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Offline Calibration Procedure of the Drift Tube Detectors

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Abstract

A detailed description of the calibration of the DT local reconstruction algorithm is reported.
After inter-channel synchronization has been verified through the appropriate hardware procedure, the time pedestal can be extracted directly from the distribution of the digi-times. Further corrections for time-of-flight and time of signal propagation are applied as soon as the three-dimensional hit position within the chamber is known. The different effects of the time pedestal miscalibration on the two main hit reconstruction algorithms are shown.
The drift velocity calibration algorithm is based on the meantimer technique and different meantimer relations for different track angles and patterns of hit cells are used. This algorithm can also be used to determine the uncertainty of the reconstructed hit position.

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1 Introduction

The barrel region of the CMS detector is equipped with a system of Drift Tube (DT) chambers [1], each one composed of two or three groups (superlayers) of four staggered layers of independent drift cells. Charged particles crossing a cell produce ionization electrons in the Ar/CO_2 gas mixture; the drift time of such electrons in a properly shaped electrostatic field is measured to determine the spatial coordinates of the ionizing particle.

Electrons produced at a time t_{ped} by the incoming particle migrate toward the anode with a velocity v_{drift} and reach the anode at a time t_{TDC}, which is the time measured by the TDC. The distance of the track with respect to the anode wire is therefore given by

$$x = \int_{t_{ped}}^{t_{TDC}} v_{drift} \cdot dt. \tag{1}$$

The measurement of the track distance from the wire (x) requires understanding of this time-space relationship.

Two algorithms are available in the CMSSW reconstruction code. The first reconstruction algorithm is based on the assumption of a constant drift velocity within the entire cell. In this case, the above formula becomes

$$x = (t_{TDC} - t_{ped}) \cdot v_{drift}^{EFF} = t_{drift} \cdot v_{drift}^{EFF} \tag{2}$$

where v_{drift}^{EFF} is the effective, average drift velocity.

The goal of the calibration procedure is in this case to determine the time pedestal (t_{ped}), which is needed to extract the drift time (t_{drift}) from the TDC measurement (t_{TDC}), and the average drift velocity v_{drift}^{EFF}.
The value of v_{drift}^{EFF} depends on the track impact angle and on the residual magnetic field. However, the detector can be subdivided in properly limited spatial regions where these parameters can be assumed approximatively constant. The calibration procedure is performed with the correspondent granularity, therefore the computed drift velocity is averaged under local variations of such parameters in each region.

The second reconstruction algorithm is based on a parameterization of the cell response [2] obtained with GARFIELD [3]. This parameterization includes the dependence on the track impact angle, α, and on the stray magnetic field $\mathbf{B}$:

$$x = f((t_{TDC} - t_{ped}), \alpha, \mathbf{B}) \tag{3}$$

In this case the only quantity to be calibrated is t_{ped}, as the dependency on the relevant parameters is already accounted for by the parameterization.

It should be noted that the residual magnetic field and the track angle also influence the intrinsic cell resolution due to their effect on the cell non-linearities. Correct estimation of the hit uncertainty is important for the track fit; for this reason, the calibration algorithm must also be able to assign correct uncertainty to the reconstructed hits.

The procedure to determine the time pedestals is described in Section 2. Section 3 introduces the calibration of the drift velocity and the assignment of the uncertainty on the hit position. Finally, Section 4 outlines the reciprocal dependence between the time pedestal and the drift velocity.

2 Calibration of the Time Pedestals

A DT measurement consists in a TDC time, which also contains contributions from other than the drift time of the ionization electrons in the cell, including

- the time-of-flight (TOF) of the muon from the interaction point to the cell;
- the propagation time of the signal along the anode wire;
- delays due to the cable length and read-out electronics;
- the time latency due to the Level-1 trigger.

These offsets must be estimated and subtracted from the TDC time during reconstruction. The jitter in the drift time deriving from the uncertainties of this procedure directly contributes to the DT resolution.

The extraction of the drift time from the TDC measurement is performed in several consecutive steps.

- *Inter-channel synchronization.*
  First, it is necessary to correct the measured TDC times for the relative difference in the signal path length to the readout electronics of each wire. This relative difference is measured for each wire by sending simultaneous (with an error smaller than 150 ps) "test-pulses" to the front-ends and computing the difference between the measured times, called t_0. This relative correction is usually between 1 and 8 ns. Once the t_0 is subtracted, the resulting TDC times for the different channels within the chamber are synchronized among them.

- *Absolute offset determination.*
  Once the channels are synchronized, it is possible to compute the absolute offset of the drift time distribution. This offset, called t_{trig} because of its dependence on the trigger latency, allows the extraction of the drift time from the TDC measurement. The t_{trig} is directly estimated from the distribution of the digi times using the procedure described in Section 2.1. Its value depends on the specific DAQ setup and is usually on the order of a few μs.

Note that the determination of these two delays does not completely solve the problem of synchronization of the digi times, as normally, due to the limited available data, the t_{trig} is computed for a group of cells together, e.g., all cells in a superlayer. In this case, the measured t_{trig} includes the average TOF and the average signal propagation time of the muons that crossed the superlayer. If the chamber is uniformly illuminated, which is the case for pp-collisions, this average TOF is approximately equivalent to that of a muon crossing the superlayer center, while the average signal propagation time is equivalent to the propagation time for a signal produced in the middle of the wire.

Therefore, further corrections for these two effects can be computed as soon as the three-dimensional hit position within the chamber is known, namely after the hits are associated into 3D track segments. Specifically, if the t_{trig} is computed for a full superlayer uniformly illuminated:

- the 3D position obtained from the segment extrapolation to the hit plane, if available, is used to correct the TOF with respect to the superlayer center;

- the hit coordinate along the wire is used to correct the propagation time with respect to the middle of the wire, assuming a propagation velocity of 0.244 m/ns, as directly measured on test-beam data [4].

These corrections can be as high as ≈ 2 ns for the TOF and ≈ 6 ns for the signal propagation delay and they can be adapted or switched off in case of different running conditions. This is the case, for example, for cosmic data, where the previous definition of the TOF can not be applied, or for test-beam data, where the chamber is usually illuminated in a relatively small region. Particular care has been taken to provide enough flexibility for such cases.

2.1 Determination of the t_{trig} Offset

Since the digi times of the different channels in a chamber have already been synchronized by subtracting the t_0 offset, the t_{trig} can be computed with every possible granularity within the chamber. The usual choice is to compute it superlayer by superlayer, as a compromise between accuracy in accounting for the average TOF and the quantity of available data.

Due to its dependency on the trigger latency, the t_{trig} pedestal must be calibrated each time the trigger configuration and synchronization change. Moreover, as it accounts for the average contribution of the TOF and the signal propagation along the anode wire, the t_{trig} also depends on the running conditions: these contributions are different if the superlayer is not illuminated uniformly, as, for example, in test-beam data taking. This has to be taken into account when using the pedestals in the reconstruction.

The pedestal can be estimated directly from the distribution of the digi times, which is usually referred as the *time box*. An example of such distribution is shown in Fig.1 for a superlayer RZ of a chamber exposed to a muon test beam.
In order to compute the pedestal it is necessary to find a feature of this distribution which can be identified in an unambiguous and automatic way. Earlier studies have shown that a suitable feature is the inflexion point of the rising edge, which can be obtained from a Gaussian fit of the derivative of the time-box distribution [5]. This method, however, is sensitive to noise and spikes due to the read-out electronics. To implement an automatic procedure to fit the drift time box in unattended mode for all the superlayers of the 250 DT chambers, we developed

a different, more robust method, based on a fit of the rising edge of the drift time distribution with the integral of the Gaussian function (the so-called *error function*):

$$f(t) = \frac{1}{2} I \left[1 + \text{erf}\left(\frac{t - \langle t \rangle}{\sigma\sqrt{2}} \right) \right], \tag{4}$$

where the normalization I, the standard deviation σ and the mean $\langle t \rangle$ are free parameters of the fit. In Fig.1 on the right an example of this fit is shown for a time box of a RZ superlayer illuminated during a muon test beam.

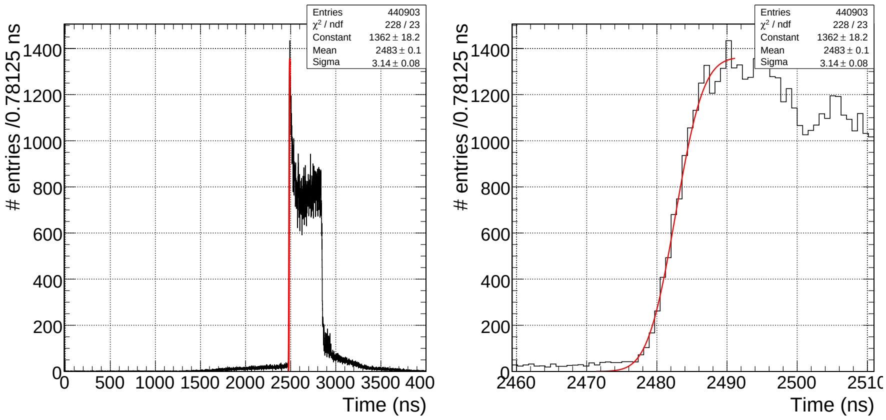


Figure 1: Distribution of the digi times of one superlayer acquired during the 2004 test beam. The rising edge of the time box (right) is fitted with the integral of a Gaussian to measure the time pedestal of the drift times (t_{trig}).

The inflexion point of the rising edge of the time box, $\langle t \rangle$, does not directly represent the time pedestal of the distribution, but can be related to it by defining

$$t_{trig} = \langle t \rangle - k \cdot \sigma, \tag{5}$$

where k is a factor that is tuned by requiring the minimization of the residuals on the reconstructed hit position, superlayer by superlayer. A typical value of the k factor is 1.3.

In order to obtain meaningful residual distributions is necessary to have a preliminary estimation of the t_{trig} at least correct to 10 ns, while the value of the t_{trig} can varies up to some microseconds depending on the trigger configuration and cable length. Therefore the fit of the time box rising edge has to be performed before the k factor optimization can be done.

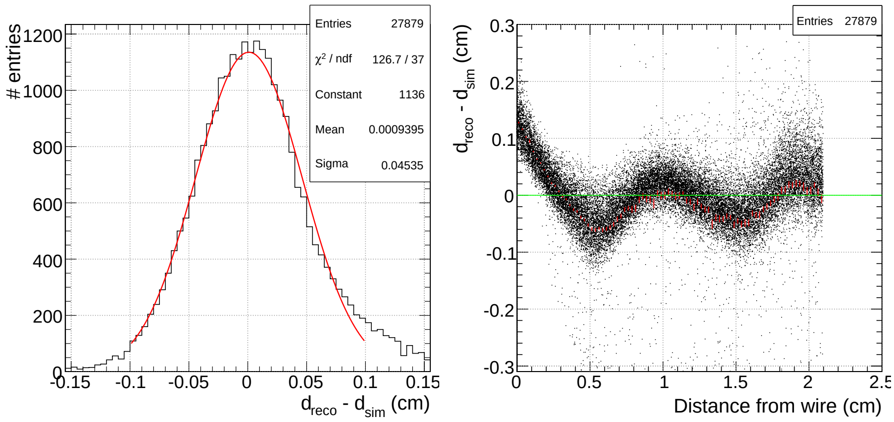
It should be noted that the optimal value of t_{trig} depends on the algorithm used in the reconstruction. In particular, the cell parameterization has a small, arbitrary, intrinsic offset deriving from the way the signal arrival time is computed in the GARFIELD simulation [2]. For this reason a fine tuning of the t_{trig} has to be done differently for the two reconstruction algorithms.

In addition, the effect of a mis-calibration of the time pedestal is different for the two reconstruction algorithms.

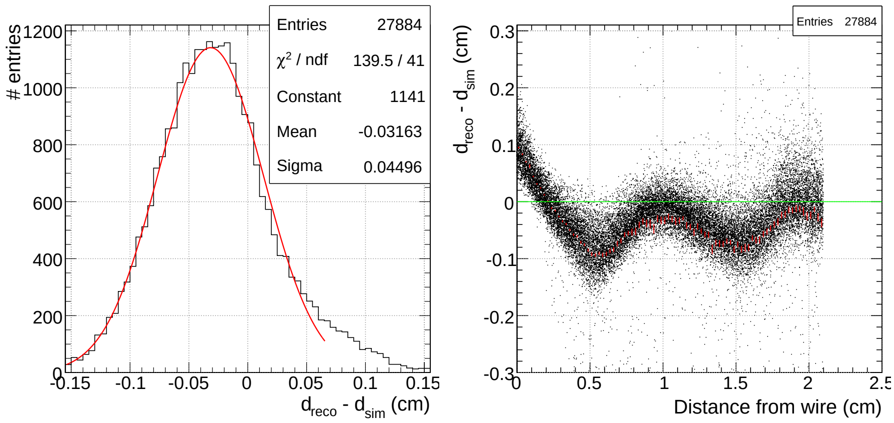
If the reconstruction is performed using a constant drift velocity over the entire cell, a t_{trig} not perfectly calibrated results in an error on the estimated drift time and therefore in a constant offset for all the reconstructed distances from the wire. This is illustrated for Monte Carlo simulated pp-collisions in Fig. 2 which shows the residuals on the distance from the wire ($|x_{reco}| - |x_{sim}|$) for two particular choices of the time pedestal: the "optimal" value and a t_{trig} mis-calibrated of $\Delta t = 6$ ns [1)]. The error on the pedestal affects the mean value of the distribution of a quantity given by $-\Delta t \cdot v_{drift}$, while the standard deviation of the Gaussian fit is essentially unaffected, being dominated by the non-linearities responsible for the modulation shown in the scatter plots of Fig. 2. This independence of σ on the actual value of t_{trig} allows t_{trig} to be optimized superlayer by superlayer by tuning the k factor of Eq. 5 to minimize the mean of the residual distribution.

Note that Fig. 2 shows the distributions obtained for all the muon tracks originating in pp-collisions recorded in the RZ superlayers of wheels ± 2, i.e., the superlayers in which the effects of non-linearity are expected to be larger, because of the bigger average values of the track incident angles with respect to the direction normal to the chambers and the larger values of the residual magnetic field in the chamber volume.

[1)] This value of the pedestal corresponds to an extreme case of mis-calibration, chosen for illustration purposes. The t_{trig} can be usually calibrated with much higher accuracy.



(a) Optimal time pedestal t_{trig} (MC truth)

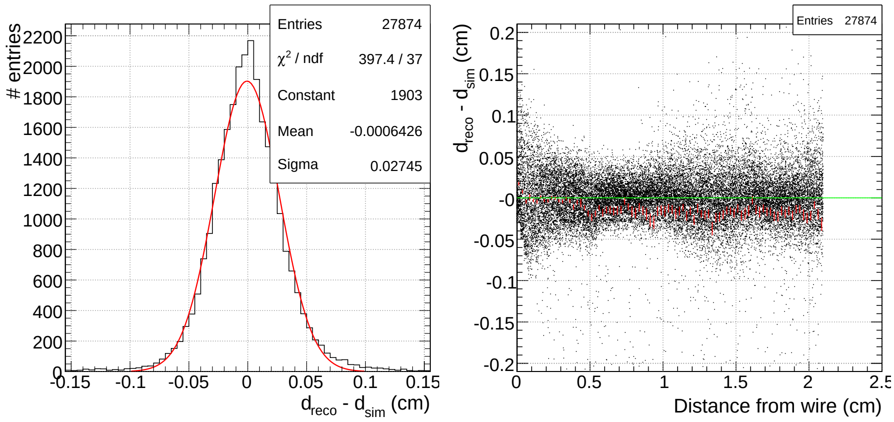


(b) Mis-calibrated time pedestal (t_{trig}+ 6 ns)

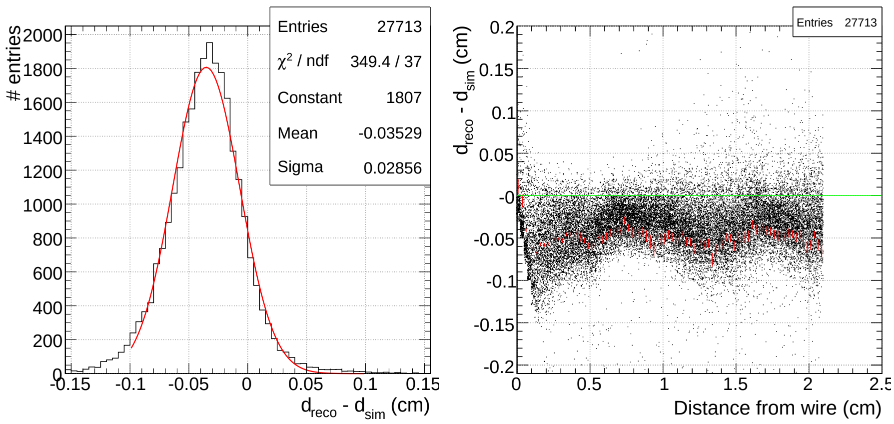
Figure 2: Residuals between the reconstructed and the simulated hit distances from the wire ($d = |x|$) for RZ superlayers in wheels ± 2. The plots on the right show the residuals as a function of the distance from the wire. The plots have been obtained using a constant drift velocity (a) with the optimal value of the t_{trig} and (b) with a t_{trig} 6 ns greater than the optimal one. No further correction for the TOF or the time of signal propagation along the wire has been applied.

The effect of a mis-calibration of the t_{trig} pedestal is more complex when the reconstruction is performed using the GARFIELD parameterization. As this parameterization accounts for the cell non-linearity as a function of the drift time, an offset in the input time does not simply produce an offset in the mean value of the residuals, but also implies that the non-linearities are accounted for incorrectly, resulting in in a wider residual distribution. This is illustrated in Fig. 3, which again shows the residuals of the reconstructed hit distances from the wire in the RZ superlayers of wheels ± 2 for the two extreme choices of the t_{trig} pedestal considered above. It can be observed that since the parameterization corrects for the non-linearities, the presence of an offset in the t_{trig} introduces artificial deviations, leading to a broadening of the residual distribution in addition to a shift of the mean value. This effect can be used for the optimization of the t_{trig} value, which can be simply performed by minimizing the residuals: the optimal t_{trig} value is the value for which the parameterization of non-linearities best fits the input data.

It should be noted that in real data the residuals will be computed with respect to the reconstructed 3D segment and this will introduce systematic effects on the k factor optimization to be studied.



(a) Optimal time pedestal t_{trig} (MC truth)



(b) Mis-calibrated time pedestal (t_{trig}+ 6 ns)

Figure 3: Residuals between the reconstructed and the simulated hit distances from the wire ($d = |x|$) for RZ superlayers in wheels ± 2. The plots on the right show the residuals as a function of the distance from the wire. The plots have been obtained using the GARFIELD parameterization with the optimal value of the t_{trig} (a) and with a t_{trig} 6 ns bigger than the optimal one (b). No further correction for the TOF or the time of signal propagation along the wire has been applied.

3 Calibration of the Drift Velocity

The drift velocity depends on many parameters, including the gas purity and condition and the electrostatic configuration of the cell. Moreover, the presence of stray magnetic field and the angle of incidence of the track (indicated with α in Fig. 4) influence the effective drift velocity. In particular the effect of the track angle is due to the fact that the electrons with smaller drift time are not the ones produced in the cell median plane. This effect has been measured and the results are reported in [1].

The working condition of the chambers will be monitored continuously and important variations are not expected among different regions of the spectrometer. The situation is different for the stray magnetic field and for the track impact angle: these parameters will vary substantially, on average, moving from chamber to chamber and also from superlayer to superlayer due to the different positions within the return yoke and to the different pseudorapidities of the impact angles in the RZ cells. For this reason, the reconstruction algorithm based on a constant drift velocity requires a calibration procedure that allows the average velocity to be found separately for different groups of cells.

To fulfill these requirements, a calibration algorithm based on the so-called *meantimer* [6] computation has been developed and is described below. This technique estimates the maximum drift time and therefore the average drift velocity in the cell. Moreover, it also measures the cell resolution, which can be used as an estimate of the uncertainties associated to each measurement.

3.1 Meantimer Technique

The meantimer formulas are relations among the drift times produced by a track in consecutive layers of a superlayer (t_i) and the maximum drift time (T_{max}) in a semi-cell (i.e. half cell), under the assumption of a constant drift velocity. Even with small deviations from this assumption, as in the case of the DTs, the average of the meantimer distribution contains information about the average drift velocity in different regions of the cell, since it is computed using drift times produced by hits all over the gas volume. The mathematical expression of the meantimer relation depends on the track angle and on the pattern of cells hit by the track. In the easiest case the track crosses a semi-column of cells i.e. the interested wires are at the same position for each couple of staggered cells. In this simple case the correspondent meantimer relation is

$$T_{max} = (t_1 + t_3)/2 + t_2 \tag{6}$$

The meantimer relations for different track angles and patterns of hit cells are listed in Table 1, using the naming convention illustrated in Fig. 4. It should be noted that not all the track geometrical configurations can be used because in some cases the relation between drift times is independent of T_{max}.

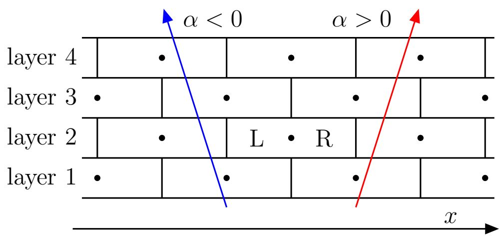


Figure 4: Schematic of a superlayer showing the track segment angle convention and the pattern of semi-cells crossed by the track.

The proper meantimer formula is chosen among those listed in Table 1 track by track, using the direction and position information provided by the three-dimensional segments in a superlayer. This implies an iterative calibration procedure, starting with values of the drift velocity and of t_{trig} that already result in efficient pattern recognition and segment reconstruction.

The meantimer is normally computed superlayer by superlayer, assuming the same effective drift velocity in all layers. It may be interesting, however, to calibrate the average drift velocity with finer granularity to take into account possible local variations within the layer quadruplet due to magnetic field inhomogeneities.
The mechanical precision of the wire and layer positions inside the superlayers is of the order of 100μm and it should be known to 10μm after the first alignment procedure. This precision corresponds to a bias of 1.8 ns (0.18 ns) on the measured drift times and it causes a different uncertainty on the T_{max} depending on the formula, the consequent error on the drift velocity is of the order of 1% (0.1%) or less.

In Section 3.1.1 the various steps of the drift velocity calibration procedure are listed.

3.1.1 Calibration Procedure

The calibration procedure of the drift velocity consists of the following steps:

- a Gaussian is fit to the meantimer distribution for each track pattern j to estimate the mean value T^j_{max}, the standard deviation σ^j_T, and the error on the mean $\sigma^j_T/\sqrt{N_j}$ (where N_j is the number of entries in the distribution);
- the weighted average of the values of T^j_{max} is computed where the weights are taken as $N_j/(\sigma^j_T)^2$:

$$\langle T_{max} \rangle = \frac{\sum_j \frac{T^j_{max}}{(\sigma^j_T)^2} N_j}{\sum_j \frac{N_j}{(\sigma^j_T)^2}}. \tag{7}$$

  This accounts for the relative importance of the different cell patterns in the computation of the maximum drift time.

- once $\langle T_{max}\rangle$ is computed it is straightforward to find the average drift velocity through the relation:

$$v_{drift} = \frac{L/2}{\langle T_{max}\rangle}; \tag{8}$$

where L is the width of the cell. The effective drift velocity computed for each superlayer is then stored in a database to be used by both the HLT and the off-line hit reconstruction.

3.2 Estimate of the Cell Resolution

The meantimer technique allows the estimation of the cell resolution and hence the uncertainties on the reconstructed distance.

The standard deviation of the meantimer distribution (σ_T^j) is a measurement of the resolution of T_{max}^j. It can be therefore used to estimate the uncertainty on the measurement of the drift times (σ_t^j) with a relation that depends on the particular formula used to compute the meantimer. In the case of tracks crossing a semi-column of cells (123LRL or 123RLR), given the meantimer relation in Table 1, the time resolution can be computed as

$$\sigma_t^j = \sqrt{\frac{2}{3}} \cdot \sigma_T^j, \tag{9}$$

which is valid under the assumption that the uncertainties are the same for all three layers used in the meantimer computation.

Since the cell resolution depends on the track angle, an average effective value is computed by averaging the different values obtained for the contributing cell patterns weighted on the number of entries in each meantimer histogram (N_j):

$$\langle\sigma_t\rangle = \frac{\sum_j \sigma_t^j \cdot N_j}{\sum_j N_j}. \tag{10}$$

The resolution of the reconstructed distance is therefore given by:

$$\sigma_d = v_{drift} \cdot \langle\sigma_t\rangle. \tag{11}$$

This value is used during the reconstruction to assign the uncertainties to the one-dimensional RecHits in the gas volume. These uncertainties include the effect of the cell non-linearities (as those shown in Fig.2) only on average, therefore their dependence on the distance from the wire cannot be taken into account with this method.

4 Interplay of Meantimer Computation and Time Pedestals Determination

Reconstruction using a constant drift velocity requires both the calibration of the time pedestals needed for synchronization and of the average drift velocity. These two tasks are not independent since on one hand the computation of the meantimer requires knowledge of the time pedestals and on the other hand fine tuning of t_{trig} is based on analysis of the residuals, which are directly affected by a mis-calibration of the drift velocity.

If the determination of t_{trig} is affected by a systematic shift Δt:

$$t'_{trig} = t_{trig} + \Delta t, \tag{12}$$

the meantimer will be consequently biased by a quantity that depends on the particular formula among those in Table 1. In the case of tracks crossing a semi-column (Table 1: 123LRL or 123RLR) we can evaluate the effect on T_{max} as

$$T'_{max} = T_{max} - 2\Delta t. \tag{13}$$

In a simplified scenario where this particular pattern is the one determining the meantimer calculation ($\langle T_{max}\rangle \approx T'_{max}$) the bias on t_{trig} determination will result in a mis-calibration of the drift velocity Δv_{drift}, which can be estimated as

$$\begin{aligned} v_{drift} + \Delta v_{drift} &= \frac{L}{2 \cdot T'_{max}} \\ &= \frac{L}{2 \cdot (T_{max} - 2\Delta t)}. \end{aligned} \tag{14}$$

To first order, this is equivalent to the following requirement:

$$2v_{drift}\Delta t - T_{max}\Delta v_{drift} = 0, \tag{15}$$

which can be considered as a calibration condition: all values of drift velocity and time pedestal that satisfy this relation will not affect the mean value of the residuals. This is strictly true only for small variations around the "optimal" values of t_{trig} and v_{drift} since larger fluctuations may affect pattern recognition efficiency and segment building. Lacking an external system for the track measurement, the segment is used as a reference for the computation of the residuals of the reconstructed drift distance.

The main sources of uncertainty in the determination of the time pedestal are the fluctuations in the mean value $\langle t \rangle$ and in the σ of the fit in the different layers of a superlayer: the intrinsic statistical error, the presence of noise before the drift time box (evidenced, e.g., by the entries shown in Fig. 1 before the starting point of the drift time box), the finite step size of the TDC (0.78 ns), and the fact that the distribution is not perfectly described by Eq. 4, which together limit the accuracy of t_{trig} determination to about 1 ns. Further systematic uncertainties come from the uncertainty of the drift velocity, as demonstrated by Eq. 15, therefore higher accuracy can only be achieved using a procedure for fine tuning of the time pedestal independent of the drift velocity.

An alternative approach consists in using the different dependences of t_{trig} mis-calibration of the various meantimer formulas listed in Table 1 to calibrate the pedestal. The differences among the values of T_{max} computed using different formulas can be used to measure the value of the mis-calibration Δt once the dependence of the meantimer on the track impact angle is well under control. This would allow t_{trig} to be tuned without relying on the residual distribution and therefore without depending on the calibration precision of the drift velocity. This alternative approach will be investigated in the future.

5 Conclusions

The calibration task is fundamental to the local reconstruction: the knowledge of the time pedestal is an unavoidable prerequisite for the computation of the drift distance, while the calibration of the average drift velocity determines the accuracy of the reconstruction.

For this reason, a robust calibration procedure has been developed with the goal of satisfying the requirements imposed by all possible running conditions: dedicated cosmic runs, test beams, and pp-collision data.

The calibration algorithms described in the present document have been tested both on the simulation and on real data acquired during commissioning, the MTCC, and the 2004 test beam. Additional documents are presently in preparation regarding these subjects.

Using the tools developed for the calibration and synchronization procedure we also studied the effect of possible mis-calibration of the pedestals and of the drift velocity on the muon track fit and thus eventually on higher level reconstructed quantities. We applied these to analysis of the systematic uncertainties while studying the physics reach of the experiment as documented in [7].

Further optimization is still possible. In particular, the accuracy of the current procedure is limited by the interdependence of the time pedestal and the drift velocity used in the reconstruction. Other methods for fine tuning of t_{trig} are under study; a procedure based on the usage of different meantimer formulas to estimate the best value of the time pedestal is the most promising.

Table 1: Meantimer equations for different track angles and patterns of hit semi-cells. The definition of the sign of the segment angle α is given in Fig. 4. The pattern is defined through four labels, one for each layer: L and R stand for left and right semi-cells, respectively. The label enclosed in parentheses refers to the layer not directly used in the T_{max} computation. Where necessary the relative positions of the hit wires of the first and the last layers in the chamber RF (x_1, x_4) are also shown. The time t_i is the measurement in the cell belonging to layer i.

ID	**Meantimer formula**	**Segment direction**	**Semi-cell pattern**
	Layers 1-2-3		
123LRL 123RLR	$T_{max} = (t_1 + t_3)/2 + t_2$	all α	LRL(L/R) RLR(L/R)

Table 1: (continued)

ID	Meantimer formula	Segment direction	Semi-cell pattern
123LLR	$T_{max} = (t_3 - t_1)/2 + t_2$	$\alpha > 0$	LLR(L/R)
123RRL		$\alpha < 0$	RRL(L/R)
123LRR	$T_{max} = (t_1 - t_3)/2 + t_2$	$\alpha > 0$	LRR(L/R)
123RLL		$\alpha < 0$	RLL(L/R)
	Layers 1-2-4		
124LRR(1)	$T_{max} = 3t_2/2 + t_1 - t_4/2$	all α	LR(L)R $x_4 < x_1$
124RLL(1)			RL(R)L $x_4 > x_1$
124LLR	$T_{max} = 3t_2/2 - t_1 + t_4/2$	$\alpha > 0$	LL(L/R)R
124RRL		$\alpha < 0$	RR(L/R)L
124LLL(1)	$T_{max} = 3t_2/2 - t_1 - t_4/2$	$\alpha > 0$	LL(R)L
124LRR(2)			LR(R)R
124RRR(1)		$\alpha < 0$	RR(L)R
124RLL(2)			RL(L)L
124RRR(2)	$T_{max} = -3t_2/2 + t_1 + t_4/2$	$\alpha > 0$	RR(L)R
124LLL(2)		$\alpha < 0$	LL(R)L
124RLR	$T_{max} = 3t_2/2 + t_1 + t_4/2$	$\alpha > 0$	RL(L/R)R
124LRL		$\alpha < 0$	LR(L/R)L
124LRL	$T_{max} = 3t_2/4 + t_1/2 + t_4/4$	$\alpha > 0$	LR(L/R)L
124RLR		$\alpha < 0$	RL(L/R)R
124LRR(3)	$T_{max} = 3t_2/4 + t_1/2 - t_4/4$	$\alpha > 0$	LR(R)R $x_4 > x_1$
124RLL(3)		$\alpha < 0$	RL(L)L $x_4 < x_1$
	Layers 1-3-4		
134LLR(1)	$T_{max} = 3t_3/2 + t_4 - t_1/2$	all α	LRLR $x_4 < x_1$
134RRL(1)			RLRL $x_4 > x_1$
134LRR	$T_{max} = 3t_3/2 - t_4 + t_1/2$	$\alpha > 0$	L(L/R)RR
134RLL		$\alpha < 0$	R(L/R)LL
134RRR(1)	$T_{max} = 3t_3/2 - t_4 - t_1/2$	$\alpha > 0$	R(L)RR
134LLL(1)		$\alpha < 0$	L(R)LL
134LLL(2)	$T_{max} = -3t_3/2 + t_4 + t_1/2$	$\alpha > 0$	L(R)LL
134RRR(2)		$\alpha < 0$	R(L)RR
134LRL	$T_{max} = 3t_3/2 + t_4 + t_1/2$	$\alpha > 0$	L(L/R)RL
134RLR		$\alpha < 0$	R(L/R)LR
134RLR	$T_{max} = 3t_3/4 + t_4/2 + t_1/4$	$\alpha > 0$	R(L/R)LR
134LRL		$\alpha < 0$	L(L/R)RL
134LLR(2)	$T_{max} = 3t_3/4 + t_4/2 - t_1/4$	$\alpha > 0$	LLLR $x_4 > x_1$
134RLR(2)		$\alpha < 0$	RRRL $x_4 < x_1$

Table 1: (continued)

ID	**Meantimer formula**	**Segment direction**	**Semi-cell pattern**
	Layers 2-3-4		
234RLR 234LRL	$T_{max} = (t_2 + t_4)/2 + t_3$	all α	(L/R)RLR (L/R)LRL
234LRR 234RLL	$T_{max} = (t_2 - t_4)/2 + t_3$	$\alpha > 0$ $\alpha < 0$	(L/R)LRR (L/R)RLL
234LLR 234RRL	$T_{max} = (t_4 - t_2)/2 + t_3$	$\alpha > 0$ $\alpha < 0$	(L/R)LLR (L/R)234RRL

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