.5	H= 1.5

Cross Sectional View

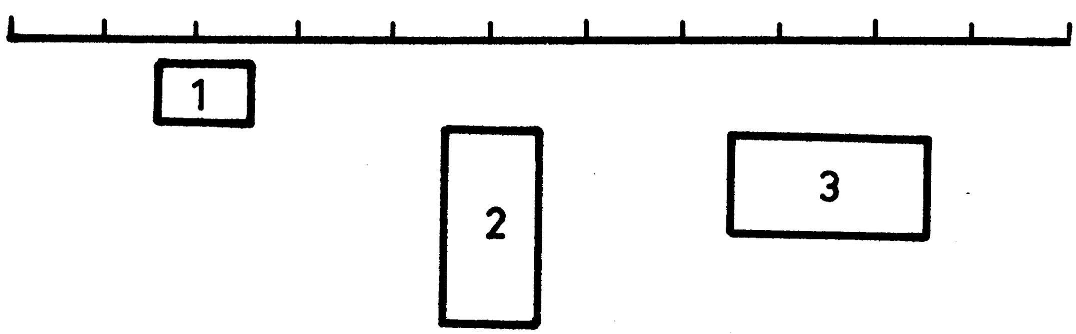


Figure 1 General Diagram of Rectangular Blocks

Format Description

The results of the computer have been programmed to be presented in final semi-graphical form. The results for the dipole-dipole array are plotted in conformance with the format on page 29. The pole-dipole results represent the response of the array for which the source is a point and the receiver a pair of electrodes one unit apart.

On page 77 are presented the results for block (1) centered at $x= 10.0$, $z= 0.70$. All the blocks are symmetrical about the profile plane so that the coordinate of the center is always $y= 0$. The spatial increments define the number of Δx, Δy and Δz increments used in the numerical integration discussed in Appendix I. The dimensions of the block are in the order (A,B,C).

For the dipole-dipole array on page 77, with the source electrodes at stations 6-7 and the receiver electrodes at 10-11, there is an 18% decrease from the background value in apparent resistivity measured. For the pole-dipole array, the effect for a source at station 8 and receiving at 12-13 is a 4% increase in apparent resistivity.

```
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1     2     3     4     5     6     7     8     9    10    11    12    13    14    15    16    17    18    19
*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*

      0     0     0     0     0     0     1    -2   -21   -16    -3    -0    -0    -0    -0    -0    -0
         0     0     0     0     0     2    -5   -18    -2    -9    -4    -1    -0    -0    -0    -0
            0     0     0     1     2    -6   -16     2     2    -5    -4    -1    -0    -0    -0
               0     0     1     3    -8   -15     4     4     3    -3    -3    -1    -0    -0
                  0     1     3    -9   -14     4     4     3     3    -2    -3    -1    -0
POLE-DIPOLE          1     3   -10   -13     5     4     3     3     3    -1    -2    -1


1     2     3     4     5     6     7     8     9    10    11    12    13    14    15    16    17    18    19
*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*

         0     0     0     0     0     1    -1   -23   -23    -1     1     0     0     0     0     0
            0     0     0     0     2    -3   -21    -7   -21    -3     2     0     0     0     0
               0     0     0     2    -4   -18     0     0   -18    -4     2     0     0     0
                  0     0     2    -6   -17     2     4     2   -17    -6     2     0     0
DIPOLE-DIPOLE        1     3    -7   -16     3     4     4     3   -16    -7     3     1
THE DATE IS MAY  11,1959.          THE TIME IS 2116.9
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        0     0     0     0     0     0     0    -0    -7    -7    -2    -0    -0    -0    -0    -0    -0
           0     0     0     0     0     1    -1   -12   -11    -8    -4    -1    -0    -0    -0    -0
              0     0     0     1     1    -2   -15   -10    -7    -7    -4    -1    -0    -0    -0
                 0     0     1     1    -3   -16    -8    -4    -4    -6    -4    -2    -0    -0
                    1     1     1    -3   -16    -6    -1    -1    -2    -5    -3    -2    -1
POLE-DIPOLE            1     1    -3   -17    -5    -0     0     0    -1    -4    -3    -2
```

```
1     2     3     4     5     6     7     8     9     10    11    12    13    14    15    16    17    18    19
*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*

           0     0     0     0     0     0     0    -4    -4     0     0     0     0     0     0     0
              0     0     0     0     1    -0   -10   -12   -10    -0     1     0     0     0     0
                 0     0     0     1    -1   -13   -12   -12   -13    -1     1     0     0     0
                    0     1     1    -2   -14   -11    -8   -11   -14    -2     1     1     0
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THE DATE IS MAY 11,1959.   THE TIME IS 2118.7

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*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*

     0     0     0     0     0     0     1     0    -6   -12    -8    -2    -0    -0    -0    -0    -0
        0     0     0     0     1     1     0   -11   -19   -16   -10    -4    -1    -0    -0    -0
           0     0     0     1     2    -0   -13   -19   -11   -10    -8    -4    -1    -0    -0
              0     0     1     2    -1   -15   -17    -6    -3    -6    -7    -4    -1    -0
                 1     1     2    -1   -15   -16    -3     0    -0    -4    -6    -3    -1
POLE-DIPOLE         2     2    -1   -15   -16    -1     2     3     1    -2    -5    -3


1     2     3     4     5     6     7     8     9     10    11    12    13    14    15    16    17    18    19
*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*

        0     0     0     0     0     0     0    -4    -9    -4     0     0     0     0     0     0
           0     0     0     0     1     0    -9   -20   -20    -9     0     1     0     0     0
              0     0     0     1     0   -12   -21   -19   -21   -12     0     1     0     0
                 0     1     2    -0   -13   -19   -12   -12   -19   -13    -0     2     1
DIPOLE-DIPOLE       1     2    -0   -14   -18    -7    -4    -7   -18   -14    -0     2
THE DATE IS MAY  11,1959.        THE TIME IS 2117.8
```

3.6 Compositing of blocks and Shifting of the Operators

In order to treat general problems of resistivity interpretation, it is necessary to combine blocks of small homogeneous regions so as to form a larger subsurface region of effectively variable conductivity. It is more convenient to discuss the large regions as being mainly horizontal or vertical in the orientation of the blocks and also their associated operators. Different configurations of blocks and the associated resistivity measurements can be constructed and lead to a large wet of combinations. Moreover the actual number of regions and measurements used to form an operator can vary so that a great deal of freedom appears to be available in forming such operators.

Those operators presented here have utilized the blocks of Section 3.4 and have been chosen both with consideration given to the desired resolution power and symmetry property of this interpretation scheme and also the field procedures commonly employed to prospect an area. Previous empirical methods of interpretation have been capable of yielding 3 pieces of information regarding the subsurface conductivity variations:

1) Determining whether features near the surface or deep are causing the anomaly,
2) Approximate lateral extent of the region and
3) Significance of the conductivity contrast

These also represent the basic values which the finite region concept will predict, but with greater detail possible because of the large number of regions used to model the subsurface. Also the relative conductivity contrasts of these regions will be determined on a quantitative basis.

In general, a profile line of measurements using the modified eltran array is made long enough to cover the entire anomaly measured so that its lateral extent may be accurately determined, and also a value assigned to the background resistivity. It is possible to construct operators that use exactly as many resistivity measurements as the field profiles do, but since the length is variable every new line might necessitate construction of a new operator. A solution to this problem of reformulating an entirely new set of regions and data points is to form an operator of a length corresponding to the minimum length line usually employed, and to shift this operator along any longer lines so that there will be some overlap in the regions modelled and the data values used. This shifting of basic operators along lines when combined with the determination of background resistivity provides a highly flexible scheme of interpretation which allows for the examination of different sections of a profile for slightly different geometries. For those operators which overlap in both their data points and blocks modelled, a comparison of those matching blocks and also the background resistivity can be made to determine the correlation of both the geology of the subsurface and the interpretation derived from the field data.

When using the limited interpretation procedure and assuming the background resistivity, the success of shifting the operator along the line may possibly be less. There must be an assumption made regarding the background resistivity and while the complete interpretation problem determines this only from the data, the limited problem must rely upon the experience and ability of the individual doing the interpretation.

Pages 83-87 present the cross-sectional views of the different operators which have been constructed from the basic blocks, and the convention for numbering the blocks and the resistivity measurements. The code used to identify the operators is: 1^{st} digit= number of blocks, 2^{nd}= Horizontal or Vertical structure, L S=least squares and the final digits the number of data points utilized.

4.1 Comparison of Forward Problem Results with Models

A completely quantitative analysis of the error in the modified first approximation forward solution is not possible for all subsurface geometries and conductivity variations. In order to determine the error it is necessary that the exact solution be known so that a comparison can be made and the relative error determined. Belluigi and Maaz (1956) have

```
1         2         3         4         5         6         7         8         9        10
*---------*---------*---------*---------*---------*---------*---------*---------*---------*
                         *****************************************
                         *         *         *         *         *
                         *         *         *         *         *
                         *    1    *    2    *    3    *    4    *
                         *         *         *         *         *
                         *****************************************
                         *                   *                   *
                         *                   *                   *
                         *         5         *         6         *
                         *                   *                   *
                         *                   *                   *
                         *****************************************



1         2         3         4         5         6         7         8         9        10
*---------*---------*---------*---------*---------*---------*---------*---------*---------*

               1         2         3         4         5         6         7

                    8         9        10        11        12        13

                        14                  15                  16
```

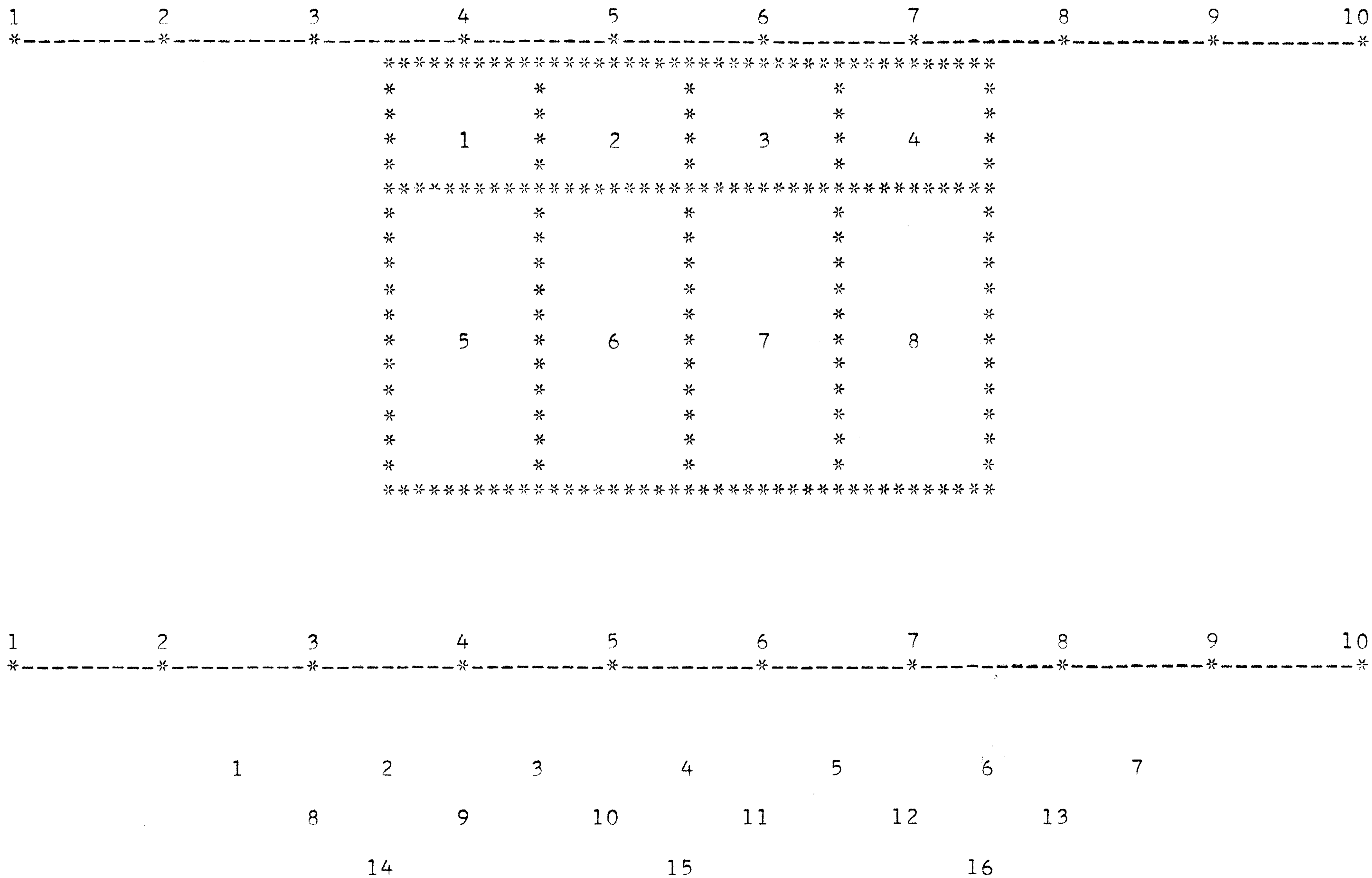
BLOCK AND DATA POINT IDENTIFICATION FOR OPERATOR 6HLS16

```
1         2         3         4         5         6         7         8         9         10
*---------*---------*---------*---------*---------*---------*---------*---------*---------*
               *************************************************************
               *         *         *         *         *         *         *
               *         *         *         *         *         *         *
               *    1    *    2    *    3    *    4    *    5    *    6    *
               *         *         *         *         *         *         *
               *************************************************************
               *                   *                   *                   *
               *                   *                   *                   *
               *         7         *         8         *         9         *
               *                   *                   *                   *
               *                   *                   *                   *
               *************************************************************


1         2         3         4         5         6         7         8         9         10
*---------*---------*---------*---------*---------*---------*---------*---------*---------*

               1         2         3         4         5         6         7

                    8         9        10        11        12        13

                        14                  15                  16
```

BLOCK AND DATA POINT IDENTIFICATION FOR OPERATOR 9HLS16

```
1         2         3         4         5         6         7         8         9         10
*---------*---------*---------*---------*---------*---------*---------*---------*---------*
               *************************************************************
               *         *         *         *         *         *         *
               *         *         *         *         *         *         *
               *    1    *    2    *    3    *    4    *    5    *    6    *
               *         *         *         *         *         *         *
               *************************************************************
               *                   *                   *                   *
               *                   *                   *                   *
               *         7         *         8         *         9         *
               *                   *                   *                   *
               *                   *                   *                   *
               *************************************************************


1         2         3         4         5         6         7         8         9         10
*---------*---------*---------*---------*---------*---------*---------*---------*---------*


               1         2         3         4         5         6         7

                    8         9         10        11        12        13

                        14        15        16        17        18

                             19                            20
```

BLOCK AND DATA POINT IDENTIFICATION FOR OPERATOR 9HLS20



BLOCK AND DATA POINT IDENTIFICATION FOR OPERATOR 8VLS16

```
1         2         3         4         5         6         7         8         9         10
*---------*---------*---------*---------*---------*---------*---------*---------*---------*
                         *****************************************
                         *         *         *         *         *
                         *         *         *         *         *
                         *    1    *    2    *    3    *    4    *
                         *         *         *         *         *
                         *****************************************
                         *         *         *         *         *
                         *         *         *         *         *
                         *         *         *         *         *
                         *         *         *         *         *
                         *         *         *         *         *
                         *    5    *    6    *    7    *    8    *
                         *         *         *         *         *
                         *         *         *         *         *
                         *         *         *         *         *
                         *         *         *         *         *
                         *         *         *         *         *
                         *****************************************
```

```
1         2         3         4         5         6         7         8         9         10
*---------*---------*---------*---------*---------*---------*---------*---------*---------*


              1         2         3         4         5         6         7

                   8         9         10        11        12        13

                       14        15        16        17        18

                           19                            20
```

BLOCK AND DATA POINT IDENTIFICATION FOR OPERATOR 8VLS20 critically examined the first approximation of Stevenson for an exponential variation of conductivity with depth. They have shown that only the first term in the series expansion of the exponential is exactly prdeicted in the Stevenson method. This particular example might seem to indicate that any first approximation solution and derived interpretation procedure would not be very accurate. However, this turns out not to be true when considering finite regions of conductivity variation.

It should be noted that when only vertical variations of conductivity occur, the modified first approximation reduces exactly to the approximation of Stevenson. For rectangular regions exact solutions are not possible and recourse must be made to the results of model experiments. The use of model results will allow a comparison of not only the forward problem solutions but also the interpretations obtained based upon the modified first approximation. In this section a comparison of the forward problem solutions and model results for a number of different geometries will be made.

Previous methods of interpretation of the apparent resistivity data obtained from the modified eltran array have utilized contours of the data on a logarithmic scale. The shape of the contours has been found to be very characteristic of the subsurface structure and a fair degree of accuracy has been possible in predicting the lateral extent of anomalous regions. However, the depth determination was rather qualitative and unless the anomaly was rather 'sharp' so that interpretation was straightforward little could definitely be said about it. The interpretation was accomplished by personnel familiar with the method of obtaining and contour plotting the profile data and model results were often used as a guide and reference for the final interpretation.

The forward problem solutions are based upon the assumption that the induced sources are bounded in magnitude and that for a finite jump in the conductivity they are determined from:

$$K_i = 2\left[\frac{\rho_o - \rho_i}{\rho_o + \rho_i}\right] \qquad 4.1.1$$

which is exactly the factor inferred to be correct in section 2.7. The solutions for the rectangular homogeneous blocks used in the modelling and also for those blocks used in forming the composite interpretation operators of this thesis were obtained by numerical integration of :

$$\bar{\varphi}(\alpha|\xi) = \frac{1}{2}\int \frac{\nabla\sigma}{\sigma}\cdot\left[\nabla\varphi_o(\alpha) + \nabla\varphi_o(\xi)\right] G \, dVol_P \qquad 4.1.2$$

This was done on an electronic digital computer by considering the surface of the block to be divided into a number of small areas and an equivalent induced point source placed at the center of each area. That only the surfaces of the regions need be considered is due to the fact that the conductivity gradient is zero elsewhere. The complete mathematical formulation of this phase of the forward problem is presented in Appendix I.

The exact comparison of the apparent resistivity profiles obtained from the modelling with that predicted by the first approximation is not possible because of experimental errors in the actual model data. The most striking feature being that the experimental results were seldom symmetrical even though the geometry of the block and array was such that they should be. This is interpretated to indicate that the positioning of the block relative to the surface electrodes was incorrect. The use of thedipole-dipole array which measures a second derivative of the primary and secondary dipole fields makes the results very sensitive to relative location of surface electrodes and blocks. Any error in the positioning of the electrodes would then be greatly magnified by the use of this array. However, if the results are averaged so as to empirically determine a set of symmetrical values a numerical comparison can be effected.

Good agreement of the contoured model results and the modified first approximation solutions have been obtained. The general character of the predicted solutions and the relative positioning of the contour levels in both horizontal and vertical blocks has been satisfactory. Moreover, the actual magnitude values of apparent resistivity predicted are in approximate agreement with the model results altered as indicated above. Four particular geometries have been chosen to be presented in this section as representative of the results of the modified approximation solutions. The format for their presentation closely parallels that of page 76 but the values given are apparent resistivity values assuming a background value of 450, which is the value assigned to the background in the modeling. The conductivity contrast of the model blocks to background has been very high or close to being 'saturated' so that the source strengths K_1 have been set equal to 2.0. The presentation of pole-dipole results is made since extremely little computational effort is required to obtain them beyond that required to produce the dipole-dipole results and some investigators have used this electrode array in field operations. It is not necessary to give the dimensions of the resistivity values since it is possible to assign the values proportional to the desired units of measurement. This is also to be done with the linear dimensions of the array and subsurface region. Effectively the results are presented in a set of dimensionless variables which are only defined when the dimensions of both the resistivity and the electrode spacing are given.

In order that a comparison be made of the actual numerical values a profile for each geometry has been derived from the model data but only for the dipole-dipole measurements which are the immediate concern of this thesis. As already indicated the numerical values of the actual model data are obtained by forcing symmetry in the model values. The diagram on the following page indicates the geometry of the four blocks whose results are presented in this section and a label that is indicative of the orientation and size of the block. The first character in the label (H or V) indicates whether the block is vertical or horizontal, the second digit the thickness of the block in units of electrode spacing and the third digit the depth to the top surface of the block in the units previously defined. This convenient manner of identifying the blocks has been in all the model work of Adler (1958). All the model blocks were 4 units long on either side of the profile plane. The conductivity contrasts between background and the block are indicated in the parenthesized ratio ie (1/200).

The following pages present the forward problem solutions and the model results for comparison of numerical values. These blcoks represent rather different geometries and hence allow an evaluation of the modified first approximation over

```
PROFILE LINE   9   10   11   12   13   14   15   16   17   SURFACE
*----*----*----*----*----*----*----*----*----*----*----*----*----*


                              ******
                              *    *
                              *    *
                              *    *
                              *    *
                              *    *
                              *    *             V-1-1  (1/150)
                              *    *
                              *    *
                              *    *
                              *    *
                              *    *
                              ******

PROFILE LINE   9   10   11   12   13   14   15   16   17   SURFACE
*----*----*----*----*----*----*----*----*----*----*----*----*----*


                           ************
                           *          *
                           *          *
                           *          *
                           *          *
                           *          *
                           *          *          V-2-1  (1/215)
                           *          *
                           *          *
                           *          *
                           *          *
                           *          *
                           ************

PROFILE LINE   9   10   11   12   13   14   15   16   17   SURFACE
*----*----*----*----*----*----*----*----*----*----*----*----*----*


                      **********************
                      *                    *     H-1-1  (1/200)
                      **********************

PROFILE LINE   9   10   11   12   13   14   15   16   17   SURFACE
*----*----*----*----*----*----*----*----*----*----*----*----*----*


                      **********************
                      *                    *
                      *                    *
                      *                    *     H-2-2  (1/155)
                      *                    *
                      **********************
```

CROSS-SECTIONAL VIEW OF MODEL BLOCKS IN PROFILE PLANE

PROBLEM M-487 NESS   SPATIAL INCREMENTS (2,16, 8)   DIMENSIONS (1.00,8.00,4.00)   CENTER AT 10.50,3.00

NUMBER OF TERMS= 1   SCALE FACTOR=450.0   CONTRAST= 2.00

```
1      2      3      4      5      6      7      8      9      10     11     12     13     14     15     16     17     18     19
*------*------*------*------*------*------*------*------*------*------*------*------*------*------*------*------*------*------*

    450    450    450    451    452    454    457    459    417    368    405    433    443    446    448    449    449
       450    451    452    454    458    463    459    380    320    344    382    420    437    443    446    448
          452    454    457    461    466    455    354    308    355    358    378    413    432    441    445
             455    459    464    467    451    336    305    373    389    376    381    411    430    439
                460    466    467    446    324    305    388    410    411    392    387    411    429
POLE-DIPOLE        467    467    443    315    306    399    425    432    427    404    394    413
```

```
1      2      3      4      5      6      7      8      9      10     11     12     13     14     15     16     17     18     19
*------*------*------*------*------*------*------*------*------*------*------*------*------*------*------*------*------*------*

          450    450    450    450    451    454    459    435    392    435    459    454    451    450    450    450
             450    451    452    454    460    462    406    333    333    406    462    460    454    452    451
                452    453    457    464    462    380    312    329    312    380    462    464    457    453
                   455    460    467    459    360    305    345    345    305    360    459    467    460
DIPOLE-DIPOLE         462    468    455    345    302    360    374    360    302    345    455    468
```

THE DATE IS MAY 11,1959.   THE TIME IS 2123.8

```
PROBLEM M-487  NESS    SPATIAL INCREMENTS ( 4,16, 8)    DIMENSIONS (2.00,8.00,4.00)   CENTER AT 10.50,3.00
                       NUMBER OF TERMS=  1               SCALE FACTOR=450.0              CONTRAST=  2.00

1     2     3     4     5     6     7     8     9     10    11    12    13    14    15    16    17    18    19
*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*

   450   450   451   452   454   458   464   457   379   313   355   412   435   443   446   448   449
      451   453   455   459   467   474   449   314   213   243   313   385   421   436   443   446
         455   458   464   473   478   437   273   189   249   272   309   373   412   431   440
            461   468   478   479   425   247   183   284   321   307   319   370   408   428
               472   482   478   416   229   182   311   366   368   337   333   371   406
POLE-DIPOLE       484   477   409   217   182   333   396   412   400   362   346   375


1     2     3     4     5     6     7     8     9     10    11    12    13    14    15    16    17    18    19
*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*

      450   450   450   451   454   460   461   411   363   411   461   460   454   451   450   450
         451   452   454   460   470   461   354   237   237   354   461   470   460   454   452
            454   458   466   477   453   313   199   197   199   313   453   477   466   458
               461   471   481   444   282   185   229   229   185   282   444   481   471
DIPOLE-DIPOLE     476   483   434   260   180   258   292   258   180   260   434   483
THE DATE IS MAY  11,1959.        THE TIME IS 2122.0
```

```
PROBLEM M-487  NESS    SPATIAL INCREMENTS ( 8,16, 2)    DIMENSIONS (4.00,8.00,1.00)    CENTER AT 10.50,1.50
                       NUMBER OF TERMS=  1              SCALE FACTOR=450.0             CONTRAST=  2.00

1     2     3     4     5     6     7     8     9     10    11    12    13    14    15    16    17    18    19
*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*

    450   450   451   452   456   462   460   398   339   313   308   366   421   440   446   448   449
       451   452   455   462   471   460   359   269   194   175   262   342   405   432   442   446
          454   458   467   476   457   340   267   210   172   254   306   347   399   428   440
             461   471   479   456   330   273   245   243   319   338   342   359   399   426
                474   481   455   324   277   266   291   405   406   382   367   370   401
POLE-DIPOLE          482   455   320   281   280   314   451   476   441   406   384   380


1     2     3     4     5     6     7     8     9     10    11    12    13    14    15    16    17    18    19
*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*

        450   450   451   452   457   461   418   374   373   374   418   461   457   452   451   450
           451   452   457   465   462   377   271   178   178   271   377   462   465   457   452
              454   461   471   460   355   258   157    65   157   258   355   460   471   461
                 464   476   458   343   263   202   147   147   202   263   343   458   476
DIPOLE-DIPOLE         479   455   335   269   233   231   290   231   233   269   335   455
THE DATE IS MAY  11,1959.          THE TIME IS 2128.2
```

PROBLEM M-487 NESS SPATIAL INCREMENTS (8,16, 4) DIMENSIONS (4.00,8.00,2.00) CENTER AT 10.50,3.00

NUMBER OF TERMS= 1 SCALE FACTOR=450.0 CONTRAST= 2.00

```
1     2     3     4     5     6     7     8     9     10    11    12    13    14    15    16    17    18    19
*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*

   450   450   451   452   454   456   454   445   432   422   419   426   436   442   446   448   448
      452   453   455   459   461   455   430   396   369   358   373   398   421   434   442   445
         455   458   463   464   452   414   366   324   304   319   351   384   411   429   438
            461   466   466   448   401   347   304   282   294   322   351   380   407   425
               468   467   445   391   336   297   284   303   324   341   360   382   405
POLE-DIPOLE       467   441   384   329   296   294   325   350   358   362   371   386
```

```
1     2     3     4     5     6     7     8     9     10    11    12    13    14    15    16    17    18    19
*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*

      450   450   451   451   453   454   452   450   449   450   452   454   453   451   451   450
         451   452   455   458   457   446   426   413   413   426   446   457   458   455   452
            454   458   462   458   434   395   356   337   356   395   434   458   462   458
               461   464   456   421   370   317   278   278   317   370   421   456   464
DIPOLE-DIPOLE     466   453   410   353   299   259   247   259   299   353   410   453
```

THE DATE IS MAY 11,1959. THE TIME IS 2126.3

```
1     2     3     4     5     6     7     8     9    10    11    12    13    14    15    16    17    18    19
*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*

   444   438   438   430   437   444   451   400   308   400   451   444   437   430   438   438
      447   447   439   448   462   483   369   205   205   369   483   462   448   439   447
         455   451   455   476   488   363   181   142   181   363   488   476   455   451
            448   484   491   509   347   201   162   162   201   347   509   491   484
```

MODEL V-1-1 (1/150)

```
1     2     3     4     5     6     7     8     9    10    11    12    13    14    15    16    17    18    19
*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*

   421   419   434   427   437   446   435   375   313   375   435   446   437   427   434   419
      455   468   465   474   510   474   298   176   176   298   474   510   474   465   468
         451   467   479   509   466   275   141   125   141   275   466   509   479   467
            482   547   546   504   272   185   155   155   185   272   504   546   547
```

MODEL V-2-1 (1/215)

```
1      2      3      4      5      6      7      8      9     10     11     12     13     14     15     16     17     18     19
*------*------*------*------*------*------*------*------*------*------*------*------*------*------*------*------*------*------*

          436    428    434    435    445    445    363    350    334    350    363    445    445    435    434    428

             445    445    449    469    448    289    186    209    209    186    289    448    469    449    445

                447    458    476    442    255    130     77    124     77     13    255    442    476    458

                   471    519    470    256    129    105    180    180    105    129    256    470    519
```

MODEL H-1-1 (1/200)

```
1      2      3      4      5      6      7      8      9     10     11     12     13     14     15     16     17     18     19
*------*------*------*------*------*------*------*------*------*------*------*------*------*------*------*------*------*------*

          457    445    445    443    445    440    439    448    439    448    439    440    445    443    445    445

             450    450    450    450    443    429    406    411    411    406    429    443    450    450    450

                455    478    474    458    418    372    346    351    346    372    418    458    474    478

                         486    446    396    328    296    282    282    296    328    396    446    486
```

MODEL H-2-2 (1/155)

PROBLEM M-487 NESS   SPATIAL INCREMENTS (4,16, 2)   DIMENSIONS (2.00,8.00,1.00)   CENTER AT 10.50,1.50

NUMBER OF TERMS= 1   SCALE FACTOR=100.0   CONTRAST= 1.00

```
1     2     3     4     5     6     7     8     9     10    11    12    13    14    15    16    17    18    19
*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*

      0     0     0     0     0     0     1     0    -6   -12    -8    -2    -0    -0    -0    -0    -0
         0     0     0     0     1     1     0   -11   -20   -16   -10    -4    -1    -0    -0    -0
            0     0     0     1     2    -0   -14   -19   -12   -11    -9    -4    -1    -0    -0
               0     1     1     2    -1   -15   -18    -7    -4    -7    -7    -4    -2    -0
                  1     2     2    -1   -16   -17    -4    -0    -0    -4    -6    -4    -2
POLE-DIPOLE          2     2    -1   -16   -17    -2     2     2     0    -3    -5    -3
```

```
1     2     3     4     5     6     7     8     9     10    11    12    13    14    15    16    17    18    19
*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*-----*

         0     0     0     0     0     0     0    -4    -9    -4     0     0     0     0     0     0
            0     0     0     0     1     0    -9   -20   -20    -9     0     1     0     0     0
               0     0     1     1     0   -12   -21   -20   -21   -12     0     1     1     0
                  0     1     2    -0   -14   -20   -13   -13   -20   -14    -0     2     1
DIPOLE-DIPOLE        1     2    -0   -15   -19    -8    -5    -8   -19   -15    -0     2
```

THE DATE IS MAY 11,1959.   THE TIME IS 2119.8

rather wide limits. In general the results are good, the best obtained for the deepest block with the least interactions of induced sources, the H-2-2 model. The poorest are for the V-1-1 model, which shows that the approximation is not good when the vertical surfaces of the rectangular regions project to the profile line at an electrode position. This failure is associated with the relative importance of the interactions of induced sources for the particular geometry. This is the reason discussed in section 3.5 why blocks whose surfaces do not project to an electrode position have been chosen for the interpretation operators.

The last page of this set of results is the predicted forward solution for a block corresponding to one of those used in the operators of section 3.5. However, the length of the block perpendicular to the profile plane was 4 units on either side rather than 3. The results are hardly different and certainly within the experimental accuracy of the 5-10% of field operations. This points out the reason for using blocks in the interpretation operators which extended 3 units on each side. It is essentially a numerical demonstration of the resolution limits of both the pole-dipole and dipole-dipole apparent resistivity measurements.

4.2 Prediction of IP Effects from Model and 1st Approximation Results

It has been possible to obtain a reasonable estimate of the apparent resistivity profile using the modified first approximation. As mentioned being closely associated with the resistivity prospecting method is the recently developed method of induced polarization. The IP effect depends upon the change in conductivity of earth material for a change in the frequency of the source current. Thus it should be possible to predict the induced polarization effects by calculating the apparent resistivity profile for two different values of the conductivities of the regions causing the polarization phenomena. The normalized difference in apparent resistivity $\Delta\rho_{Aj}/\rho_{Aj}$ will then be a measure of the polarization properties of the subsurface region. It is not the intent of this thesis to discuss the relative merits of the frequency or time methods of prospecting and interpreting the IP effects.

As pointed out in section 1.8 the IP effect in the frequency domain can be measured as a decrease in ρ_A as the frequency is increased. However, the % change in ρ_A from low to high frequencies when normalized by the background value may be very small and within the experimental error so that accurate determinations of $\Delta\rho_A$ are not possible. For example, with a background resistivity of 100 and a measured ρ_A of 5, a 25% decrease in ρ_A from 5 to 4 would only correspond to a 1% change when normalized by the background. This 1% is what any model experiment would have to measure in order to determine the IP effects. Thus the 5°-10% errors of model experiments may often be much too large to use for the accurate prediction of $\Delta\rho_{Aj}/\rho_{Aj}$.

This section will indicate how model results can be combined with the approximate results to predict the magnitude of the IP effects. The non-linear behavior of the final current flow may be important because of strong interactions of induced sources. In these cases the linear approximation prediction of ρ_A would be less accurate than the model results even considering the experimental error. The approach will be to use the model results to indicate the low frequency apparent resistivity profile and then to utilize the approximate results for the same geometry to calculate the change in apparent resistivity as the frequency and effectively the conductivity of the regions is increased. It may be necessary to symmetrize the model results when the geometry dictates that this should be the case.

The assumption is made that the low frequency values have been obtained for the ρ_A profile for a certain geometry and conductivity contrast. It is shown in Appendix III that the apparent resistivity is a homogeneous function of the degree 1 in the specific resistivities. This means that if all the specific resistivities are multiplied by the scalar value t then the apparent resistivities will be multiplied by t also. That is, letting ρ_i represent the specific resistivity of the i^{th} region

$$\rho_A(t\rho_i) = t\,\rho_A(\rho_i) \qquad 4.2.1$$

As a result of this homogeneity of ρ_A the following equation can be written for N + 1 regions, assuming ρ_0 the background:

$$\rho_{Aj} = \sum_{i=0}^{N} \rho_i \frac{\partial \rho_{Aj}}{\partial \rho_i} \qquad 4.2.2$$

This relates the specific resistivities to the apparent resistivities by their effect on each ρ_{Aj} measurement. This equation is exact but relates the apparent resistivity to the specific resistivities.

It is desired to obtain an exact relation between the change in ρ_{Aj} to the change in ρ_i . This can be derived as follows: take the total differential of 4.2.2 and obtain:

$$d\rho_{Aj} = \sum_{i=0}^{N} \left[d\rho_i \frac{\partial \rho_{Aj}}{\partial \rho_i} + \rho_i d\left(\frac{\partial \rho_{Aj}}{\partial \rho_i}\right)\right] \qquad 4.2.3$$

thus

$$d\rho_{Aj} = \sum_{i=0}^{N} \left[d\rho_i \frac{\partial \rho_{Aj}}{\partial \rho_i} + \rho_i \sum_{k=0}^{N} \left(\frac{\partial^2 \rho_{Aj}}{\partial \rho_i \partial \rho_k}\right) d\rho_k \right]$$

Differentiating the exact expression 4.2.2 with respect to ρ_k and holding the remaining ρ_i constant then

$$\frac{\partial \rho_{Aj}}{\partial \rho_k} = \frac{\partial \rho_{Aj}}{\partial \rho_k} + \sum_{i=0}^{N} \rho_i \left(\frac{\partial^2 \rho_{Aj}}{\partial \rho_i \partial \rho_k}\right) \qquad 4.2.4$$

This implies however that:

$$\sum_{i=0}^{N} \rho_i \left(\frac{\partial^2 \rho_{Aj}}{\partial \rho_i \, \partial \rho_k} \right) = 0 \qquad 4.2.5$$

and thus interchanging the summations in 4.2.3 and using 4.2.5 the exact relation desired is derived:

$$\partial \rho_{Aj} = \sum_{i=0}^{N} \frac{\partial \rho_{Aj}}{\partial \rho_i} \, d\rho_i \qquad 4.2.6$$

This equation will be useful for small changes in the as:

$$\Delta \rho_{Aj} \approx \sum_{i=0}^{N} \frac{\partial \rho_{Aj}}{\partial \rho_i} \, \Delta \rho_i \qquad 4.2.7$$

In order to utilize this equation it is necessary to determine $\left(\frac{\partial \rho_{Aj}}{\partial \rho_i} \right)$ from the modified first approximation forward problem solutions.

Instead of using the logarithym of the conductivity contrast between the region and background for K_i, the definition of K_i in Eq. 4.1.1 is assumed to be correct for the induced source strength. Thus it is possible to write:

$$\frac{\partial \rho_{Aj}}{\partial \rho_i} = \left(\frac{\partial K_i}{\partial \rho_i} \right) \left(\frac{\partial \rho_{Aj}}{\partial K_i} \right) \qquad 4.2.8$$

and using Eq. 4.1.1 this is equal to:

$$\frac{\partial \rho_{Aj}}{\partial \rho_i} = \frac{-4 \rho_0}{[\rho_0 + \rho_i]^2} \left(\frac{\partial \rho_{Aj}}{\partial K_i} \right) \qquad 4.2.9$$

Now the values A_{ij} introduced in Section 3.3 represented the normalized effects on the j^{th} apparent resistivity measurement due to the i^{th} region and thus can be interpreted as

$$\frac{1}{\rho_o}\left(\frac{\partial \rho_{Aj}}{\partial K_i}\right) \approx A_{ij} \qquad 4.2.10$$

This will determine the values of A_{ij} for $i = 1$ to N but an interpretation and evaluation of A_{oj} must be made.

Recalling that the linear approximation effectively ignores the presence of other regions when computing the effect for a particular region, consider the case of a homogeneous region with no conductivity contrasts so that $N = 0$. Then in this case from 4.2.2

$$\rho_{Aj} = \rho_o \frac{\partial \rho_{Aj}}{\partial \rho_o} \equiv \rho_o$$

but this must also be given by:

$$\rho_{Aj} = -\left(\frac{\partial \rho_{Aj}}{\partial K_i}\right)$$

and these two equations lead to:

$$\frac{1}{\rho_o}\left(\frac{\partial \rho_{Aj}}{\partial K_i}\right) = -1 = A_{oj} \qquad 4.2.11$$

Notice that this last result is valid only for the first approximation in which the linearity of the solution in the regions is correct.

Thus with a knowledge of the modified first approximation factors A_{ij} the change in ρ_{Aj} can be computed from a change in the ρ_i as:

$$\Delta\rho_{Aj} \simeq \sum_{i=0}^{N} \left[\frac{-4\rho_0^2}{(\rho_0+\rho_i)^2} A_{ij} \right] \Delta\rho_i \qquad 4.2.12$$

This result is valid only when the change in specific resistivity is small as indicated in equation 4.2.7. Finally the high frequency ρ_{Aj}^{H} is determined from:

$$\rho_{Aj}^{H} = \rho_{Aj}^{L} + \Delta\rho_{Aj} \qquad 4.2.13$$

where ρ_{Aj}^{L} represents the model apparent resistivity measurement.

It is very important to note that in 4.2.12 when $\rho_0 >> \rho_i$ then the equation approaches:

$$\Delta\rho_{Aj} \simeq \sum_{i=0}^{N} \left[-4 A_{ij} \Delta\rho_i \right]$$

and this indicates that even when the conductivity contrasts between ρ_0, ρ_i for low frequency measurements are very large, the high frequency measurements can still detect the changes in the ρ_i . This is true provided that the A_{ij} parameters are sufficiently large, which implies that the region to be sampled for polarizability must be capable of detection in a normal resistivity survey even if its contrast or strength factor is not great. Eq. 4.2.14 explains what has always been one of the very powerful but previously inexplicable properties of IP prospecting: regardless of the existing low frequency conductivity contrasts, the high frequency measurements always have been capable of detecting and resolving a polarized subsurface region. This property has proven to be one of the most fortunate attributes that has been essentially built right into the IP prospecting method.

4.3 Interpretation Results for Model Data

The initial attempt at interpretation of resistivity profiles was made on model data for rectangular blocks, including those models used in section 4.1 for comparison of the forward problem solution. This section presents some of the results of the interpretation operators using apparent resistivity data from model experiments. The method of presentation of the results has been to graphically represent the profile plane and the outlines of the rectangular blocks and to insert within each block the numerical value of the determined K_i value. In addition the actual data used in the interpretation have been reproduced in the familiar two-dimensional plot introduced in section 1.10.

The labels are for the most part self-explanatory, but one important point is that the stations indicated on the profile lines go from 1 to 10 for the data utilized. Thus if the label states that the stations used were 7/16 then 7 in the model corresponds to 1 in the plot and 8 to 2 etc.. In all the model experiments the blocks were centered about station 12.5 so that data for stations 8/17 are symmetrical as are the interpreted results while those sets of data for stations 7/16 represent a shifting by one unit to the left of the model data. The RMS ERROR represents the fractional root mean square error of the apparent resistivity fit: that is

$$\text{RMS ERROR} = \frac{1}{\rho_0}\sqrt{\frac{1}{M}\sum_{j=1}^{M}(\rho_{Aj}-\rho_j)^2} \qquad 4.3.1$$

where ρ_{Aj} is predicted and ρ_j measured.

A plot of the factor K_i in Equation 4.1.1 is presented on the following page as a function of (σ_i/σ_0) and (ρ_i/ρ_0). K_i is positive for a conducting region relative to background and negative for a resistive region. The following pages present the results of the operators 6HLS16, 9HLS16 and 8VLS16. The model results treated by these operators have all had background values of 450 and the operators have had to determine a background value from the 16 data points given. The results for the most part are strikingly good for example the 9HLS16 result for the H-2-1A (1/13) model on page 121 and the 8VLS16 result for the V-2-1A (1/3.8) model. As anticipated the best results occur for blocks with small conductivity

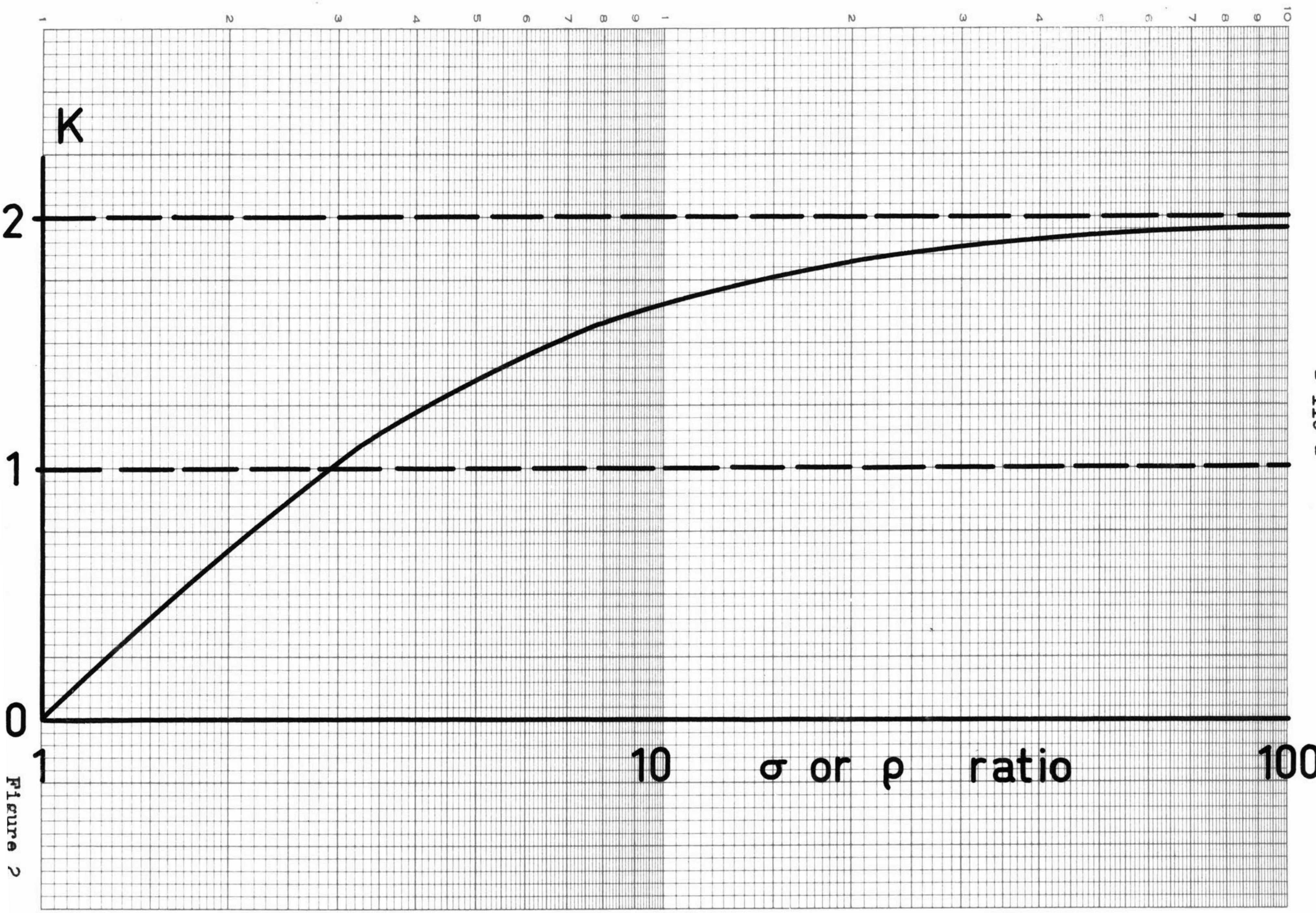


Figure 2

```
2         3         4         5         6         7         8         9
*---------*---------*---------*---------*---------*---------*---------*
               *****************************************
               *         *         *         *         *
               *         *         *         *         *
               *  -0.47  *   0.07  *   0.07  *  -0.47  *
               *         *         *         *         *
               *****************************************
               *                   *                   *
               *                   *                   *
               *                   *                   *
               *        2.31       *        2.31       *
               *                   *                   *
               *                   *                   *
               *****************************************


2         3         4         5         6         7         8         9
*---------*---------*---------*---------*---------*---------*---------*

    445       360       350       330       350       360       445

         290       186       210       210       186       290

              130                 120                 130
```

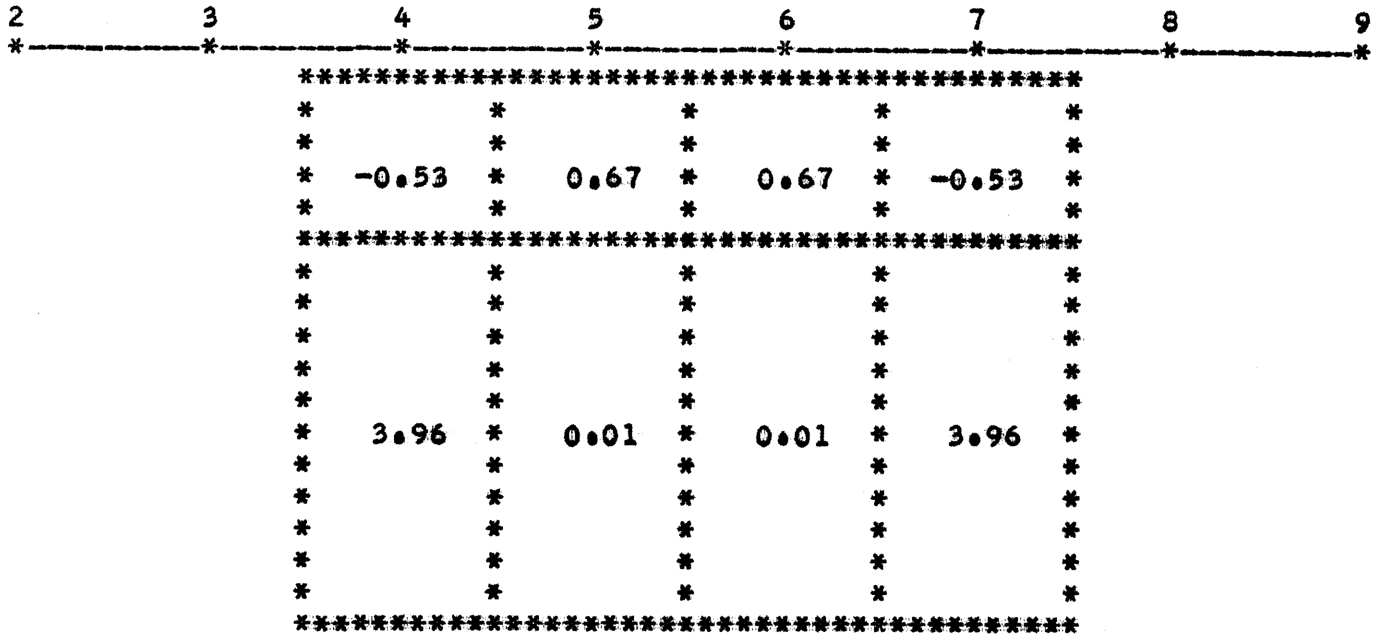
LINE NO= H-1-1A (1/200) BACKGROUND= 388

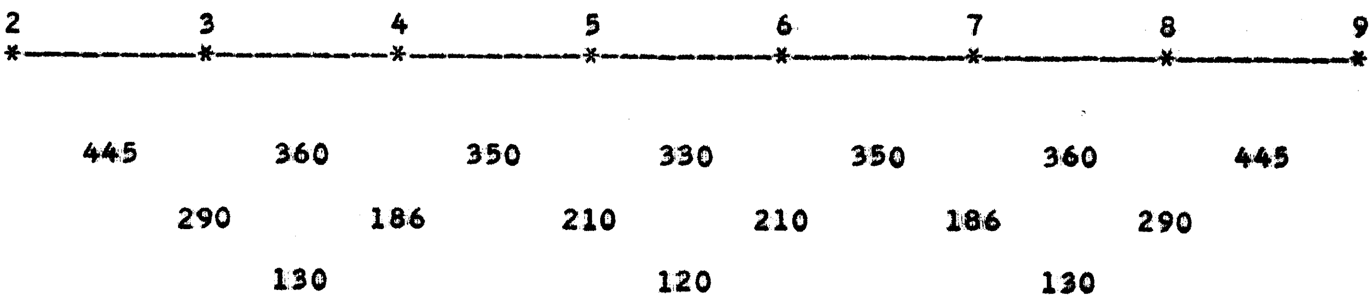
STATIONS= 8/17 OPERATOR= 6HLS16

LOCATION= MODEL RMS ERROR= 0.11 DATE= 3/23/59

```
2         3         4         5         6         7         8         9
*---------*---------*---------*---------*---------*---------*---------*
     *************************************************************
     *         *         *         *         *         *         *
     *         *         *         *         *         *         *
     *  -0.02  *  -0.04  *   0.47  *   0.47  *  -0.04  *  -0.02  *
     *         *         *         *         *         *         *
     *************************************************************
     *                   *                   *                   *
     *                   *                   *                   *
     *                   *                   *                   *
     *       2.16        *       1.27        *       2.16        *
     *                   *                   *                   *
     *                   *                   *                   *
     *************************************************************


2         3         4         5         6         7         8         9
*---------*---------*---------*---------*---------*---------*---------*

    445       360       350       330       350       360       445

         290       186       210       210       186       290

              130                 120                 130
```

```
LINE  NO=  H-1-1A  (1/200)                          BACKGROUND=   471
STATIONS=  8/17                                     OPERATOR=  9HLS16
LOCATION=  MODEL            RMS ERROR=  0.04        DATE=     3/23/59
```





LINE NO= H-1-1A (1/200) BACKGROUND= 418

STATIONS= 8/17 OPERATOR= 8VLS16

LOCATION= MODEL RMS ERROR= 0.07 DATE= 3/23/59

```
2         3         4         5         6         7         8         9
*---------*---------*---------*---------*---------*---------*---------*
               *****************************************
               *         *         *         *         *
               *         *         *         *         *
               *  -0.52  *  -0.18  *   0.11  *  -0.05  *
               *         *         *         *         *
               *****************************************
               *                   *                   *
               *                   *                   *
               *                   *                   *
               *        1.68       *        2.87       *
               *                   *                   *
               *                   *                   *
               *****************************************


2         3         4         5         6         7         8         9
*---------*---------*---------*---------*---------*---------*---------*

    450       445       360       350       330       350       360
         447       290       186       210       210       186
              255                  75                  75

LINE  NO=  H-1-1A  (1/200)                             BACKGROUND=   408
STATIONS=  7/16                                        OPERATOR=  6HLS16
LOCATION=  MODEL             RMS ERROR=  0.12          DATE=     3/23/59
```

```
2         3         4         5         6         7         8         9
*---------*---------*---------*---------*---------*---------*---------*
     **************************************************************
     *         *         *         *         *         *         *
     *         *         *         *         *         *         *
     *  -0.18  *  -0.02  *   0.00  *   0.11  *  -0.01  *  -0.12  *
     *         *         *         *         *         *         *
     **************************************************************
     *                   *                   *                   *
     *                   *                   *                   *
     *                   *                   *                   *
     *        0.75       *        2.15       *        2.46       *
     *                   *                   *                   *
     *                   *                   *                   *
     **************************************************************


2         3         4         5         6         7         8         9
*---------*---------*---------*---------*---------*---------*---------*

    450       445       360       350       330       350       360

         447       290       186       210       210       186

              255                 75                  75


LINE  NO=  H-1-1A  (1/200)                             BACKGROUND=   440

STATIONS=  7/16                                        OPERATOR=  9HLS16

LOCATION=  MODEL              RMS ERROR=  0.08         DATE=     3/23/59
```

```
2         3         4         5         6         7         8         9
*---------*---------*---------*---------*---------*---------*---------*
              *****************************************
              *         *         *         *         *
              *         *         *         *         *
              *  0.25   *  -0.77  *  1.33   *  -0.87  *
              *         *         *         *         *
              *****************************************
              *         *         *         *         *
              *         *         *         *         *
              *         *         *         *         *
              *         *         *         *         *
              *         *         *         *         *
              * -0.87   *  5.24   * -3.20   *  6.55   *
              *         *         *         *         *
              *         *         *         *         *
              *         *         *         *         *
              *         *         *         *         *
              *         *         *         *         *
              *****************************************

2         3         4         5         6         7         8         9
*---------*---------*---------*---------*---------*---------*---------*

     450       445       360       350       330       350       360

          447       290       186       210       210       186

               255                 75                  75
```

LINE NO= H-1-1A (1/200) BACKGROUND= 416

STATIONS= 7/16 OPERATOR= 8VLS16

LOCATION= MODEL RMS ERROR= 0.09 DATE= 3/23/59

```
2         3         4         5         6         7         8         9
*---------*---------*---------*---------*---------*---------*---------*
               *****************************************
               *         *         *         *         *
               *         *         *         *         *
               *  -0.31  *  0.05   *  0.05   *  -0.31  *
               *         *         *         *         *
               *****************************************
               *                   *                   *
               *                   *                   *
               *                   *                   *
               *        2.13       *        2.13       *
               *                   *                   *
               *                   *                   *
               *****************************************


2         3         4         5         6         7         8         9
*---------*---------*---------*---------*---------*---------*---------*

    450       375       363       360       363       375       450

         320       215       195       195       215       320

              180                 135                 180
```

```
LINE  NO=  H-2-1A  (1/13)                              BACKGROUND=   406

STATIONS=  8/17                                        OPERATOR=  6HLS16

LOCATION=  MODEL            RMS ERROR=  0.08           DATE=      3/23/59
```

```
2         3         4         5         6         7         8         9
*---------*---------*---------*---------*---------*---------*---------*
     *************************************************************
     *         *         *         *         *         *         *
     *         *         *         *         *         *         *
     *  0.08   *  0.08   *  0.35   *  0.35   *  0.08   *  0.08   *
     *         *         *         *         *         *         *
     *************************************************************
     *                   *                   *                   *
     *                   *                   *                   *
     *                   *                   *                   *
     *       1.68        *       1.59        *       1.68        *
     *                   *                   *                   *
     *                   *                   *                   *
     *************************************************************
```

```
2         3         4         5         6         7         8         9
*---------*---------*---------*---------*---------*---------*---------*

    450       375       363       360       363       375       450

         320       215       195       195       215       320

              180                 135                 180
```

LINE NO= H-2-1A (1/13) BACKGROUND= 483

STATIONS= 8/17 OPERATOR= 9HLS16

LOCATION= MODEL RMS ERROR= 0.03 DATE= 3/23/59

```
2        3        4        5        6        7        8        9
*--------*--------*--------*--------*--------*--------*--------*
              *************************************
              *        *        *        *        *
              *        *        *        *        *
              * -0.28  *  0.47  *  0.47  * -0.28  *
              *        *        *        *        *
              *************************************
              *        *        *        *        *
              *        *        *        *        *
              *        *        *        *        *
              *        *        *        *        *
              *        *        *        *        *
              *  2.96  *  0.62  *  0.62  *  2.96  *
              *        *        *        *        *
              *        *        *        *        *
              *        *        *        *        *
              *        *        *        *        *
              *        *        *        *        *
              *************************************


2        3        4        5        6        7        8        9
*--------*--------*--------*--------*--------*--------*--------*

    450      375      363      360      363      375      450
         320      215      195      195      215      320
              180               135               180
```

LINE NO= H-2-1A (1/13) BACKGROUND= 429

STATIONS= 8/17 OPERATOR= 8VLS16

LOCATION= MODEL RMS ERROR= 0.06 DATE= 3/23/59

```
2         3         4         5         6         7         8         9
*---------*---------*---------*---------*---------*---------*---------*
               *****************************************
               *         *         *         *         *
               *         *         *         *         *
               *  -0.45  *   0.02  *   0.01  *  -0.03  *
               *         *         *         *         *
               *****************************************
               *                   *                   *
               *                   *                   *
               *                   *                   *
               *        1.17       *        2.71       *
               *                   *                   *
               *                   *                   *
               *****************************************



2         3         4         5         6         7         8         9
*---------*---------*---------*---------*---------*---------*---------*

    450       450       375       363       360       363       375

         459       320       215       195       195       215

              300                 140                 140
```

```
LINE  NO=  H-2-1A  (1/13)                                   BACKGROUND=   414

STATIONS=  7/16                                             OPERATOR=  6HLS16

LOCATION=  MODEL               RMS ERROR=  0.10             DATE=      3/23/59
```

```
2         3         4         5         6         7         8         9
*---------*---------*---------*---------*---------*---------*---------*
     *************************************************************
     *         *         *         *         *         *         *
     *         *         *         *         *         *         *
     *  -0.06  *  -0.01  *   0.12  *   0.04  *  -0.01  *   0.04  *
     *         *         *         *         *         *         *
     *************************************************************
     *                   *                   *                   *
     *                   *                   *                   *
     *                   *                   *                   *
     *         0.31      *         2.03      *         2.14      *
     *                   *                   *                   *
     *                   *                   *                   *
     *************************************************************


2         3         4         5         6         7         8         9
*---------*---------*---------*---------*---------*---------*---------*

    450       450       375       363       360       363       375

         459       320       215       195       195       215

              300                 140                 140
```

LINE NO= H-2-1A (1/13) BACKGROUND= 445

STATIONS= 7/16 OPERATOR= 9HLS16

LOCATION= MODEL RMS ERROR= 0.05 DATE= 3/23/59

```
2            3            4            5            6            7            8            9
*------------*------------*------------*------------*------------*------------*------------*
                  *************************************************
                  *            *            *            *            *
                  *            *            *            *            *
                  *    0.31    *   -0.65    *    1.09    *   -0.79    *
                  *            *            *            *            *
                  *************************************************
                  *            *            *            *            *
                  *            *            *            *            *
                  *            *            *            *            *
                  *            *            *            *            *
                  *            *            *            *            *
                  *   -1.62    *    5.05    *   -2.77    *    5.86    *
                  *            *            *            *            *
                  *            *            *            *            *
                  *            *            *            *            *
                  *            *            *            *            *
                  *            *            *            *            *
                  *************************************************


2            3            4            5            6            7            8            9
*------------*------------*------------*------------*------------*------------*------------*

      450          450          375          363          360          363          375

            459          320          215          195          195          215

                  300                       140                       140


LINE  NO=  H-2-1A  (1/13)                                       BACKGROUND=   417

STATIONS=  7/16                                                 OPERATOR=  8VLS16

LOCATION=  MODEL                  RMS ERROR=  0.07              DATE=      3/23/59
```

```
2         3         4         5         6         7         8         9
*---------*---------*---------*---------*---------*---------*---------*
              *****************************************
              *         *         *         *         *
              *         *         *         *         *
              *  -0.46  *   0.04  *   0.04  *  -0.46  *
              *         *         *         *         *
              *****************************************
              *                   *                   *
              *                   *                   *
              *                   *                   *
              *        2.49       *        2.49       *
              *                   *                   *
              *                   *                   *
              *****************************************


2         3         4         5         6         7         8         9
*---------*---------*---------*---------*---------*---------*---------*

    447       370       360       350       360       370       447

         285       175       173       173       175       285

              130                 110                 130


LINE  NO=  H-2-1B  (1/190)                               BACKGROUND=   395

STATIONS=  8/17                                          OPERATOR=  6HLS16

LOCATION=  MODEL              RMS ERROR=  0.11           DATE=     3/23/59
```

```
2         3         4         5         6         7         8         9
*---------*---------*---------*---------*---------*---------*---------*
     *************************************************************
     *         *         *         *         *         *         *
     *         *         *         *         *         *         *
     *  0.03   *  -0.02  *  0.42   *  0.42   *  -0.02  *  0.03   *
     *         *         *         *         *         *         *
     *************************************************************
     *                   *                   *                   *
     *                   *                   *                   *
     *                   *                   *                   *
     *       2.13        *       1.59        *       2.13        *
     *                   *                   *                   *
     *                   *                   *                   *
     *************************************************************


2         3         4         5         6         7         8         9
*---------*---------*---------*---------*---------*---------*---------*

    447       370       360       350       360       370       447

         285       175       173       173       175       285

              130                 110                 130


LINE  NO=  H-2-1B  (1/190)                          BACKGROUND=   483

STATIONS=  8/17                                     OPERATOR=  9HLS16

LOCATION=  MODEL            RMS ERROR=  0.03        DATE=     3/23/59
```

```
2         3         4         5         6         7         8         9
*---------*---------*---------*---------*---------*---------*---------*
               *****************************************
               *         *         *         *         *
               *         *         *         *         *
               *  -0.47  *   0.61  *   0.61  *  -0.47  *
               *         *         *         *         *
               *****************************************
               *         *         *         *         *
               *         *         *         *         *
               *         *         *         *         *
               *         *         *         *         *
               *         *         *         *         *
               *   3.84  *   0.36  *   0.36  *   3.84  *
               *         *         *         *         *
               *         *         *         *         *
               *         *         *         *         *
               *         *         *         *         *
               *         *         *         *         *
               *****************************************


2         3         4         5         6         7         8         9
*---------*---------*---------*---------*---------*---------*---------*

    447       370       360       350       360       370       447

         285       175       173       173       175       285

              130                 110                 130
```

LINE NO= H-2-1B (1/190) BACKGROUND= 425

STATIONS= 8/17 OPERATOR= 8VLS16

LOCATION= MODEL RMS ERROR= 0.08 DATE= 3/23/59

```
2         3         4         5         6         7         8         9
*---------*---------*---------*---------*---------*---------*---------*
               *****************************************
               *         *         *         *         *
               *         *         *         *         *
               *  -0.63  *  -0.03  *   0.06  *  -0.04  *
               *         *         *         *         *
               *****************************************
               *                   *                   *
               *                   *                   *
               *                   *                   *
               *        1.67       *        2.95       *
               *                   *                   *
               *                   *                   *
               *****************************************
```

```
2         3         4         5         6         7         8         9
*---------*---------*---------*---------*---------*---------*---------*

    450       447       370       360       350       360       370

         465       285       175       173       173       175

              245                 100                 100
```

LINE NO= H-2-1B (1/190) BACKGROUND= 412

STATIONS= 7/16 OPERATOR= 6HLS16

LOCATION= MODEL RMS ERROR= 0.12 DATE= 3/23/59

```
2         3         4         5         6         7         8         9
*---------*---------*---------*---------*---------*---------*---------*
     ***************************************************************
     *         *         *         *         *         *         *
     *         *         *         *         *         *         *
     *  -0.01  *  -0.08  *   0.18  *   0.11  *   0.02  *   0.06  *
     *         *         *         *         *         *         *
     ***************************************************************
     *                   *                   *                   *
     *                   *                   *                   *
     *                   *                   *                   *
     *        0.68       *        2.20       *        2.43       *
     *                   *                   *                   *
     *                   *                   *                   *
     ***************************************************************


2         3         4         5         6         7         8         9
*---------*---------*---------*---------*---------*---------*---------*

     450       447       370       360       350       360       370

          465       285       175       173       173       175

               245                 100                 100
```

LINE NO= H-2-1B (1/190) BACKGROUND= 460

STATIONS= 7/16 OPERATOR= 9HLS16

LOCATION= MODEL RMS ERROR= 0.07 DATE= 3/23/59

```
2         3         4         5         6         7         8         9
*---------*---------*---------*---------*---------*---------*---------*
              *****************************************
              *         *         *         *         *
              *         *         *         *         *
              *  0.34   *  -0.91  *   1.45  *  -1.07  *
              *         *         *         *         *
              *****************************************
              *         *         *         *         *
              *         *         *         *         *
              *         *         *         *         *
              *         *         *         *         *
              *         *         *         *         *
              * -1.91   *   6.63  *  -4.12  *   7.22  *
              *         *         *         *         *
              *         *         *         *         *
              *         *         *         *         *
              *         *         *         *         *
              *         *         *         *         *
              *****************************************


2         3         4         5         6         7         8         9
*---------*---------*---------*---------*---------*---------*---------*

    450       447       370       360       350       360       370
         465       285       175       173       173       175
              245                 100                 100
```

LINE NO= H-2-1B (1/190) BACKGROUND= 414

STATIONS= 7/16 OPERATOR= 8VLS16

LOCATION= MODEL RMS ERROR= 0.08 DATE= 3/23/59

```
2         3         4         5         6         7         8         9
*---------*---------*---------*---------*---------*---------*---------*
               *****************************************
               *         *         *         *         *
               *         *         *         *         *
               *  0.30   *  0.40   *  0.40   *  0.30   *
               *         *         *         *         *
               *****************************************
               *                   *                   *
               *                   *                   *
               *                   *                   *
               *        1.29       *        1.29       *
               *                   *                   *
               *                   *                   *
               *****************************************


2         3         4         5         6         7         8         9
*---------*---------*---------*---------*---------*---------*---------*

    443       451       400       308       400       451       443

          483       369       204       204       369       483

               363                 142                 363
```

LINE NO= V-1-1 (1/150) BACKGROUND= 522

STATIONS= 8/17 OPERATOR= 6HLS16

LOCATION= MODEL RMS ERROR= 0.08 DATE= 3/30/59

```
2         3         4         5         6         7         8         9
*---------*---------*---------*---------*---------*---------*---------*
     *************************************************************
     *         *         *         *         *         *         *
     *         *         *         *         *         *         *
     *  0.23   *  0.28   * -0.04   * -0.04   *  0.28   *  0.23   *
     *         *         *         *         *         *         *
     *************************************************************
     *                   *                   *                   *
     *                   *                   *                   *
     *                   *                   *                   *
     *       -0.50       *       3.03        *      -0.50        *
     *                   *                   *                   *
     *                   *                   *                   *
     *************************************************************


2         3         4         5         6         7         8         9
*---------*---------*---------*---------*---------*---------*---------*

    443       451       400       308       400       451       443

         483       369       204       204       369       483

              363                 142                 363


LINE  NO=  V-1-1  (1/150)                             BACKGROUND=   472

STATIONS=  8/17                                       OPERATOR=  9HLS16

LOCATION=  MODEL            RMS ERROR=  0.04          DATE=      3/30/59
```

```
2         3         4         5         6         7         8         9
*---------*---------*---------*---------*---------*---------*---------*
               *****************************************
               *         *         *         *         *
               *         *         *         *         *
               *  0.39   *  0.03   *  0.03   *  0.39   *
               *         *         *         *         *
               *****************************************
               *         *         *         *         *
               *         *         *         *         *
               *         *         *         *         *
               *         *         *         *         *
               *         *         *         *         *
               * -0.63   *  2.58   *  2.58   * -0.63   *
               *         *         *         *         *
               *         *         *         *         *
               *         *         *         *         *
               *         *         *         *         *
               *         *         *         *         *
               *****************************************


2         3         4         5         6         7         8         9
*---------*---------*---------*---------*---------*---------*---------*

    443       451       400       308       400       451       443

         483       369       204       204       369       483

              363                 142                 363


LINE  NO=  V-1-1  (1/150)                              BACKGROUND=   472

STATIONS=  8/17                                        OPERATOR=  8VLS16

LOCATION=  MODEL             RMS ERROR=  0.05          DATE=     3/30/59
```

```
2         3         4         5         6         7         8         9
*---------*---------*---------*---------*---------*---------*---------*
               *****************************************
               *         *         *         *         *
               *         *         *         *         *
               *   0.19  *  2.04   *  2.04   *  0.19   *
               *         *         *         *         *
               *****************************************
               *                   *                   *
               *                   *                   *
               *                   *                   *
               *       1.86        *       1.86        *
               *                   *                   *
               *                   *                   *
               *****************************************


2         3         4         5         6         7         8         9
*---------*---------*---------*---------*---------*---------*---------*

    495       400        52        55        52       400       495

         365        6         24        24         6       365

               13                  80                  13


LINE  NO=  V-2-0  (1/160)                                    BACKGROUND=   469

STATIONS=  8/17                                              OPERATOR=  6HLS16

LOCATION=  MODEL              RMS ERROR=  0.13               DATE=     3/30/59
```

```
2         3         4         5         6         7         8         9
*---------*---------*---------*---------*---------*---------*---------*
     *************************************************************
     *         *         *         *         *         *         *
     *         *         *         *         *         *         *
     *   0.47  *   0.60  *   2.13  *   2.13  *   0.60  *   0.47  *
     *         *         *         *         *         *         *
     *************************************************************
     *                   *                   *                   *
     *                   *                   *                   *
     *                   *                   *                   *
     *        1.77       *        0.68       *        1.77       *
     *                   *                   *                   *
     *                   *                   *                   *
     *************************************************************


2         3         4         5         6         7         8         9
*---------*---------*---------*---------*---------*---------*---------*

    495       400        52        55        52       400       495

         365        6        24        24         6       365

              13                   80                   13


LINE  NO=  V-2-0  (1/160)                             BACKGROUND=   650

STATIONS=  8/17                                       OPERATOR=  9HLS16

LOCATION=  MODEL             RMS ERROR=  0.10         DATE=      3/30/59
```

```
2         3         4         5         6         7         8         9
*---------*---------*---------*---------*---------*---------*---------*
               *****************************************
               *         *         *         *         *
               *         *         *         *         *
               *  0.10   *  2.47   *  2.47   *  0.10   *
               *         *         *         *         *
               *****************************************
               *         *         *         *         *
               *         *         *         *         *
               *         *         *         *         *
               *         *         *         *         *
               *         *         *         *         *
               *  3.54   * -0.32   * -0.32   *  3.54   *
               *         *         *         *         *
               *         *         *         *         *
               *         *         *         *         *
               *         *         *         *         *
               *         *         *         *         *
               *****************************************


2         3         4         5         6         7         8         9
*---------*---------*---------*---------*---------*---------*---------*

    495       400        52        55        52       400       495

         365         6        24        24         6       365

              13                  80                  13


LINE  NO=  V-2-0  (1/160)                         BACKGROUND=   508

STATIONS=  8/17                                    OPERATOR=  8VLS16

LOCATION=  MODEL          RMS ERROR=  0.10         DATE=      3/30/59
```

```
2         3         4         5         6         7         8         9
*---------*---------*---------*---------*---------*---------*---------*
               *****************************************
               *         *         *         *         *
               *         *         *         *         *
               *  -0.05  *   0.18  *   0.18  *  -0.05  *
               *         *         *         *         *
               *****************************************
               *                   *                   *
               *                   *                   *
               *                   *                   *
               *        1.11       *        1.11       *
               *                   *                   *
               *                   *                   *
               *****************************************


2         3         4         5         6         7         8         9
*---------*---------*---------*---------*---------*---------*---------*

    426       432       391       366       391       432       426

         436       348       273       273       348       436

              331                 241                 331
```

LINE NO= V-2-1A (1/3.8) BACKGROUND= 449

STATIONS= 8/17 OPERATOR= 6HLS16

LOCATION= MODEL RMS ERROR= 0.03 DATE= 3/30/59

```
2         3         4         5         6         7         8         9
*---------*---------*---------*---------*---------*---------*---------*
     *************************************************************
     *         *         *         *         *         *         *
     *         *         *         *         *         *         *
     *  0.08   *  0.02   * -0.01   * -0.01   *  0.02   *  0.08   *
     *         *         *         *         *         *         *
     *************************************************************
     *                   *                   *                   *
     *                   *                   *                   *
     *                   *                   *                   *
     *        0.06       *        1.97       *        0.06       *
     *                   *                   *                   *
     *                   *                   *                   *
     *************************************************************


2         3         4         5         6         7         8         9
*---------*---------*---------*---------*---------*---------*---------*

    426       432       391       366       391       432       426

         436       348       273       273       348       436

              331                 241                 331


LINE  NO=  V-2-1A  (1/3.8)                            BACKGROUND=   438

STATIONS=  8/17                                       OPERATOR=  9HLS16

LOCATION=  MODEL             RMS ERROR=  0.02         DATE=      3/30/59
```

2 3 4 5 6 7 8 9

0.01	0.04	0.04	0.01
0.14	1.58	1.58	0.14

2 3 4 5 6 7 8 9

426		432		391		366		391		432		426
	436		348		273		273		348		436	
		331				241				331		

LINE NO= V-2-1A (1/3.8)

STATIONS= 8/17

LOCATION= MODEL

RMS ERROR= 0.01

BACKGROUND= 431

OPERATOR= 8VLS16

DATE= 3/30/59

```
2        3        4        5        6        7        8        9
*--------*--------*--------*--------*--------*--------*--------*
             *************************************
             *        *        *        *        *
             *        *        *        *        *
             *  0.00  *  0.43  *  0.43  *  0.00  *
             *        *        *        *        *
             *************************************
             *                 *                 *
             *                 *                 *
             *                 *                 *
             *      1.67       *      1.67       *
             *                 *                 *
             *                 *                 *
             *************************************


2        3        4        5        6        7        8        9
*--------*--------*--------*--------*--------*--------*--------*

   445      435      363      313      363      435      445
        474      298      176      176      298      474
            275               125               275
```

LINE NO= V-2-1B (1/215) BACKGROUND= 486

STATIONS= 8/17 OPERATOR= 6HLS16

LOCATION= MODEL RMS ERROR= 0.06 DATE= 3/30/59

```
2        3        4        5        6        7        8        9
*--------*--------*--------*--------*--------*--------*--------*
    *****************************************************
    *        *        *        *        *        *        *
    *        *        *        *        *        *        *
    *  0.27  *  0.12  *  0.18  *  0.18  *  0.12  *  0.27  *
    *        *        *        *        *        *        *
    *****************************************************
    *                 *                 *                 *
    *                 *                 *                 *
    *                 *                 *                 *
    *      0.03       *      3.01       *      0.03       *
    *                 *                 *                 *
    *                 *                 *                 *
    *****************************************************


2        3        4        5        6        7        8        9
*--------*--------*--------*--------*--------*--------*--------*

   445      435      363      313      363      435      445

        474      298      176      176      298      474

             275               125               275
```

LINE NO= V-2-1B (1/215) BACKGROUND= 479

STATIONS= 8/17 OPERATOR= 9HLS16

LOCATION= MODEL RMS ERROR= 0.03 DATE= 3/30/59

```
2         3         4         5         6         7         8         9
*---------*---------*---------*---------*---------*---------*---------*
               *****************************************
               *         *         *         *         *
               *         *         *         *         *
               *   0.09  *   0.19  *   0.19  *   0.09  *
               *         *         *         *         *
               *****************************************
               *         *         *         *         *
               *         *         *         *         *
               *         *         *         *         *
               *         *         *         *         *
               *         *         *         *         *
               *   0.10  *   2.55  *   2.55  *   0.10  *
               *         *         *         *         *
               *         *         *         *         *
               *         *         *         *         *
               *         *         *         *         *
               *         *         *         *         *
               *****************************************


2         3         4         5         6         7         8         9
*---------*---------*---------*---------*---------*---------*---------*

    445       435       363       313       363       435       445
         474       298       176       176       298       474
              275                 125                 275
```

LINE NO= V-2-1B (1/215) BACKGROUND= 452

STATIONS= 8/17 OPERATOR= 8VLS16

LOCATION= MODEL RMS ERROR= 0.03 DATE= 3/30/59

contrasts when the linear approximation neglecting interactions is valid. The background values are close to 450 or within 10% for the majority of them. A relatively large number of model results has been included so that a fairly complete evaluation of the interpretation operators can be made for known geometries. As indicated the strength factor which has been suggested is that of Eq. 4.1.1 and that there are very few interpretations which yield strength factors much greater than this. See page128 for an interpretation by 8VLS16 which demonstrates this. There are analytic solutions such as the sphere in a plane field for which the Ki is bounded by 3.0. In general it appears that the interpretation of the K_i must be made on the basis of an equation showing saturation.

Certain patterns of behavior of the operators is readily evidenced from the results. If there is a deep region which in reality extends over or into 3 or more of the lower blocks such as H-1-1A, stations 8/17 for operators 9HLS16 or 8VLS16 then the interior block's K_i is always depressed in value while the exterior K_i which contain the actual boundary of the block are increased. See pages 112,113 ,119,124 and also 125 for examples of this bshavior. This property of the interpretation scheme is evidenced in all those pertinent examples and is probably representative of the final distribution of sources after interaction has taken place. The modified first approximation appears to be able to fit the data best when it over-and under-compensates certain blocks such as on page 116. The amount of such compensation depends directly upon the actual strength of the region. Since the operators appear to always operate in this very predictable manner, due account can be taken in utilizing their interpretations on field data.

The interpretation results for the K_i when considered from the point of view of a bounded strength factor are extremely good in picking out the conducting regions even when the model and operator blocks did not have correspondance of boundaries. Pages 114 and 120 represent such cases. Moreover, the use of vertical and horizontal operators on regions that were the exact opposite in structure did not yield necessarily poor results although the magnitude of the K_i in this case greatly exceeded the assumed bound of 2.0. Refer to pages 113 and 130 for examples of this type of interpretation result.

The RMS error does not appear to be as useful an indication of appropriate fit and operator as does the inspection of the magnitude of the K_i. For those results which represented shifting of the operator along the line so that results were not symmetrical but the blocks corresponded better a significant improvement in the interpretation was made. Pages 115 and 121 illustrate this improvement of the interpretations.

Again the RMS error did not prove to ve a good figure of merit to evaluate the operator. Although the RMS error is not a unique factor that will allow an evaluation of the validity of the interpretation to be made, a certain idea of the magnitude of fitting error is obtained from these results which will be useful as a reference in the interpretations on actual field data.

4.4 Interpretation Results for theoretical solutions to vertical layers

Additional comparisons of the interpretation results using the 8VLS16 operator have been made on theoretical solutions for vertical layers. The profile line was oriented perpendicular to the vertical surfaces separating the homogeneous regions and a conducting middle layer in a uniform background $\rho_0 = 100$ has been the target. The actual location of the middle layer is well detected for thicknesses greater than or equal to one unit of the electrode spacing interval. See pages 145 and 143 for such interpretations. When the middle layer is thiner than one unit there is a problem of resolution that the blocks and measurements cannot hope to correctly delineate, but the interpretation does pick out the appropriate blocks. A very thin vertical layer is interpreted properly on page 149 . In these cases the middle layer extended from station 5.5 to the right 0.1, 0.2 or 0.4

units as the individual geometry specified. The remaining cases all represent vertical layers with bounding surfaces at points midway between stations. That is, the layer 4 units thick on page 153 extended from stations 3.5 to 7.5.

As in section 4.3 the interpretation results are good, although the magnitude of the K_1 often exceeds the proposed limit of 2.0. This is no doubt associated with the fact that the theoretical ρ_A results are for infinite regions rather than finite regions and the first approximation forward solutions for vertical layers are very poor because of the particular geometry. It is not to be expected that the vertical operators for vertical regions will be as good as the horizontal operators for horizontal regions. A comparison of the interpretation results for the model V-2-0 on page 134 and the vertical layer two units thick on page 151 is remarkably similar. This fact is an example of the limits of the resolution power of the dipole-dipole array regarding regions that extend outside the immediate volume of the profile line. Moreover the over-under compensation result for the deeper blocks is again well evidenced (see page 152) in all those cases for which a conducting region actually extended over or into 3 or more blocks.

The following pages present the results for a few of the vertical layers which were tested by the 8VLS16 operator and essentially display the range of results obtained. Included is a set of results for the same geometry (1 unit thick middle

```
2           3           4           5           6           7           8           9
*-----------*-----------*-----------*-----------*-----------*-----------*-----------*
                 *****************************************
                 *           *           *           *           *
                 *           *           *           *           *
                 *   0.06    *   0.32    *   2.24    *   0.62    *
                 *           *           *           *           *
                 *****************************************
                 *           *           *           *           *
                 *           *           *           *           *
                 *           *           *           *           *
                 *           *           *           *           *
                 *           *           *           *           *
                 *   0.62    *  -0.64    *   3.60    *  -0.41    *
                 *           *           *           *           *
                 *           *           *           *           *
                 *           *           *           *           *
                 *           *           *           *           *
                 *           *           *           *           *
                 *****************************************

2           3           4           5           6           7           8           9
*-----------*-----------*-----------*-----------*-----------*-----------*-----------*

     102         107         98          20          20          98         107
           113         96          22          40          22          96
                 94                      42                      23

LINE  NO=  VERT LAYER  (1/10)                                BACKGROUND=   109
STATIONS=  THICKNESS= 1.0                                    OPERATOR=  8VLS16
LOCATION=  THEORETICAL      RMS ERROR=  0.05                 DATE=      3/30/59
```

```
2          3          4          5          6          7          8          9
*----------*----------*----------*----------*----------*----------*----------*
               *****************************************
               *         *         *         *         *
               *         *         *         *         *
               *  0.08   *  0.44   *  2.19   *  0.84   *
               *         *         *         *         *
               *****************************************
               *         *         *         *         *
               *         *         *         *         *
               *         *         *         *         *
               *         *         *         *         *
               *         *         *         *         *
               *  0.78   * -0.86   *  4.68   * -0.61   *
               *         *         *         *         *
               *         *         *         *         *
               *         *         *         *         *
               *         *         *         *         *
               *         *         *         *         *
               *****************************************


2          3          4          5          6          7          8          9
*----------*----------*----------*----------*----------*----------*----------*


    102        108        99         11         11         99        108

         116        97         13         24         13         97

              96                    24                    13


LINE  NO=  VERT LAYER  (1/20)                            BACKGROUND=   113

STATIONS=  THICKNESS= 1.0                                OPERATOR=  8VLS16

LOCATION=  THEORETICAL       RMS ERROR=  0.07            DATE=      3/30/59
```

```
2         3         4         5         6         7         8         9
*---------*---------*---------*---------*---------*---------*---------*
              *****************************************
              *         *         *         *         *
              *         *         *         *         *
              *  0.09   *  0.52   *  2.12   *  0.97   *
              *         *         *         *         *
              *****************************************
              *         *         *         *         *
              *         *         *         *         *
              *         *         *         *         *
              *         *         *         *         *
              *         *         *         *         *
              *  0.92   * -1.07   *  5.42   * -0.76   *
              *         *         *         *         *
              *         *         *         *         *
              *         *         *         *         *
              *         *         *         *         *
              *         *         *         *         *
              *****************************************

2         3         4         5         6         7         8         9
*---------*---------*---------*---------*---------*---------*---------*

    102       109       99        6         6         99        109
         118       99        8         15        8         99
              97                  12                  6

LINE   NO=  VERT LAYER  (1/40)                        BACKGROUND=   115
STATIONS=  THICKNESS= 1.0                             OPERATOR=  8VLS16
LOCATION=  THEORETICAL       RMS ERROR=  0.08         DATE=     3/30/59
```

```
2          3          4          5          6          7          8          9
*----------*----------*----------*----------*----------*----------*----------*
               *****************************************
               *         *         *         *         *
               *         *         *         *         *
               *  0.18   *  0.31   *  0.30   *  0.27   *
               *         *         *         *         *
               *****************************************
               *         *         *         *         *
               *         *         *         *         *
               *         *         *         *         *
               *         *         *         *         *
               *         *         *         *         *
               * -0.55   *  0.78   *  1.49   * -0.91   *
               *         *         *         *         *
               *         *         *         *         *
               *         *         *         *         *
               *         *         *         *         *
               *         *         *         *         *
               *****************************************


2          3          4          5          6          7          8          9
*----------*----------*----------*----------*----------*----------*----------*

    100        103         98         65         94        103        101

          105         96         73         73         93        106

               96                    80                    92

LINE  NO=  VERT LAYER  (1/10)                            BACKGROUND=   104
STATIONS=  THICKNESS= 0.1                                OPERATOR=  8VLS16
LOCATION=  THEORETICAL        RMS ERROR=  0.05           DATE=     3/30/59
```

```
2         3         4         5         6         7         8         9
*---------*---------*---------*---------*---------*---------*---------*
              *****************************************
              *         *         *         *         *
              *         *         *         *         *
              *  0.16   *  0.43   *  0.49   *  0.31   *
              *         *         *         *         *
              *****************************************
              *         *         *         *         *
              *         *         *         *         *
              *         *         *         *         *
              *         *         *         *         *
              *         *         *         *         *
              * -0.41   *  0.68   *  2.37   * -1.08   *
              *         *         *         *         *
              *         *         *         *         *
              *         *         *         *         *
              *         *         *         *         *
              *         *         *         *         *
              *****************************************


2         3         4         5         6         7         8         9
*---------*---------*---------*---------*---------*---------*---------*

    101       104        96        54        85       106       101
         108        94        62        62        82       110
              93                   69                  81
```

LINE NO= VERT LAYER (1/10) BACKGROUND= 106

STATIONS= THICKNESS= 0.2 OPERATOR= 8VLS16

LOCATION= THEORETICAL RMS ERROR= 0.06 DATE= 3/30/59

```
2          3          4          5          6          7          8          9
*----------*----------*----------*----------*----------*----------*----------*
               *****************************************
               *         *         *         *         *
               *         *         *         *         *
               *  -0.24  *   0.76  *   0.94  *   0.43  *
               *         *         *         *         *
               *****************************************
               *         *         *         *         *
               *         *         *         *         *
               *         *         *         *         *
               *         *         *         *         *
               *         *         *         *         *
               *   1.20  *  -1.57  *   4.76  *  -1.64  *
               *         *         *         *         *
               *         *         *         *         *
               *         *         *         *         *
               *         *         *         *         *
               *         *         *         *         *
               *****************************************


2          3          4          5          6          7          8          9
*----------*----------*----------*----------*----------*----------*----------*

    101        105         95         44         52        112        102
          110         92         51         51         50        119
                90                    57                    50

LINE  NO=  VERT LAYER  (1/10)                              BACKGROUND=   107
STATIONS=  THICKNESS= 0.4                                  OPERATOR=  8VLS16
LOCATION=  THEORETICAL        RMS ERROR=  0.06             DATE=      3/30/59
```

2	3	4	5	6	7	8	9

0.02	2.35	2.35	0.02
2.55	-0.03	-0.03	2.55

2	3	4	5	6	7	8	9

108 99 17 10 17 99 108

99 17 20 20 17 99

14 35 14

LINE NO= VERT LAYER (1/10) BACKGROUND= 114

STATIONS= THICKNESS= 2.0 OPERATOR= 8VLS16

LOCATION= THEORETICAL RMS ERROR= 0.11 DATE= 3/30/59

```
2           3           4           5           6           7           8           9
*-----------*-----------*-----------*-----------*-----------*-----------*-----------*
                  *****************************************
                  *           *           *           *           *
                  *           *           *           *           *
                  *   0.58    *   1.50    *   3.28    *   0.91    *
                  *           *           *           *           *
                  *****************************************
                  *           *           *           *           *
                  *           *           *           *           *
                  *           *           *           *           *
                  *           *           *           *           *
                  *           *           *           *           *
                  *   0.10    *   4.24    *  -6.74    *   6.88    *
                  *           *           *           *           *
                  *           *           *           *           *
                  *           *           *           *           *
                  *           *           *           *           *
                  *           *           *           *           *
                  *****************************************

2           3           4           5           6           7           8           9
*-----------*-----------*-----------*-----------*-----------*-----------*-----------*

     108         100          18          9          9          18         100
           99          17          15          10          15          17
                 17                      19                      15
```

LINE NO= VERT LAYER (1/10) BACKGROUND= 106

STATIONS= THICKNESS= 3.0 OPERATOR= 8VLS16

LOCATION= THEORETICAL RMS ERROR= 0.13 DATE= 3/30/59

```
2         3         4         5         6         7         8         9
*---------*---------*---------*---------*---------*---------*---------*
               *****************************************
               *         *         *         *         *
               *         *         *         *         *
               *   1.56  *   2.50  *   2.50  *   1.56  *
               *         *         *         *         *
               *****************************************
               *         *         *         *         *
               *         *         *         *         *
               *         *         *         *         *
               *         *         *         *         *
               *         *         *         *         *
               *   4.63  *  -2.90  *  -2.90  *   4.63  *
               *         *         *         *         *
               *         *         *         *         *
               *         *         *         *         *
               *         *         *         *         *
               *         *         *         *         *
               *****************************************


2         3         4         5         6         7         8         9
*---------*---------*---------*---------*---------*---------*---------*

    100        14        10         9        10        14        100

          19        14         9         9        14        19

               16                  10                  16
```

```
LINE  NO=  VERT LAYER  (1/10)                              BACKGROUND=    95

STATIONS=  THICKNESS= 4.0                                  OPERATOR=  8VLS16

LOCATION=  THEORETICAL       RMS ERROR=  0.15              DATE=     3/30/59
```

layer) with different conductivity contrasts. The operator has modified the interpreted K_i values in a consistent manner as the conductivity contrast increases although the magnitude of the K_i are greater than 2.0. It appears that the vertical operator is capable of resolving the geometry of the subsurface with approximately the same success as the horizontal operator but that the K_i values are less susceptible to interpretation by Eq. 4.1.1. There is however a strong indication that the results should still be interpretated on the basis of a bounded source strength so that the relative significance of the K_i is not linearly dependent upon their values.

In concluding this section it is useful to point out that there are certain properties which the operators possess which require a familiarity with their results for known geometries. This is necessary in order that results obtained for field data be properly interpreted with regards the tendency of the linear approximation to compensate for the non-linear data with which it must work by modifying adjacent blocks' K_i values so as to best fit the data. Also there is a possibility that the apparent resistivity values which are predicted from the final interpretation K_i and ρ_0 may be negative. This implies an over-shoot in the first approximation which is a result of the compensation necessary to best fit the data by the linear theory. A negative apparent resistivity implies that there would also be a certain configuration of source and receiver which would lead to a zero value of ρ_{Aj} . This property of the linear theory isrepresentative of the importance of non-linear interactions of induced sources which determine the final current flow.

4.5 Interpretation of Field Data with Horizontal Operators

The success of the interpretation scheme developed on the basis of finite sized homogeneous regions in the subsurface and tested on both model and theoretical solutions has lead to application on actual field data. The evaluation of the results of such an interpretation depend critically upon the amount of geological control which is available and also the experience of the personnel doing the empirical interpretation in similar geological areas. This section presents the results of a resistivity survey in one area using the 6HLS16 and 9HLS16 operators. The plan view on the following page indicates the relative location of the profile lines and stations so that correlation of the operator results can be made. There are essentially two smaller areas within the large area of interest and these have been prospected somewhat differently. Field Data 1 represents the area which was surveyed with the aid of a systematic grid of lines and stations while Field Data 3 refers to the area with no grid but simply three related lines. These were profiled in the sequence A then B and C as it became evident as the results were obtained that larger spacing intervals would be required to detect the deep

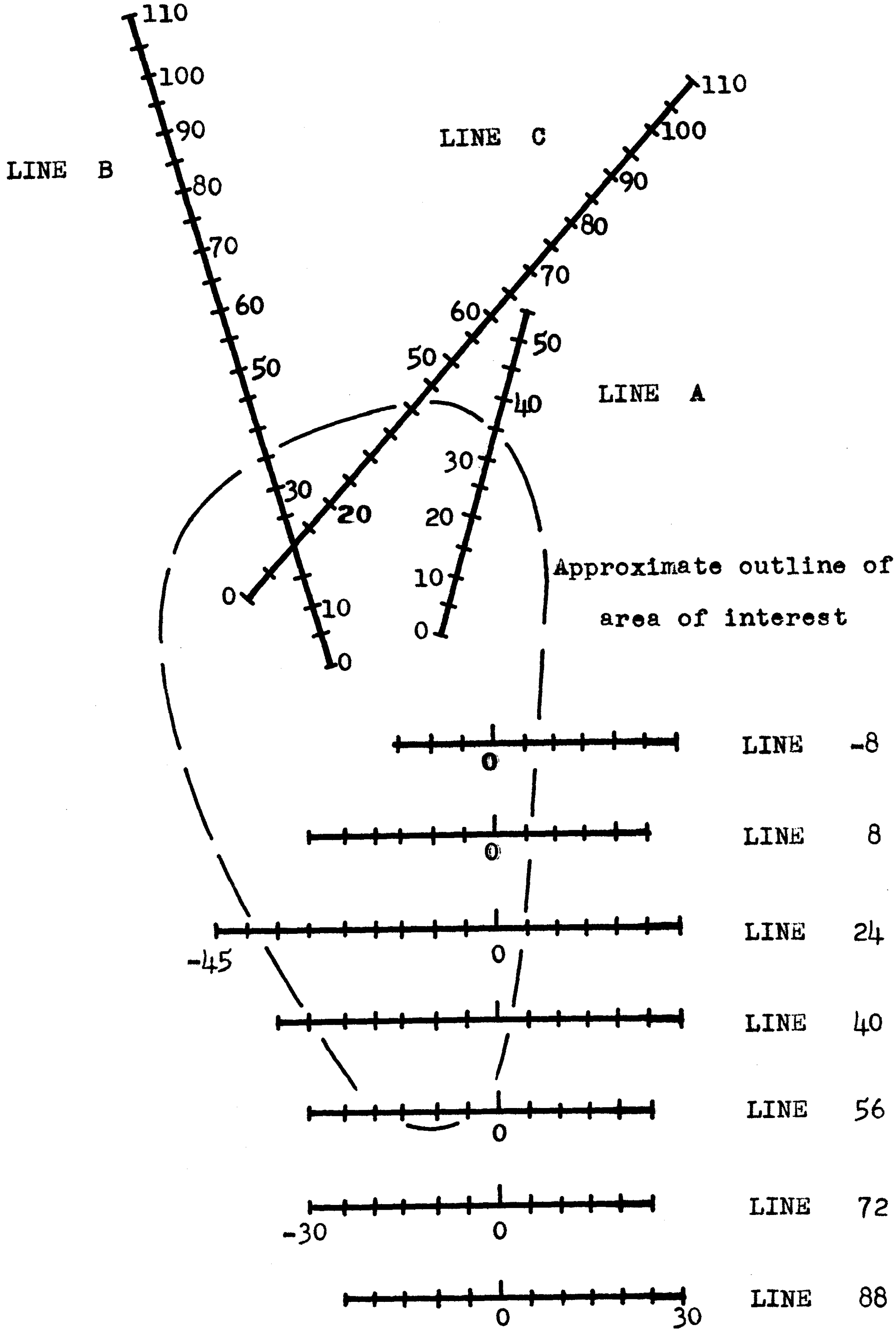


Figure 3 Plan View of Profile Lines for Field Data Number 1 and 3

anomalous region. For the grid lines (88, 72, 56, 40, 24 and -8) and line A the spacing intervals were the same while for B and C the interval was doubled.

Both operators were applied to the entire set of resistivity data in order to test their relative merits. All of the lines but -8 were longer than the length of the basic operators and thus required a shifting along the lines to cover them completely. The 6HLS16 and 9HLS16 interpretation results are very satisfactory and are in close agreement with each other and the known geological information available. The anomalous region is somewhat oval shaped and in general is approximately one unit deep or more but rises to the surface in the vicinity of lines 24, 8 and -8.

The complete set of results for the 9HLS16 operator are presented and a summary of the shifting of the operator along each line has also been prepared. Those blocks which overlap are seen to correlate fairly well with each other and the shifting of the operator reproduces consistently the resistivity structure of the subsurface. An example of this for 9HLS16 is seen on page 175 . A summary of the results for the 6HLS16 operator over the same lines has also been prepared and a comparison with those of the 9HLS16 results are in complete accord. Compare pages 175 and 178 . The manner of presenting the summary results has been to use separate

```
2         3         4         5         6         7         8         9
*---------*---------*---------*---------*---------*---------*---------*
     *************************************************************
     *         *         *         *         *         *         *
     *         *         *         *         *         *         *
     *  -0.34  *   0.97  *   0.83  *   0.95  *   0.83  *   0.39  *
     *         *         *         *         *         *         *
     *************************************************************
     *                   *                   *                   *
     *                   *                   *                   *
     *                   *                   *                   *
     *        0.99       *       -1.01       *        0.89       *
     *                   *                   *                   *
     *                   *                   *                   *
     *************************************************************


2         3         4         5         6         7         8         9
*---------*---------*---------*---------*---------*---------*---------*

    190       120       110       134        72       150       110

         100       165       153       100       136       137

              125                 123                 110
```

LINE NO= 88 | BACKGROUND= 187

STATIONS= -25/20 | OPERATOR= 9HLS16

LOCATION= FIELD DATA 1 | RMS ERROR= 0.14 | DATE= 3/23/59

```
2         3         4         5         6         7         8         9
*---------*---------*---------*---------*---------*---------*---------*
     *************************************************************
     *         *         *         *         *         *         *
     *         *         *         *         *         *         *
     *  0.95   *  0.47   *  1.06   *  0.60   * -0.19   *  0.04   *
     *         *         *         *         *         *         *
     *************************************************************
     *                   *                   *                   *
     *                   *                   *                   *
     *                   *                   *                   *
     *       0.37        *      -0.54        *       2.57        *
     *                   *                   *                   *
     *                   *                   *                   *
     *************************************************************


2         3         4         5         6         7         8         9
*---------*---------*---------*---------*---------*---------*---------*

    110       134        72       150       110       157       129
         153       100       136       137        91        96
              123                 110                  54

LINE  NO=  88                                         BACKGROUND=   178
STATIONS=  -15/30                                     OPERATOR=  9HLS16
LOCATION=  FIELD DATA 1     RMS ERROR=  0.13          DATE=     3/23/59
```

```
2         3         4         5         6         7         8         9
*---------*---------*---------*---------*---------*---------*---------*
     *************************************************************
     *         *         *         *         *         *         *
     *         *         *         *         *         *         *
     *   1.39  *   0.48  *   0.96  *   1.43  *   1.10  *   0.85  *
     *         *         *         *         *         *         *
     *************************************************************
     *                   *                   *                   *
     *                   *                   *                   *
     *                   *                   *                   *
     *        0.73       *       -0.04       *        1.46       *
     *                   *                   *                   *
     *                   *                   *                   *
     *************************************************************


2         3         4         5         6         7         8         9
*---------*---------*---------*---------*---------*---------*---------*

     200       121       190       128       91        121       220
          190       120       154       90        44        144
               180                 101                 113
```

LINE NO= 72

STATIONS= -30/15

LOCATION= FIELD DATA 1

RMS ERROR= 0.05

BACKGROUND= 295

OPERATOR= 9HLS16

DATE= 3/23/59

```
2         3         4         5         6         7         8         9
*---------*---------*---------*---------*---------*---------*---------*
     *************************************************************
     *         *         *         *         *         *         *
     *         *         *         *         *         *         *
     *  0.36   *  1.17   *  1.01   *  0.49   * -0.45   *  1.05   *
     *         *         *         *         *         *         *
     *************************************************************
     *                   *                   *                   *
     *                   *                   *                   *
     *                   *                   *                   *
     *       0.73        *       0.90        *       0.76        *
     *                   *                   *                   *
     *                   *                   *                   *
     *************************************************************


2         3         4         5         6         7         8         9
*---------*---------*---------*---------*---------*---------*---------*

    190       128        91       121       220       158       150

         154        90        44       144       146       155

              101                 113                 132
```

LINE NO= 72 BACKGROUND= 222

STATIONS= -20/25 OPERATOR= 9HLS16

LOCATION= FIELD DATA 1 RMS ERROR= 0.10 DATE= 3/23/59

```
2         3         4         5         6         7         8         9
*---------*---------*---------*---------*---------*---------*---------*
     *************************************************************
     *         *         *         *         *         *         *
     *         *         *         *         *         *         *
     *  -0.89  *  0.46   *  1.31   *  1.53   *  0.55   *  0.87   *
     *         *         *         *         *         *         *
     *************************************************************
     *                   *                   *                   *
     *                   *                   *                   *
     *                   *                   *                   *
     *        0.79       *      -0.04        *       1.82        *
     *                   *                   *                   *
     *                   *                   *                   *
     *************************************************************


2         3         4         5         6         7         8         9
*---------*---------*---------*---------*---------*---------*---------*

    160       167        81        34        78        59        98

         125       103        45        61        40        41

               85                  75                  40
```

LINE NO= 56 BACKGROUND= 146

STATIONS= -30/15 OPERATOR= 9HLS16

LOCATION= FIELD DATA 1 RMS ERROR= 0.10 DATE= 3/23/59

```
2         3         4         5         6         7         8         9
*---------*---------*---------*---------*---------*---------*---------*
     *************************************************************
     *         *         *         *         *         *         *
     *         *         *         *         *         *         *
     *  1.28   *  1.26   *  0.68   *  0.83   *  0.14   * -0.11   *
     *         *         *         *         *         *         *
     *************************************************************
     *                   *                   *                   *
     *                   *                   *                   *
     *                   *                   *                   *
     *          0.68     *          0.92     *          1.40     *
     *                   *                   *                   *
     *                   *                   *                   *
     *************************************************************


2         3         4         5         6         7         8         9
*---------*---------*---------*---------*---------*---------*---------*

     81        34        78        59        98       115       125

          45        61        40        41        67        98

               75                  40                  57
```

LINE NO= 56 BACKGROUND= 132

STATIONS= -20/25 OPERATOR= 9HLS16

LOCATION= FIELD DATA 1 RMS ERROR= 0.09 DATE= 3/23/59

```
2         3         4         5         6         7         8         9
*---------*---------*---------*---------*---------*---------*---------*
     *****************************************************************
     *         *         *         *         *         *         *
     *         *         *         *         *         *         *
     *  -0.48  *  -0.20  *   0.72  *   0.43  *   1.18  *   1.21  *
     *         *         *         *         *         *         *
     *****************************************************************
     *                   *                   *                   *
     *                   *                   *                   *
     *                   *                   *                   *
     *        1.97       *        1.19       *        2.68       *
     *                   *                   *                   *
     *                   *                   *                   *
     *****************************************************************


2         3         4         5         6         7         8         9
*---------*---------*---------*---------*---------*---------*---------*

    110       131        69        97        37        26        49

          94        58        45        17        16         8

               34                   7                   8
```

LINE NO= 40 BACKGROUND= 119

STATIONS= -35/10 OPERATOR= 9HLS16

LOCATION= FIELD DATA 1 RMS ERROR= 0.13 DATE= 3/23/59

```
2         3         4         5         6         7         8         9
*---------*---------*---------*---------*---------*---------*---------*
     *************************************************************
     *         *         *         *         *         *         *
     *         *         *         *         *         *         *
     *  -0.75  *  -0.39  *   1.55  *   0.56  *   0.41  *   0.84  *
     *         *         *         *         *         *         *
     *************************************************************
     *                   *                   *                   *
     *                   *                   *                   *
     *                   *                   *                   *
     *        2.44       *        1.64       *        2.18       *
     *                   *                   *                   *
     *                   *                   *                   *
     *************************************************************


2         3         4         5         6         7         8         9
*---------*---------*---------*---------*---------*---------*---------*

     69        97        37        26        49        33        51

          45        17        16         8        17        27

               7                   8                  11

LINE  NO=  40                                        BACKGROUND=    76
STATIONS=  -25/20                                    OPERATOR=  9HLS16
LOCATION=  FIELD DATA 1     RMS ERROR=  0.11         DATE=     3/13/59
```

```
2         3         4         5         6         7         8         9
*---------*---------*---------*---------*---------*---------*---------*
     *************************************************************
     *         *         *         *         *         *         *
     *         *         *         *         *         *         *
     *  1.71   *  0.69   *  0.97   *  1.32   *  0.62   * -0.21   *
     *         *         *         *         *         *         *
     *************************************************************
     *                   *                   *                   *
     *                   *                   *                   *
     *                   *                   *                   *
     *       2.22        *       1.33        *       0.14        *
     *                   *                   *                   *
     *                   *                   *                   *
     *************************************************************


2         3         4         5         6         7         8         9
*---------*---------*---------*---------*---------*---------*---------*

     37        26        49        33        51        106       119

          16         8        17        27        63        114

               8                   11                  61
```

LINE NO= 40 | BACKGROUND= 114

STATIONS= -15/30 | OPERATOR= 9HLS16

LOCATION= FIELD DATA 1 | RMS ERROR= 0.12 | DATE= 3/23/59

```
2         3         4         5         6         7         8         9
*---------*---------*---------*---------*---------*---------*---------*
     *************************************************************
     *         *         *         *         *         *         *
     *         *         *         *         *         *         *
     *  -0.21  *  -0.00  *   0.88  *   0.56  *   0.82  *   2.25  *
     *         *         *         *         *         *         *
     *************************************************************
     *                   *                   *                   *
     *                   *                   *                   *
     *                   *                   *                   *
     *        2.26       *        0.91       *        2.79       *
     *                   *                   *                   *
     *                   *                   *                   *
     *************************************************************


2         3         4         5         6         7         8         9
*---------*---------*---------*---------*---------*---------*---------*


    110        82        74        66        45         6         8

          66        35        42        13         3         2

               28                  5                   1
```

LINE NO= 24 BACKGROUND= 106

STATIONS= -45/0 OPERATOR= 9HLS16

LOCATION= FIELD DATA 1 RMS ERROR= 0.13 DATE= 3/25/59

```
2         3         4         5         6         7         8         9
*---------*---------*---------*---------*---------*---------*---------*
     *************************************************************
     *         *         *         *         *         *         *
     *         *         *         *         *         *         *
     *  -0.23  *  0.03   *  1.38   *  1.90   *  1.30   *  2.37   *
     *         *         *         *         *         *         *
     *************************************************************
     *                   *                   *                   *
     *                   *                   *                   *
     *                   *                   *                   *
     *       2.70        *       0.70        *       1.67        *
     *                   *                   *                   *
     *                   *                   *                   *
     *************************************************************


2         3         4         5         6         7         8         9
*---------*---------*---------*---------*---------*---------*---------*

    74        66        45         6         8         2         2
         42        13         3         2         1         1
               5                   1                   1
```

LINE NO= 24 BACKGROUND= 80

STATIONS= -35/10 OPERATOR= 9HLS16

LOCATION= FIELD DATA 1 RMS ERROR= 0.16 DATE= 3/25/59

```
2         3         4         5         6         7         8         9
*---------*---------*---------*---------*---------*---------*---------*
     *************************************************************
     *         *         *         *         *         *         *
     *         *         *         *         *         *         *
     *   0.02  *   1.65  *   2.01  *   2.24  *   1.39  *   1.43  *
     *         *         *         *         *         *         *
     *************************************************************
     *                   *                   *                   *
     *                   *                   *                   *
     *                   *                   *                   *
     *        3.04       *       -1.18       *        2.44       *
     *                   *                   *                   *
     *                   *                   *                   *
     *************************************************************


2         3         4         5         6         7         8         9
*---------*---------*---------*---------*---------*---------*---------*

     45        6         8         2         2         6         15

          3         2         1         1         2         3

               1                   1                   1

LINE  NO=  24                                          BACKGROUND=    46

STATIONS=  -25/20                                      OPERATOR=  9HLS16

LOCATION=  FIELD DATA 1      RMS ERROR=  0.14          DATE=      3/25/59
```

```
2         3         4         5         6         7         8         9
*---------*---------*---------*---------*---------*---------*---------*
     *************************************************************
     *         *         *         *         *         *         *
     *         *         *         *         *         *         *
     *   2.21  *   1.40  *   1.85  *   2.11  *   0.56  *   0.25  *
     *         *         *         *         *         *         *
     *************************************************************
     *                   *                   *                   *
     *                   *                   *                   *
     *                   *                   *                   *
     *        1.85       *        0.23       *        2.00       *
     *                   *                   *                   *
     *                   *                   *                   *
     *************************************************************


2         3         4         5         6         7         8         9
*---------*---------*---------*---------*---------*---------*---------*

     8         2         2         6         15        61        70

          1         1         2         3         10        57

               1                   1                   1
```

LINE NO= 24 BACKGROUND= 91

STATIONS= -15/30 OPERATOR= 9HLS16

LOCATION= FIELD DATA 1 RMS ERROR= 0.17 DATE= 3/25/59

```
2         3         4         5         6         7         8         9
*---------*---------*---------*---------*---------*---------*---------*
     *************************************************************
     *         *         *         *         *         *         *
     *         *         *         *         *         *         *
     *  -1.20  *   0.16  *   1.75  *   1.92  *   1.42  *   1.50  *
     *         *         *         *         *         *         *
     *************************************************************
     *                   *                   *                   *
     *                   *                   *                   *
     *                   *                   *                   *
     *         2.39      *         0.36      *         2.01      *
     *                   *                   *                   *
     *                   *                   *                   *
     *************************************************************


2         3         4         5         6         7         8         9
*---------*---------*---------*---------*---------*---------*---------*


     85        84        28         5         3        11        24

          50        22         7         3         2         3

               10                   3                   1
```

LINE NO= 8 BACKGROUND= 74

STATIONS= -30/15 OPERATOR= 9HLS16

LOCATION= FIELD DATA 1 RMS ERROR= 0.13 DATE= 3/25/59

```
2         3         4         5         6         7         8         9
*---------*---------*---------*---------*---------*---------*---------*
     *************************************************************
     *         *         *         *         *         *         *
     *         *         *         *         *         *         *
     *   1.04  *   1.63  *   2.32  *   1.44  *   0.97  *   0.22  *
     *         *         *         *         *         *         *
     *************************************************************
     *                   *                   *                   *
     *                   *                   *                   *
     *                   *                   *                   *
     *        2.05       *      -0.59        *       2.99        *
     *                   *                   *                   *
     *                   *                   *                   *
     *************************************************************


2         3         4         5         6         7         8         9
*---------*---------*---------*---------*---------*---------*---------*

     28         5         3         1        24        14        45
           7         3         2         3         7         8
                3                   1                   5
```

LINE NO= 8 BACKGROUND= 50

STATIONS= -20/25 OPERATOR= 9HLS16

LOCATION= FIELD DATA 1 RMS ERROR= 0.12 DATE= 3/25/59

```
2        3        4        5        6        7        8        9
*--------*--------*--------*--------*--------*--------*--------*
     ***************************************************************
     *         *         *         *         *         *         *
     *         *         *         *         *         *         *
     *  1.85   *  0.86   *  2.05   *  2.03   *  1.34   *  2.02   *
     *         *         *         *         *         *         *
     ***************************************************************
     *                   *                   *                   *
     *                   *                   *                   *
     *                   *                   *                   *
     *       2.32        *      -0.29        *       1.88        *
     *                   *                   *                   *
     *                   *                   *                   *
     ***************************************************************


2        3        4        5        6        7        8        9
*--------*--------*--------*--------*--------*--------*--------*

     32       354      86       26       49       103      100

         84       5        10       10       43       92

              1                  6                  32
```

```
LINE  NO=  -8                                     BACKGROUND=   957

STATIONS=  -15/30                                 OPERATOR=  9HLS16

LOCATION=  FIELD DATA 1      RMS ERROR=  0.22     DATE=      3/25/59
```

3	4	5	6	7	8	9	10	11	12
-0.34	0.97	0.83	0.95	0.83	0.39				
		0.95	0.47	1.06	0.60	-0.19	0.04		

3–4	5–6	7–8	9–10	11–12
0.99	-1.01	0.80		
	0.37	-0.54	2.57	

BACKGROUND=	187	178
RMS ERROR=	0.14	0.13

LOCATION= FIELD DATA 1 STATIONS= -25/30 LINE NO= 88 OPERATOR= 9HLS16

3	4	5	6	7	8	9	10	11	12
1.39	0.48	0.96	1.43	1.10	0.85				
		0.36	1.17	1.01	0.49	-0.45	1.05		

3–4	5–6	7–8	9–10	11–12
0.73	-0.04	1.46		
	0.73	0.90	0.76	

BACKGROUND=	295	222
RMS ERROR=	0.05	0.10

LOCATION= FIELD DATA 1 STATIONS= -30/25 LINE NO= 72 OPERATOR= 9HLS16

```
       3         4         5         6         7         8         9        10        11        12
-------*---------*---------*---------*---------*---------*---------*---------*---------*---------*------
*****************************************************************************************************
*         *         *         *         *         *         *         *         *         *         *
*  -0.89  *  0.46   *  1.31   *  1.53   *  0.55   *  0.87   *         *         *         *         *
*         *         *         *         *         *         *         *         *         *         *
*         *         *  1.28   *  1.26   *  0.68   *  0.83   *  0.14   * -0.11   *         *         *
*****************************************************************************************************
*                   *                   *                   *                   *                   *
*       0.79        *      -0.04        *       1.82        *                   *                   *
*                   *                   *                   *                   *                   *
*                   *       0.68        *       0.92        *       1.40        *                   *
*                   *                   *                   *                   *                   *
*****************************************************************************************************

BACKGROUND=                   146                 132

RMS  ERROR=                   0.10                0.09

          LOCATION=  FIELD DATA 1        STATIONS=  -30/25   LINE  NO=  56    OPERATOR=  9HLS16
```



```
       3         4         5         6         7         8         9        10        11        12
-------*---------*---------*---------*---------*---------*---------*---------*---------*---------*------
*****************************************************************************************************
*         *         *         *         *         *         *         *         *         *         *
*  -0.48  * -0.20   *  0.72   *  0.43   *  1.18   *  1.21   *         *         *         *         *
*         *         * -0.75   * -0.39   *  1.55   *  0.56   *  0.41   *  0.84   *         *         *
*         *         *         *         *  1.71   *  0.69   *  0.97   *  1.32   *  0.62   * -0.21   *
*****************************************************************************************************
*                   *                   *                   *                   *                   *
*       1.97        *       1.19        *       2.68        *                   *                   *
*                   *       2.44        *       1.64        *       2.18        *                   *
*                   *                   *       2.22        *       1.33        *       0.14        *
*                   *                   *                   *                   *                   *
*****************************************************************************************************

BACKGROUND=                   119                 76                  114

RMS  ERROR=                   0.13                0.11                0.12

          LOCATION=  FIELD DATA 1        STATIONS=  -35/30   LINE  NO=  40    OPERATOR=  9HLS16
```

```
       3         4         5         6         7         8         9        10        11        12
-------*---------*---------*---------*---------*---------*---------*---------*---------*---------*-------
  *****************************************************************************************************
  *  -0.21  *  -0.00  *   0.88  *   0.56  *   0.82  *   2.25  *         *         *         *         *
  *         *         *  -0.23  *   0.03  *   1.38  *   1.90  *   1.30  *   2.37  *         *         *
  *         *         *         *         *   0.02  *   1.65  *   2. 1  *   2.24  *   1.39  *   1.43  *
  *         *         *         *         *         *         *   2.21  *   1.40  *   1.85  *   2.11  *
  *****************************************************************************************************
  *                   *                   *                   *                   *                   *
  *        2.26       *        0.91       *        2.79       *                   *                   *
  *                   *        2.70       *        0.70       *        1.67       *                   *
  *                   *                   *        3.04       *       -1.18       *        2.24       *
  *                   *                   *                   *        1.85       *        0.23       *
  *****************************************************************************************************

  BACKGROUND=                 106                  80                  46                  91

  RMS  ERROR=                 0.13                0.16                0.14                0.17

            LOCATION=  FIELD DATA 1       STATIONS=  -45/30   LINE  NO=  24    OPERATOR=  9HLS16
```

```
       3         4         5         6         7         8         9        10        11        12
-------*---------*---------*---------*---------*---------*---------*---------*---------*---------*-------
  *****************************************************************************************************
  *         *         *         *         *         *         *         *         *         *         *
  *  -1.20  *   0.16  *   1.75  *   1.92  *   1.42  *   1.50  *         *         *         *         *
  *         *         *         *         *         *         *         *         *         *         *
  *         *         *   1.04  *   1.63  *   2.32  *   1.44  *   0.97  *   0.22  *         *         *
  *****************************************************************************************************
  *                   *                   *                   *                   *                   *
  *        2.39       *        0.36       *        2.01       *                   *                   *
  *                   *                   *                   *                   *                   *
  *                   *        2.05       *       -0.59       *        2.99       *                   *
  *                   *                   *                   *                   *                   *
  *****************************************************************************************************

  BACKGROUND=                  74                  50
  RMS  ERROR=                 0.13                0.12

            LOCATION=  FIELD DATA 1       STATIONS=  -30/25   LINE  NO=  8     OPERATOR=  9HLS16
```

```
       3         4         5         6         7         8         9        10        11        12
-------*---------*---------*---------*---------*---------*---------*---------*---------*---------*---------
  *****************************************************************************************************
  *         *         *         *         *         *         *         *         *         *         *
  *   0.96  *   0.72  *   0.42  *   0.80  *         *         *         *         *         *         *
  *         *         *         *         *         *         *         *         *         *         *
  *         *         *   0.25  *   0.50  *  -0.07  *  -0.66  *         *         *         *         *
  *****************************************************************************************************
  *                   *                   *                   *                   *                   *
  *       -0.62       *        0.40       *                   *                   *                   *
  *                   *                   *                   *                   *                   *
  *                   *       -0.44       *        1.80       *                   *                   *
  *                   *                   *                   *                   *                   *
  *****************************************************************************************************

  BACKGROUND=        163                 135

  RMS  ERROR=        0.16                0.19

            LOCATION=  FIELD DATA 1       STATIONS=  -25/30   LINE  NO=  88    OPERATOR=  6HLS16
```

```
       3         4         5         6         7         8         9        10        11        12
-------*---------*---------*---------*---------*---------*---------*---------*---------*---------*---------
  *****************************************************************************************************
  *         *         *         *         *         *         *         *         *         *         *
  *   0.41  *   0.48  *   0.87  *   0.93  *         *         *         *         *         *         *
  *         *         *         *         *         *         *         *         *         *         *
  *         *         *   0.79  *   0.57  *   0.37  *  -0.52  *         *         *         *         *
  *****************************************************************************************************
  *                   *                   *                   *                   *                   *
  *       -0.03       *        1.42       *                   *                   *                   *
  *                   *                   *                   *                   *                   *
  *                   *        1.37       *        0.53       *                   *                   *
  *                   *                   *                   *                   *                   *
  *****************************************************************************************************

  BACKGROUND=        202                 176

  RMS  ERROR=        0.15                0.13

            LOCATION=  FIELD DATA 1       STATIONS=  -30/25   LINE  NO=  72    OPERATOR=  6HLS16
```

```
      3         4         5         6         7         8         9        10        11        12
------*---------*---------*---------*---------*---------*---------*---------*---------*---------*------
 *****************************************************************************************************
 *         *         *         *         *         *         *         *         *         *         *
 *  -0.03  *   1.10  *   1.50  *   1.20  *         *         *         *         *         *         *
 *         *         *         *         *         *         *         *         *         *         *
 *         *         *   1.73  *   0.58  *   0.40  *  -0.54  *         *         *         *         *
 *****************************************************************************************************
 *                   *                   *                   *                   *                   *
 *        0.34       *        1.12       *                   *                   *                   *
 *                   *                   *                   *                   *                   *
 *                   *        0.50       *        1.82       *                   *                   *
 *                   *                   *                   *                   *                   *
 *****************************************************************************************************

 BACKGROUND=        140                 106

 RMS  ERROR=        0.19                0.15

           LOCATION=  FIELD DATA 1        STATIONS=  -30/25   LINE  NO=  56    OPERATOR=  6HLS16
```

```
      3         4         5         6         7         8         9        10        11        12
------*---------*---------*---------*---------*---------*---------*---------*---------*---------*------
 *****************************************************************************************************
 *         *         *         *         *         *         *         *         *         *         *
 *  -1.16  *   0.22  *   0.04  *   1.77  *         *         *         *         *         *         *
 *         *         *  -1.23  *   1.33  *   0.38  *   0.64  *         *         *         *         *
 *         *         *         *         *   1.52  *   0.96  *   1.19  *   0.01  *         *         *
 *****************************************************************************************************
 *                   *                   *                   *                   *                   *
 *        2.09       *        2.78       *                   *                   *                   *
 *                   *        2.80       *        2.35       *                   *                   *
 *                   *                   *        2.52       *        0.55       *                   *
 *                   *                   *                   *                   *                   *
 *****************************************************************************************************

 BACKGROUND=         95                  64                  103

 RMS  ERROR=        0.24                0.19                 0.24

           LOCATION=  FIELD DATA 1        STATIONS=  -35/30   LINE  NO=  40    OPERATOR=  6HLS16
```

```
        3         4         5         6         7         8         9        10        11        12
--------*---------*---------*---------*---------*---------*---------*---------*---------*---------*-------
   *****************************************************************************************************
   * -0.49   *  0.54   *  0.19   *  1.88   *         *         *         *         *         *         *
   *         *         * -0.74   *  0.93   *  1.94   *  2.44   *         *         *         *         *
   *         *         *         *         *  1.90   *  1.68   *  1.94   *  2.22   *         *         *
   *         *         *         *         *         *         *  2.55   *  1.84   *  1.75   * -0.34   *
   *****************************************************************************************************
   *                   *                   *                   *                   *                   *
   *        1.81       *        2.75       *                   *                   *                   *
   *                   *        2.64       *        1.28       *                   *                   *
   *                   *                   *        1.23       *        0.95       *                   *
   *                   *                   *                   *        1.18       *        2.02       *
   *****************************************************************************************************

   BACKGROUND=         89                  61                  34                  61

   RMS  ERROR=       0.33                0.32                0.31                0.31

             LOCATION=  FIELD DATA 1       STATIONS=  -45/30   LINE  NO=  24    OPERATOR=  6HLS16
```

```
        3         4         5         6         7         8         9        10        11        12
--------*---------*---------*---------*---------*---------*---------*---------*---------*---------*-------
   *****************************************************************************************************
   *         *         *         *         *         *         *         *         *         *         *
   * -0.48   *  1.45   *  1.92   *  2.32   *         *         *         *         *         *         *
   *         *         *         *         *         *         *         *         *         *         *
   *         *         *  2.18   *  2.14   *  0.95   *  0.99   *         *         *         *         *
   *****************************************************************************************************
   *                   *                   *                   *                   *                   *
   *        1.89       *        1.37       *                   *                   *                   *
   *                   *                   *                   *                   *                   *
   *                   *        0.92       *        1.77       *                   *                   *
   *                   *                   *                   *                   *                   *
   *****************************************************************************************************

   BACKGROUND=         68                  35

   RMS  ERROR=       0.26                0.26

             LOCATION=  FIELD DATA 1       STATIONS=  -30/25   LINE  NO=  8     OPERATOR=  6HLS16
```

```
2         3         4         5         6         7         8         9
*---------*---------*---------*---------*---------*---------*---------*
     *************************************************************
     *         *         *         *         *         *         *
     *         *         *         *         *         *         *
     *  1.47   *  0.11   *  0.07   *  0.30   * -0.07   * -1.15   *
     *         *         *         *         *         *         *
     *************************************************************
     *                   *                   *                   *
     *                   *                   *                   *
     *                   *                   *                   *
     *        1.43       *        0.87       *        2.19       *
     *                   *                   *                   *
     *                   *                   *                   *
     *************************************************************


2         3         4         5         6         7         8         9
*---------*---------*---------*---------*---------*---------*---------*

    100        91        149       177       147       205       226

         106        78        107       113       117       135

               97                  80                  75


LINE  NO=  A                                       BACKGROUND=   187

STATIONS=  0/45                                    OPERATOR=  9HLS16

LOCATION=  FIELD DATA 3      RMS ERROR=  0.06      DATE=      3/25/59
```

```
2         3         4         5         6         7         8         9
*---------*---------*---------*---------*---------*---------*---------*
     *************************************************************
     *         *         *         *         *         *         *
     *         *         *         *         *         *         *
     *  -0.75  *  -0.25  *  -0.40  *  -1.52  *  -1.69  *  -0.41  *
     *         *         *         *         *         *         *
     *************************************************************
     *                   *                   *                   *
     *                   *                   *                   *
     *                   *                   *                   *
     *        1.97       *        1.75       *        3.39       *
     *                   *                   *                   *
     *                   *                   *                   *
     *************************************************************


2         3         4         5         6         7         8         9
*---------*---------*---------*---------*---------*---------*---------*

    149       177       147       205       226       206       100

         107       113       117       135       139       77

               80                  75                  60
```

LINE NO= A   BACKGROUND= 151

STATIONS= 10/55   OPERATOR= 9HLS16

LOCATION= FIELD DATA 3   RMS ERROR= 0.13   DATE= 3/25/59

```
2         3         4         5         6         7         8         9
*---------*---------*---------*---------*---------*---------*---------*
     *************************************************************
     *         *         *         *         *         *         *
     *         *         *         *         *         *         *
     *  -0.35  *  -0.22  *   0.79  *   1.44  *   0.35  *  -0.72  *
     *         *         *         *         *         *         *
     *************************************************************
     *                   *                   *                   *
     *                   *                   *                   *
     *                   *                   *                   *
     *        2.93       *        0.71       *        1.08       *
     *                   *                   *                   *
     *                   *                   *                   *
     *************************************************************


2         3         4         5         6         7         8         9
*---------*---------*---------*---------*---------*---------*---------*

    110       125        80        55        49       146       125
          56        33        44        56        68        95
               23                  39                  57

LINE  NO=  B                                       BACKGROUND=   126
STATIONS=  0/90                                    OPERATOR=  9HLS16
LOCATION=  FIELD DATA 3      RMS ERROR=  0.09      DATE=     3/25/59
```

```
2         3         4         5         6         7         8         9
*---------*---------*---------*---------*---------*---------*---------*
     *************************************************************
     *         *         *         *         *         *         *
     *         *         *         *         *         *         *
     *  0.35   *  1.07   *  0.50   * -0.39   * -0.61   *  0.57   *
     *         *         *         *         *         *         *
     *************************************************************
     *                   *                   *                   *
     *                   *                   *                   *
     *                   *                   *                   *
     *        2.30       *       -0.41       *        3.06       *
     *                   *                   *                   *
     *                   *                   *                   *
     *************************************************************


2         3         4         5         6         7         8         9
*---------*---------*---------*---------*---------*---------*---------*


     80        55        49        146       125       89        80

          44        56        68        95        62        48

               39                  57                  57
```

LINE NO= B

STATIONS= 20/110

LOCATION= FIELD DATA 3 RMS ERROR= 0.12

BACKGROUND= 118

OPERATOR= 9HLS16

DATE= 3/25/59

```
2         3         4         5         6         7         8         9
*---------*---------*---------*---------*---------*---------*---------*
     *************************************************************
     *         *         *         *         *         *         *
     *         *         *         *         *         *         *
     *  -0.80  *  -0.16  *   0.16  *   0.48  *   0.96  *   1.01  *
     *         *         *         *         *         *         *
     *************************************************************
     *                   *                   *                   *
     *                   *                   *                   *
     *                   *                   *                   *
     *        2.27       *        1.25       *        0.96       *
     *                   *                   *                   *
     *                   *                   *                   *
     *************************************************************


2         3         4         5         6         7         8         9
*---------*---------*---------*---------*---------*---------*---------*

    135       106       110        76        63        57        64
          70        65        55        35        53        54
              46                  26                  58
```

```
LINE  NO=  C                                     BACKGROUND=   113
STATIONS=  0/90                                  OPERATOR=  9HLS16
LOCATION=  FIELD DATA 3     RMS ERROR=  0.07     DATE=     3/25/59
```

```
2         3         4         5         6         7         8         9
*---------*---------*---------*---------*---------*---------*---------*
     *************************************************************
     *         *         *         *         *         *         *
     *         *         *         *         *         *         *
     *  -0.57  *  0.12   *  1.19   *  0.82   *  0.44   *  1.14   *
     *         *         *         *         *         *         *
     *************************************************************
     *                   *                   *                   *
     *                   *                   *                   *
     *                   *                   *                   *
     *       2.52        *      -0.34        *       0.81        *
     *                   *                   *                   *
     *                   *                   *                   *
     *************************************************************
```

```
2         3         4         5         6         7         8         9
*---------*---------*---------*---------*---------*---------*---------*

    110        76         63        57         64         55         60

          55         35        53         54         52         59

               26                   58                    63
```

LINE NO= C

STATIONS= 20/110

LOCATION= FIELD DATA 3   RMS ERROR= 0.06

BACKGROUND= 97

OPERATOR= 9HLS16

DATE= 3/25/59

3	4	5	6	7	8	9	10	11	12
1.47	0.11	0.07	0.30	-0.07	-1.15				
		-0.75	-0.25	-0.40	-1.52	-1.69	-0.41		

1.43	0.87	2.19		
	1.97	1.75	3.39	

BACKGROUND=	187	151
RMS ERROR=	0.06	0.13

LOCATION= FIELD DATA 3 STATIONS= 0/55 LINE NO= A OPERATOR= 9HLS16

3	4	5	6	7	8	9	10	11	12
-0.35	-0.22	0.79	1.44	0.35	-0.72				
		0.35	1.07	0.50	-0.39	-0.61	0.57		

3–4	5–6	7–8	9–10	11–12
2.93	0.71	1.08		
	2.30	-0.41	3.06	

BACKGROUND= 126 118

RMS ERROR 0.09 0.12

LOCATION= FIELD DATA 3 STATIONS= 0/110 LINE NO= B OPERATOR= 9HLS16

3	4	5	6	7	8	9	10	11	12
-0.80	-0.16	0.16	0.48	0.96	1.01				
		-0.57	0.12	1.19	0.82	0.44	1.14		

3–4	5–6	7–8	9–10	11–12
2.27	1.25	0.96		
	2.52	-0.34	0.81	

BACKGROUND= 113 97

RMS ERROR= 0.07 0.06

LOCATION= FIELD DATA 3 STATIONS= 0/110 LINE NO= C OPERATOR= 9HLS16

3	4	5	6	7	8	9	10	11	12
0.67	-0.13	0.04	-0.70						
		-0.62	-0.57	-1.58	-1.60				

1.46	1.90			
	2.09	3.06		

BACKGROUND=	172	146
RMS ERROR=	0.25	0.21

LOCATION= FIELD DATA 3 STATIONS= 0/55 LINE NO= A OPERATOR= 6HLS16

```
        3         4         5         6         7         8         9        10        11        12
-------*---------*---------*---------*---------*---------*---------*---------*---------*---------*-------
    *****************************************************************************************************
    *         *         *         *         *         *         *         *         *         *         *
    *  -0.17  *   0.61  *   1.34  *  -0.01  *         *         *         *         *         *         *
    *         *         *         *         *         *         *         *         *         *         *
    *         *         *   0.93  *  -0.48  *  -1.53  *  -0.96  *         *         *         *         *
    *****************************************************************************************************
    *                   *                   *                   *                   *                   *
    *        2.62       *        0.77       *                   *                   *                   *
    *                   *                   *                   *                   *                   *
    *                   *        1.42       *        1.95       *                   *                   *
    *                   *                   *                   *                   *                   *
    *****************************************************************************************************

    BACKGROUND=         121                   84

    RMS  ERROR=         0.15                  0.25

              LOCATION=  FIELD DATA 3        STATIONS=  0/110    LINE  NO=  B     OPERATOR=  6HLS16
```



```
        3         4         5         6         7         8         9        10        11        12
-------*---------*---------*---------*---------*---------*---------*---------*---------*---------*-------
    *****************************************************************************************************
    *         *         *         *         *         *         *         *         *         *         *
    *  -0.68  *  -0.06  *   0.29  *   1.23  *         *         *         *         *         *         *
    *         *         *         *         *         *         *         *         *         *         *
    *         *         *  -0.41  *   0.74  *   0.48  *   0.71  *         *         *         *         *
    *****************************************************************************************************
    *                   *                   *                   *                   *                   *
    *        2.22       *        1.29       *                   *                   *                   *
    *                   *                   *                   *                   *                   *
    *                   *        1.81       *       -0.10       *                   *                   *
    *                   *                   *                   *                   *                   *
    *****************************************************************************************************

    BACKGROUND=         100                   79

    RMS  ERROR=         0.16                  0.16

              LOCATION=  FIELD DATA 3        STATIONS=  0/110    LINE  NO=  C     OPERATOR=  6HLS16
```

horizontal lines within the blocks for the results along different segments of each line.

The overall results of the interpretation operators are excellant and certain additional details of the resistivity structure are obtained which were not capable of resolution by empirical interpretation by skilled personnel. The sharp cut-off in the anomaly to the right on line 40 summarized on page 175 was not predicted in the original interpretation. There appears to be no doubt that the interpretations by either operator are consistent with the known geology and that the interpretation operators have yielded valid results. It should be noted that in general the area is a rather straight forward problem for a skilled interpretor and that the evaluation of the operator results has been made partially on the basis of this empirical interpretation.

4.6 Interpretation of Field Data with Vertical Operator

The great success of any interpretation scheme is not to simply yield correct results for those areas in which the skilled interpretor is sufficiently good but to be able to properly interpret those areas in which he would fail. Fortunately field results were readily available for such an area which had geological control. It was known that the regional structure consisted of a number of parallel layers dipping almost vertically so that it indicated a vertical operator should be employed. Attempts to empirically interpret the field data were rather indefinite and an interpretation with the 8VLS16 operator was made.

The following page presents the plan view of the profile lines and stations employed in the set of data referred to as Field Data 4. Only two lines were available for study in this immediate area but they were sufficiently long so that a total of 7 applications of the operator could be made. A summary of the results of both lines is also presented on pages 200-1 and the results are extremely good. As seen on both lines there are two thin highly conducting zones dipping to the Northwest at approximately 45°. The first intersects the surface around station 1E while the second intersects the surface around 4E. The correlation of the results along each line and the consistency from one line to the other clearly shows the exact location and orientation of the two regions.

The geological information available states that a zone approximately 1 unit thick dips to the Northwest at 35° and intersects the lines at station 1E. In addition there is possibly a second zone parallel to this first one but to the East. Certainly the agreement between the geology and predicted resistivity structure is very good. It is seen that there may well be an indication of a third or fourth zone intersecting the lines at stations 6E and 8E but there is insufficient data to definitely interpret the eastern extremity of the lines.

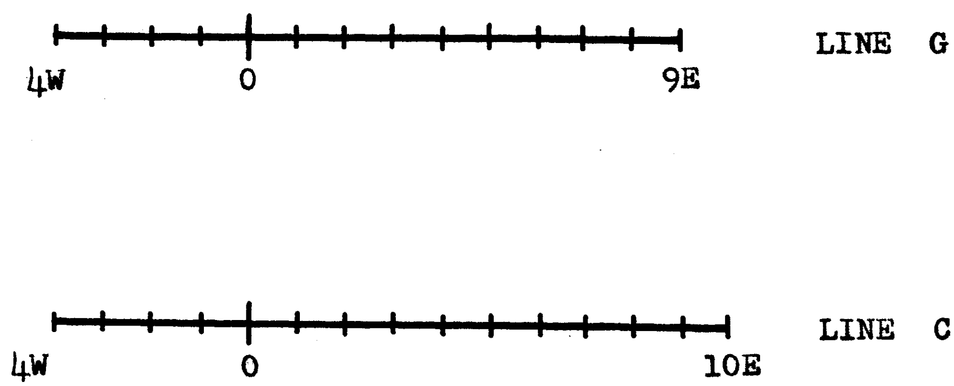


Figure 4 Plan View of Profile Lines for
Field Data Number 4

```
2         3         4         5         6         7         8         9
*---------*---------*---------*---------*---------*---------*---------*
               *****************************************
               *         *         *         *         *
               *         *         *         *         *
               *  -0.23  *   1.18  *   2.32  *  -0.80  *
               *         *         *         *         *
               *****************************************
               *         *         *         *         *
               *         *         *         *         *
               *         *         *         *         *
               *         *         *         *         *
               *         *         *         *         *
               *   1.70  *   0.57  *  -2.37  *   2.79  *
               *         *         *         *         *
               *         *         *         *         *
               *         *         *         *         *
               *         *         *         *         *
               *         *         *         *         *
               *****************************************


2         3         4         5         6         7         8         9
*---------*---------*---------*---------*---------*---------*---------*

    524       393       503       217       532       309       415
         278       338       400       605       294       314
              252       305      1045       274       299
                   246                           280

LINE  NO=  C                                           BACKGROUND=   654
STATIONS=  4W/5E                                       OPERATOR=  8VLS20
LOCATION=  FIELD DATA 4      RMS ERROR=  0.32          DATE=     4/21/59
```

```
2          3          4          5          6          7          8          9
*----------*----------*----------*----------*----------*----------*----------*
                ******************************************
                *         *         *         *         *
                *         *         *         *         *
                *  2.34   *  -0.19  *  0.20   *  1.97   *
                *         *         *         *         *
                ******************************************
                *         *         *         *         *
                *         *         *         *         *
                *         *         *         *         *
                *         *         *         *         *
                *         *         *         *         *
                * -2.20   *  0.07   *  2.90   * -1.11   *
                *         *         *         *         *
                *         *         *         *         *
                *         *         *         *         *
                *         *         *         *         *
                *         *         *         *         *
                ******************************************


2          3          4          5          6          7          8          9
*----------*----------*----------*----------*----------*----------*----------*

    503        217        532        309        415        211        265
         400        605        294        314        420        311
             1045       274        299        300        609
                  473                                430
```

LINE NO= C BACKGROUND= 630

STATIONS= 2W/7E OPERATOR= 8VLS20

LOCATION= FIELD DATA 4 RMS ERROR= 0.25 DATE= 4/21/59

```
2         3         4         5         6         7         8         9
*---------*---------*---------*---------*---------*---------*---------*
               *****************************************
               *         *         *         *         *
               *         *         *         *         *
               *  -0.73  *   1.60  *   0.84  *   0.25  *
               *         *         *         *         *
               *****************************************
               *         *         *         *         *
               *         *         *         *         *
               *         *         *         *         *
               *         *         *         *         *
               *         *         *         *         *
               *   1.35  *  -2.03  *   0.78  *  -0.10  *
               *         *         *         *         *
               *         *         *         *         *
               *         *         *         *         *
               *         *         *         *         *
               *         *         *         *         *
               *****************************************

2         3         4         5         6         7         8         9
*---------*---------*---------*---------*---------*---------*---------*

    532       309       415       211       265       328       336

         294       314       420       311       298       312

              299       300       609       320       299

                   282                           304

LINE  NO=  C                                          BACKGROUND=   410

STATIONS=  0/9E                                       OPERATOR=  8VLS20

LOCATION=  FIELD DATA 4      RMS ERROR=  0.19         DATE=      4/21/59
```

```
          3         4         5         6         7         8         9
*---------*---------*---------*---------*---------*---------*---------*
               *****************************************
               *         *         *         *         *
               *         *         *         *         *
               *  1.12   *  0.82   *  0.76   *  0.03   *
               *         *         *         *         *
               *****************************************
               *         *         *         *         *
               *         *         *         *         *
               *         *         *         *         *
               *         *         *         *         *
               *         *         *         *         *
               * -1.72   *  1.24   * -2.26   *  2.15   *
               *         *         *         *         *
               *         *         *         *         *
               *         *         *         *         *
               *         *         *         *         *
               *         *         *         *         *
               *****************************************

2         3         4         5         6         7         8         9
*---------*---------*---------*---------*---------*---------*---------*

     309       415       211       265       328       336       350

          314       420       311       298       312       384

               300       609       320       299       338

                    430                           294
```

LINE NO= C BACKGROUND= 411

STATIONS= 1W/10E OPERATOR= 8VLS20

LOCATION= FIELD DATA 4 RMS ERROR= 0.14 DATE= 4/21/59

```
2                   3                   4                   5                   6                   7                   8                   9
*-------------------*-------------------*-------------------*-------------------*-------------------*-------------------*-------------------*
                             *****************************************
                             *                   *                   *                   *                   *
                             *                   *                   *                   *                   *
                             *       0.66        *       0.14        *       1.76        *      -2.22        *
                             *                   *                   *                   *                   *
                             *****************************************
                             *                   *                   *                   *                   *
                             *                   *                   *                   *                   *
                             *                   *                   *                   *                   *
                             *                   *                   *                   *                   *
                             *                   *                   *                   *                   *
                             *       2.17        *       0.40        *      -4.14        *       4.25        *
                             *                   *                   *                   *                   *
                             *                   *                   *                   *                   *
                             *                   *                   *                   *                   *
                             *                   *                   *                   *                   *
                             *                   *                   *                   *                   *
                             *****************************************


2                   3                   4                   5                   6                   7                   8                   9
*-------------------*-------------------*-------------------*-------------------*-------------------*-------------------*-------------------*

         513                 230                 332                 436                 778                 391                 523

                   325                 256                 426                 855                 356                 395

                             383                 383                 790                 386                 369

                                       500                                                           410

LINE  NO=  G                                                                                     BACKGROUND=   549

STATIONS=  4W/5E                                                                                 OPERATOR=  8VLS20

LOCATION=  FIELD DATA 4          RMS ERROR=  0.25                                                DATE=      4/21/59
```

```
2          3          4          5          6          7          8          9
*----------*----------*----------*----------*----------*----------*----------*
               *****************************************
               *         *         *         *         *
               *         *         *         *         *
               *   1.37  *  -2.08  *   0.18  *   0.86  *
               *         *         *         *         *
               *****************************************
               *         *         *         *         *
               *         *         *         *         *
               *         *         *         *         *
               *         *         *         *         *
               *         *         *         *         *
               *  -4.35  *   3.30  *   1.37  *  -0.18  *
               *         *         *         *         *
               *         *         *         *         *
               *         *         *         *         *
               *         *         *         *         *
               *         *         *         *         *
               *****************************************


2          3          4          5          6          7          8          9
*----------*----------*----------*----------*----------*----------*----------*


    332        436        778        391        523        296        370

          426        855        356        395        428        328

               790        386        369        313        518

                    362                                367

LINE  NO=  G                                           BACKGROUND=   504

STATIONS=  2W/7E                                       OPERATOR=  8VLS20

LOCATION=  FIELD DATA 4      RMS ERROR=  0.21          DATE=      4/21/59
```

```
2         3         4         5         6         7         8         9
*---------*---------*---------*---------*---------*---------*---------*
               *****************************************
               *         *         *         *         *
               *         *         *         *         *
               *  -0.28  *   1.26  *   0.42  *   0.81  *
               *         *         *         *         *
               *****************************************
               *         *         *         *         *
               *         *         *         *         *
               *         *         *         *         *
               *         *         *         *         *
               *         *         *         *         *
               *   1.95  *  -1.23  *   1.43  *   0.14  *
               *         *         *         *         *
               *         *         *         *         *
               *         *         *         *         *
               *         *         *         *         *
               *         *         *         *         *
               *****************************************

2         3         4         5         6         7         8         9
*---------*---------*---------*---------*---------*---------*---------*

    778       391       523       296       370       371       280
         356       395       428       328       438       343
              369       313       518       377       366
                   300                           329

LINE  NO=  G                                          BACKGROUND=   564
STATIONS=  0/9E                                       OPERATOR=  8VLS20
LOCATION=  FIELD DATA 4      RMS ERROR=  0.20         DATE=     4/21/59
```

3	4	5	6	7	8	9	10	11	12
-0.23	1.18	2.32	-0.80						
		2.34	-0.19	0.20	1.97				
				-0.73	1.60	0.84	0.25		
					1.12	0.82	0.76	0.03	

3	4	5	6	7	8	9	10	11	12
1.70	0.57	-2.37	2.79						
		-2.20	0.07	2.90	-1.11				
				1.35	-2.03	0.78	-0.10		
					-1.72	1.24	-2.26	2.15	

BACKGROUND= 654 630 410 411

RMS ERROR= 0.32 0.25 0.19 0.14

LOCATION= FIELD DATA 4 STATIONS= 4W/10E LINE NO= C OPERATOR= 8VLS20

3	4	5	6	7	8	9	10	11	12
0.66	0.14	1.76	-2.22						
		1.37	-2.08	0.18	0.86				
				-0.28	1.26	0.42	0.81		
******	******	******	******	******	******	******	******	******	******
2.17	0.40	-4.14	4.24						
		-4.35	3.30	1.37	-0.18				
				1.95	-1.23	1.43	0.14		

BACKGROUND 549 504 564

RMS ERROR= 0.25 0.21 0.20

LOCATION= FIELD DATA 4 STATIONS= 4W/9E LINE NO= G OPERATOR= 8VLS20

There is little doubt that the interpretation operator has proven to be very helpful in interpreting a rather complicated structure.

4.7 Final Conclusions Regarding Resistivity Interpretation

The success of the interpretation operators developed in this research has been tested for a wide variety of subsurface structures. In all cases the results have been consistent with the information available regarding the actual resistivity structure. The linear approximation requires that some 'over-shooting' of the apparent resistivities predicted occur to best fit the ρ_A data by least squares analysis. This is due to the non-linear interactions of the induced sources which the real data represents and which the approximation must fit as best it can. It has been observed that if the real structure at depth extends horizontally into 3 or more blocks then a depression of the K_i values for the deep interior blocks occurs while the K_i s of the deep exterior blocks are increased. This is a predictable behavior which can be partially eliminated by the use of either a shorter operator and/or a shifting of the operator being used. The idea of a shorter operator would be to use the 6HLS16 rather than the 9HLS16 for the same data.

The ability to properly interpret complicated apparent resistivity profiles is of great help in the possible applications of theresistivity prospecting method for it allows not only a detection but also a resolution of the structure to be made. The results presented have all been obtained for the complete interpretation operator which determines the background ρ_o in addition to the K_1. Work has aldo been done on applying the limited operators which necessitate that the background be estimated by the individual responsible for the final interpretation. The operators $([A^T][A])^{-1}[A^T]$ for 6HLS16, 9HLS16, 9HLS20, 8VLS16 and 8VLS20 are presented in Appendix II. Their results are in good agreement with the ones herein presented and the final conclusion reached regarding this method of interpretation is that it is a valid quantitative approach to the direct interpretation problem. The results obtained still require the evaluation by adequately trained personnel but much greater detail is possible in delineating the subsurface structure.

There is no question that this procedure is an approximate one but the results are sufficiently good so that practical applications can be made. The results have been obtained only for the dipole-dipole array but the concept can be applied to any other electrode array. Because of the success of this interpretation procedure it is anticipated that resistivity methods will be of much greater help in geophysical prospecting in the future. This is basically a direct method of interpretation after the operator has once been chosen. The choice of operator should be based on the available geological control about the general structural relations in the area. However, the use of an operator that does not correspond to the general trends of the subsurface will not lead to erroneous conclusions. It will however be readily apparent from the magnitudes of the K_i that a different operator should possibly be employed.

Because of the speed and economy with which an electronic computer can process field data there is no reason why a number of different operators cannot be used for the same data. This will lead to a set of interpretations which can be compared with each other and the best subsurface representation determined. The RMS Error does not appear to be as good a figure of merit to judge the applicability of an operator as the overall range of the K_i. It is suggested that the K_i determined be interpreted on the basis of a bounded source strength such as 4.1.1 and that more than one operator be employed in areas of little geological control. Finally it must be realized that the results obtained refer to the subsurface resistivity structure and this may depart from the conventional geological structure. In all interpretations obtained with these operators due account must be made of the geological information available as an independent check on the tentative structures inferred.

4.8 Applicability of Interpretation for IP Data

An attempt has been made to extend the linear approximation approach of resistivity interpretation to determining the subsurface polarizable regions from induced polarization measurements. The results however have not been successful and it is concluded that the interpretation of induced polarization surveys may not be made by any direct method of interpretation based on linear approximation theory. As already pointed out although ρ_A may change by 25% at a given station configuration this may only represent a 1% change when referred to the background ρ_0. Thus any method which uses effectively the high frequency ρ_{Aj}^H profile to determine the corresponding K_i or ΔK_i would only illustrate the stability of the operator to slight changes in the ρ_{Aj} from the low frequency value.

It is necessary to develop a method which works directly with the change in ρ_{Aj} and relates this to the $\Delta\rho_i$ for each of the subsurface regions. Eq. 4.2.12 is one form of such an equation but as might be expected it requires the knowledge of the individual ρ_i. Since the strength factor in 4.1.1 used to form 4.2.12 is bounded by 2.0 there is no way of properly interpreting the ρ_i from those K_i greater than 2.0 and some manner of scaling them must be introduced so that an actual finite ρ_i may be abstracted from each K_i determined.

Then a least squares fitting of the $\Delta\rho_{kj}$ knowing the A_{ij} and ρ_i would yield the $\Delta\rho_i$. However, this entire procedure has required the introduction of a non-linear strength factor which is bounded and shows saturation. At present the linear interpretation procedure does not adequately treat this non-linear strength factor and this difficulty must be properly resolved before any advance on the IP interpretation by direct methods can be made.

4.9 Suggestions for further work and Summary of Assumptions

The interpretation system developed has been based upon the concept of a subsurface region consisting of a number of finite sized homogeneous volumes. The geometry and relative location of these regions has been fixed prior to the interpretation by least squares fitting of the modified first approximation forward solutions. The compositing of the volumes by linear superposition has been possible because the approximation used is linear in the effect of each volume on the measured data. A transformation of the variable to be fitted from potential to apparent resistivity has eliminated the need to integrate field data and any errors introduced by such operations.

The modified first approximation has preserved the very essential property of symmetry which must be present in the measurements and utilization of symmetric sets of data points and subsurface regions has maintained this symmetry. It is very important that this property be consistently contained within the interpretation procedure so that correct interpretations of subsurface geometrical relations can be made. There is good reason to interpret the resistivity measurements and interpreted K_1 on the basis of the bounded strength factor and especially on such as 4.1.1. However, if such a factor is used then due account must be made of the error introduced at adjacent surfaces of the homogeneous regions if simple linear compositing of the regions is done. Finally the induced polarization direct interpretation must await the development of the satisfactory treatment of the strength factors so that the ρ_i may be properly determined for each region in order that Equations 4.2.7 and 4.2.8 be utilized.

Certainly additional work must be done on applying this method of resistivity interpretation to more field data and preferable some with greater geological control. The author feels that the method is sufficiently well developed to apply practically to resistivity interpretation and that a good place to begin would be the already available field data. There is nothing unique about the array nor data point distribution that has been utilized but it has been consistent with past field operational procedures. The array, block configuration, data points and their number can be varied to specifically satisfy a particular geometry. The resolution limits must always be kept in mind and too much detailed information must not be the goal of the interpretation.

The saturation phenomenon for induced sources which is evidenced in the bounded strength factor for the regions can only adequately be treated in the interpretation operators by the solution of a non-linear set of equations. The procedure would be to calculate the effect of each surface bounding the homogeneous subsurface regions rather than the effect of the region. Then by fitting the data to these effects it would be possible to include the correct strength factor between adjacent surfaces of neighboring regions. That is, if simple linear superposition of the regions' effects are made then an error in the equivalent induced source at such an interface arises. Let σ_1 represent the conductivity of region 1 and σ_2 that of the neighboring region. The strength factor to utilize if assuming 4.1.1 to be correct would be:

$$K = 2\left[\frac{\sigma_1 - \sigma_2}{\sigma_1 + \sigma_2}\right] \qquad 4.9.1$$

However linear superposition places the sum of the two sources computed when each region is considered independently:

$$\Delta K_L = \frac{4(\sigma_1 - \sigma_2)}{(\sigma_1 + \sigma_0)(\sigma_2 + \sigma_0)} \qquad 4.9.2$$

Thus the error is equal to the difference between 4.9.1 and 4.9.2 and is given by:

$$\Delta K - K = \left[\frac{\sigma_0(\sigma_1 + \sigma_2) - \sigma_0^2 - \sigma_1\sigma_2}{(\sigma_1 + \sigma_0)(\sigma_2 + \sigma_0)} \right] K \qquad 4.9.3$$

It is seen that this error is zero only when $\sigma_1 = \sigma_2$ or either σ_1 or σ_2 is equal to the background.

In this treatment of the saturation phenomenon the variables become the actual conductivities once an assumption regarding the correct form of K_i has been made. The set of equations relating the apparent resistivity effects to the A_{ij} will not be linear in these variables and it may prove to be a difficult task to solve such a set of equations. Possibly an iterative procedure which utilized the linear solution for the K_i considering the regions as a unit would lead to a method of solving the set of equations. Starting from this initial solution for the ρ_i modifications would be made so that a better fit of the data would result. The effects of each surface would be used as the influence parameters taking into account the strength factor, Eq. 4.9.1, for adjacent regions. As yet this approach has not been tried.

Some effort has been made on investigating numerically the convergence of the Stevenson series expansion solution for the forward problem. A complete development is not available but preliminary results indicate that the convergence of the solution may be rather slow. The higher order terms in the expansion Eq. 2.3.3 were calculated from the interactions of the induced proimary sources on the conducting region. The amount of computation necessary to effect such a solution is an order of magnitude greater than that required for the initial linear solution. The results have been evaluated on the basis of whether or not symmetry of the solution is improved by including the higher order terms. Symmetry does not appear to be improved for the one geometry considered to second order, but this question has not been completely resolved. The question of convergence of the forward solution of Stevenson has not been capable of analytical treatment and any numerical work along this line will prove very helpful in justifying the method implied. It appears on the basis of intuitive arguments that the convergence of the series must occur and how rapidly this convergence takes place and when does symmetry appear may be treated in this numerical approach.

Appendix I Numerical Evaluation of Forward Problem

The numerical solution of Eq. 4.1.2 is obtained by an approximate integration over the surface of the homogeneous subsurface region. The surface is considered to be subdivided into a large number of small areas and the induced surface source due to the primary point source calculated. An equivalent point source is then placed at the center of the area and the secondary potential resulting from these induced point sources determined. The surface integral is thus replaced by a double summation which is developed in the following paragraphs.

Reference to the general diagram of rectangular blocks on page 75 will be made. Cartesian coordinates are chosen with the x axis coinciding with the profile line, the y axis horizontal and symmetrical about the profile plane and the positive z axis directed vertically down-wards with $z = 0$ the surface of the half-space. The center of the body is assumed to be at depth H below the surface of the half-space and at $x = D$, $y = 0$. Also for simplicity in the formulation let the half-lengths of the body be defined as: $a = A/2$, $b = B/2$ and $c = C/2$. Thus the surface integral has to be computed over the 6 surfaces $x = D \pm a$, $y = \pm b$ and $z = H \pm c$ and it proves economical in both the expressions and actual computation of the surface integral to consider these surfaces two at a time in the sequence 1,2, --6.

Let ξ represent the source position and α the point at which the potential is measured, both points on the profile line so that the coordinates $y=z=0$. Now the surface is considered to be subdivided into a number of small rectangular surface areas with L,M and N representing respectively the number in the x,y and z directions. Thus $\Delta x = A/L$, $\Delta y = B/M$ and $\Delta z = C/N$. K represents the magnitude of $\nabla\sigma/\sigma$, or any other expression assumed to be correct for the strength of the induced sources.

Normalize the primary potential φ_0 so that $\varphi_0 = 1/R$ where R represents the distance from the source point to the point (x,y,z). Finally l,m and n will represent the positions of the small areas in a sequential numbering in the x,y,z directions. Thus the secondary potential due to surfaces 1 and 2 when $x = D \pm a$ is given by the double summation:

$$\sum_{n=1}^{N}\sum_{m=1}^{M/2}\left\{\frac{\pm K(\pm a + D - \xi)\,\Delta y\,\Delta z}{\left[(\pm a + D - \xi)^2 + (m\Delta y - \tfrac{1}{2}\Delta y)^2 + (H - c + n\Delta z - \tfrac{1}{2}\Delta z)^2\right]^{3/2}} \cdot \frac{1}{\pi}\left[\frac{1}{\left[(\alpha \mp a - D)^2 + (m\Delta y - \tfrac{1}{2}\Delta y)^2 + (H - c + n\Delta z - \tfrac{1}{2}\Delta z)^2\right]^{1/2}}\right]\right\} \qquad \text{I.1}$$

The secondary potential due to surfaces 3 and 4 are given by:

$$\sum_{1}^{L}\sum_{1}^{N}\left\{\frac{+K(b)\,\Delta x\,\Delta z}{\left[(-a + D + l\Delta x - \tfrac{1}{2}\Delta x - \xi)^2 + b^2 + (H - c + n\Delta z - \tfrac{1}{2}\Delta z)^2\right]^{3/2}} \cdot \frac{1}{\pi}\left[\frac{1}{\left[(\alpha + a - D - l\Delta x + \tfrac{1}{2}\Delta x)^2 + b^2 + (H - c + n\Delta z - \tfrac{1}{2}\Delta z)^2\right]^{1/2}}\right]\right\} \qquad \text{I.2}$$

and those of surfaces 5 and 6 by:

$$\sum_{1}^{L}\sum_{1}^{M/2}\left\{\frac{\pm K(H\pm c)\,\Delta y\,\Delta x}{\left[(-a+D+l\Delta x-\frac{1}{2}\Delta x-\xi)^2+(m\Delta y-\frac{1}{2}\Delta y)^2+(H\pm c)^2\right]^{3/2}}\cdot\right. \qquad \text{I.3}$$

$$\left.\frac{1}{\pi}\left[\frac{1}{\left[(\alpha+a-D-l\Delta x+\frac{1}{2}\Delta x)^2+(m\Delta y-\frac{1}{2}\Delta y)^2+(H\pm c)^2\right]^{1/2}}\right]\right\}$$

The net effect is the sum of these 6 summations and represents the normalized secondary potential due to the subsurface rectangular block.

The errors involved in approximating the surface integral by the discrete summation cannot be determined accurately. Exact solutions cannot be found analytically for the error terms. They are proportional to the product of second derivatives of the functional being integrated and the spacing intervals chosen as:

$$E \approx hk\left[mh^2\frac{\partial^2 f}{\partial x_1^2}\bigg|_{\xi_1,\eta_1}+nk^2\frac{\partial^2 f}{\partial x_2^2}\bigg|_{\xi_2,\eta_2}\right] \qquad \text{I.4}$$

where h and k represent the spacing intervals and m and n the number of the intervals respectively in the x_1 and x_2 directions. Numbrical computation of the secondary potentials for a range of L,M and N however led to certain conclusions regarding the relative size of the intervals which would lead to the proper answer. As the spacing decreased to half the electrode interval spacing the results asymptotically approached a constant value. Although the dimensions of the small areas have been half the electrode intervals it is necessary to refine the spacing when blocks are near the surface.

The computer program utilized carried approximately 8 digit accuracy in floating point form. The effects of the individual contributions range only over a few orders of magnitude so that roundoff errors are not important and the results are accurate within the truncation error of the discrete summation. As previously indicated this possibility has been numerically investigated and results indicate that this error will be less than ½% as long as the spatial dimensions of the small areas are on the on the order of half the electrode intervals.

The computation of the higher order terms in the series expansion of Stevenson are calculated by considering each induced point source as a 'primary source'. The effects of these 'primary sources' are to modify the existing induced sources. This interaction can be continued to as high an order desired.

The ΔV_s for any electrode array using point sources is then readily calculated by appropriately combining the primary plus secondary potentials. Multiplication by the necessary geometrical factor leads to the ρ_A for the subsurface region under consideration.

Appendix II <u>Numerical Evaluation of the Inverse Problem</u>

The numerical solution to the interpretation problem in resistivity prospecting which utilizes the concept of finite sized homogeneous subregions is most easily effected with the use of matrix notation and manipulations. The following pages present the results of machine calculation of the matrix operator for the interpretation operators 6HLS16, 9HLS16, 9HLS20, 8VLS16 and 8VLS20. The operator matrix is given by:

$$\left([A^T][A]\right)^{-1}[A^T] \qquad \text{II.1}$$

The convention to be followed in determining the final results of the K_i from the B_j has already been defined in the block and data point definitions on pages 83-87. The necessary explanatory material for the use of these matrices is presented in the text. In order to present the result in a convenient form, the matrices have been compressed in the horizontal direction so that data points 1-8 then 9-16 for block 2, etc. for the 16 data point operators. The 20 data point operators have been compressed as 1-7, 8-14, and 15-20.

OPERATOR MATRIX

-7.681E-02	-1.604E 00	-1.385E 00	-3.491E-02	4.580E-03	4.726E-02	7.211E-03	-1.117E 00
6.309E-01	-5.358E-01	2.344E-01	2.813E-02	-3.749E-02	1.273E 00	-8.141E-02	1.962E-02
9.206E-02	4.333E-02	-1.645E 00	-1.117E 00	5.356E-01	-4.865E-02	-4.412E-02	1.870E-01
-5.325E-01	1.274E 00	-1.604E 00	1.432E-01	-2.574E-02	-1.039E-01	9.133E-01	-5.582E-02
-4.412E-02	-4.865E-02	5.356E-01	-1.117E 00	-1.645E 00	4.333E-02	9.206E-02	-2.574E-02
1.432E-01	-1.604E 00	1.274E 00	-5.325E-01	1.870E-01	-5.582E-02	9.133E-01	-1.039E-01
7.211E-03	4.726E-02	4.580E-03	-3.491E-02	-1.385E 00	-1.604E 00	-7.681E-02	-3.749E-02
2.813E-02	2.344E-01	-5.358E-01	6.309E-01	-1.117E 00	1.962E-02	-8.141E-02	1.273E 00
-4.268E-03	9.533E-01	1.652E 00	7.560E-01	-1.187E-01	2.177E-02	1.656E-02	1.412E-01
-1.333E 00	-1.390E 00	8.129E-01	4.401E-01	2.694E-01	-2.179E 00	-1.420E 00	6.160E-01
1.656E-02	2.177E-02	-1.187E-01	7.560E-01	1.652E 00	9.533E-01	-4.268E-03	2.694E-01
4.401E-01	8.129E-01	-1.390E 00	-1.333E 00	1.412E-01	6.160E-01	-1.420E 00	-2.179E 00

OPERATOR MATRIX

-2.301E 00	-1.398E 00	5.773E-01	-1.446E-02	2.124E-02	7.086E-03	2.431E-03	1.087E 00
-1.203E 00	2.429E-01	-4.041E-01	-3.330E-02	2.238E-01	8.856E-01	1.441E-01	-1.189E-01
6.819E-01	-1.061E 00	-1.734E 00	1.136E-02	2.152E-01	3.349E-02	-1.168E-01	-8.377E-01
1.015E 00	-9.413E-01	3.438E-01	2.828E-02	-4.724E-02	1.164E 00	-3.663E-01	1.582E-01
1.631E-01	1.093E-01	-1.786E 00	-1.113E 00	3.962E-01	-1.173E-01	7.688E-02	2.313E-01
-8.181E-01	1.245E 00	-1.073E 00	2.164E-01	-5.012E-01	-2.978E-01	1.023E 00	5.627E-02
7.688E-02	-1.173E-01	3.962E-01	-1.113E 00	-1.786E 00	1.093E-01	1.631E-01	-5.012E-01
2.164E-01	-1.073E 00	1.245E 00	-8.181E-01	2.313E-01	5.627E-02	1.023E 00	-2.978E-01
-1.168E-01	3.349E-02	2.152E-01	1.136E-02	-1.734E 00	-1.061E 00	6.819E-01	-4.724E-02
2.828E-02	3.438E-01	-9.413E-01	1.015E 00	-8.377E-01	1.582E-01	-3.663E-01	1.164E 00
2.431E-03	7.086E-03	2.124E-02	-1.446E-02	5.773E-01	-1.398E 00	-2.301E 00	2.238E-01
-3.330E-02	-4.041E-01	2.429E-01	-1.203E 00	1.087E 00	-1.189E-01	1.441E-01	8.856E-01
8.021E-01	8.476E-01	1.173E 00	4.824E-01	-7.912E-01	-1.425E-01	2.733E-01	-2.336E 00
-9.281E-01	2.641E-01	1.480E 00	-1.730E-02	-6.662E-01	-2.533E 00	-1.554E-01	1.052E-01
-6.291E-01	4.545E-01	1.380E 00	7.204E-01	1.380E 00	4.545E-01	-6.291E-01	1.897E 00
2.806E-02	-1.741E 00	-1.741E 00	2.806E-02	1.897E 00	-1.885E-01	-2.194E 00	-1.885E-01
2.733E-01	-1.425E-01	-7.912E-01	4.824E-01	1.173E 00	8.476E-01	8.021E-01	-6.662E-01
-1.730E-02	1.480E 00	2.641E-01	-9.281E-01	-2.336E 00	1.052E-01	-1.554E-01	-2.533E 00

OPERATOR MATRIX

-2.035E 00	-1.299E 00	3.916E-01	-3.809E-02	-4.733E-02	-4.870E-02	3.223E-02
1.055E 00	-9.133E-01	3.097E-01	-3.657E-01	4.672E-02	1.506E-01	9.710E-01
-7.750E-01	2.850E-01	-9.158E-02	-1.874E-02	5.896E-01	-1.034E-01	
4.304E-01	-9.866E-01	-1.094E 00	5.998E-02	2.517E-03	9.373E-02	-1.229E-01
-7.821E-01	7.670E-01	-1.384E 00	3.368E-01	-6.771E-02	2.432E-01	7.874E-01
6.474E-01	-6.863E-01	7.403E-02	8.312E-02	3.317E-01	-2.060E-02	
-5.418E-02	2.535E-01	-8.741E-01	-1.093E 00	-2.183E-01	-9.843E-02	1.804E-01
9.619E-02	-1.059E 00	7.017E-01	-8.227E-01	2.031E-01	-1.763E-01	-7.769E-01
5.828E-01	7.670E-01	-1.637E-01	1.638E-01	6.715E-01	-2.342E-01	
1.804E-01	-9.843E-02	-2.183E-01	-1.093E 00	-8.741E-01	2.535E-01	-5.418E-02
-1.763E-01	2.031E-01	-8.227E-01	7.017E-01	-1.059E 00	9.619E-02	1.638E-01
-1.637E-01	7.670E-01	5.828E-01	-7.769E-01	-2.342E-01	6.715E-01	
-1.229E-01	9.373E-02	2.517E-03	5.998E-02	-1.094E 00	-9.866E-01	4.304E-01
2.432E-01	-6.771E-02	3.368E-01	-1.384E 00	7.670E-01	-7.821E-01	8.312E-02
7.403E-02	-6.863E-01	6.474E-01	7.874E-01	-2.060E-02	3.317E-01	
3.223E-02	-4.870E-02	-4.733E-02	-3.809E-02	3.916E-01	-1.299E 00	-2.035E 00
1.506E-01	4.672E-02	-3.657E-01	3.097E-01	-9.133E-01	1.055E 00	-1.874E-02
-9.158E-02	2.850E-01	-7.750E-01	9.710E-01	-1.034E-01	5.896E-01	
1.031E 00	8.142E-01	2.195E-01	4.577E-01	1.730E-01	-4.201E-02	8.392E-02
-2.125E 00	-8.665E-01	8.123E-01	1.027E 00	-1.703E-01	-9.779E-01	-2.134E 00
-4.445E-01	-1.889E-01	4.455E-01	-2.241E-01	-6.704E-01	6.033E-01	
-3.451E-01	6.680E-02	6.561E-01	6.138E-01	6.561E-01	6.680E-02	-3.451E-01
1.405E 00	6.240E-01	-1.003E 00	-1.003E 00	6.240E-01	1.405E 00	6.254E-01
-9.737E-01	-9.821E-01	-9.737E-01	6.254E-01	-9.829E-01	-9.829E-01	
8.392E-02	-4.201E-02	1.730E-01	4.577E-01	2.195E-01	8.142E-01	1.031E 00
-9.779E-01	-1.703E-01	1.027E 00	8.123E-01	-8.665E-01	-2.125E 00	-2.241E-01
4.455E-01	-1.889E-01	-4.445E-01	-2.134E 00	6.033E-01	-6.704E-01	

OPERATOR MATRIX

-3.125E-01	-2.061E 00	-2.039E 00	3.665E-02	7.767E-01	5.673E-01	2.091E-01	1.151E 00
1.202E 00	-1.664E 00	4.926E-01	-8.290E-02	-1.505E 00	7.937E-01	-1.319E-01	7.877E-01
3.517E-01	6.897E-01	-8.177E-01	-1.057E 00	-2.918E-01	-6.505E-01	-2.725E-01	-2.212E 00
-1.305E 00	2.289E 00	-1.874E 00	2.785E-01	1.681E 00	3.105E-01	8.571E-01	-8.152E-01
-2.725E-01	-6.505E-01	-2.918E-01	-1.057E 00	-8.177E-01	6.897E-01	3.517E-01	1.681E 00
2.785E-01	-1.874E 00	2.289E 00	-1.305E 00	-2.212E 00	-8.152E-01	8.571E-01	3.105E-01
2.091E-01	5.673E-01	7.767E-01	3.665E-02	-2.039E 00	-2.061E 00	-3.125E-01	-1.505E 00
-8.290E-02	4.926E-01	-1.664E 00	1.202E 00	1.151E 00	7.877E-01	-1.319E-01	7.937E-01
9.532E-01	2.755E 00	4.114E 00	5.134E-01	-3.280E 00	-2.072E 00	-7.421E-01	-9.528E 00
-4.258E 00	3.837E 00	7.147E-01	2.516E-01	5.224E 00	-5.992E-01	-1.126E 00	-2.602E 00
-1.364E 00	-2.308E 00	-2.723E 00	6.045E-01	4.393E 00	3.180E 00	1.208E 00	1.285E 01
2.650E 00	-6.824E 00	2.158E 00	-4.006E-01	-8.647E 00	-4.299E 00	-1.248E 00	4.499E 00
1.208E 00	3.180E 00	4.393E 00	6.045E-01	-2.723E 00	-2.308E 00	-1.364E 00	-8.647E 00
-4.006E-01	2.158E 00	-6.824E 00	2.650E 00	1.285E 01	4.499E 00	-1.248E 00	-4.299E 00
-7.421E-01	-2.072E 00	-3.280E 00	5.134E-01	4.114E 00	2.755E 00	9.532E-01	5.224E 00
2.516E-01	7.147E-01	3.837E 00	-4.258E 00	-9.528E 00	-2.602E 00	-1.126E 00	-5.992E-01

PERATOR MATRIX

-1.844E-01	-1.495E 00	-7.955E-01	5.148E-02	-3.327E-01	1.233E-01	7.997E-02
-1.803E-02	9.192E-01	-1.986E 00	5.464E-01	1.605E-01	-1.449E-01	8.567E-01
7.226E-01	-2.790E-01	-3.071E-01	5.416E-01	6.225E-01	-5.604E-01	

2.712E-01	4.814E-01	-1.420E 00	-1.027E 00	5.381E-01	-2.441E-01	-1.965E-01
-1.258E 00	-1.150E 00	2.163E 00	-2.229E 00	1.134E-01	1.095E 00	8.669E-02
-1.884E-01	6.264E-01	4.413E-01	-8.511E-01	-1.035E-01	6.136E-01	

-1.965E-01	-2.441E-01	5.381E-01	-1.027E 00	-1.420E 00	4.814E-01	2.712E-01
1.095E 00	1.134E-01	-2.229E 00	2.163E 00	-1.150E 00	-1.258E 00	-8.511E-01
4.413E-01	6.264E-01	-1.884E-01	8.669E-02	6.136E-01	-1.035E-01	

7.997E-02	1.233E-01	-3.327E-01	5.148E-02	-7.955E-01	-1.495E 00	-1.844E-01
-1.449E-01	1.605E-01	5.464E-01	-1.986E 00	9.192E-01	-1.803E-02	5.416E-01
-3.071E-01	-2.790E-01	7.226E-01	8.567E-01	-5.604E-01	6.225E-01	

4.496E-01	1.004E 00	-2.033E-01	5.799E-01	1.487E 00	5.765E-02	-2.511E-01
-4.214E 00	-3.186E 00	4.099E 00	-5.337E-01	-7.574E-01	7.005E-01	-1.396E 00
-2.332E 00	-1.554E 00	2.179E 00	-2.244E 00	-1.207E 00	2.798E 00	

-7.231E-01	-3.522E-01	2.447E 00	4.517E-01	-1.943E 00	2.067E-01	5.893E-01
5.707E 00	1.383E 00	-6.530E 00	4.324E 00	9.315E-01	-3.382E 00	-2.881E 00
1.951E 00	-6.097E-02	-3.373E 00	4.426E 00	1.490E 00	-3.992E 00	

5.893E-01	2.067E-01	-1.943E 00	4.517E-01	2.447E 00	-3.522E-01	-7.231E-01
-3.382E 00	9.315E-01	4.324E 00	-6.530E 00	1.383E 00	5.707E 00	4.426E 00
-3.373E 00	-6.097E-02	1.951E 00	-2.881E 00	-3.992E 00	1.490E 00	

-2.511E-01	5.765E-02	1.487E 00	5.799E-01	-2.033E-01	1.004E 00	4.496E-01
7.005E-01	-7.574E-01	-5.337E-01	4.099E 00	-3.186E 00	-4.214E 00	-2.244E 00
2.179E 00	-1.554E 00	-2.332E 00	-1.396E 00	2.798E 00	-1.207E 00	

Appendix III <u>Homogeneity of Apparent Resistivity in the specific resistivities</u>

The fact that the apparent resistivity is a homogeneous function of degree one in the specific resistivities was first brought to the attention of the author in an oral presentation (1958) of Dr. Harold Seigel. A modified proof of that given by him is presented in the following.

The apparent resistivity measurement is defined as:

$$\rho_A = \frac{V}{I} F \tag{III.1}$$

where F is a geometrical factor depending only upon the electrode array employed and I is the current inserted into the ground with V the voltage measured. The voltage can be written as the line integral of the electric field between the two points of measurement:

$$V = \int \vec{E} \cdot d\vec{s} \tag{III.2}$$

Now equation 4.1.2 governs the flow of current within the region and is reproduced here as:

$$\sigma \nabla \cdot \vec{E} + \vec{E} \cdot \nabla \sigma = q$$

A scalar multiplication of the $\vec{E}$ field by t and a corresponding division of the conductivity by t retains the same form of this as:

$$(\sigma/t)\nabla\cdot(t\vec{E}) + (t\vec{E})\cdot\nabla(t/\sigma) = q \qquad \text{III.3}$$

Thus the current flow lines will be exactly the same but the effective conductivity will be σ/t and the effective electric field $t\vec{E}$. Hence any multiplication of the specific resistivities will be accompanied by a multiplication of the electric field by the same value. Finally, considering the case of apparent resistivity for a given resistivity distribution and that for a resistivity t times as great leads to:

$$\rho_A(\rho_i) = \frac{F}{I}\int \vec{E}\cdot d\vec{s} \qquad \text{III.4}$$

$$\rho_A(t\rho_i) = \frac{F}{I}\int t\vec{E}\cdot d\vec{s} \qquad \text{III.5}$$

Combining equations III.4 and III.5 the final result is symbollically represented as:

$$\rho_A(t\rho_i) = t\,\rho_A(\rho_i) \qquad \text{III.6}$$

Appendix IV. <u>Remarks concerning the use of computers</u>

Throughout this entire thesis the use of a high-speed digital electronic computer, the IBM 704, has been made. It is only recently that such machines have been produced for general use by industrial organizations. The speed of these machines is phenomenal and the costs have rapidly spiralled downwards. This thesis investigation would not have been possible without the use of these machines, not because they are capable of operations that humans with desk calculators cannot perform but because of their speed and accuracy in performing the immense number of very routine computations necessary.

It is to be noted that the machines of today serve in a great variety of ways. Primarily they are computational devices but certain operations allow them to be instructed to translate from one language to another. Thus a sequence of commands can be written in a rather symbolic form and the machines used to translate them into the vary basic instructions which any machine must eventually use. In addition to this added phase of application the manner of presenting the results may also be automated. Some machines have cathode ray tubes as a part of the output equipment and graphical or numerical data and results may be displayed and photographed.

In this particular thesis the actual printing of the results has been controlled by a special format which requires no further drafting or tabulation to use as a final form for presentation. The output devices and translation ability form extremely flexible and useful additions to the basic hardware of the computers.

For specific examples of the time and costs involved in the use of the 704 on certain phases of this thesis the following tabulation is made:

Phase	Time to Compute	Cost (@ $360/ hr.)
Forward Problem	2.5 minutes	$15. 00
Complete Interpretation	0.2 "	$1.20
Limited Interpretation	0.2 "	$1.00

These costs are extremely low and represent somewhat conservative estimates of the actual time required.

There has been a tremendous growth of the computer industry and also the applications of them to a wide variety of problems. For problems of interpretation and also forward problems in Geophysics the computers will allow treatments and approaches to be tried that heretofore have been prohibitive because of the man-hours required. There is no doubt that the areas of possible application is geophysics are as great, if not greater, than in any other discipline. A word of warning for forward problem solutions: an extremely large collection of exact solutions will not be of assistance in interpretation but only a hinderance in their use simply because of the quantity. The fact that machines can solve problems is not sufficient justification for obtaining a large set of such results.

In conclusion it should be noted that throughout the country many organizations unable to finance such computers are readily able to rent a varying amount of time from those larger firms able to carry such a large investment in these machines. This essentially provides a supply of computer time for all those parties interested in using these machines in their particular problems.

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BIOGRAPHICAL NOTE

The author was born in Springfield, Massachusetts on April 15, 1933 and moved to Meriden, Connecticut a few years later. Graduating from the Meriden High School in 1951 he entered MIT the same year. He received The B.S. degree in Geophysics in June 1955 and entered the MIT Graduate School that fall. During the summers between school years the author has been employed by industrial concerns in both petroleum and mining geophysical exploration and research.