Gravity Cutoff in Theories with Large Discrete Symmetries

Gia Dvali,[1,2] Michele Redi,[3] Sergey Sibiryakov,[1,4] and Arkady Vainshtein[1,5]

[1]*Theory Group, Physics Department, CERN, CH-1211 Geneva 23, Switzerland*
[2]*Center for Cosmology and Particle Physics, Department of Physics, New York University, New York, New York 10003, USA*
[3]*Institut de Théorie des Phénomenes Physiques, EPFL, CH-1015, Lausanne, Switzerland*
[4]*Institute for Nuclear Research of the Russian Academy of Sciences, 60th October Anniversary prospect, 7a, 117312 Moscow, Russia*
[5]*Fine Theoretical Physics Institute, School of Physics and Astronomy, University of Minnesota, 116 Church Street SE, Minneapolis, Minnesota 55455, USA*

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We set an upper bound on the gravitational cutoff in theories with exact quantum numbers of large N periodicity, such as Z_N discrete symmetries. The bound stems from black hole physics. It is similar to the bound appearing in theories with N particle species, though *a priori*, a large discrete symmetry does not imply a large number of species. Thus, there emerges a potentially wide class of new theories that address the hierarchy problem by lowering the gravitational cutoff due to the existence of large $Z_{10^{32}}$-type symmetries.

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Introduction.—Black hole (BH) physics is a powerful tool for extracting nonperturbative information about microscopic structure of the theory. As examples of such use of BHs one may list the argument about violation of continuous global symmetries in gravitational theories [1], the bound on the entropy of bounded systems [2], and the constraints on possible violation of Lorentz invariance [3,4]. Yet another example [5] is the restriction on the number N and mass M of particle species, which in the case of stable species and large N reads

$$NM^2 \lesssim M_P^2, \tag{1}$$

up to a factor that scales as $\sim \ln N$. Here M_P is the Planck mass.

As also shown in [5], the same BH bound applies even to a single species of mass M that carries an exactly conserved quantum number (not associated with any long-range classical gauge force) of periodicity N. An example of such a quantum number can be a discrete gauge symmetry Z_N [6,7] or a quantum hair under some massive integer spin field [8]. In what follows, we shall investigate this situation. Thus, unless otherwise stated, N will denote periodicity of Z_N (or of some other exact quantum number) and not the number of particle species, and the bound (1) should be understood accordingly.

The above bound is applicable, in particular, to scalar fields and implies that masses of N scalar fields, or of a single scalar field charged under a Z_N symmetry, are automatically limited by $M_P/\sqrt{N}$. By naturalness arguments, the presence of light scalar fields suggests the existence of some new stabilizing physics at that scale. For the case of many species this statement was made rigorous in [9] where it was shown that in a theory with large number of species the gravitational cutoff comes down to $M_P/\sqrt{N}$. (This is consistent with earlier perturbative arguments [10,11].) The purpose of this Letter is to prove a similar statement for the case of large discrete symmetry.

Black hole argument.—Consider a (scalar) field ϕ of mass M transforming under a discrete symmetry Z_N,

$$\phi \mapsto \phi e^{i(2\pi/N)k} \qquad k = 0, \ldots, N-1. \tag{2}$$

We assume that this Z_N symmetry is exact; i.e., it is not violated at any scale. A straightforward way to ensure this is to declare that Z_N is a gauge symmetry. However, for our reasoning it is unimportant what underlying physics guarantees exactness of Z_N. We then make the two following assumptions: (a) the particle ϕ has the largest charge to mass ratio among all the particles carrying Z_N charge, and (b) there are no BH remnants.

We are going to prove that there is a bound on the cutoff Λ of the low-energy theory,

$$\Lambda \leq M_P^2/(NM). \tag{3}$$

If $M \sim M_P/\sqrt{N}$ this bound coincides with the bound

$$\Lambda \leq M_P/\sqrt{N} \tag{4}$$

implied by naturalness. However, in general, the bound (3) is weaker than (4). We show that the stronger bound (4) is obtained if one makes an additional assumption that the property of negative heat capacity of BHs persists in the high-energy theory.

Let us proceed to the proof of Eq. (3). One performs the same thought experiment [5] as in establishing the bound (1). Take a macroscopic (arbitrarily large) BH and throw a number $\sim N$ of ϕ particles into it. In this way we endow the BH with Z_N charge of order N. Then one waits for the BH in question to evaporate. Since the Z_N symmetry is exact at all scales and there are no remnants, the BH eventually has to return the exact amount of the swallowed charge. Indeed, if the returned charge were not equal to the original one, the BH would mediate a process that violates Z_N explicitly, in contradiction with our assumption.

The crucial point is that as long as the BH is the usual Schwartzschild BH it cannot give out any Z_N charge. Indeed, radiation of Schwartzschild BH is thermal and contains as many ϕ particles as the antiparticles. Thus, to return back the Z_N charge, the properties of BH must get modified when it reaches a certain size, $R_{\rm BH} \sim \Lambda^{-1}$. This implies the existence of a new physics at the scale Λ; in other words, Λ is a cutoff of the low-energy theory.

Consider the BH that has just reached the cutoff scale. The mass of the BH at this moment must be sufficient to produce $\sim N$ of ϕ quanta,

$$M_{\rm BH} \geq NM. \tag{5}$$

On the other hand, the BH mass and size are still related at this moment by the standard Schwartzschild expression,

$$M_{\rm BH} \sim R_{\rm BH} M_P^2. \tag{6}$$

Combining Eqs. (5) and (6) one obtains the bound (3).

Notice that the above proof is UV insensitive in the sense that it does not depend on the precise nature of BHs that are smaller than the cutoff scale Λ^{-1}. All we have used is the conservation of energy which is entirely a large-distance constraint.

The bound can be improved if we make an additional assumption that the property of negative heat capacity of BHs persists in the high-energy theory. More precisely, we assume that the BH, after it reaches the size $R_{\rm BH} \sim \Lambda^{-1}$ corresponding to the Hawking temperature $T_H \sim \Lambda$, continues to radiate preferentially into modes with energies equal or higher than Λ. Then, Eq. (5) is replaced by $M_{\rm BH} \geq N\Lambda$. When combined with (6) it yields the stronger bound (4).

The above assumption about the BH spectrum appears to be natural. Its violation would imply very unusual properties of small BHs: they should be very cold and decay into quanta with inverse momenta greatly exceeding the size of the BH. Although we cannot exclude such a possibility, we conclude that under reasonable assumptions about the properties of small BHs the bound on the cutoff scale is (4).

It is worth comparing the argument presented in this section with the case of large number of species [9]. In the latter case the existence of the low cutoff can already be established in perturbation theory by considering the graviton propagator. The loop corrections to the propagator are amplified by the large number of species and the perturbative expansion goes out of control precisely at the scale $M_P/\sqrt{N}$ signaling that the cutoff is reduced to this value. The same conclusion can also be inferred directly from BH physics. The Hawking radiation of a BH of size $(M_P/\sqrt{N})^{-1}$ is drastically amplified as it can radiate N species. As a consequence, such BH would have a lifetime of order of its size and therefore it is not a classical object as in ordinary general relativity. On the other hand, in the case of a single field charged under a large discrete symmetry, the perturbation theory does not show any sign of breaking down. Similarly, there is no indication of the radiation of a BH of the size $(M_P/\sqrt{N})^{-1}$ blowing up. Nevertheless as we have shown the consistency of the theory requires the presence of a low cutoff. The argument that enables one to establish the existence of the cutoff is intrinsically nonperturbative and uses in an essential way the BH physics. This is reminiscent of the "gravity as the weakest force" conjecture [12]. It would be interesting to explore a possible connection between this conjecture and our work.

Explicit examples.—In this section we consider a few examples of theories with large discrete symmetries.

(1) Consider a $U(1)$ gauge symmetry with two scalar fields, ϕ and χ. The field ϕ has a unit charge e while the charge of the field χ is Ne. Let the χ field develop a nonzero vacuum expectation value (VEV), thus breaking the $U(1)$ symmetry down to Z_N. The latter symmetry acts on ϕ according to Eq. (2). The field ϕ is assumed to be lighter than the other fields, so it is the only degree of freedom at low energies.

It is important to notice that setting the ratio of charges of the fields χ and ϕ to a rational number (which we, for simplicity, took to be an integer) is not a fine-tuning. Rather, this is required by the bound (1). Indeed, if the ratio of charges were an irrational number, the effective discrete symmetry would be Z_∞, which is impossible.

In this theory the Z_N charge inside a given volume of space can be monitored in the following way. Because of the nontrivial topological structure of the vacuum manifold (noncontractible loops) there are cosmic strings in this theory, around which the phase of the χ VEV winds by 2π multiple. These cosmic strings contain a unit flux of the gauge field. This allows one to monitor the Z_N charge of a system through the Aharonov-Bohm effect in the scattering of the cosmic strings from the system [6,7]. In particular, if the system collapses into a BH, the latter has a quantum Z_N hair [6,7] that the Aharonov-Bohm effect can probe.

The black hole proof given above implies that gravity in this model must be modified at distances $(M_P/\sqrt{N})^{-1}$. Indeed, from the proof it is clear that BHs with the size smaller than $(M_P/\sqrt{N})^{-1}$ have to acquire hair capable of producing Z_N-charge asymmetry in the BH evaporation. On the other hand, such hair are impossible in Einstein's general relativity. At the classical level this follows from the no-hair theorems [1]. Quantum effects do not help either. Indeed, the existence of quantum hair leads to polarization of vacuum around BHs with Z_N charge [7]. This vacuum polarization is sensitive to the Z_N charge of the BH and, *a priori*, can contribute to the asymmetry of the evaporation. However, at weak coupling, $Ne \ll 1$, the effect is exponentially suppressed [7] and is unable to produce the necessary asymmetry. Thus we conclude that the physics responsible for the cutoff at $M_P/\sqrt{N}$ must involve gravity in an essential way.

(2) The existence of a large discrete symmetry may be accompanied by the presence of a large number of species in the theory. Then, the latter property, by itself, implies a low gravitational cutoff [9]. This point is illustrated by the following example.

Consider an $SU(2)$ gauge theory with two scalar fields, ϕ_j and $\chi_{j_1 j_2,\ldots,j_N}$, transforming as a fundamental and N-rank symmetric tensors, respectively. Here $j = 1, 2$ and $j_k = 1, 2$, $k = 1, \ldots, N$ are fundamental indices. We assume that the field χ acquires a VEV of only one component $\chi_{11,\ldots,1}$. This VEV breaks the continuous $SU(2)$ symmetry down to a discrete Z_N factor, under which $\phi_1 \mapsto \phi_1 e^{i(2\pi/N)}$ and $\phi_2 \mapsto \phi_2 e^{-i(2\pi/N)}$.

One may be tempted to apply our argument to show that the gravitational cutoff in this theory is low using the field ϕ_1 (or ϕ_2) in the proof. However, it would be incorrect: the proof given in this Letter is not directly applicable to this case. The reason is that the theory contains particles with arbitrarily large Z_N charges and so assumption (a) of the proof is violated. The states with large Z_N charges are the components of the field χ. Indeed, a component $\chi_{j_1 j_2,\ldots,j_N}$ with n indices equal to 1 and remaining $N - n$ indices equal to 2 carry $2n - N$ units of the Z_N charge. Correspondingly, a discrete charge of arbitrary $2n - N < N$ number of the ϕ_1 fields can be recycled by a BH into a single χ quantum. The corresponding gauge invariant operator has the form

$$\bar{\phi}^{j_1} \ldots \bar{\phi}^{j_{2n}} \chi_{j_1,\ldots,j_n a_1,\ldots,a_{N-n}} \chi_{j_{n+1},\ldots,j_{2n} b_1,\ldots,b_{N-n}} \epsilon^{a_1 b_1} \ldots \epsilon^{a_{N-n} b_{N-n}}. \tag{7}$$

However, the gravitational cutoff is still lowered down to $M_P/\sqrt{N}$ in this model. This is due to the fact that the theory contains N species which are the N components of the symmetric tensor χ. Thus we again find in this example that large discrete symmetry implies cutoff $M_P/\sqrt{N}$, though, in this case, indirectly, through a large number of species.

(3) The bound (4) has been obtained without references to the explicit structure of the theory of quantum gravity. Hence, by consistency, it should be satisfied in the string theory. Here we propose a simple example which shows that this is indeed the case. Consider the setup where the Z_N group is generated by an isometry of compact space in string theory compactification. We take the string coupling to be of order one so that the 10-dimensional Planck mass is set by the string scale M_S. Consider now a compactification on $T_6 \times \mathcal{M}_4$, where T_6 is a 6-dimensional torus and $\mathcal{M}_4$ is the 4-dimensional Minkowski space. The isometry group of this space is $U(1)^6$. We wish now to break one of the $U(1)$'s down to Z_N. Let the radius of the corresponding circle be R. We assume the radii of the other tori to be of order the string length. Then the relation between the 4-dimensional Planck mass and the string scale is

$$M_P^2 = M_S^2(RM_S). \tag{8}$$

Let us break the $U(1)$ isometry on the R circle down to Z_N by creating N fixed points around the circle. Alternatively this can be done by placing N identical branes and requiring the exact symmetry under cyclic shifts. Since the distance between the fixed points or the branes is bounded by the string scale, the maximal number of them that can be fitted on the circle is $N \leq RM_S$. Recalling that M_S is the cutoff of the low-energy effective theory, one sees that (8) reproduces the bound (4).

Implication for the hierarchy problem.—The results of this Letter shed new light on the proposal [5] to solve the hierarchy problem by postulating a large discrete symmetry with $N \sim 10^{32}$. We find that in this case the gravitational cutoff of the theory is not far from the weak scale; thus, the latter is automatically stabilized. A generic prediction of this solution to the hierarchy problem is the appearance of strong gravitational physics not far from the weak scale. From the experimental point of view this physics is expected to manifest itself in softening of the scattering amplitudes at energies above the scale $M_P/\sqrt{N}$. We now briefly discuss some aspects of implementing the above idea in model building.

From the constructive point of view it is desirable to have an explicit mechanism ensuring the hierarchy between the Planck and the weak scales. In the standard model (SM) the Higgs boson cannot transform under any exact symmetry, so it is impossible to give the Z_N charge to the Higgs boson itself to apply the bound (1) directly to its mass. Thus the idea is to ascribe the Z_N charge to some other fields whose mass gets contributions from the Higgs VEV. The bound (1) on the mass of these particles then implies the bound on the Higgs VEV.

Let us stress that in pursuing this strategy one should be careful not to run into conflict with the low gravitational cutoff. To illustrate what we mean, let us consider the following example. One can identify large Z_N with the subgroups of the existing global symmetries of the SM that would appear exact in the absence of gravity. Ignoring gravity, the SM has two classically exact continuous global symmetries that account for baryon and lepton number conservations. Thus one possibility is to declare that the Z_N symmetry in question is a subgroup of some combination of baryon and lepton number symmetries. It is most straightforward to embed Z_N into the $B - L$ symmetry because the latter is automatically anomaly free. Then, from Eq. (1) the bound on N is $M_P^2/m_\nu^2 \gtrsim 10^{54}$ where $m_\nu \lesssim$ eV is the mass of the lightest neutrino. Postulating the Z_N symmetry with $N \sim 10^{54}$ we would prevent the Higgs VEV from being larger than 10–100 TeV since large Higgs VEV would make neutrino heavier [13] than the BH upper bound for $N \sim 10^{54}$. However, according to the results of this Letter, such a large N would lower the gravitational cutoff below the weak scale, in contradiction with the observations. If we want the cutoff at an acceptable level, we have to chose $N \sim 10^{32}$. In this case the direct BH bound on the Higgs VEV is much higher than the bound on the cutoff.

The reader may question why one should worry about applying the BH bound (1) directly to the Higgs VEV, given the fact that in the above example with $N \sim 10^{32}$ the cutoff is at the needed level. Seemingly, the latter would suffice to solve the hierarchy problem. The point is that, in general, the cutoff controls the radiative stability of the weak scale but need not necessarily constrain its tree-level value. Correspondingly, if the tree-level value is large, the physical scale will also be large even though the radiative corrections are small. Hence, the small cutoff does not necessarily guarantee the smallness of the physically observable weak scale, whereas the direct BH bound on the mass (1) does.

To make our reasoning more transparent, it is useful to make a parallel with a much more familiar example of the low-energy supersymmetry. The cutoff that controls the radiative corrections to the Higgs boson mass is the supersymmetry breaking scale in the observable sector, $m_{\rm susy} \sim$ TeV. However, smallness of this cutoff cannot explain why there is no large tree-level contribution to the Higgs boson mass. The latter puzzle is the essence of the celebrated μ problem. Thus, in order to solve the hierarchy problem in supersymmetry, smallness of $m_{\rm susy}$ is not enough. One needs an additional mechanism that would guarantee smallness of μ. In our case the analog of $m_{\rm susy}$ is the low gravity cutoff $M_P/\sqrt{N}$. However, the physical weak scale is restricted by the BH bound on the particle masses. Whenever we can directly apply this bound to the weak scale, the hierarchy problem is solved, with no need of any further assumptions about the tree-level masses versus cutoff.

An example when there is no large discrepancy between the cutoff and the direct BH bound on the Higgs VEV is obtained in the following way. One introduces a scalar S transforming under Z_N symmetry and having no charge under the SM gauge group. This scalar gets a mass from the Higgs VEV through the following coupling in the Lagrangian, $-\lambda H^* H S^* S - M^2 S^* S$, where $\lambda \sim 1$ is the coupling constant and M^2 is the bare mass. Equation (1) yields a bound on the total mass of S,

$$\lambda H^* H + M^2 \lesssim M_P^2/N,$$

which for $\lambda > 0$, $M^2 > 0$ translates into the bound on the Higgs VEV $\langle H \rangle \lesssim M_P/\sqrt{N}$. The latter is of the same order as the bound on the cutoff.

To avoid confusion, let us stress that the solution of the hierarchy problem considered above, based on the existence of a discrete symmetry Z_N with large N, is physically different from the explanation of the hierarchy considered in Refs. [9,14] where the large number N was the number of particle species. In particular, the arguments presented in [14] to show that the many-species scenario can simultaneously solve the strong CP problem are not directly applicable to the case of large discrete symmetry. It would be interesting to understand whether an alternative argument exists that could explain the smallness of the strong CP parameter in the large Z_N case as well.

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