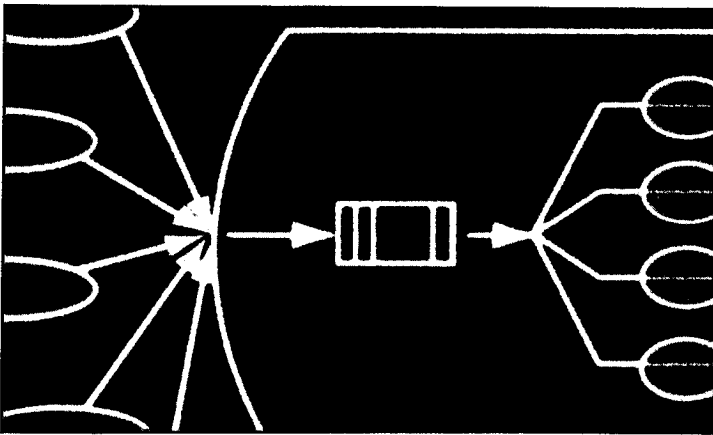

J. MEDHI



**STOCHASTIC
MODELS IN
QUEUEING THEORY**

S E C O N D E D I T I O N

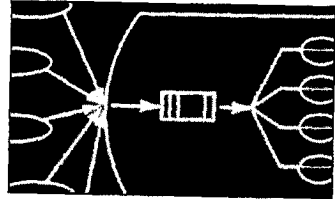


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