Study on influence of two-phase filtration transformation on formation of zones of undeveloped oil reserves

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In order to study the process of fluid filtration during flooding of an oil field, article uses Rapoport – Lis model of non-piston oil displacement by water. During plane-radial filtration in a homogeneous formation, radii of disturbance zones are determined with and without taking into account the end effect. Influence of changes in value of capillary pressure gradient on distribution of water saturation coefficient in the non-piston displacement zone for high and low permeability reservoirs is revealed. Application of an element model for a five-point injection and production well placement system showed that, using traditional flooding technology, flat-radial fluid filtration is transformed into rectilinear-parallel. At solving equation of water saturation, Barenblatt method of integral relations was used, which allows determining the transformation time. By solving the saturation equation for rectilinear-parallel filtration, change in the value of water saturation coefficient at bottomhole of production well for an unlimited and closed deposit is determined. It is shown that an increase in water cut coefficient of a production well is possible only for a closed formation. To determine coefficient of water saturation in a closed deposit, a differential equation with variable coefficients is obtained, an iterative solution method is proposed. In the element of the five-point system, oil-saturated zones not covered by development were identified. For channels of low filtration resistance, conditions for their location in horizontal and vertical planes are established. It is shown that, at maintaining formation pressure, there is an isobar line in formation, corresponding to initial formation pressure, location of which determines direction of fluid crossflow rates. Intensity of crossflows affects application efficiency of hydrodynamic, physical and chemical, thermal and other methods of enhanced oil recovery.

Key words: Rapoport – Lis model; Barenblatt method; water saturation coefficient; transformation of filtration process; residual mobile reserves of oil

Introduction. Model of oil displacement by water has been well studied in classical works on underground hydrodynamics, nevertheless, study of oil recovery process during flooding is one of the most difficult tasks in design and control of oil field development. This is due to fact that when water is injected in bottomhole formation zone (BHFZ) of injection well, an end effect (EE) takes place. Water accumulates in a region of radius r_k, in which water saturation coefficient increases to a certain value of s_k. When this value is exceeded, water enters the formation, process of two-phase filtration and non-piston oil displacement by water begins. Water flows at internal boundary of EE region, and at external boundary water does not flow out, but oil does. Study of filtration process is complicated by a nonlinear boundary condition on external region of EE [14]. Because of it end effect is neglected, coefficient of water saturation at bottomhole of injection well takes limit (maximum) value s^*. Porosity and permeability properties (PPP) of the formation, depending on saturation coefficients, geological structure of deposit, location of wells affect change in nature of the fluid flow in formation and, therefore, development of mobile oil reserves.

Statement of the problem. Existing problem of undeveloped residual mobile reserves (RMR) of oil is due to physical properties of the fluids, implemented development system, macroheterogeneity of the filtration parameters in lateral and thickness of oil reservoir [2, 8, 9, 12, 15, 16, 18]. Surface phenomena occurring at phase boundary, capillary pressure gradient, hydrophilicity or hydrophobicity of reservoir (microheterogeneity) significantly affect deviation of actual development indicators from design technological ones [7, 11]. To eliminate discrepancy between the indicators, technologies (methods) of enhanced oil recovery (EOR) are used. When selecting and evaluating the effectiveness of corresponding method, it is necessary to take into account the change

(transformation) of the filtration processes and influence of the fluid saturation degree in reservoir capacitive space.

Methodology. Physical models for multiphase filtration of non-piston oil displacement are considered in [4, 6, 10, 13]. From equations of continuity, motion and rheological equations, a system of partial differential equations is obtained, solution of which is reduced to determining the distribution of saturation and phase pressure coefficients in disturbance region – two-phase filtration. To connect water saturation coefficient with pressure, an additional closing equation is introduced. All considered models can be divided into two groups: taking into account and not taking into account the capillary pressure gradient. Solution of differential equations systems, as a rule, is carried out by approximate numerical methods. In the present work, to solve Rapoport – Lis equations, Barenblatt method of integral relations is used [1]. Transformation of filtration zones in process of developing a production object (PO) is considered. Obtained approximate analytical solutions of water saturation coefficient values allow identifying the zones with existence of two-phase filtration and residual oil.

Discussion. Results of the research. To study transformation of filtration processes, a model of flooding a homogeneous reservoir is considered. With commissioning of injection well, a radial non-piston displacement of oil by water occurs. Pressure at the bottomhole of injection well is greater than initial formation and bottomhole pressure of production well. Over time, flat-radial filtration transforms into rectilinear-parallel filtration between injection and production wells [17]. To determine values of water saturation coefficient in the model proposed by Rapoport – Lis, a special case of which is Buckley – Leverett model, formation and fluids are considered incompressible. Task is reduced to solving the differential equation for saturation coefficient taking into account capillary pressure at phase boundary and corresponds to a rigid elastic water drive regime:

$$m_0 \frac{\partial s}{\partial t} + W(t)\frac{\partial f(s)}{\partial x} + \frac{k_0}{\mu_2} div\left[k_2^*(s)f(s)\left(\frac{\partial p_k}{\partial s}\frac{\partial s}{\partial x} + \Delta\rho g\sin\alpha + \Delta g_0 \right) \right] = 0, \qquad (1)$$

where m_0 – open porosity coefficient; s – water saturation coefficient in a two-phase filtration zone; $W(t)$ – total phases' filtration rate; x – coordinate, for radial filtration x is replaced by r; $f(s)$ – Buckley – Leverett function; k_0 – absolute permeability coefficient; $k_2^*(s)$ – relative phase permeability (RPP) of oil reservoir; p_k – capillary pressure; $\Delta\rho = \rho_2 - \rho_1$, ρ_2 and ρ_1 – oil and water densities; g – acceleration of gravity; $\Delta g_0 = g_2 - g_1$, g_2 and g_1 – initial pressure gradients during oil displacement by water in a two-phase filtration zone.

Buckley – Leverett function is determined experimentally:

$$f(s) = \frac{k_1(s)}{k_1^*(s) + \mu_0 k_2^*(s)}, \quad \mu_0 = \frac{\mu_1}{\mu_2}, \qquad (2)$$

where μ_1, μ_2 – dynamic viscosity coefficients of water and oil; $k_1^*(s)$, $k_2^*(s)$ – reservoir RPP for water and oil.

Equation (1) contains time derivative of the function $s(x,t)$, and operator div corresponds to an equation of parabolic type. Therefore, to solve (1), Barenblatt method of integral relations can be used [1].

Origin of coordinates will be compatible with bottomhole of injection well. Radius of disturbance zone of two-phase filtration is denoted by $R(r, t)$. Since the change in water saturation coefficient corresponds to change in pressure of water injected into reservoir, solution is found in the form [6]

$$s(r,t) = s_0 + \eta(t)\Delta s \frac{f_1(r, R(t))}{f_1(r_w, R(t))}; \quad s = s^* - s_0, \qquad (3)$$

where s_0 – coefficient of residual water saturation (bound water); s^* – water saturation limit value; r_w – well radius. Heaviside unit function $\eta(t)$ equals 0 at $t = 0$ and 1 at $t > 1$:

$$f_1(r, R(t)) = -n \ln \frac{R(t)}{r} - \left(\frac{r}{R(t)}\right)^n + 1; \quad f_1(r_w, R(t)) = -n \ln \frac{R(t)}{r_w} - \left(\frac{r_w}{R(t)}\right)^n + 1. \tag{4}$$

Note that $f_1(r, R(t)) < 0$ for $r_w \le r \le R(t)$.

If the end effect is neglected, then function $s(r, t)$ satisfies following boundary conditions:

$$s(r,0) = s_0, \; s(r_w, t) = s^*; \quad s(R(t), t) = s_0, \left.\frac{\partial s}{\partial r}\right|_{r=R(t)} = 0. \tag{5}$$

Pressure at the boundary of two-phase filtration zone $R(t)$ is equal to initial pressure in deposit p_0.

With a flat radial displacement of oil by water, equation (1) takes form

$$m_0 \frac{\partial s}{\partial t} = -\frac{q_s}{r} \frac{\partial f(s)}{\partial r} - \frac{k_0}{\mu_2} \frac{1}{r} \frac{\partial}{\partial r}\left[r k_2^*(s) f(s)\left(\frac{\partial p_k}{\partial s}\frac{\partial s}{\partial r} + \Delta\rho g \sin\alpha + \Delta g_0\right)\right], \tag{6}$$

where $q_s = \dfrac{Q(t)}{2\pi h}$, $Q(t)$ – flow rate of injection well; h – formation thickness.

To determine the radius of two-phase filtration zone $R(t)$, (6) is integrated over r in the interval from r_w to $R(t)$. Thus

$$-\frac{\Delta s n^2}{4(n+2)} \frac{d}{dt}\left(\frac{R^2}{f_1(r_w, R)}\right) = -\frac{q_s(t)}{m_0} f(s)\Big|_{r_w}^{R} - \frac{k_0}{m_0 \mu_2}\left[r k_2^*(s)\left(\frac{\partial p_k}{\partial s}\frac{\partial s}{\partial r} + \Delta\rho g \sin\alpha + \Delta g_0\right)\right]\Big|_{r_w}^{R}. \tag{7}$$

It follows from boundary conditions (5) that, for $r = R(t)$ water saturation coefficient $s = s_0$. RPP of reservoir takes maximum value $k_2^*(s_0) = k_{2\max}^*$, and Buckley – Leverett function $f(s_0) = 0$. At $r = r_w$, $s = s^*$, $k_2^*(s^*) = 0$ Buckley – Leverett function $f(s^*) = 1$. As a result, there is an ordinary differential equation:

$$\frac{d}{dt}\left(\frac{R^2(t)}{f_1(r_w, R(t))}\right) = -\frac{4(n+2)}{n^2 m_0 \Delta s} q_s(t). \tag{8}$$

If $q_s = $ const after separation of variables and integration, a transcendental equation for determining $R(t)$ is obtained:

$$R(t) = \sqrt{r_c^2 - \frac{4(n+2)}{m_0 \Delta s n^2} f_1(r_w, R(t)) q_s t} \approx \sqrt{r_w^2 + \frac{4(n+2)}{m_0 \Delta s n}\left(\ln \frac{R(t)}{r_w} - \frac{1}{n}\right) q_s t}. \tag{9}$$

Consequently, under given boundary conditions (5), capillary pressure and RPP do not explicitly affect the radius of two-phase filtration zone.

However, if end effect of radius r_k is not neglected, then second boundary condition (5) at bottomhole of injection well will take the form

$$s(r_k, t) = s_k < s^*; \quad r_k > r_w.$$

At $r = r_k$ and $s = s_k$ RPP of reservoir for oil $k_2^*(s_k) \ne 0$, and Buckley – Leverett function $f(s_k) < 1$.

After integration in the interval $[r_k, R(t)]$ right part of equation (7) takes following form

$$\frac{q_s}{m_0}f(s_k)+\frac{k_0 k_2^*(s_k)}{m_0\mu_2}r_k f(s_k)\left(g\Delta\rho\sin\alpha+\Delta g+\frac{\partial p_k}{\partial s}\bigg|_{s=s_k}\frac{\partial s}{\partial r}\bigg|_{r_k}\right)=\frac{q_s}{m_0}f(s_k)(1+A_k);$$

$$A_k=\frac{k_0 k_2^*(s_k)}{q_s\mu_2}r_k\left(g\Delta\rho\sin\alpha+\Delta g+\frac{\partial p_k}{\partial s}\bigg|_{s=s_k}\frac{\partial s}{\partial r}\bigg|_{r_k}\right).$$

At end effect $s(r,t)$ and its derivative are equal to

$$s(r,t)=s_0+\eta(t)\Delta s_k\frac{f_1(r,R(t))}{f_1(r_k,R(t))};\ \ \Delta s_k=s_k-s_0;$$

$$\frac{\partial s}{\partial r}=\frac{n\Delta s}{rf_1(r_w,R(t))}\left(1-\frac{r_k^n}{R^n(t)}\right).$$

For $\alpha=0$ and $\Delta g=0$

$$A_k=\frac{k_0 k_2^*(s_k)}{q_s\mu_2}\frac{n\Delta s_k}{f_1(r_k,R)}\frac{\partial p_k}{\partial s}\bigg|_{s_k}. \tag{10}$$

After substituting (10) in right part of equation (7), a differential equation is obtained that differs from (8) by its right part:

$$\frac{d}{dt}\left(\frac{R^2(t)}{f_1(r_w,R(t))}\right)=-\frac{4(n+2)}{n^2 m_0\Delta s}f(s_k)q_s(t)(1+A)_k. \tag{11}$$

From formula (10) it can be seen that parameter A_k depends on technological parameter q_s, phase permeability of oil, derivative of capillary pressure with respect to s at point s_k, radius of a two-phase filtration zone R. According to the data given in [3], capillary pressure derivative with respect to s is equal to tangent of inclination angle to axis s (tgα) in interval $s_0<s<s^*$. The highest values of capillary pressure derivative are observed in vicinity of water saturation coefficient s_0 corresponding to front of oil displacement by water, distanced from bottomhole of injection well (Fig.1). Derivative values vary in range $-10<$tg$\alpha<-8$. At the bottomhole of injection well, a change in capillary pressure for a high permeable reservoir (curve 1) has practically no effect on oil displacement: tg$\alpha\approx-0.1$. For a low permeable reservoir (curve 2) tg$\alpha\approx-1$.

Indeed, let us consider variation interval of perturbation zone radius $4\le R\le 200$ м. For a high permeable reservoir at following parameters: $Q=300$ m³/day; $k_0=200$ mD; $k_2^*=0.6$; $\mu_2=2$ mPa·s; tg$\alpha=-0.1$ MPa; $\Delta s_k=0.72$ values of parameter A_k change in the interval $0.01\le A_k\le 0.06$. For a low permeable reservoir at set parameters: $Q=8$ m³/day; $k_0=4$ mD; $k_2^*=0.2$; $\mu_2=2$ mPa·s; tg$\alpha=-1$ MPa; $\Delta s_k=0.27$ values of parameter A_k change in the interval $0.03\le A_k\le 0.16$. With increasing radius $R(t)$ parameter A_k decreases. As $A_k<1$, then to determine the radius of perturbation zone, formula (9) can be used.

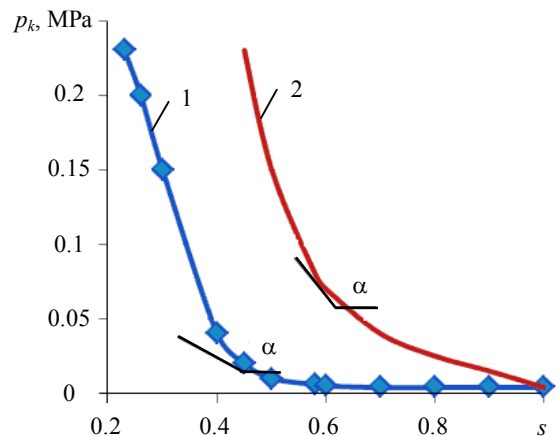*Example 1.* Given: $r_w=0.1$ m; $h=10$ m; $s_0=0.2$; $s^*=0.8$; $m_0=0.2$; $Q=300$ m³/day.

Fig.1. Capillary pressure dependence on coefficient of water saturation

1 – for a high permeable reservoir;
2 – for a low permeable reservoir

Calculation results are shown in Table 1.

Let us consider an element of operational object consisting of injection and production wells (Fig.2). While inequality $R(t) + R_2(t) < L$ holds, where L – distance between wells; $R_2(t)$ – radius of disturbance zone 2 for production well, in which only oil is filtered, there is flat radial filtration. Since injection pressure is greater than initial formation pressure and bottomhole pressure of production well, bulk of injected water will move to perforation zone of production well. Filtration becomes rectilinear-parallel. Pressure at interface of zones 1 and 2 (points C_i) is equal to initial formation pressure p_0. Another part of water will flow at a speed V_1 to zone 1. In disturbance region 2 of production well, oil will flow into linear filtration channel at a speed V_2. Moreover, flow speeds will decrease over time. Note that, under given boundary conditions, points C_i are motionless.

Transformation time t^* is determined from condition [6, 7]

$$R(t^*) + R_2(t^*) = L.\tag{12}$$

According to well-known formulas

$$R_2(t) = \sqrt{C\chi_2 t}; \ \chi_2 = \frac{k_0 k_2^*}{\mu_2 \beta_2^*},$$

where C = const.

Substituting R_2 and expression (9) into (12), transformation time from transcendental equation is obtained:

$$t^* = \frac{L^2}{C\chi_2\left[1 + \sqrt{1 + \dfrac{4(n+2)q_s}{C\chi_2 m_0 n \Delta s}\left(\ln\dfrac{R(t^*)}{r_w} - \dfrac{1}{n}\right)}\right]^2}.\tag{13}$$

Table 1

Determination of values for disturbance zone radius by formula (9) for different values n

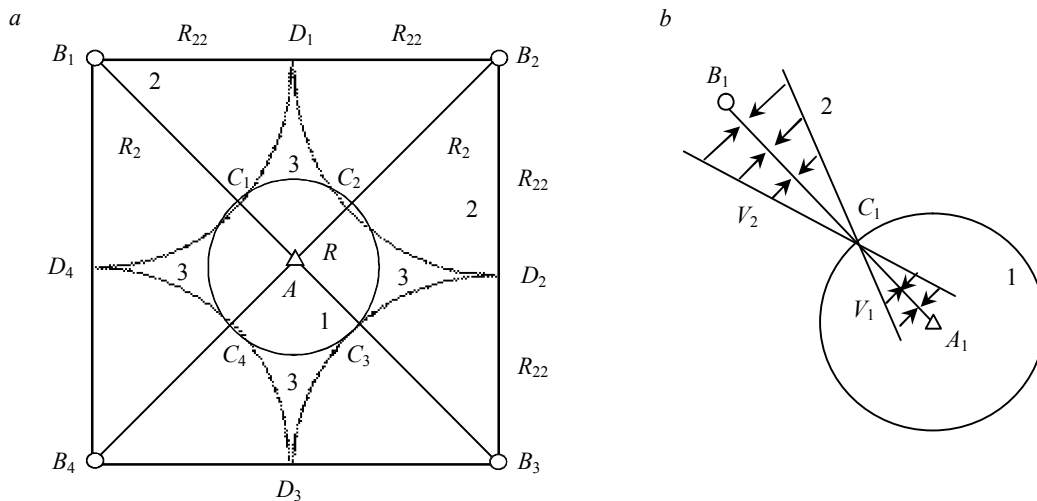| t, days	$R(t)$, m			
	$n = 1$	$n = 2$	$n = 3$	$n = \infty$
0.01	3	3	3	2.2
0.1	14	12	11	8
1	50	42	39	30
2	73	61	56	44
10	176	147	135	105
100	607	506	464	361
365	1210	1006	923	718

Fig.2. Transformation scheme of the filtration process for element of inverse five-point flooding system:
a: R – radius of zone 1 of two-phase filtration of an injection well A; R_2, R_{22} – radii of disturbance zones 2 for production wells B_i; 3 – zone (region), not involved in flooding;
b: V_1, V_2 – fluid flow speeds in zones 1 and 2; 1 – injection well area A_1

Example 2. Given: $r_w = 0.1$ m; $h = 10$ m; $s_0 = 0.2$; $s^* = 0.8$; $m_0 = 0.2$; $Q = 300$ m^3/day; $L = 400$ m; $k_0 = 100$ mD; $k_2^* = 0.6$; $\mu_2 = 2$ mPa·s; $\beta_2^* = 10^{-3}$ 1/MPa; $L = 400$ m; $C = 12$

Calculation results t^* according to formula (13) are given in Table 2.

For $n = 2$, distance from injection well to transformation boundary is $R = 80$ m, pressure at the boundary is p_0. In zone 1, two-phase filtration takes

Table 2

Determination of transformation start time by formula (13) for different values *n*

t^*, days	R, m		
	$n = 1$	$n = 2$	$n = 3$
3	92		
3.3		80	
3.4			75

place – oil displacement by water according to Rapoport – Lis model. Disturbance zone radius of production well, in which single-phase oil filtration exists, is 320 m. Since commissioning of injection well, time of changing the nature of the filtration process for given PPP and specified technological parameters does not exceed 4 days. Since pressure at bottomhole of production wells is less than pressure at other points in deposit, flat-radial filtration is transformed into rectilinear-parallel. Flooding of production wells will begin through channels A_1B_i.

To describe linear displacement of oil by water, equation (1) is used. Origin of coordinates coincides with bottomhole of injection well. Boundary conditions in this case change, since on interval $[x_w, R]$ a change in coefficient of water saturation has already occurred in formation, and at bottomhole of injection well for $x = x_w = r_w$ coefficient of water saturation $s = s^*$. Over time at border $x = R$ coefficient of water saturation will increase: $s > s_0$. Therefore consider $s(x, t)$ on two intervals: $s_1(x, t)$ in interval $x_w \le x \le l(t) \le R$ and $s_2(x,t)$ in interval $R < x \le l(t)$.

For interval $x_w \le x \le l(t) \le R$ solution is found in the form

$$s_1(x,t) = s_R(x,R) + c\frac{x - x_w}{l(t)} + b\frac{(x - x_w)^2}{l^2(t)}; \quad s_R(x,R) = s_0 + \Delta s\frac{f_1(x,R)}{f_1(x_w,R)};$$

$$f_1(x,R) = -n\ln\frac{R}{x} - \left(\frac{x}{R}\right)^n + 1; \quad f_1(x_w,R) = -n\ln\frac{R}{x_w} - \left(\frac{x_w}{R}\right)^n + 1. \tag{14}$$

For $R < x \le l(t)$

$$s_2(x,t) = a_0 + a_1\frac{x - R}{l(t)} + a_n\frac{(x - R)^n}{l^n(t)}; \quad n \ge 2. \tag{15}$$

Here R – constant, corresponding to n (Table 2); c, b, a_0, a_1, a_n – unknown parameters determined from boundary conditions and conditions for merging of functions s_1 and s_2.

Boundary conditions for function s_2 are set at the boundary of displacement front:

$$s_2(l(t),t) = s_0; \quad \left.\frac{\partial s_2}{\partial x}\right|_{x=l(t)} = 0. \tag{16}$$

Conditions for merging of functions s_1 and s_2 for $x = R$ will be

$$s_1(R,t) = s_2(R,t); \quad \frac{\partial s_1}{\partial x} = \frac{\partial s_2}{\partial x}; \quad \frac{\partial^2 s_1}{\partial x^2} = \frac{\partial^2 s_2}{\partial x^2}. \tag{17}$$

From a joint solution of system (16), (17) for $n = 2$ parameters a_i, b, c are

$$a_0 = s_0 + a_2\left(1 - \frac{R}{l}\right)^2; \quad a_1 = -2a_2\left(1 - \frac{R}{l}\right); \quad a_2 = -\frac{2\Delta s}{f_1(x_w,R)};$$

$$b = a_2 + \frac{2\Delta s}{f_1(x_w,R)}\left(\frac{l}{R}\right)^2; \quad c = -2a_2 - \frac{4\Delta s}{f_1(x_w,R)}\frac{l}{R}. \tag{18}$$

After substitution of (18) into (15) we get

$$s_2(x,t) = s_0 + a_2 \frac{(l(t)-x)^2}{l^2(t)}. \tag{19}$$

For practical testing of determining the coefficients of water saturation and watering at bottomhole of production well, let us consider the case $R < x \le l(t)$, $s = s_2$.

After substitution of s_2 into equation (1) and subsequent integration over x in range from R to $l(t)$, we obtain a differential equation for determining linear size of two-phase filtration zone $l(t)$:

$$\frac{d}{dt}\left[\frac{(l-R)^3}{l^2}\right] = \frac{3W(t)}{a_2 m_0} f(\bar{s})\left[1 + A_{kl} \frac{(l-R)}{l^2}\right];$$

$$A_{kl} = \frac{4\Delta s k_0 k_2^*(\bar{s})}{\mu_2 W(t) f_1(x_w,R)} \left.\frac{\partial p_k}{\partial s}\right|_{\bar{s}}. \tag{20}$$

Here $\bar{s}$ is value of water saturation coefficient corresponding to $x = R$. It should be noted, that $\bar{s} \ge s_0$ is variable value, which will increase over time. Buckley – Leverett function varies in the interval $0 < f(\bar{s}) < 1$ and also increases.

Verification calculation of parameter A_{kl} shows that for various values included in formula, its order does not exceed 10^{-4}, and from (20) with increasing $l(t)$ the second term in square bracket tends to zero. Therefore, equation for determining $l(t)$ has the form

$$\frac{d}{dt}\left[\frac{(l(t)-R)^3}{l(t)^2}\right] = \frac{3W(t)}{a_2 m_0} f(\bar{s});$$

$$\left[\frac{(l(t)-R)^3}{l(t)^2}\right] = \int_R^{l(t)} \frac{3W(t)}{a_2 m_0} f(\bar{s})dt = \Phi(t). \tag{21}$$

From where cubic equation is obtained

$$l^3 - (3R + \Phi)l^2 + 3R^2 l - R^3 = 0. \tag{22}$$

Using the Cardano method, $l(t)$ is found:

$$l(t) = R + \frac{1}{3}\Phi(t)\left[1 + \sqrt[3]{1 + \left(\frac{R}{\Phi(t)}\right)^{1,5}} + \sqrt[3]{1 - \left(\frac{R}{\Phi(t)}\right)^{1,5}}\right]. \tag{23}$$

Value $\bar{s}$ is determined from formula (19), which for $x = R$ will be

$$\bar{s} = s_0 + a_2 \frac{(l(t)-R)^2}{l(t)^2}, \quad l > R. \tag{24}$$

To obtain an approximate analytical solution, we estimate the function $\Phi(t)$, included in right part of (21). Let total phase filtration rate $W(t)$ be constant. Then, using mean value theorem, we can write

$$\Phi(t) = \frac{3W}{a_2 m_0} \int_0^t f(\bar{s})dt \approx \frac{3W}{a_2 m_0} f(\bar{\bar{s}})t; \quad s_0 < \bar{\bar{s}} < \bar{s} < s^*. \tag{25}$$

It follows from (24) that value $\bar{s}$ depends on time, consequently, $\bar{\bar{s}}$ also increases over time t. Limit value

$$\bar{s} = s_0 + a_2 = s_0 + \frac{\Delta s}{\ln\dfrac{R}{x_w} - 0.5}. \tag{26}$$

For $s_0 = 0.2$; $\Delta s = 0.6$ and $R = 80$ m value of $\bar{s} = 0.2485$. Thus, Buckley – Leverett function $f(s)$ in the integrand expression changes insignificantly (a weakly changing function) and its removal beyond the sign of integral is justified. In the future we will take $\bar{\bar{s}} = \bar{s}$.

Note, that at $t \to \infty$ formula (23) will take a simple form:

$$l(t) = R + \Phi(t) = R + \frac{3W}{a_2 m_0} f(\bar{\bar{s}})t. \quad (27)$$

Table 3

Dependence of change in the size of linear filtering zone $l(t)$ on time in an infinite formation

t, days	$l(t)$, m (23)	$l(t)$, m (27)	$x > R$	$s_2(x,t)$ (19)
0.1	80.1	80.03	81	0.2924
1	80.9	80.3	100	0.2913
2	82	81	300	0.2803
10	89	83	361	0.2771
100	169	117	500	0.2701
365	405	394	800	0.2560
3650	3333	3332	3333	0.2000

From (23) and (27) it follows that after transformation, size of two-phase filtration zone $l(t)$ linearly depends on time. Formula (23) is valid for short times, and for $R \ll \Phi(t)$ (27) can be used.

Values of Buckley – Leverett function $f(s)$ are determined if the formulas for dependence of RPP on coefficient of water saturation are known. According to research of Chen Jung-xiang [16]

$$k_1^*(s) = \left(\frac{s - 0.2}{0.8}\right)^{3,5}; \ 0.2 \leq s \leq 1; \ k_2^*(s) = \left(\frac{0.85 - s}{0.85}\right)^{2,8}(1 + 2.4s); \ 0 \leq s \leq 0.85. \quad (28)$$

In this case for $\bar{s} = 0.2485$ Buckley – Leverett function equals $f(\bar{s}) = 1.81 \cdot 10^{-4}$.

Table 3 shows the results of calculating $l(t)$ for different values of time.

Second and third columns show calculated values of $l(t)$ in time interval from 0.1 days to 10 years, calculated by the formulas (23) and (27). In fourth column, the x coordinate values are from 81 to 3333 m, in fifth column there are calculated values of water saturation coefficient in this interval. Note, that if $x = L$ – distance between wells, then from formula (19) it follows that for $l(t) \to \infty$, $s(L,t) = s_0 + a_2, x$ – constant value. This is contrary to practice, since water cut increases, therefore, at bottomhole of production well, proportion of water in fluid flow should increase. Accordingly, size of filtration zone should be limited to $l_k = l(t_k)$, which is determined from geometric structure of the deposit.

Pressure distribution in confined deposits was considered in [1, 16]. By analogy, change in coefficient of water saturation in a closed limited deposit is found in the form

$$s_3(x,t) = D(t)s_2(x,l_k) = D(t)\left[s_0 + a_2\left(\frac{l_k - x}{l_k}\right)^2\right]; \ R \leq x \leq l_k; \ D(0) = 1. \quad (29)$$

Unknown function $D(t)$ is defined after substitution (29) in equation (1) and after integration in the interval $[R, l_k]$ we get differential equation:

$$\frac{dD}{dt} = G - BD, \quad (30)$$

where

$$G = \frac{W[f(s_R) - f(s_k)]}{m_0 H}; \ H = (l_k - R)\left[s_0 + \frac{1}{3}a_2\left(\frac{l_k - R}{l_k}\right)^2\right];$$

$$B = \frac{1}{m_0 H} \frac{2a_2 k_0 k_2^*(s_R)f(s_R)}{\mu_2} \frac{(l_k - R)}{l_k^2} \left.\frac{\partial p_k}{\partial s}\right|_{s_R}; \ B < 0. \quad (31)$$

Here s_R corresponds to value of water saturation coefficient at $x = R$, s_k corresponds to $x = l_k$, $s^* > s_R > s_k > s_0$. For the Buckley – Leverett functions, relation $f(s_k) < f(s_R) \leq 1$ is true.

It follows from (29) that function $D(t)$ is limited above, its maximum value $D_{max} = s^*/s_0$.

For a constant total filtration rate $W(t) = W$ and limited time-dependent functions G, B result of (30) will be

$$D(t) = \frac{1}{B}\left[G - (G - B)e^{-Bt}\right].$$ (32)

Expanding the exponent in a row, we obtain an approximate solution:

$$D(t) = 1 + (G - B)t.$$ (33)

For calculation, following iterative scheme is proposed:
1. By known values l_k and R from first formula (31) parameter H is found.
2. Set time values t and parameter $D(t) = 1{,}01$.
3. From (29) we define

$$s_{31}(L, t) = D(t)s_2(L, l_k) = D(t)\left[s_0 + a_2\left(\frac{l_k - L}{l_k}\right)^2\right]; \quad R \le L \le l_k;$$

$$s_R = s_{31}(R, t) = Ds_2(R, l_k) = D(t)\left[s_0 + a_2\left(\frac{l_k - R}{l_k}\right)^2\right];$$

$$s_{lk} = s_{31}(l_k, t) = Ds_2(l_k, l_k) = D(t)s_0,$$

where L – distance between wells.

4. Calculate values of RPP coefficients and Buckley – Leverett function by the formulas (28) and (2) for s_R and s_{lk}.

5. From Fig.1 or functional dependence we find $\left.\dfrac{\partial p_k}{\partial s}\right|_{s_R}$, by formulas (31) we define parameters B, G.

6. We calculate new value of coefficient s_{32}. If $[s_{31} - s_{32}] > 10^{-3}$, then repeat the process from point 3) iterative scheme.

Setting a new value of time, we determine increase in coefficient of water saturation $s_3(L, t)$ at bottomhole of production well.

To determine water cut ratio σ on channel A_1B_1 we use the formula

$$\sigma = \frac{v}{W}; \quad W = v + v_2,$$

where v – water filtration rate; v_2 – oil filtration rate; W – total fluid flow rate at bottomhole of production well. If W is constant, then terms change, oil filtration rate decreases, and water filtration rate increases.

Water filtration rate is expressed through total rate and depends on capillary pressure gradient [15]:

$$v = f(s)\left[W + k_0\frac{k_2^*(s)}{\mu_2}\frac{\partial p_k}{\partial s}\frac{\partial s}{\partial x}\right].$$

Water cut coefficient

$$\sigma = f(s)(1 + A); \quad A = k_0\frac{k_2^*(s)}{W\mu_2}\left.\frac{\partial p_k}{\partial s}\right|_{sL}\left.\frac{\partial s}{\partial x}\right|_L = -D(t)k_0\frac{k_2^*(s)}{W\mu_2}\frac{2a_2(l_k - L)}{l_k^2}\left.\frac{\partial p_k}{\partial s}\right|_{sL}; \quad \frac{\partial p_k}{\partial s} < 0.$$ (34)

For small values s, close to s_0, RPP $k_2^*(s)$ takes values, close to maximum, and Buckley – Leverett function is small. At $s \to s^*$ function $f(s) \to 1$, and $k_2^*(s) \to 0$. Therefore, for large values of water saturation coefficient at bottomhole of production well, Buckley – Leverett function corresponds to water cut coefficient of well fluid. Thus, experimental work on determination of RPP values plays an important role in modeling of technological indicators and choice of well operation mode.

Time of partial watering of fluid t_L for well B_1 through channel A_1B_1 corresponds to value of water saturation coefficient, defined by formula (29), at $t = t_L$ and $x = L$. After substitution in (33) we get

$$t_L = \frac{D(t_L) - 1}{G}; \quad D(t_L) = \frac{s^*}{s_0 + a_2\left(\dfrac{l_k - L}{l_k}\right)^2}.$$

(35)

Parameter G is defined from formula (31) at $f(s_R) = 1$, $k_2^*(s^*) = 0$.

Example 3. Given: $Q = 200$ m³/day; $h = 10$ m; $R = 80$ m; $L = 400$ m; $l_k = 1000$ m; for value of function $f(s_k) = 0.5$ time of fluid watering, calculated with formula (35) is $t_L = 6.3$ days. If value of Buckley – Leverett function will be $f(s_k) = 0.9$, then $t_L = 31.7$ days. If filtration zone size is $l_k = 3000$ m, then for value of Buckley – Leverett function $f(s_k) = 0.5$ and $f(s_k) = 0.9$ values of fluid watering time $t_L = 16.1$ days and $t_L = 90$ days respectively.

Filtration zone size l_k depends on geological structure of deposit and significantly affects time of watering. In Rapoport – Lis model, porosity coefficient is constant, fluids and reservoir are incompressible, and therefore, linear filtration channels can be neglected. Consequently, in areas 3 (Fig.2, *a*) there will remain oil reserves not involved in development.

To determine oil reserves not involved in development in zones 3 we define t_{22} – contact time of disturbance zones of production wells B_1B_2 in point D_1.

Distance between production wells $B_1B_2 = L_2 = L\sqrt{2}$. Radius of oil filtration zone R_{22} for homogeneous reservoir

$$R_{22} = \frac{L_2}{2} = \frac{\sqrt{2}}{2}L.$$

(36)

Denote by t_{22} time corresponding to reaching of point D_1, – interface between zones of filtration of production wells:

$$R_{22} = \sqrt{C\chi_2 t_{22}},$$

(37)

where χ_{22} – coefficient of oil piezoconductivity; C – numerical coefficient.

Radius R_2 and transformation start time t^* satisfy the relation

$$R_2 = L - R = \sqrt{C\chi_2 t^*}.$$

(38)

From formulas (36) and (37) we get

$$t_{22} = t^* \frac{R_{22}^2}{R_2^2} = t^* \frac{L^2}{2(L - R)^2}.$$

(39)

We use data of example 2 and calculation results of Table 2. For $L = 400$ m, $R = 80$ m, $t^* = 3.3$ days and $R_2 = 320$ m from (38) and (39) we get $t_{22} = 2.6$ days, $R_{22} = 283$ m. Consequently, disturbance zones of two production wells are in contact at a point D_1 earlier, than they connect at point C_1 of disturbance zone of injection and production wells.

We make an essential remark. Formation pressure in point D_1 at $t_{22} = 2.6$ days is p_0, which will further decrease. Therefore, in time interval $t_{22} \le t \le t^*$ in zone 3 oil filtration will begin in direction of point D_1. If due to "proximity" of values time t_{22} and t^* is neglected, it is possible to determine oil reserves in areas not covered by flooding.
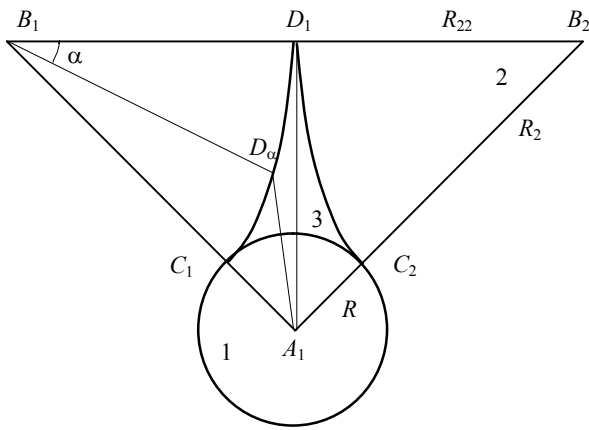
Fig.3. Fragment of an operational object element
for determining the area of zone 3. Arbitrary point
$D_\alpha \in$ curve $D_1 C_1$. Angle $0 \le \alpha \le 45°$.

So, at time t^* three zones with different saturation patterns form in the PO element of the five-point system (Fig.2).

1. In vicinity of injection well first zone has the shape of a circle with radius R. Saturation pattern – water and oil.

2. Four zones 2, each of which has the shape of a closed region bounded by curved lines $C_i D_i$. Saturation pattern – oil.

3. Four zones 3, bounded by curved lines $C_i C_j D_i$, not covered by development, saturated with oil, pressure in which is equal to initial pressure p_0.

Denote the area of zone 3 by S_3. Consider an isosceles right triangle $B_1 B_2 A_1$, whose legs are equal L. From Fig.2 determine the area of zone 3:

$$S_3 = S_\Delta - S_{A_1 C_1 C_2} - 2S_\alpha = \frac{1}{2} L^2 - \frac{\pi R^2}{4} - 2S_\alpha, \tag{40}$$

where S_Δ – area of a triangle $A_1 B_1 B_2$; $S_{A_1 C_1 C_2}$ – area of a sector $A_1 C_1 C_2$; S_α – area of a figure $B_1 D_1 C_1$ with center in point B_1, limited by arc $D_1 C_1$ with central angle $\alpha_0 = 45°$. Curvature radius $B_1 C_1$ changes from R_{22} to R_2. Area of zone S_3 depends on the accuracy of determining (calculating) area S_α.

Formula for determining S_α for a variable radius of curvature has the form

$$S_\alpha = \frac{1}{2} \int_0^{\alpha_0} r^2 d\alpha. \tag{41}$$

Consider several options for calculating the area of a figure $B_1 D_1 C_1$ (Fig.3). As can be seen from the figure, arc $D_1 C_1$ is convex with respect to the center B_1. At changing the angle in the interval $0 \le \alpha \le \pi/4$ radius R_α increases from $R_{22} = 283$ m to $R_2 = 320$ m. This condition is satisfied by function

$$B_1 D_\alpha = R_\alpha = L \frac{\sin\left(45° - \alpha\right)}{\cos\alpha} + (L - R)\operatorname{tg}\alpha. \tag{42}$$

For interval $0 \le \alpha \le \pi/4$ distance from injection well A_1 to point D_α is determined by ratio

$$A_1 D_\alpha = L \frac{\sin\left(\frac{\pi}{4} - \alpha\right)}{\cos\alpha} + R\operatorname{tg}\alpha. \tag{43}$$

It follows from formulas (42) and (43), that curvature radius R_α and distance $A_1 D_\alpha$ to injection well A_1 depends on angle α. Calculation results are presented in Table 4.

Example 4. Given: $R = 80$ m; $L = 400$ m; $R_2 = 320$ m; $R_{22} = 283$ m; $0 \le \alpha \le \pi/4$.

Table 4

Calculated values of curvature radii of the figure $B_1 D_1 C_1$ and distance $A_1 D_\alpha$, m

Parameters	Calculation number						
	1	2	3	4	5	6	7
α, rad	0	0.087	0.175	0.349	0.524	0.698	0.785
R_α, (42)	283	286	289	296	304	314	320
$A_1 D_\alpha$, (43)	283	265	247	209	166	113	80

After substitution of (42) in (41) area $B_1 D_1 C_1$ is

$$S_\alpha = \frac{1}{2}\left[\frac{\pi}{4}\frac{L^2}{2} + \sqrt{2}L\left[R - \left(\frac{\sqrt{2}}{2}+1\right)L\right]\frac{1}{2}\ln 2 + \left[R - \left(\frac{\sqrt{2}}{2}+1\right)L\right]^2\left(1-\frac{\pi}{4}\right)\right] = 28682\,\text{m}^2. \quad (44)$$

If, to determine area S_α, we take average radius r_{av}, equal to half-sum of radii $R_2 = 320$ m and $R_{22} = 283$ m, then

$$S_\alpha == \frac{\pi r_{av}^2}{8} = 35661\,\text{m}^2.$$

Thus, approximation of section $B_1 D_1 C_1$ to sector of constant average radius gives an overestimated value of area, and, as follows from formula (40), a smaller value of area of zone 3, which will lead to an incorrect estimate of reserves.

Element of PO $B_1 B_2 B_3 B_4$ (see Fig.2) contains four zones 3, each with area S_3. Volume of oil (geological reserves)

$$V_3 = 4 S_3 h m_0 (1 - s_0). \quad (45)$$

To determine volume of recoverable oil reserves in zone 3 we take coefficient of oil displacement by water

$$\eta_1 = \frac{s_{in.oil} - s_{res.oil}}{s_{in.oil}} = \frac{s^* - s_0}{1 - s_0},$$

where $s_{in.oil} = 1 - s_0$ – coefficient of initial oil saturation; $s_{res.oil} = 1 - s^*$ – coefficient of residual oil saturation; s_0 – coefficient of residual water saturation; s^* – limiting coefficient of water saturation at which oil filtration stops.

Volume of recoverable oil reserves

$$V_{3iz} = \eta_1 \eta_2 V_3 = 4\eta_2 S_3 h m_0 (s^* - s_0), \quad (46)$$

where η_2 – coefficient of formation coverage by flooding.

Example 5. Take $m_0 = 0.2$; $h = 10$ m; $s_0 = 0.2$; $s^* = 0.8$; $\eta_2 = 1$. Remaining data will be taken from example 3.

From formulas (40), (45), (46) we get $S_3 = 17613$ m^2, $V_3 = 12721$ m^3, $V_{3iz} = 84540$ m^3. If input flow rate of well that penetrated zone 3 is 50 m^3/day, then time spent on extraction of mobile reserves V_{3iz}, is 4 years, if boundaries of zone 3 do not change. Volume of geological oil reserves in PO element is 512000 m^3. Share of unextracted reserves will be 0.22.

However, over time, there will be an intrusion of water in zone 3, in which two-phase filtration will occur, the oil flow rate will fall.

Let us consider how filtration processes change over time with simultaneous operation of wells. Following remark is appropriate. Due to given boundary conditions in PO element, following inequality is true:

$$p_{B_1} < p_{D_1} < p_{D\alpha} < p_{C_1} = p_{G_1} = p_0 < p_{A_1}, \quad (47)$$

where arbitrary point $G_\alpha \in$ zone 3.

Condition (47) is satisfied from the moment the zone formation begins. Consequently, flooding of zones 2 and 3 will occur unevenly. With growth of time $t > t^* > t_{22}$ in zone 3, two-phase filtration begins, boundary $C_1 G_\alpha$ of which is limited by an isobar with initial pressure p_0. Single-phase filtration zone 2 is deformed, water injected into formation will begin to flow through the boundary along channel $C_1 D_\alpha$.

Thus, in a reservoir, which is permeability homogeneous in Oxy plane, linear filtration will occur in zones 3 and 2. Values of saturation coefficients and pressures in filtration channels will differ from initial values at beginning of operation.

Consider the filtering process in element of PO in form of a channels system connecting injection A_1 and production B_1 well.

Article [5] shows that watering of well fluid happens in presence of low filtration resistance (LFR) channels in formation. Cases of LFR channels' location along the thickness of reservoir and in the planes Oxy are considered. To determine channel length, parameters ω_j, were introduced, depending on concentration of indicator peaks and fixation time, j is indicator fixation number. For the element of the five-point development system considered above, we have

$$L_j = \omega_j L = A_1 D_\alpha B_1 = \frac{L}{\cos\alpha}\left[2\sin\left(\frac{\pi}{4} - \alpha\right) + \sin\alpha\right]. \tag{48}$$

From which

$$\omega_j = \frac{2\sin\left(\frac{\pi}{4} - \alpha\right) + \sin\alpha}{\cos\alpha}. \tag{49}$$

Angle $\alpha = 45°$ corresponds to a straight line $A_1 B_1 = L$ – the shortest distance between injection and production wells, parameter $\omega_j = 1$. Angle $\alpha = 0°$ c corresponds to the greatest distance $A_1 D_1 B_1 = L\sqrt{2}$, parameter $\omega_j = \sqrt{2}$. Therefore, LFR channels that satisfy the conditions

$$L \le L_j \le L\sqrt{2};\ 1 \le \omega_j \le \sqrt{2}, \tag{50}$$

lie in horizontal Oxy planes. If conditions (50) are not satisfied, then LFR channels are located in a vertical plane.

In Tables 3-5 of article [5] cases of horizontal arrangement of LFR channels L_j for three given parameters ω_j are considered.

Table 5 shows values ω_j and L_j from article [5, see Table 3] and dependencies ω_j and $A_1 D_\alpha B_1$ on angle α.

Table 5

Comparison of parameters ω_j, L_j for $L = 505$ m [5] with calculated values (48) (49)

Peaks j	ω_j [5]	$L_j = \omega_j L$, m [5]	α, deg.	ω_j (49)	$A_1 D_\alpha B_1$ (48)
1	1	505	45	1	505
2	1.44	707	30	1.17	593
3	2.6	1313	20	1.26	638
4	3.8	1919	10	1.34	677
5	5.4	2828	0	1.41	714

Obviously, for a set parameter $\omega_j = t_j / t_1$ there are two channels in horizontal plane L_j: $j = 1$ and $j = 2$. Remaining LFR channels are located in a vertical plane. Note that methodology for calculating the volume of pore channels, necessary to apply alignment of injectivity profile is given in the article [5].

Each model has its own limitations, its own field of application, including the Buckley – Leverett model. Confirmation of reliability is compliance of calculated and actual field indicators [7]. It is known that watering of a production well does not occur "abruptly", but gradually increases with increasing water saturation coefficient.

If formation is layered-inhomogeneous, consists of interlayers (seams) of different permeability, then for each ith seam, its own zones 1, 2, 3 are formed. Filtration transformation time for each interlayer will be different, determined by formulas (9), (12), (13). In absence of crossflows between seams (within framework of accepted Rapoport – Lis model), following boundary conditions are possible: flow rate of water injected into formation should be distributed in proportion to conductivity coefficients $k_i h_i$ of seams [8, 15] or it should be assumed that bottomhole pressure is constant in all

seams [11]. Note that each case requires a separate study. LFR channels correspond to highly permeable seams (HPS). For stationary filtration, presence of water "tongues" in the most permeable sections of formation, which is carried out through HPS, has been established. Change in water saturation coefficient is determined by formulas (29), (31), (33).

Original method for studying the distribution of residual mobile oil reserves in a laterally heterogeneous formation was proposed by M.I.Maksimov in a monograph [8]. It is shown that, depending on location of zones with permeability of 40, 200, and 1000 mD, a "development self-regulation" takes place in an area heterogeneous formation. In our opinion, this is a consequence of fluid flows (see Fig.2, *b*), individual for each zone.

In the 50s of the twentieth century. V.N.Shchelkachev [18] formulated conditions (criteria) for the use of forced fluid extraction (FFE) recommended for PO at a late stage of development: fluid water cut of at least 90 %, high productivity coefficients, collector stability, etc. We believe that large value of water cut coefficient is not a decisive criterion for determining the late fourth stage of PO.

As it is known, FFE technology consists in a phased change in operating modes of production and injection wells, in creation of high-pressure gradients in formation, allowing low-permeability interlayers, dead end and stagnant zones to be involved in development. This leads to a change in directions of the filtration flows, and presence of a hydrodynamic connection between differently permeable seams involves developing low-permeability differences, i.e. cyclic flooding. Therefore, there is a combined unstationary flooding. Existence of regions 3, formed as a result of filtration flows transformation and containing mobile unextracted oil reserves, indicates the need for oil recovery enhancement methods (EOR) that were not previously used in this PO. There are techniques that determine density of oil reserves in areas not covered by development, in particular the Voronoi method. In numerous scientific works of recent years [15, 16], commissioning of second wellbores with a shallow and horizontal end and multi-stage hydraulic fracturing are recommended. Length of the wellbore in reservoir, as follows from above calculations, should not exceed the radius of drainage zone R_2 of production well (Fig.2, *b*). Methods for calculating the horizontal drain in formation are quite fully considered in literature [15, 16].

Thus, a change in direction of the filtration flows leads to creation of linear filtration channels between production wells and formation of zones not covered by development (Fig.3). Obviously, this must be taken into account at designing and arranging second wellbores for efficient development of oil reserves.

Since bottomhole pressure of injection wells exceeds initial formation pressure p_0, and bottomhole pressure of production wells, on the contrary, is less than p_0, then isobar line corresponding to it – G_0 will exist in the formation. Moreover, existence of G_0 line does not depend on chosen physical model. Location of this line in space depends on boundary conditions and affects formation of filtration flows. On the other hand, isobar line G_0 limits efficient effect of all existing EOR: hydrodynamic, physical and chemical, gas, heat, fracturing, zones of efficient operation of horizontal wells endings, etc. Presence of fluid crossflows in horizontal plane (see Fig.2, *b*), as well as flows in hydrodynamically interconnected seams in layered-heterogeneous formations affect efficiency of the technologies used.

Conclusion

1. Study of traditional flooding technology using Rapoport – Lis model of non-piston oil displacement by water for plane-radial filtration taking and taking not into account the end effect found that changing the capillary pressure gradient practically does not affect distribution of saturation coefficients in two-phase filtration zone.

2. Simulation of simultaneous operation of injection and production wells revealed that plane-radial filtration is transformed into a straight-parallel flow. Time of this process and distribution of water saturation coefficients for rectilinear-parallel filtration in an infinite and final (closed) formation are determined.

3. On example of considered model, formation zones containing 22 % of unextracted oil reserves are identified. Under real conditions of development of an operational object, results that are more negative can be obtained.

4. For a layered-heterogeneous formation without taking into account crossflows between seams, a scheme for determining water saturation coefficients in each interlayer is proposed.

5. Criteria for possible location of low filtration resistance channels in lateral and formation thickness are established.

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