Instanton induced open string superpotentials and branes at singularities

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ABSTRACT: We study different aspects of the non-perturbative superpotentials induced by Euclidean $E3$-branes on systems of $D3/D7$-branes located at Abelian orbifold singularities. We discuss in detail how the induced couplings are consistent with the $U(1)$ symmetries carried by the $D3/D7$ branes. We construct different compact and non-compact examples, and show phenomenologically relevant couplings like μ-terms or certain Yukawa couplings generated by these E3 instantons. Some other novel effects are described. We show an example where $E3$ instantons combine with standard gauge instantons to yield new multi-instanton effects contributing to superpotential, along the lines of ref. [28]. In the case of non-SUSY Z_N tachyon-free singularities it is shown how $E3$-instantons give rise to non perturbative scalar couplings including exponentially suppressed scalar bilinears.

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Contents

1. Introduction

Euclidean brane instantons (see e.g. [1]) provide the leading non-perturbative contribution to several important quantities in string compactifications. For instance D-brane instantons in type II compactifications (or F/M-theory duals) have led to (in some cases very explicit) proposals to lift 4d moduli which are otherwise flat directions of the theory in the classical supergravity approximation. In addition, instantons arising from euclidean Dp-branes wrapped on $(p+1)$-cycles (henceforth Ep-branes), intersecting the 4d spacefilling D-branes yielding the gauge group, provide the leading contribution to perturbatively forbidden superpotential couplings of the relevant 4d effective gauge theory [2, 3] (see also [4 – 6] and [7 – 14] for related applications).

One of the main problems in improving the understanding and potential applications of these instantons lies in the difficulty in constructing explicit models where the instantons have the appropriate number of zero modes to lead to contributions to the non-perturbative superpotential. In particular, the non-vanishing of such contributions depends on the presence of non-trivial couplings for all fermion zero modes (charged or uncharged under the 4d gauge group) except the two 4d $N = 1$ Goldstinos. These couplings are difficult to evaluate in general Calabi-Yau compactifications, or even for exactly solvable but non free-field CFTs. Such couplings can however be explicitly discussed in compactifications on toroidal orientifolds (see e.g. [15, 16] for explicit examples), or for instantons from branes wrapped on compact cycles in systems of D-branes at singularities (see e.g. [6, 17–21]).

In this paper we focus on instanton effects from euclidean branes wrapped on non-compact cycles in local models of D3/D7-branes at singularities. More specifically we consider effects from E3-brane instantons wrapped on holomorphic 4-cycles passing though the singular point (and thus intersecting the 4d spacefilling D-brane system). Such instanton effects become physical when the local model is embedded in a full-fledged compactification, but many of its properties (and in particular the structure of 4d charged fields involved in the effective vertex they produce) depend crucially only on the local model (plus some mild assumption about behaviour at infinity). Note that E3-brane instantons may not be the only instanton effects in such global compactifications, but they are the most generic, in the following sense. For instance, E5-branes wrapped on the whole internal space, and carrying stable holomorphic world-volume gauge bundles, provide additional instantons. However, note that in general such instantons are BPS only at particular loci in moduli space, away from which they cross lines of marginal stability We thus stick to the simpler situation of E3-brane instantons, and to compact examples where they are BPS all over the moduli space.

We moreover concentrate on the case of abelian orbifold singularities, given the simple and powerful CFT description of the instanton zero modes and their interactions, and the relative ease to embed such models (at least for low order orbifolds) in toroidal orientifold compactifications. Similar ideas can be applied for other non-orbifold but toric singularities, using dimer diagram techniques, as we sketch in an appendix.

A further motivation to consider this kind of system is phenomenological. Indeed, one of the most attractive possibilities to embed the Standard Model in string theory is via systems of D-branes at singularities, since they naturally lead to world-volume chiral gauge theories. In fact, several realizations of semirealistic models have been proposed [22–27]. It is therefore a natural question to consider the structure of field theory operators that can be induced by instanton effects in this setup. As we explain in next section, the fact that the most promising singularities leading to semi-realistic models are not orientifold singularities implies that the only D-brane instantons contributing to the superpotential are (except for brane instantons with gauge theory interpretation) those wrapped on non-compact cycles passing through the singularity, and through orientifold planes away from the latter.

We provide the general formalism to study such effects for general supersymmetric abelian orbifold singularities, and illustrate it with a set of explicit examples lead-

ing to a rich pattern of physical phenomena (supersymmetry breaking, generation of mass terms,. . .) and interesting superpotential couplings forbidden in perturbation theory (quark and/or lepton Yukawa couplings, Higgs μ-terms, etc). In addition, we also describe possible effects of brane instantons in (non-tachyonic) non-supersymmetric orbifold models, which have not been described in the literature, and argue that they generate potentials for the 4d charged scalars.

There is another interesting aspect to this class of $E3$ instantons. It has been recently pointed out in [28] that instantons with additional neutral fermion zero modes may still contribute to a non-perturbative superpotential if the extra zero modes are lifted by another instanton. In particular we find that $E3$ instantons may combine with standard $E(-1)$ gauge instantons to induce novel superpotential couplings and provide an explicit compact $\mathbb{Z}_3$ orientifold example in which this phenomenon takes place.

The paper is organized as follows. In section 2 we describe the general features of E3-brane instantons for systems of D3/D7-branes at $\mathbb{C}^3/\mathbb{Z}_N$ local singularities, in particular we provide the explicit description of the structure of their charged and neutral zero modes. In section 3 we discuss the instanton action and its coupling to RR fields, and show the gauge invariance of the instanton effective vertex under gauge transformations of the 4d spacefilling gauge D-branes. In section 4 we provide explicit examples of non-compact and compact models and interesting instanton effects, including examples of phenomenologically interesting couplings in semi-realistic examples of [22]. We also describe there a realization of the new phenomenon of superpotential contributions from multi-instantons [28]. We also discuss the generation of Fayet SUSY breaking induced by $E3$ instantons. Section 5 describes the novel situation of instanton effects for systems of D-branes at (non-tachyonic) non-supersymmetric singularities. Section 6 contains our final remarks. Appendix A describes the computation of the 4d field theory on D3/D7-brane systems at orbifold singularities. In appendix B we present a compact $\mathbb{Z}_7$ toroidal orientifold example. Appendix C sketches the generalization of the instanton effects from E3-branes on 4-cycles to general toric singularities, based on dimer diagram techniques.

2. Euclidean E3-brane instantons at $\mathbb{C}^3/\mathbb{Z}_N$ singularities

2.1 Generalities

In this article we consider euclidean D-brane instanton effects on systems of $D3$- and $D7$-branes at $\mathbb{R}^6/\mathbb{Z}_N$ singularities. We summarize the basic formalism to compute the spectrum and interactions on the world-volume of $D3$- and $D7$- branes at Abelian singularities in the appendix (see [22] for more details and references). We will mostly concentrate on the case of supersymmetric singularities, and on euclidean instantons contributing to the non-perturbative superpotential.[1] Later on we will briefly discuss the case of euclidean instantons at $\mathcal{N} = 0$ non-supersymmetric singularities, and comment on the resulting non-perturbative interactions.

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[1]We denote Ep a $(p + 1)$-dimensional euclidean D-brane wrapped on a $(p + 1)$-cycle.

In order to contribute to the superpotential, one has to focus on BPS D-branes. Hence there are two classes of euclidean D-brane instantons which can contribute to the superpotential in these systems, $E(-1)$ and $E3$ Euclidean branes.[2] When such euclidean branes wrap the same cycle and have the same Chan-Paton transformation properties as some of the 4d-spacefilling branes in the background, they can be interpreted as gauge instantons. Otherwise they correspond to genuine stringy effects, without field theory interpretation, which we hence dub stringy instantons. We will be most interested in the latter.

$E(-1)$ instanton induced superpotentials have been discussed mostly for the case of the conifold or the orbifolded conifold in [6, 17, 18, 20, 21] (these can be regarded as bound states of E3-E1-E(-1) on compact cycles in the blownup limit). As emphasized in [18, 16, 8], both $E(-1)$ and $E3$ stringy instantons on Calabi-Yau compactifications generically have at least four neutral fermionic zero modes, and thus cannot contribute to the superpotential. The two extra fermion zero modes are Goldstinos of an accidental enhancement of $N = 1$ to $N = 2$ in the Ep-Ep open string sector. A simple way (and seemingly the only one in perturbative models) to reduce the number of universal zero modes to two is to consider instantons mapped to themselves under the orientifold action. For a $O(1)$ CP symmetry one obtains the required number of zero modes in that sector (clearly, there may be additional fermion zero modes in other sectors).

Focusing on systems of D-branes at singularities, for a $E(-1)$ instanton to induce a non-perturbative superpotential term involving charged chiral multiplets from the D-branes at the singularity, the $E(-1)$ must also sit at the singularity, which thus must also be fixed under the orientifold action (thus, it is an orientifolded singularity). This is the case considered e.g. in [6, 17, 21]. In orbifold language, the instanton corresponds to a fractional $E(-1)$-brane, with Chan-Paton phases not associated to any of the background D3-branes (i.e. corresponding to an unoccupied node in the quiver). Geometrically, it corresponds to Euclidean $E1$-brane wrapping a collapsed 2-cycle at the singularity on which no 4d-spacefilling brane wraps. These instantons give rise generically to genuine stringy effects.

Non-perturbative effects have been suggested to play a key role in semi-realistic string models of particle physics, in order to generate interesting couplings which are absent in perturbation theory, due to perturbatively exact global U(1) symmetries [2, 3, 8]. There exist systems of D3/D7-branes at singularities leading to such semi-realistic models [22 – 27]. It would be interesting to study the possible appearance of non-perturbative effects from stringy instantons in this class of models. However, such models are obtained for branes systems sitting at orbifold (not orientifold) singularities. Indeed, as argued in [22], models from $D3$-branes sitting at orientifold singularities suffer from a generic difficulty in yielding realistic spectra. The problem arises because the orientifold projection removes from the spectrum the diagonal U(1) which is always anomaly free and is crucial to obtain correct hypercharge assignments. This means that in the class of realistic models based on branes at singularities not fixed under the orientifold action, $E(-1)$ instantons can never give rise to non-perturbative superpotentials involving phenomenologically relevant chiral

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[2]Here we refer to the dimension of the brane as described in the parent space. Since we work with fractional branes, in the quotient space they in general correspond, in the geometric large volume limit, to E3/E1/E(−1) bound states.

fields like those of quarks and leptons.

This caveat is nicely avoided by E3-brane instantons, wrapping a 4-cycle passing through the singularity. The E3-brane can be mapped to itself by an orientifold action leading to orientifold planes away from the relevant singularity. The orientifold will thus map the relevant singularity to a mirror image singularity. The net effect is that the system at the singularity is insensitive to the orientifold action and can reproduce one of the semi-realistic models of [22–24, 27]. The instanton is fixed under the orientifold action, and can have $O(1)$ CP symmetry and lead to just two fermion zero modes. Since it also intersects the D-branes at the singularity, the corresponding superpotential can lead to interesting SM operators.

From the phenomenological perspective this is one of the motivations to study the effects of $E3$ rather than $E(-1)$ instantons. Nevertheless, we will keep an open mind and consider examples of non-perturbative effects on systems of D-branes at orientifold singularities as well, even though they are less promising from the model building point of view.

It is interesting to consider the above kind of configurations from the viewpoint of the local physics near the (non-orientifold) singularity. We have a D3/D7-brane system at a non-orientifold singularity, and an euclidean E3-instanton wrapped on a non-compact 4-cycle. On the E3-E3 open string spectrum, there are four fermion fields from the accidental $N = 2$ susy in this sector. However, these fields propagate on the non-compact 4-cycle of the instanton. The existence or not of fermion zero modes for the spacetime instanton is determined, from the local model viewpoint, by the boundary conditions at infinity for these fields. For E3-branes with $O(1)$ CP symmetry, the boundary condition for two of the fermion fields is that they vanish at infinity, and thus removes their corresponding zero mode.

Given their simplicity and their general applicability, it is thus convenient to study first the local models. As is clear form the above, one should nevertheless be careful with the discussion of modes supported on non-compact cycles, since the corresponding zero mode spectrum is sensitive to boundary conditions at infinity.

We thus concentrate on the effects coming from $E3$ instantons (see also [15, 16]). We are going to consider euclidean 3-branes $E3^r$ located at a $\mathbb{R}^6/\mathbb{Z}_N$ singularity and wrapping a 4-cycle Σ_4^r transverse to the D3-branes. Our analysis here will be local and we will assume that eventually the singularity is embedded into a compact manifold so that the action of the instantons is finite and the $D7^s$ branes give rise to physical gauge bosons and not merely to a flavour symmetry. We also assume that the E3 branes do not touch further branes which could give rise to extra zero modes to be included in the analysis. Compact examples with these characteristics will be provided later on.

We consider instantons $E3^r$, r=1,2,3 which wrap a 4-cycle transverse to the rth complex plane (thus defined by $z_r = 0$). The CP factors for these $E3^r$ will be of the form (see the appendix for notation)

$$\gamma_{\theta,E3^r} = \mathrm{diag}\,(\mathrm{I}_{v_0^r}, e^{2\pi i/N}\mathrm{I}_{v_1^r}, \dots, e^{2\pi i(N-1)/N}\mathrm{I}_{v_{N-1}^r}) \qquad (2.1)$$

up to an overall phase which depends on the existence or not of vector structure [29]. In order to get the correct number of neutral zero modes for the instantons, eventually
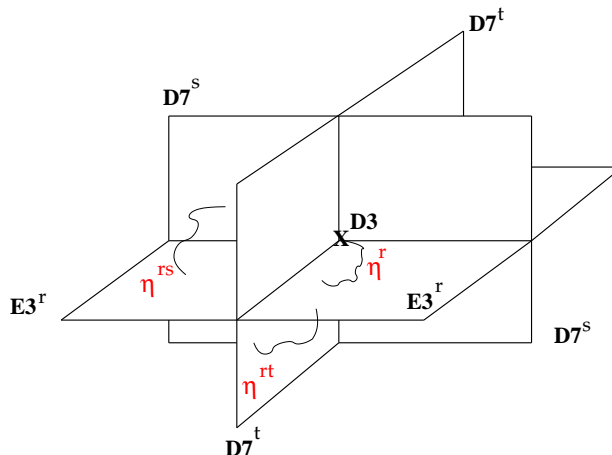
Figure 1: Pictorial view of the instanton $E3^r$ and $D7^{s,t}$-branes going through a singularity where a stack of $D3$-branes are located. The $E3^r - D3$ and $E3^r - D7^{s,t}$ fermionic zero modes (η^r and η^{rs}, η^{rt} respectively) are also shown.

we will be interested in an orientifold projection giving rise to an $O(1)$ CP symmetry. Since orientifolds act by conjugating the Chan-Paton phases (see [30, 31] for a few $\mathbb{C}^2/\mathbb{Z}_N$ exceptional cases where it does not) this is obtained for $v_0^r = 1$, $v_i^r = 0$ with $i \neq 0$. This already implies that appropriate instantons exist only in models with vector structure, where the overall phase mentioned above is unity and the Chan-Paton matrix is exactly as in (2.1). This already excludes a large class of models, in particular most of the even order orbifolds (compact even order orbifolds are difficult to construct due to the very stringent RR tadpole cancellation conditions [32], see however [33, 34]).

It is easy to show that for orbifolds acting on all three complex planes (i.e. not of the form $\mathbb{C}^2/\mathbb{Z}_N \times \mathbb{C}$, the E3-E3 open string sector reduces to a universal sector. It always contains four bosonic zero modes, corresponding to the 4d translational Goldstones associated to the instanton position in 4d Minkowski space. The fermionic zero mode content depends on the orientifold action on the instanton, ss discussed in [8]: for U(1) instantons, there are 4 fermion zero modes; for $O(1)$ instantons, there are 2 fermion zero modes; for Sp(2) instantons, there are 2 singlet fermion zero modes, and 6 fermion zero modes transforming as two triplets. Since the possible mechanisms to lift the additional zero modes in the U(1), Sp(2) cases require ingredients beyond those in our configurations, we will focus on the case of $O(1)$ instantons.

Let us now discuss in turn the different types of charged fermionic zero modes which will appear in a configuration with both D3- and $D7^s$-branes. For the CP matrices of these branes we use the notation defined in the appendix. A pictorial view of the zero modes is given in figure 1.

2.2 E3-D3 instanton zero modes

This sector is fully localized in the non-compact orbifold dimensions, so it is insensitive to boundary conditions at infinity.

Open strings in the $E3^r$-$D3$ sector give rise to no bosonic zero modes. This is due to the fact that there are altogether 8 space-time components with mixed DN boundary conditions, which lift up the zero energy of the bosonic states. On the other hand there is a fermionic state from the R-state in the complex direction with DD boundary conditions. The gauge quantum numbers and multiplicities of these fermions is totally analogous to the ones of the $D7^r$-$D3$ states (see eq. (A.5) in the appendix), i.e.

D3 − E3$_\mathbf{r}$, E3$_\mathbf{r}$ − D3 Fermions
$$
\begin{aligned}
\sum_{i=0}^{N-1}\left[\,(n_i,\overline{v}^r{}_{i-\frac{1}{2}a_r})+(v_i^r,\overline{n}_{i-\frac{1}{2}a_r})\,\right] & \quad a_r \text{ even}\\
\sum_{i=0}^{N-1}\left[\,(n_i,\overline{v}^r{}_{i-\frac{1}{2}(a_r+1)})+(v_i^r,\overline{n}_{i-\frac{1}{2}(a_r+1)})\,\right] & \quad a_r \text{ odd}
\end{aligned}
\tag{2.2}
$$

where again r denotes the complex plane transverse to the $E3^r$ instanton and a_r is defined in appendix A. There will be couplings between two such fermionic zero modes η^{3-E3_r}, η^{E3_r-3} and $D3 - D3$ chiral superfields of the form (see eq. (A.6))

$$
\sum_{i=0}^{N-1} \mathrm{Tr}\left(\Phi_{i,i+a_r}^r \quad \eta_{i+a_r,i+\frac{a_r}{2}}^{3-E3_r} \quad \eta_{i+\frac{a_r}{2},i}^{E3_r-3}\right)\ .
\tag{2.3}
$$

2.3 E3-D7 instanton zero modes

If there are $D7^s$ branes which are passing through the singularity the $E3^r$ instantons will necessarily intersect them and there will generically be further instanton fermionic zero modes. The structure of the intersection and of the possible zero mode depends on the 4-cycles wrapped by the E3- and the D7-branes. We are going to consider here $r \neq s$ where the intersection is on the complex plane transverse to the directions r, s (namely $z_r = z_s = 0$) and we briefly discuss the $r = s$ case below. The worldvolumes are non-compact, hence the actual existence of zero modes depends on boundary conditions at infinity. Equivalently, if we consider that the singularity is eventually embedded into a CY manifold, the instanton and D7-branes have finite volume on the CY, and the E3-D7 spectrum depends on the details of the compactification. To give a concrete example, one may consider that in a global toroidal embedding of the model there may be Wilson lines on the D7-brane along the relevant 2-torus, such that certain of these E3-D7 zero modes are projected out. Likewise, the E3 instanton may have also Wilson lines which may project out some E3-D7 states. Note that in the case of of $O(1)$ instantons one can still have a discrete Z_2 Wilson line, which automatically removes all E3-D7 zero modes.

In studying the local model, we consider this not to be the case, and will abuse language by denoting the E3-D7 fields as zero modes. Thus we will need to remove these fields if necessary in concrete examples which involve ingredient projecting out such zero modes (like the Wilson lines mentioned above).

The $E3^r - D7^s$ sector has 8 space-time dimensions with DN boundary conditions. Thus there are no bosonic zero modes. Since there is a twisted complex plane with NN boundary conditions, the multiplicities and quantum numbers of the fermion zero modes is analogous to that of $D7^r - D7^s$ systems. One thus gets fermionic zero modes given by:

D7$^\mathbf{s}$ − E3$_\mathbf{r}$, E3$_\mathbf{r}$ − D7$^\mathbf{s}$ Fermion
$$
\begin{aligned}
\sum_{i=0}^{N-1}\left[\,(u_i^s,\overline{v}^r{}_{i-\frac{1}{2}a_t})+(v_i^r,\overline{u}^s{}_{i-\frac{1}{2}a_t})\,\right] & \quad a_t \text{ even}\\
\sum_{i=0}^{N-1}\left[\,(u_i^s,\overline{v}^r{}_{i-\frac{1}{2}(a_t+1)})+(v_i^r,\overline{u}^s{}_{i-\frac{1}{2}(a_t+1)})\,\right] & \quad a_t \text{ odd}
\end{aligned}
\tag{2.4}
$$

with $t \neq s \neq r \neq t$. We will denote them η^{rs}, η^{sr} with two superindices indicating they come from the overlap of $D7^s$ and $E3^r$ branes.

These $E3^r - D7^s$ fermionic zero modes have three types of couplings to chiral fields on 33 and 37 sectors:

- $(E3^r - D3)(D3 - D7^s)(D7^s - E3^r)$

  This is a coupling between a chiral superfield in a $D3 - D7^s$ sector to two fermionic zero modes from $E3^r - D3$ and $D7^s - E3^r$ respectively.

$$\sum_{r \neq s} \eta^r \ \Phi^{(37_s)} \ \eta^{rs} \tag{2.5}$$

- $(E3^r - D7^s)(D7^s - D7^s)(D7^s - E3^r)$

  Let us assume that the fields in the $D7^s$-$D7^s$ sector, which propagate on the non-compact z_s 2-plane, have boundary conditions leading to zero modes (we implicitly make this assumption in forthcoming similar analysis). Then there is a coupling between a chiral superfield in a $D7^s - D7^s$ sector to two fermionic zero modes from $E3^r - D7^s$ and $D7^s - E3^r$ respectively.

$$\epsilon^{rst} \ \eta^{rs} \ \Phi_t^{(77_s)} \ \eta^{sr} \tag{2.6}$$

- $(E3^r - D7^s)(D7^s - D7^t)(D7^t - E3^r)$

  This is a variation of the previous one but with η^{rs} zero modes coupling chiral fields in a mixed $D7^s - D7^t$ sector.

$$\eta^{rt} \ \Phi^{(7_t 7_s)} \ \eta^{sr} \tag{2.7}$$

with $r \neq s \neq t \neq r$.

Note that in the case of the last two couplings, a vev for the chiral fields $\Phi_t^{(77_s)}$ and/or $\Phi^{(7_t 7_s)}$ would give mass to $E3^r - D7^s$ fermion zero modes. In such a case one has to integrate out appropriately those fermionic zero modes to obtain the correct effective instanton action. Such vevs may be triggered in the presence of non-vanishing FI-terms for $U(1)$'s living on the worldvolume of D7-branes (in fact this is equivalent to the appearance of an insertion of the corresponding field in the effective 4d superpotential, taking a constant value if the field acquires a vev).

Similar analysis can be carried out for $r = s$ for different Chan-Paton actions for E3- and D7-branes, namely for E3- and D7-branes which wrap the same 4-cycle, but carry different world-volume gauge bundles. In that case one can check there are again no massless bosonic zero modes but three copies of fermionic zero modes $(E3^r - D7^r)_i$, $i = 1, 2, 3$. They give rise to couplings to $(D7^r - D7^r)_j$ as well as $(D3 - D7^r)$ $D = 4$ fields analogous to those just discussed. We will not discuss them in more detail here. Finally, as we said, the case of overlapping E3- and D7-branes with same Chan-Paton action correspond to brane instantons with interpretation as standard gauge theory instantons.

3. Action and charges of the instantons

The action of these instantons has two pieces corresponding to a global piece which depends on the 4-cycle $\Sigma 4^r$ wrapped by the $E3^r$ (which depends on untwisted Kahler moduli T_i) and a local piece depending on the twisted moduli ϕ_k at the singularity. Thus the classical action of the instanton should have the two pieces

$$S_{E3^r} \;=\; S_{E3^r}^{unt} \;+\; S_{E3^r}^{\text{twist}} \;=\; T_{\Sigma_4}^r \;+\; \sum_{k=0}^{N-1} d_k^r \, \phi_k \tag{3.1}$$

Here T^r will be some linear combination of untwisted Kahler moduli characteristic of the cycle of the $E3^r$ in the bulk. Its real part is controlled by the volume of the wrapped toroidal 4-cycle. The ϕ_k are closed string twisted moduli associated to the singularity. In the orbifold configuration, the background values of the twisted moduli are zero, $\phi_k = 0$, so the instanton amplitude is controlled by ReT^r. The coefficients d_k^r will be computed below. As we have just seen, there are additional pieces in the action coming from couplings among bifundamental 33 chiral fields $\Phi_r^{\alpha\beta}$ and the $E3^r - D3$ zero modes η_i^α, η_j^β of the general form

$$S'_{E3^r} \;=\; \sum_{i,j,r} c_{ij}^r \, \eta_i^\alpha \Phi_r^{\alpha\beta} \eta_j^\beta \tag{3.2}$$

Integration over the fermionic zero modes gives rise to a non-perturbative superpotential

$$e^{-S_{E3r}} \int [d\eta^\alpha][d\eta^\beta] \; e^{-\sum_{i,j,r} c_{ij}^r \, \eta_i^\alpha \Phi_r^{\alpha\beta} \eta_j^\beta} \;\propto\; e^{-S_{E3r}} \; det(\Phi_r) \quad . \tag{3.3}$$

Let us now check that this induced operator is invariant under the gauged $U(1)$ symmetries on the 4d spacefilling D3-branes (Note that these arguments are valid even if the instanton has additional zero modes, and thus leads to a higher F-term in the 4d effective action).[3]

As shown in [35] it is only the twisted moduli which are shifted under the $U(1)$ gauge symmetries living on the D3-,D7-branes. In particular consider a stack of $D3$-branes living at a $\mathbb{R}^6/\mathbb{Z}_N$ singularity. Consider the $U(1)_a$ group associated to one of the $U(n_a)$ factors with CP matrix λ_a. Following the rules in [36], it was shown in [35] that there are couplings

$$\sum_{k=0}^{N-1} \sqrt{c_k} \; Tr(\, \gamma_{\theta^k}^{D3} \, \lambda_a) \times (F_a \wedge B_k) \tag{3.4}$$

where B_k are RR 2-forms in the kth twisted sector, thus associated to the singularity. The twist CP matrix $\gamma_{\theta^k}^{D3}$ is defined in Appendix A. The B_k are Poincare dual to scalars $b_k = Im\phi_k$ which then transform under a $U(1)_a$ gauge transformation of parameter $\Lambda(x)_a$ like

$$b_k \;\longrightarrow\; b_k \;+\; \sqrt{c_k} \, Tr(\, \gamma_{\theta^k}^{D3} \, \lambda_a) \, \Lambda(x)_a \tag{3.5}$$

---

[3]This kind of computation is a particular case of the general argument in the appendix of [3], which applied to general systems of D-branes and instantons. From this viewpoint, the discussion below amounts to a careful computation of the couplings of the branes to the RR fields in the orbifold CFT.

The value of the c_k coefficients is given below. Now, the twisted piece of the action of the $E3^r$ instanton may be obtained from the corresponding DBI+CS action. In particular the coupling to the imaginary part of the twisted field is topological, and may be inferred from the known couplings of the $F \wedge F$ term of a $D7^r$ brane to the twisted RR fields b_k. This is because a 4d $F \wedge F$ background on a $D7^r$-brane must carry the same topological couplings as an $E3^r$-brane. Notice however that the D7-brane is in general not present in the configuration, and we merely use it as a trick to obtain the couplings, which can be computed by other techniques. The couplings for the D7-branes were also found in [35] to be given by

$$\frac{1}{N} \sum_{k=0}^{N-1} \sqrt{c_k} \, Tr(\, (\gamma_{\theta^k}^{D7^r})^{-1} \, \lambda_i^2) \, b_k \, \times (F_i \wedge F_i) \tag{3.6}$$

Including also the real part of ϕ_k, mentioned above, one then has for the $E3^r$ instanton action

$$S_{E3^r} \; = \; T_{\Sigma_4}^r \; + \; \frac{1}{N} \sum_{k=0}^{N-1} \sqrt{c_k} \, Tr(\, (\gamma_{\theta^k}^{E3^r})^{-1} \, (\lambda_b)^2) \, \phi_k \tag{3.7}$$

with λ_b a CP matrix of the $E3^r$. One can then compute how the instanton action transforms under a $U(1)_a$ symmetry. For a $E3^r$ instanton transverse to the r-th complex plane one has [35]

$$c_k \; = \; 2 \, sin(\pi k a_r/N) \; = \; -i \, (\, \alpha^{\frac{k a_r}{2}} \, - \, \alpha^{-\frac{k a_r}{2}}) \tag{3.8}$$

with $\alpha = exp(i2\pi/N)$. The coefficients $\sqrt{c_k}$ of the coupling of the D-branes to the RR fields can be obtained by factorization of a cylinder diagram [37].

With the CP γ_{θ^k} matrices defined above one then obtains:

$$S_{E3^r} \; \longrightarrow \; S_{E3^r} \; + \; i\frac{n_a v_b}{N} \sum_{k=0}^{N-1} c_k \, \alpha^{ak} \, \alpha^{-bk} \, \Lambda(x)_a \tag{3.9}$$

$$= \; S_{E3^r} \; + \; \frac{n_a v_b}{N} \sum_{k=0}^{N-1} (\, \alpha^{k(\frac{a_r}{2}+a-b)} \, - \, \alpha^{k(-\frac{a_r}{2}+a-b)} \,) \, \Lambda(x)_a \tag{3.10}$$

$$= \; S_{E3^r} \; + \; n_a v_b \, (\delta_{b,a+\frac{a_r}{2}} \, - \, \delta_{b,a-\frac{a_r}{2}}) \, \Lambda(x)_a \tag{3.11}$$

Given the above noted relation between E3^r-branes and D7^r-brane instantons, ie 4d $F \wedge F$ backgrounds on D7-branes, this computation is quite analogous to the way the Green-Schwarz mixed anomaly cancellation takes place between a $U(1)$ from $D3$ branes and gauge groups from $D7$'s. Therefore, the appearance of the Kronecker deltas, which count the number of multiplets in the mixed D7/E3-D3 sector, is not a surprise.

One can now easily check that the charge transformation obtained is just opposite to the total $U(1)_a$ charge of $E3^r - D3$ fermionic zero modes transforming like (we are taking a_r even):

$$(n_{b+\frac{a_r}{2}}, \bar{v}_b) \; + \; (v_b, \bar{n}_{b-\frac{a_r}{2}}) \; . \tag{3.12}$$

This implies that the complete instanton effective vertex (3.3), which includes the exponential term and the insertions of 4d fields due to the integration of charged fermion zero modes, is indeed gauge invariant.

If the $E3^r$ instanton intersects some $D7^s$ brane, in general its action will also transform under U(1)'s living on the $D7^s$ branes. In general a U(1)$_c$ gauge symmetry inside a $D7^s$ brane will couple to twisted moduli like

$$\sum_{k=0}^{N-1} \sqrt{c_k} \, Tr(\, \gamma_{\theta^k}^{D7^s} \, \lambda_c) \times (F_c \wedge B_k) \tag{3.13}$$

We are considering $s \neq r$ (for $r = s$ a similar discussion can be carried out) and now the c_k factor will be given by $c_k = 2 \sin(\pi k a_t / N)$ with $t \neq r \neq s \neq t$. Thus z_t is the complex direction with NN boundary conditions in the $E3^r - D7^s$ system. Now the RR twisted fields transform with respect to a $D7^s$ U(1)$_c$ like

$$b_k \longrightarrow b_k + \sqrt{c_k} \, Tr(\, \gamma_{\theta^k}^{D7^s} \, \lambda_c) \, \Lambda(x)_c \tag{3.14}$$

and c_k is as indicated above. One can again repeat the analysis we made for $D3$-branes and find that under a U(1)$_c$ gauge transformation the action transforms like

$$S_{E3^r} \longrightarrow S_{E3^r} + u_c v_b \, (\delta_{b,c+\frac{a_t}{2}} - \delta_{b,c-\frac{a_t}{2}}) \, \Lambda(x)_c \ . \tag{3.15}$$

One can then check that this transformation corresponds to the opposite of the overall U(1)$_c$ charge of $E3^r - D7^s$ fermionic zero modes transforming like:

$$(u_{c+\frac{a_t}{2}}, \bar{v}_c) + (v_c, \bar{u}_{c-\frac{a_t}{2}}) \tag{3.16}$$

which we already discussed are indeed present, see (2.4). Thus again one recovers a fully gauge invariant operator. In the presence of $D7^s$ branes though the induced superpotentials will involve chiral fields from all 33, 37 and 77 sectors, as we will see in the specific examples.

Note that the superpotential will in general be induced only if the number of universal neutral fermion zero modes of the $E3^r$ instanton is two, providing us for the superspace measure. This is guaranteed if there is in addition an orientifold projection and the instanton has an $O(1)$ CP factor. In this connection note that all the above expressions were obtained for the case of branes at orbifold (not orientifold) singularities. The same expressions may be used for the orientifold case simply recalling the mapping between branes with α^k and α^{N-k} CP factors upon the orientifold action.

4. Applications

4.1 A local $SU(3)_c \times SU(3)_L \times SU(3)_R$ model from $D3$-branes at a $\mathbb{Z}_3$ singularity

Consider the simplest case, in which no $D7^s$ branes are present and we just have a stack of $D3$ branes at a singularity. Then twisted RR tadpole conditions dictate (if the $D3$'s are away from orientifold planes) that $Tr\gamma_{\theta^k,3} = 0$ (for all twists θ^k with the origin as only fixed point). The gauge group has then the structure $\Pi_{i=0}^{N-1} U(n)$ with Nn the total number of $D3$'s. A phenomenologically interesting example is the case with $N = n = 3$ in which we sit at a Z_3 singularity and the gauge group is $U(3)_c \times U(3)_L \times U(3)_R$. This structure contains the SM gauge group and it has been termed 'trinification' in the unified

model-building literature [38]. There are three generations with chiral multiplets from the 33 sector transforming like

$$3(3, \bar{3}, 1) \; + \; 3(\bar{3}, 1, 3) \; + \; 3(1, 3, \bar{3}) \tag{4.1}$$

These three multiplets (in 3 copies) contain respectively the left-handed quarks, right-handed quarks and the lepton/Higgs fields (plus additional vectorlike leptons). Now, the perturbative superpotential is given by

$$W \; = \; \sum_{r,s,t=1}^{3} \; \epsilon_{rst} \; (3, \bar{3}, 1)^r (\bar{3}, 1, 3)^s (1, 3, \bar{3})^t \; . \tag{4.2}$$

Note that, with the Higgs multiplets inside $(1, 3, \bar{3})$, this includes some Yukawa couplings for the quarks. However there are no lepton Yukawa couplings[4] since they would require the presence of $(1, 3, \bar{3})^3$ couplings, which are forbidden by the $U(1)_L \times U(1)_R$ gauge symmetry.

Let us assume that there is a $E3^r$ which has $O(1)$ CP symmetry and goes through this Z_3 singularity. It will have its CP factor =1. The $D3$ twist CP matrix will be

$$\gamma_{\theta,3} = \mathrm{diag}\,(\mathrm{I}_3, \alpha \mathrm{I}_3, \alpha^2 \mathrm{I}_3) \tag{4.3}$$

Take for definiteness $a_r = -2$ then there are $E3^r - D3$ zero modes transforming like

$$\eta^r \; = \; (1, 1, 3) \; ; \; \bar{\eta}^r \; = \; (1, \bar{3}, 1) \tag{4.4}$$

which have couplings to the r-th lepton chiral field

$$\eta^r \; (1, 3, \bar{3})^r \; \bar{\eta}^r \; . \tag{4.5}$$

Upon integration of these charged zero modes a superpotential coupling

$$W_{\mathrm{leptons}}^r \; = \; e^{-S_{E3^r}} \; \epsilon_{abc} \epsilon_{def} \; (1, 3^a, \bar{3}^d)^r (1, 3^b, \bar{3}^e)^r (1, 3^c, \bar{3}^f)^r \tag{4.6}$$

is obtained for the r-th generation of leptons. From eq. (3.11) one obtains

$$S_{E3^r} \; \longrightarrow \; S_{E3^r} \; + \; 3\Lambda_{U(1)_L} \; - \; 3\Lambda_{U(1)_R} \tag{4.7}$$

so that indeed the operator is fully gauge invariant. Instantons $E3^s$ transverse to the other two complex planes would give rise to leptonic Yukawa couplings for the other two generations. This is a simple example of how this class of instantons may give rise to superpotential couplings of phenomenological interest. Note that these couplings are presumably suppressed with respect to the quark one, but need not be negligibly small. Indeed, the SM gauge couplings in this model are given by the inverse of the real part of the 4-dimensional dilaton field S (plus a twisted moduli piece analogous to the second term in eq. (3.7)). These are totally independent from the instanton action (3.7) which is rather controlled by a combination of untwisted Kahler moduli T^r which may be relatively small without any phenomenological constraint dictated by the observed values of the SM couplings.

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[4]The absence of some perturbative either quark or lepton Yukawa couplings is a quite general property in semirealistic models of branes at singularities. See the left-right symmetric example in section 4.5.

4.2 A global tadpole free GUT-like example

Our previous example was a local model from $D3$-branes at a singularity. We would like now to show in a simple example how the generation of open string superpotentials can take place in simple, globally consistent (RR tadpole free) compact models. We will take the simplest compact Type IIB 4-D orientifold which one can build, a Z_3 orientifold with O3-planes [39, 40] (for a discussion of D-brane instantons in the $\mathbb{Z}_3$ orientifold, with a different distribution of D-branes, see [15]). Similar effects should be found at more complicated toroidal orientifolds.

Let us consider type IIB on the $T^6/\mathbb{Z}_3$ orbifold, modded out by the orientifold action $\Omega(-1)^{F_L} R_1 R_2 R_3$, with R_i being a reflection on the ith plane. There are 64 orientifold three-planes (O3-planes), which are localized at points in the internal space. To cancel their untwisted RR charge we need a total of 32 D3 branes. There are also 27 orbifold fixed points which may be labeled by integers (m, n, p), $m, n, p = 0, \pm 1$. Among these 27 points, only the origin $(0, 0, 0)$ is fixed under the orientifold action, hence it is an orientifold singularity. The cancellation of tadpoles at this point requires

$$3 \, \text{Tr} \, \gamma_{\theta,3} + (\text{Tr} \, \gamma_{\theta,7} - \text{Tr} \, \gamma_{\theta,\bar{7}}) = -12 \tag{4.8}$$

In our case with no $D7$-branes present the condition is $\text{Tr} \, \gamma_{\theta,3} = -4$. An SU(6) GUT model may be constructed in the following way. We can locate 14 $D3$-branes at the orientifold plane at the origin with CP twist matrix

$$\gamma_{\theta,3} = \text{diag} \, (\alpha I_6, \alpha^2 I_6, I_2) \tag{4.9}$$

and the remaining 18 $D3$ e.g. in the bulk (e.g. in 3 orbifold/orientifold invariant groups of 6 $D3$ branes), away from the origin in any of the tori. The orientifold operation exchanges D3-branes with CP factors α and α^2. The gauge group is $U(6) \times O(2)$ with chiral fermion content

$$3(\overline{15}, 0) \;+\; 3(6, +1) \;+\; 3(6, -1) \; . \tag{4.10}$$

These representations decompose as $\overline{15} = \overline{10} + \overline{5}$ and $6 = 5 + 1$ under the SU(5) subgroup of SU(6), so the model contains three standard SU(5) generations $\overline{10} + 5$ and three sets of $5 + \overline{5}$ Higgs fields. With the usual SM embedding in SU(5), the model has lepton and D-quark Yukawa couplings from quiver couplings of the form $(\overline{15}, 0)(6, 1)(6, -1)$. However U-quark Yukawa couplings would be contained in $\overline{15}\,\overline{15}\,\overline{15}$ couplings which are perturbatively forbidden by the $U(1)$ symmetry.

Now these compact orientifolds admit BPS euclidean branes $E3^r$ which wrap two 2-tori and are transverse to the r-th torus. If they sit at the origin the projection will be such that, if $\gamma_{\theta, E3^r} = 1$, the CP symmetry will be $O(1)$ and hence the number of neutral instanton zero modes will be adequate to create a superpotential.[5] There is one multiplet

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[5]These $E3^r$ instantons will in general contain in their worldvolume other orbifold fixed points without D3-branes. This means that the instanton action will also contain pieces involving these other twisted fields. However the latter are inert under the $U(1)$ transformations of the D3-branes at the orientifold plane and hence those extra pieces do not play any role in our discussion.

of charged fermion zero modes for each $E3^r$

$$\eta_a^r \;=\; (6_a, 0)^r \tag{4.11}$$

coupling to the antisymmetric chiral fields $\Phi^r = \overline{15}^r$ like

$$\eta_a^r \eta_b^r \Phi_{ab}^r \quad . \tag{4.12}$$

Upon integrating over zero modes a cubic superpotential is generated

$$W_6 \;=\; \sum_r \; e^{-S_{E3^r}} \; \epsilon^{abcdef} \Phi_{ab}^r \Phi_{cd}^r \Phi_{ef}^r \tag{4.13}$$

which includes the U-quark Yukawas which were perturbatively absent. The instanton action transforms like

$$S_{E3^r} \;\longrightarrow\; S_{E3^r} \;-\; 6\Lambda_{U(1)_3} \tag{4.14}$$

so that again the operator is fully gauge invariant.

Note that the size of these Yukawas will depend on the corresponding action which has the form previously discussed (3.7) with now the T^r being actually the untwisted Kahler moduli of this orientifold. Thus the actual values of the couplings is sensitive to the overall sizes of the different dimensions.

This GUT model is not fully realistic since as it stands it lacks the required Higgs multiplets to do the breaking down to the SM. Still it exemplifies in a global tadpole free model how instantons may give rise to phenomenologically interesting couplings. In the context of the compact Z_3 orientifold the generation of such terms was recently pointed out in [16]. Instanton induced Yukawa couplings in an SU(5) model from a local intersecting $D6$-brane configuration were also considered recently in [13].

Other globally consistent compact orientifold models are expected to present the same type of instanton induced couplings. As an additional example we discuss the case of the $\mathbb{Z}_7$ orientifold model in an appendix.

4.3 An example with multi-instanton superpotential

Let us consider the same $\mathbb{Z}_3$ toroidal orientifold, but with a different distribution of D3-branes. It illustrates a phenomenon in [28], in which instantons with additional fermion zero modes (beyond the two required Goldstinos) can still contribute to the superpotential, if the extra zero modes are lifted (or soaked up) by another instanton. We dub this phenomenon "instanton symbiosis".

Consider the same $T^6/\mathbb{Z}_3$ orientifold as above, and introduce a set of D3-branes at the origin, with Chan-Paton orbifold action $\gamma_\theta = \mathrm{diag}\,(e^{2\pi i/3}I_4, e^{4\pi i/3}I_4)$. This leads to a U(4) gauge theory with three chiral multiplets in the two-index antisymmetric representation, and no superpotential. The remaining 24 D3-branes can be located as four stacks of bulk D3-branes, and are irrelevant to our discussion.

Let us focus on the possible non-perturbative effects for the local configuration at the $\mathbb{Z}_3$ orientifold singularity a the origin. There is an $O(1)$ instanton, corresponding to an euclidean D-brane filling the unoccupied node in the quiver (that is a fractional

E(−1)-brane with Chan-Paton orbifold action $\gamma_{\theta,E(-1)}$. It has two neutral fermion zero modes, and charged fermion zero modes (in 3 copies) in the fundamental of U(4), with cubic couplings to the 4d chiral multiplets in the **6**. It gives rise to a non-perturbative superpotential of very high order in the 4d chiral multiplets, which will not interest us. In addition, there is a U(1) instanton, arising from E(−1)-branes with Chan-Paton action $\gamma_{\theta,E(-1)} = \mathrm{diag}\,(e^{2\pi i/3}, e^{4\pi i/3})$. It corresponds to a gauge theory instanton, so its effects are more suitably analyzed using field theory arguments. Forgetting about the U(1), which is massive by BF couplings, the 4d gauge theory can be regarded (since SO(6) $\simeq$ SU(4))as an SO(N_c) theory with $N_f = N_c - 3$ flavors in the vector representation [41], for $N_c = 6$, hence $N_f = 3$. The case $N_f = N_c - 3$ is somewhat analogous to the case $N_f = N_c + 1$ for SU(N_c) SQCD. In particular, instantons have extra zero modes beyond the two Goldstinos, which are not lifted. These instantons do not generate a superpotential term, but rather induce higher order F-terms, as in [42].

In addition there may be effects from $O(1)$ instantons described by euclidean E3$_r$-branes. These are similar to those considered in the previous section, and lead to non-perturbative mass terms $m_r \sim M_s\, e^{-T_r}$ for the chiral multiplets in the **6**. This model and the above discussion have already appeared in [15].

In the following we would like to have a closer look at the effects of the E3$_r$, and argue that they have a non-trivial effect on the gauge instantons and implement the non-perturbative lifting of zero modes advocated in [28]. This example thus illustrates another interesting application of E3$_r$-brane instantons, and shows that the effects in [28] naturally arise already in familiar toroidal orientifold models.

From the spacetime viewpoint, the mass terms induced by the E3-brane instantons have a non-trivial effect on the dynamics of the SU(4) gauge theory. Indeed in the far infrared one can integrate out the massive flavours and be left with a pure SU(4) theory, which has the familiar non-perturbative gaugino condensate superpotential

$$W \sim \Lambda'^3 \qquad (4.15)$$

where the pure SYM scale Λ' in the IR is related to the UV scale Λ by matching of scales $\Lambda^9 \prod_r m_r = \Lambda'^{12}$. In our case the UV scale is $\Lambda = M_s e^{-1/(9 g_Y M^2)} = M_s e^{-S/9}$, where the factor of 9 arises from the beta function (proportional to $3(N_c - 2) - N_f$), and where S is the modulus giving the gauge coupling of the U(4) theory at high scales (essentially the 4d dilaton, plus corrections from twisted moduli). Hence we have

$$W \sim \left(e^{-S} \prod_r e^{-T_r} \right)^{\frac{1}{4}} M_s^3 \qquad (4.16)$$

The above argument shows that there is a non-trivial effect of the E3$_r$ instantons on the E(−1)-brane gauge instantons, so that the latter can induce a superpotential. In fact, this is a particular case of the non-perturbative lifting of fermion zero modes in [28], where the E3-brane instantons induce a lifting of the additional fermion zero modes of the E(−1)-brane instantons. The overall process corresponds to a multi-instanton process in spacetime, where all fermionic external legs of the E(−1)-brane instanton vertex, except

two, are soaked up by the E3$_r$-brane instanton vertex. The interpretation as a multi-instanton process agrees nicely with the exponential dependence of (4.16) on the moduli.

Let us sketch a simplified version of the mechanism. In the simultaneous presence of the instantons, there are fermion zero modes $\lambda_r, \tilde{\lambda}_r$ in the E3$_r$-E(-1) sector. These couple to different pairs of the extra fermion zero modes in the E(-1)-D3 sectors, which we denote $\chi_r, \tilde{\chi}_r$ via a quartic interaction

$$S_{\text{inst}} = \sum_r \lambda_r \tilde{\lambda}_r \chi \tilde{\chi}_r \tag{4.17}$$

Notice that the r subindex for the λ's denotes the open string sector, while for the χ's it simply denotes the set of λ's to which they couple. The above argument is somewhat sloppy, since there are no microscopic quartic interactions in orbifolds. However they can be regarded as effective interactions upon integrating over bosonic zero modes from open strings between the different instantons (see [28] for details). We stick to this simplified discussion.

Upon integrating over the fermion zero modes $\lambda_r, \tilde{\lambda}_r$ of the E3$_r$-brane instantons, we find they induce an effective interaction on the world-volume action of the E(-1)$_r$-instanton.

$$\delta S_{\text{E}(-1)} = \sum_r e^{-T_r} \chi_r \tilde{\chi}_r \tag{4.18}$$

So the additional fermion zero modes of the E(-1)-brane instanton are lifted. In other words, one can pull down insertions of $\delta S_{\text{E}(-1)}$ to soak up the integrations over the fermionic collective variables $\chi, \tilde{\chi}$. Integrating over the latter, the E(-1)-brane instanton thus leads to a 4d non-perturbative superpotential term precisely of the form (4.16). The power of 1/4 arises from the fact that we are dealing with fractional instantons.[6] One can check that the complete superpotential is invariant under transformations of the U(1) symmetry in U(4). This is correlated with the fact that the complete 4-instanton system has zero intersection number (namely zero net number of chiral fermion zero modes) with the D3-branes.

A similar discussion would apply to other examples where euclidean brane instantons have been argued to modify the infrared dynamics of gauge theory sectors e.g. [6, 17].

4.4 Instanton induced SUSY breaking

The same Z_3 orientifold considered above may be used to construct a compact model in which SUSY-breaking a la Fayet may be implemented along the lines of the recent work [21]. In the present case we locate 8 $D3$-branes with CP twist matrix

$$\gamma_{\theta,3} = \text{diag}\left(\alpha I_4, \alpha^2 I_4\right) \tag{4.19}$$

at the orientifold point at the origin. This is enough to cancel twisted tadpoles. We then locate the remaining 24 $D3$-branes e.g. in the bulk (in 4 orbifold/orientifold invariant

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[6]In more precise terms, the standard gauge theory instanton contributes to a 4d correlator involving four pairs for fermions, out of which one can extract two-fermion correlators by clustering, as is familiar in the description of the gaugino condensate superpotential in $N = 1$ pure SYM.

groups of 6 $D3$ branes), away from the origin in any of the tori. The gauge group from the branes at the origin is U(4) with three chiral fields Φ^r ($r = 1, 2, 3$) transforming like $(6, -2)^r$ under SU(4) × U(1). There is one multiplet of charged fermion zero modes for each $E3^r$

$$\eta^r = (4, 1)^r \tag{4.20}$$

coupling to the chiral fields like

$$\epsilon^{abcd} \eta_a^r \eta_b^r \Phi_{cd}^r \ . \tag{4.21}$$

Upon integrating over zero modes a superpotential is generated

$$W_6 = \sum_r e^{-S_{E3^r}} \epsilon^{abcd} \Phi_{ab}^r \Phi_{cd}^r \tag{4.22}$$

which gives mass terms to the three 6-plet matter fields. These are the mass terms which were mentioned in the previous section. Again, the actual values of the masses is sensitive to the overall sizes of the different dimensions. Now, there is also a U(1) D-term potential of the form

$$V_{\mathrm{U}(1)} = \frac{1}{\lambda} \left(\sum_r -2|\Phi^r|^2 + \xi \right)^2 \tag{4.23}$$

where ξ is a (field dependent) FI-term.[7] Assume that there is some mechanism stabilizing all kahler moduli (twisted and untwisted), so that ξ will be a fixed parameter. The structure is now that of a variation of the Fayet-Iliopoulos mechanism for SUSY-breaking. Indeed, minimization of the D-term requires some of the 6-plets Φ^r to get a vev. But such a vev would give rise to a non-vanishing vacuum energy, due to the instanton-induced mass terms. Hence SUSY is spontaneously broken a la Fayet. The scale of SUSY breaking is of order:

$$F_{\Phi^r} \simeq e^{-S_{E3^r}} \sqrt{\xi} \tag{4.24}$$

with Φ^r the scalar with the largest instanton suppression $e^{-S_{E3^r}}$. Note that this could be used as the SUSY-breaking sector of a trinification model like that in the previous subsection, with gauge group U(3) × U(3) × U(3) and three generations.

The present example differs from [21] in several ways. The latter considered a $\mathbb{Z}_3$ orbifold of the conifold singularity, where the effects come from euclidean fractional $E1$ instantons, rather than Euclidean $E3$ instantons. Our example is also a global tadpole-free model, rather than a local one.

4.5 Yukawas and μ-terms in a Left-Right symmetric model

As we already mentioned, a typical drawback of semirealistic constructions from D-branes at singularities is that some Yukawa couplings are perturbatively absent. We would like to show that E3-brane instantons can cure this pathology, and provide an specific example in a semirealistic model. Here we consider a semirealistic left-right symmetric local Z_3 model.

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[7]In fact, supersymmetry relates this to the BF coupling, discussed above, from which such FI-term is given by $\xi = \frac{1}{2} \sum_{k=0}^{N-1} \sqrt{c_k} \, Tr(\, (\gamma_{\theta^k}^{D3} - \gamma_{\theta^{N-k}}^{D3}) \, \lambda_a) \phi_k.$

Matter fields	Q_3	Q_L	Q_R	$Q_{U_1^i}$	$Q_{U_2^i}$	$B-L$
33 sector						
$3(3,2,1)$	1	-1	0	0	0	1/3
$3(\bar{3},1,2)$	-1	0	1	0	0	-1/3
$3(1,2,2)$	0	1	-1	0	0	0
$\mathbf{37}_r$ sector						
$(3,1,1)$	1	0	0	-1	0	-2/3
$(\bar{3},1,1)$	-1	0	0	0	1	2/3
$(1,2,1)$	0	1	0	0	-1	-1
$(1,1,2)$	0	0	-1	1	0	1
$\mathbf{7_r 7_r}$ sector						
$3(1)'$	0	0	0	1	-1	0

Table 1: Spectrum of $SU(3) \times SU(2)_L \times SU(2)_R$ model. We present the quantum numbers under the $U(1)^9$ groups. The first three $U(1)$'s arise from the D3-brane sector. The next two come from the D7$_r$-brane sectors, and are written as a single column with the understanding that $\mathbf{37}_r$ fields are charged under $U(1)$ factors in the $\mathbf{7_r 7_r}$ sector.

This is possibly the phenomenologically most attractive model among the ones constructed in [22].[8] Let us review it here (see [43] for further discussion).

We consider the D3-brane Chan-Paton embedding

$$\gamma_{\theta,3} = \mathrm{diag}\,(I_3, \alpha I_2, \alpha^2 I_2) \qquad (4.25)$$

The corresponding tadpoles can be canceled for instance by D7$_r$-branes, $r = 1,2,3$ with the symmetric choice $u_0^r = 0$, $u_1^r = u_2^r = 1$. The gauge group on D3-branes is $U(3) \times U(2)_L \times U(2)_R$, while each set of D7$_r$-branes contains $U(1)^2$. The combination

$$Q_{B-L} = -2 \left(\frac{1}{3}\,Q_3 \ + \frac{1}{2}\,Q_L \ + \frac{1}{2}\,Q_R \right) \qquad (4.26)$$

is non-anomalous, and in fact behaves as $B-L$. The spectrum for this model, with the relevant $U(1)$ quantum numbers is given in table 1. We can see that the color triplets from the 37_r sectors can become massive after the singlets of the $\mathbf{7_r 7_r}$ sector acquire a nonvanishing vev, leaving a light spectrum really close to left-right theories considered in phenomenological model-building, with no chiral exotics.

Although it is possible to embed this configuration in globally consistent compact models (see section 4.3 in [22]), we will restrict to considering the local model. We will

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[8]It is easy to show that E3-brane instantons in the alternative Standard Model-like examples also considered in [22] lead to an unpaired set of D3-E3 fermion zero modes, which therefore lead to a vanishing amplitude. This unpairing can be regarded as a global $O(1)$ anomaly on the instanton world-volume, and signals the presence of some uncanceled charge in the non-compact model. In a global compactification this charge should cancel, which implies (regarding the instanton as a probe of discrete tadpoles, as in [44]) that these instantons necessarily have extra charged fermion zero modes from other sources, and thus necessarily lead to extra unwanted insertions in the 4d amplitude, thus rendering this case less interesting.
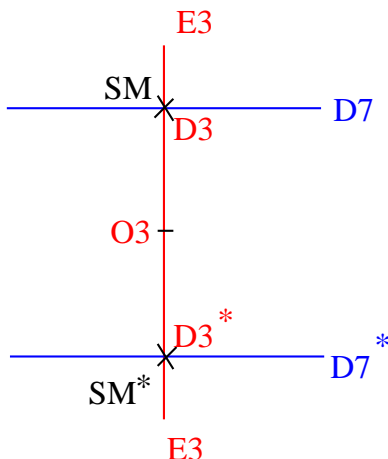
Figure 2: Pictorial view of the geometry of the $D3,D7$ semirealistic orientifold model discussed in the text. The instanton $E3^r$ pass through both the $D3$-branes giving rise to the SM group and an orientifold $O3$ plane. The $D3$ and $D7$ branes have orientifold mirror branes $D3^*$, $D7^*$ whereas the $E3$ instanton is orientifold invariant.

consider E3^r-brane instantons, wrapping non-compact directions, and implicitly assume that they are fixed under some orientifold projection in such a way that they have an $O(1)$ CP symmetry (see figure 2). Recall that we consider that the orientifold action does not fix the orbifold point, but relates it (and the D3/D7-brane system) to some mirror image $\mathbb{Z}_3$ singularity. The $E3$ will go both through the SM $D3$-branes and their mirrors.

The structure of charged zero modes in this model is as follows:

- **$E3^r$-$D3$ sector**

$$\eta_L^r \;=\; (1,\bar{2},1)_{(-1,0;0,0)}^r \;\; ; \;\; \eta_R^r \;=\; (1,1,2)_{(0,1;0,0)}^r \tag{4.27}$$

where the subindices show the charges under $U(1)_L \times U(1)_R \times U(1)_1^r \times U(1)_2^r$. Here $U(1)_{1,2}$ are the $U(1)$'s associated to each of the $D7^r$ branes which are present.

- **$E3^r$-$D7^s$ sector**

$$\eta_1^{rs} \;=\; (1,1,1)_{(0,0;-1,0)}^{rs} \;\; ; \;\; \eta_2^{rs} \;=\; (1,1,1)_{(0,0;0,1)}^{rs} \tag{4.28}$$

where here $r \neq s$.

As mentioned above, some or all these extra zero modes could be absent depending on the boundary conditions of the $D7^s$ branes at infinity. For instance, they are automatically absent for instantons with a turned on discrete $O(1) \equiv \mathbb{Z}_2$ Wilson line (a possibility easily implemented in toroidal orientifold models). We nevertheless keep them for the moment, with the understanding that the corresponding insertions could be absent for instantons with a built-in mechanism to remove E3-D7 zero modes.

Note that for fixed r, since we have $s \neq r$, there are altogether 2+2+2(1+1)=8 zero modes. Now the couplings of these zero modes to the chiral fields is as follows:

$$
\begin{aligned}
S^r_{\text{charged}} &= \eta^r_L (1, 2, \bar{2})^r \eta^r_R + \tag{4.29} \\
&+ \sum_{s \neq r} \eta^r_L (1, 2, 1)^s \eta^{rs}_2 \ + \sum_{s \neq r} \eta^r_R (1, 1, \bar{2})^s \eta^{rs}_1 \ + \tag{4.30} \\
&+ \epsilon_{rst} \, \eta^{rs}_1 \Phi^{7_s 7_s}_t \eta^{rs}_2 \ + \ + \epsilon_{rst} \, \eta^{rs}_1 \Phi^{7_s 7_t}_t \eta^{tr}_2 \tag{4.31}
\end{aligned}
$$

These zero mode couplings lead to a number of interesting 4d superpotential couplings for chiral fields. Note that since for each E3^r-brane instanton there are 8 zero modes, the superpotentials are going to be quartic in chiral fields, we will not get directly bilinears. This is due to the presence of the $D7$-branes which are required to cancel local twisted tadpoles. In particular one gets:

- **μ-terms**

  Such Higgs mass terms are forbidden perturbatively by the $\mathrm{U}(1)$ symmetries of the theory. Instantons may generate superpotentials

  $$
  W^r_\mu = e^{-S_{E3^r}} (1, 2, \bar{2})^r (1, 2, \bar{2})^r \Phi^{7_s 7_s}_t \Phi^{7_t 7_t}_s \ , \ \ r \neq s \neq t \neq r \tag{4.32}
  $$

  If the singlets $\Phi^{7_s 7_s}_t, \Phi^{7_t 7_t}_s$ get a non-vanishing expectation value, this gives rise to a mass term for the r-th Higgs multiplet. These vevs may be given without breaking SUSY by switching on FI-terms in the $\mathrm{U}(1)$'s of the $D7$-branes, which are given by the twisted moduli. Note that each instanton contributes to a particular Higgs multiplet. The masses (μ-terms) will depend both on the value of the instanton action and on the vevs of the $\Phi^{7_s 7_s}_t$ fields.

  An alternative to the presence of the Φ^{77} chiral field insertions is that the compact model was such that there are additional projections (e.g. due to Wilson lines on the D7-branes or a $\mathbb{Z}_2$ Wilson line on the E3-brane instanton) which remove some zero modes in the E3-D7 open string sector. From the viewpoint of the non-compact example, this would be regarded as the non-existence of instanton fermion zero modes in that sector, due to boundary condition at infinity.

- **Lepton Yukawa couplings**

  In this model perturbative Lepton Yukawa couplings are forbidden by the $\mathrm{U}(1)_L \times \mathrm{U}(1)_R$ symmetry (see table 1). Instantons however can generate superpotentials:

  $$
  W_Y(leptons)^r = e^{-S_{E3^r}} (1, 2, \bar{2})^r (1, 2, 1)^s (1, 1, \bar{2})^s \Phi^{7_t 7_t}_s \ , \ \ r \neq s \neq t \neq r \tag{4.33}
  $$

  $$
  W_Y(leptons)^{\prime r} = e^{-S_{E3^r}} (1, 2, \bar{2})^r (1, 2, 1)^s (1, 1, \bar{2})^t \Phi^{7_s 7_t}_s \ , \ \ r \neq s \neq t \neq r \tag{4.34}
  $$

  From eqs. (3.11), (3.15) one obtains that the instanton action transforms like

  $$
  S_{E3^r} \ \longrightarrow \ S_{E3^r} \ + \ 2\Lambda_{\mathrm{U}(1)_L} \ - \ 2\Lambda_{\mathrm{U}(1)_R} \ + \ \sum_{s \neq r} (\Lambda_{\mathrm{U}(1)^s_1} \ - \ \Lambda_{\mathrm{U}(1)^s_2}) \tag{4.35}
  $$

so that the above superpotentials are indeed gauge invariant.

Note that the second superpotential involves non-diagonal $D7^s$-$D7^t$ chiral fields. As in the case of μ-terms , lepton Yukawa couplings are obtained if fields $\Phi^{7_t 7_t}_s$ and/or $\Phi^{7_s 7_t}$ get webs, which may be triggered by $D7$ FI-terms. Note that the flavour structure is then controlled both by the values of the instanton actions as well as the singlet field insertions. In particular, if one wants to get non-negligible Yukawa couplings the exponential suppression should be small and the inserted vevs of singlets large. On the other hand this has to be done with care because otherwise all three Higgs multiplets could get too large masses from the μ-terms. It should be interesting to explore these phenomenological issues further.

A couple of comments are in order. Firstly, in the above discussion we have considered the contribution of instantons $E3^r$ with $r \neq s$. It is equally easy to compute the contribution of instantons with $r = s$ but different Chan-Paton actions for $E3-$ and $D7-$branes, i.e. for $E3-$ and $D7-$branes wrapping the same 4-cycle but carrying different CP twist. As we mentioned the contribution of those instantons gives rise to additional fermionic zero modes. In the particular model under consideration that would imply some additional insertions of chiral fields $(D7^r D7^r)_r$ in the above couplings. We have refrained from including those in order to make more clear the physics.

Secondly, we have assumed above that the E3-instanton only intersects the D3-branes where the SM sits. Things could however be a bit more tricky upon compactification. For instance, in simple toroidal orientifolds the E3-brane may be passing in addition through the origin, which is fixed under the orientifold and orbifold actions. The local twisted tadpoles at that point are $\mathrm{Tr}\,\gamma_{\theta^k,3} = -4$, $k = 1, 2$, and require the presence of a non-trivial set of D3-branes at such point. The simplest choice is 8 D3-branes with $\gamma_{\theta,3} = (\alpha \mathrm{I}_4, \alpha^2 \mathrm{I}_4)$ and gauge group $\mathrm{U}(4)$. Then there would be extra fermion zero modes from $D3-E3$ sectors, 4-plets of $\mathrm{U}(4)$. This would mean that all operators discussed above would be multiplied by factors of the form $\epsilon_{abcd} 6^{ab} 6^{cd}$ involving $\mathrm{U}(4)$ antisymmetrics. Thus these operators should also get large vevs if the superpotentials above are to be present. Again, such vevs can be triggered by the FI-terms associated to the blow-up modes of these other singularities. We skip this discussion, which would be very model dependent, and simply point out that there may exist global compactifications where the E3-brane does not intersect other D3-brane sectors, so this potential complication would be absent.

5. Instantons and non-supersymmetric $\mathbb{C}^3/\mathbb{Z}_N$ singularities

In the case of SUSY models the number of universal fermionic zero modes is often larger than two, which implies that no non-perturbative charged open string operators are induced by $E3^r$ instantons. As we mentioned in the case of orientifolds the number of zero modes is the minimal set of two only for $O(1)$ instantons. This places important constraints on the possible non-perturbative effects on these models.

A drastic modification of this problem is to consider D3-brane systems at non-SUSY orbifold singularities. In that case the orbifold projection removes all the universal fermion

zero modes from the E3-E3 sector (in agreement with the fact that, since the geometry does not preserve any supersymmetry, there are no Goldstinos on the E3-brane). Thus, there are no extra θ^α fermionic zero modes. It is also important to point out that despite the lack of supersymmetry there are E3-branes without tachyonic instabilities, which ensures that they are good saddle points of the semiclassical theory.

On the other hand there will be in general charged fermionic zero modes coming from open strings in the $E3^r - D3$ sectors, coupling to 4d matter scalars. Integration over those charged zero modes leads to insertions of 4d matter scalar fields, thus generating non-perturbative scalar interactions. Such operators will be generated for any CP structure of the instanton i.e., also for U(1) instantons and hence no orientifold projection is required.

Large classes of non-supersymmetric models may be constructed locating $D3$-branes at non-SUSY Z_N singularities (see ref. [22].).[9] A subset of those models are free of tachyons in the closed string sector,[10] and also in the open string sector, and it is very easy to obtain models with 3 quark-lepton generations. One expects that $E3^r$ instantons may yield new non-perturbative couplings in these models.

Let us first summarize some facts about $D3$-branes on non-SUSY orbifold singularities taken from [22]. We consider a stuck of $D3$-branes at a $\mathbb{R}^6/\Gamma$ singularity with $\Gamma \subset \mathrm{SU}(4)$ where, for simplicity, we take $\Gamma = \mathbb{Z}_N$. The $\mathbb{Z}_N$ action on fermions is given by the matrix in eq. (A.1) and that on bosons by the matrix (A.2). For the CP twist matrices of D3-branes we consider the general embedding given by the matrix

$$\gamma_{\theta,3} = \mathrm{diag}\,(\mathrm{I}_{n_0}, e^{2\pi i/N}\mathrm{I}_{n_1}, \ldots, e^{2\pi i(N-1)/N}\mathrm{I}_{n_{N-1}}) \tag{5.1}$$

where I_{n_i} is the $n_i \times n_i$ unit matrix, and $\sum_i n_i = n$. The matter spectrum in the 33 sector is

$$
\begin{array}{ll}
\text{Vectors} & \prod_{i=0}^{N-1} \mathrm{U}(n_i) \\
\text{Complex Scalars} & \sum_{r=1}^{3} \sum_{i=0}^{N-1} (n_i, \overline{n}_{i-b_r}) \\
\text{Fermions} & \sum_{\alpha=1}^{4} \sum_{i=0}^{N-1} (n_i, \overline{n}_{i+a_\alpha})
\end{array} \tag{5.2}
$$

Note that the spectrum is non-SUSY for $a_4 \neq 0$. One recovers $N = 1$ SUSY for $b_1 + b_2 + b_3 = 0$, which implies $a_4 = 0$.

In general there may be present $D7^r$ branes transverse to the r-th complex plane locally. There may be CP matrices:

$$\gamma_{\theta,7_r} = \mathrm{diag}\,(\,\mathrm{I}_{u_0}, e^{2\pi i/N}\mathrm{I}_{u_1}, \ldots, e^{2\pi i(N-1)/N}\mathrm{I}_{u_{N-1}}) \tag{5.3}$$

where we are assuming b_r = even. Then one finds matter fields (for b_r even):

$$
\begin{array}{lll}
b_r = \text{even} \rightarrow & \text{Fermions} & \sum_{i=0}^{N-1} \left[\,(n_i, \overline{u}_{i+\frac{1}{2}b_r}) + (u_i, \overline{n}_{i+\frac{1}{2}b_r})\,\right] \\
& \text{Complex Scalars} & \sum_{i=0}^{N-1} \left[\,(n_i, \overline{u}_{i-\frac{1}{2}(b_s+b_t)}) + (u_i, \overline{n}_{i-\frac{1}{2}(b_s+b_t)})\,\right]
\end{array} \tag{5.4}
$$

with $r \neq s \neq t \neq r$.

---

[9]Compact non-SUSY orientifolds were discussed in [45].

[10]In particular it is easy to check that Z_N singularities with $a_1 = a_2 = a_3 = 1$ and $a_4 = -3$ (see the appendix for notation) have no tachyons in any twisted sector.

We now consider the presence of Euclidean $E3^r$ instantons passing through the singularity. As in the SUSY case there will be charged fermionic zero modes from $E3^r - D3$ open strings. Those may be obtained from the corresponding fermionic zero modes from open strings in the $D7^r - D3$ sector. Let the CP twist matrix of the instanton be

$$\gamma_{\theta, E3^r} = \mathrm{diag}\,(\mathrm{I}_{v_0^r}, e^{2\pi i/N} \mathrm{I}_{v_1^r}, \ldots, e^{2\pi i (N-1)/N} \mathrm{I}_{v_{N-1}^r}) \tag{5.5}$$

Then there will be fermionic zero modes from $E3^r - D3$ strings transforming like (for b_r even):

$$(n_{c-\frac{b_r}{2}}, \overline{v}_c) \;+\; (v_c, \overline{n}_{c+\frac{b_r}{2}}) \tag{5.6}$$

These fermionic zero modes have couplings to scalars Φ in the $D3 - D3$ sector (see eq. (5.2))

$$(n_{c-\frac{b_r}{2}}, \overline{v}_c)\; \Phi_{(n_{c+\frac{b_r}{2}}, \overline{n}_{c-\frac{b_r}{2}})}\;(v_c, \overline{n}_{c+\frac{b_r}{2}}) \tag{5.7}$$

Integration over the charged fermionic zero modes will give rise to determinant couplings among the scalars Φ of the form

$$e^{-S_{E3^r}} \;\times\; det(\Phi_{(n_{c+\frac{b_r}{2}}, \overline{n}_{c-\frac{b_r}{2}})}) \tag{5.8}$$

Note that this is a purely bosonic coupling. The coupling will be gauge invariant due to the $\mathrm{U}(1)$ transformation of the euclidean action, as in the SUSY case. Indeed the derivation of the $\mathrm{U}(1)$ gauge transformations described in section 4 applies also to the case of non-SUSY singularities (and works in full analogy with the cancellation of mixed $\mathrm{U}(1)$ anomalies in D3/D7-brane systems at non-supersymmetric orbifolds in [22], for reasons already explained).

We thus see that Euclidean $E3$ instantons on this class of non-SUSY Abelian singularities can give rise to non-perturbative purely bosonic couplings. Note that the fact that universal fermionic zero modes are projected out in this class of non-SUSY singularities makes that no fermionic operators are generated.

In section 3.5 of [22], an explicit semirealistic model based on a non-SUSY Z_5 singularity is presented. It is a 3-generation left-right symmetric model with gauge group $\mathrm{U}(3) \times \mathrm{U}(2)_L \times \mathrm{U}(2)_R \times \mathrm{U}(1)^2$ (before some $\mathrm{U}(1)$'s get massive by combining with some twisted RR fields). In that model one can see that e.g. an euclidean E3-brane $\mathrm{U}(1)$ instanton with CP matrix = 1 gives rise to a B-term bilinear in the Higgs multiplet $(2_L, 2_R)$, i.e. a term of the form

$$e^{-S_{ins}}\;(\Phi_{(2_L, 2_R)} \Phi_{(2_L, 2_R)}) \;+\; h.c.. \tag{5.9}$$

It is interesting to remark that this term gives an example of a scalar bilinear term which is protected against perturbative loop corrections without supersymmetry. Indeed, loop corrections can only give rise to (quadratically divergent) corrections to bilinears of the form $|\Phi_{(2_L, 2_R)}|^2$ but not to terms such as (5.9) which are protected by the perturbatively exact $\mathrm{U}(1)$ symmetry. Hence this term can be hierarchy small compared with the UV cutoff in a completely natural way, with the small scale generated by the exponential suppression of the instanton.

6. Final comments

In this paper we have studied different aspects of the non-perturbative superpotentials induced by $E3$ euclidean instantons in systems with D3/D7-branes sitting at Abelian orbifold singularities. Much of the recent work on induced superpotentials from stringy instantons has been formulated in the context of Type IIA orientifolds with charged matter fields at intersecting D6-branes. The generation of superpotentials in this case requires D6-branes wrapping rigid 3-cycles and $O(1)$ orientifold projections. The construction of globally consistent examples within this class has shown to be challenging. We have shown how in the case of IIB with D3/D7-branes at singularities finding $E3$ instantons with the required fermion zero mode content is much easier. Furthermore the couplings of the fermionic zero modes to the chiral $D = 4$ fields, which are at the origin of the superpotentials, do not require CFT amplitude computations but rather come directly given by the singularity quiver diagrams. We have also sketched some of the aspects which appear in the case of superpotentials induced by $E3$ instantons in systems of $D3$-branes sitting at general toric singularities.

In the systems here studied both E3-D3 and E3-D7 fermionic zero modes may contribute to the amplitudes. The transformation properties of the $E3$ instantons under the $U(1)$ symmetries of both $D3$ and $D7$-branes are nicely compensated by the charges of the 4-D fields appearing in the induced operator. In this way operators perturbatively forbidden by the $U(1)$ symmetries are generated by the instantons.

Semirealistic models may be constructed using $D3/D7$ systems located at Abelian orbifold singularities. We have shown how operators with potential phenomenological interest can be generated in this context. Some Yukawa couplings are often perturbatively forbidden in semirealistic models of branes at singularities. We have presented examples in which forbidden lepton Yukawa couplings are generated due to instanton effects. We also presented a global tadpole free SU(6) GUT example in which u-quark Yukawa couplings are generated by these $E3$ euclidean instantons. In a more realistic three generations $SU(3) \times SU(2)_L \times SU(2) \times U(1)_{B-L}$ example both a Higgs bilinear (μ-term) and lepton Yukawa couplings can be generated. In this example the generation of these terms will typically require the insertion of vevs for $D7 - D7$ massless chiral fields.

The examples considered have an unbroken gauged $U(1)_{B-L}$ symmetry which forbids the generation of Majorana neutrino masses, which is one of the possible interesting applications of instanton induced superpotentials. It would be interesting to look for semirealistic models in which the $U(1)_{B-L}$ gauge boson becomes massive by combining with some closed string scalar field so that Majorana neutrino masses could be generated.

Other possible application of these instanton induced couplings is to supersymmetry breaking. Indeed, as remarked in [21] instanton generated bilinear couplings combined with $U(1)$ D-terms may lead to SUSY breaking a la Fayet (for fixed closed string moduli). We have shown an explicit global tadpole free example of this class based on the Z_3 compact orientifold. More realistic models involving this SUSY breaking mechanism should be worth studying. On the other hand we have found that certain non-perturbative scalar couplings are expected to be generated in the case of systems of $D3$-branes sitting at (tachyon free)

non-SUSY orbifold singularities. This includes the generation of exponentially suppressed scalar bilinears which get no perturbative loop corrections.

One interesting feature of $E3$ instantons that we have described is how they can combine with standard gauge instantons to provide new non-perturbative superpotentials. We have shown how this effect described in [28] can take place even in simple toroidal orientifold settings. We leave a full exploration of these novel effects for future work.

Acknowledgments

A. Type IIB branes at Abelian orbifold singularities

To fix notation we review in this appendix the basic formalism to compute the spectrum and interactions on the world-volume of D3- and D7-branes at $\mathbb{R}^6/\mathbb{Z}_N$ singularities [22]. Consider a set of n D3-branes at a $\mathbb{R}^6/\Gamma$ singularity with $\Gamma \subset \mathrm{SU}(4)$ where, for simplicity, we take $\Gamma = \mathbb{Z}_N$. Before the projection, the world-volume field theory on the D3-branes is a $\mathcal{N} = 4$ supersymmetric $\mathrm{U}(n)$ gauge theory. In terms of component fields, the theory contains $\mathrm{U}(n)$ gauge bosons, four adjoint fermions transforming in the $\mathbf{4}$ of the $\mathrm{SU}(4)_R$ $\mathcal{N} = 4$ R-symmetry group, and six adjoint real scalar fields transforming in the $\mathbf{6}$.

The $\mathbb{Z}_N$ action on fermions is given by a matrix

$$\mathbf{R_4} = \mathrm{diag}\,(e^{2\pi i a_1/N}, e^{2\pi i a_2/N}, e^{2\pi i a_3/N}, e^{2\pi i a_4/N}) \tag{A.1}$$

with $a_1 + a_2 + a_3 + a_4 = 0 \bmod N$. The action of $\mathbb{Z}_N$ on scalars can be obtained from the definition of the action on the $\mathbf{4}$, and it is given by the matrix

$$\mathbf{R_6} = \mathrm{diag}\,(e^{2\pi i b_1/N}, e^{-2\pi i b_1/N}, e^{2\pi i b_2/N}, e^{-2\pi i b_2/N}, e^{2\pi i b_3/N}, e^{-2\pi i b_3/N}) \tag{A.2}$$

with $b_1 = a_2 + a_3$, $b_2 = a_1 + a_3$, $b_3 = a_1 + a_2$. Scalars can be complexified, the action on them being then given by $\mathbf{R}_{esc} = \mathrm{diag}\,(e^{2\pi i b_1/N}, e^{2\pi i b_2/N}, e^{2\pi i b_3/N})$. When $b_1 + b_2 + b_3 = 0$, we have $a_4 = 0$ and the $\mathbb{Z}_N$ action is in $\mathrm{SU}(3)$. This case corresponds to a supersymmetric singularity. The fermions with $\alpha = 4$ transforming in the adjoint representation of $\mathrm{U}(n_i)$ become gauginos, while the other fermions transform in the same bifundamental representations as the complex scalars. The different fields fill out complete vector and chiral multiplets of $\mathcal{N} = 1$ supersymmetry.

The action of the $\mathbb{Z}_N$ generator θ will be embedded on the Chan-Paton indices. In order to be more specific we consider the general embedding given by the matrix

$$\gamma_{\theta,3} = \mathrm{diag}\,(\mathrm{I}_{n_0}, e^{2\pi i/N}\mathrm{I}_{n_1}, \ldots, e^{2\pi i(N-1)/N}\mathrm{I}_{n_{N-1}}) \tag{A.3}$$

where I_{n_i} is the $n_i \times n_i$ unit matrix, and $\sum_i n_i = n$. Analogously for a $D7^r$ which is transverse to the z^r complex coordinate:

$$\gamma_{\theta,7} = \text{diag}\left(I_{u_0}, e^{2\pi i/N} I_{u_1}, \ldots, e^{2\pi i(N-1)/N} I_{u_{N-1}}\right) \tag{A.4}$$

Then the chiral open string spectrum in the $N = 1$ case with a $D7^r$ transverse to the z^r complex coordinate is:

$$
\begin{array}{lll}
\mathbf{33} & \text{Vector mult.} & \prod_{i=0}^{N-1} U(n_i) \\
& \text{Chiral mult.} & \sum_{i=0}^{N-1} \sum_{s=1}^{3} (n_i, \overline{n}_{i+a_s}) \\
\mathbf{37_r, 7_r 3} & \text{Chiral mult.} & \sum_{i=0}^{N-1} \left[(n_i, \overline{u}_{i-\frac{1}{2}a_r}) + (u_i, \overline{n}_{i-\frac{1}{2}a_r}) \right] \quad a_r \text{ even} \\
& & \sum_{i=0}^{N-1} \left[(n_i, \overline{u}_{i-\frac{1}{2}(a_r+1)}) + (u_i, \overline{n}_{i-\frac{1}{2}(a_r+1)}) \right] \ a_r \text{ odd}
\end{array}
\tag{A.5}
$$

All chiral fields transform as bifundamentals. We denote $\Phi^r_{i,i+a_s}$ the 33 chiral multiplet in the representation $(n_i, \overline{n}_{i+a_s})$. We also denote (assuming $a_r =$ even for concreteness) $\Phi^{(37_r)}_{i,i-\frac{1}{2}a_r}$, $\Phi^{(7_r 3)}_{i,i-\frac{1}{2}a_r}$ the 37_r and $7_r 3$ chiral multiplets in the $(n_i, \overline{u}_{i-\frac{1}{2}a_r})$, $(u_i, \overline{n}_{i-\frac{1}{2}a_r})$. With this notation, the interactions are encoded in the superpotential

$$W = \sum_{r,s,t=1}^{3} \epsilon_{rst} \, \text{Tr}\left(\Phi^r_{i,i+a_r} \Phi^s_{i+a_r,i+a_r+a_s} \Phi^t_{i+a_r+a_s,i} \right) + \sum_{i=0}^{N-1} \text{Tr}\left(\Phi^3_{i,i+a_r} \Phi^{(37_r)}_{i+a_r,i+\frac{1}{2}a_r} \Phi^{(7_r 3)}_{i+\frac{1}{2}a_r,i} \right) \tag{A.6}$$

There are in general local twisted RR tadpoles. The conditions for their cancellation is

$$\left[\prod_{r=1}^{3} 2\sin(\pi k b_r/N) \right] \text{Tr}\, \gamma_{\theta^k,3} + \sum_{r=1}^{3} 2\sin(\pi k b_r/N)\, \text{Tr}\, \gamma_{\theta^k,7_r} = 0 \tag{A.7}$$

A.1 The $\mathbb{C}^3/\mathbb{Z}_3$ case

Most of the examples mentioned in the main text make use of this singularity so that for convenience we summarize here this case. The open string chiral spectrum is given by

$$
\begin{array}{ll}
\mathbf{33} & U(n_0) \times U(n_1) \times U(n_2) \\
& 3\left[(n_0, \overline{n}_1) + (n_1, \overline{n}_2) + (n_2, \overline{n}_0) \right] \\
\mathbf{37_r, 7_r 3} & (n_0, \overline{u}^r_1) + (n_1, \overline{u}^r_2) + (n_2, \overline{u}^r_0) + \\
& + (u^r_0, \overline{n}_1) + (u^r_1, \overline{n}_2) + (u^r_2, \overline{n}_0)
\end{array}
\tag{A.8}
$$

The superpotential terms are

$$W = \sum_{i=0}^{2} \sum_{r,s,t=1}^{3} \epsilon_{rst} \text{Tr}\left(\Phi^r_{i,i+1} \Phi^s_{i+1,i+2} \Phi^t_{i+2,i} \right) + \sum_{i=0}^{2} \sum_{r=1}^{3} \text{Tr}\left(\Phi^r_{i,i+1} \Phi^{37_r}_{i+1,i+2} \Phi^{7_r 3}_{i+2,i} \right) \tag{A.9}$$

eeqa In this $\mathbb{C}^3/\mathbb{Z}_3$ singularity it is possible to consider the generic case of $D7^\beta$-branes, with world-volume defined by $\sum_r \beta_r Y_r = 0$, which preserve the $\mathcal{N} = 1$ supersymmetry of the configuration for arbitrary complex β_r. The 37^β, $7^\beta 3$ spectra are as above, but the superpotential is $W = \sum_i \sum_r \beta_r \text{Tr}\left(\Phi^r \Phi^{37^\beta} \Phi^{7^\beta 3} \right)$, with fields from a single mixed sector coupling to 33 fields from all complex planes.

The twisted tadpole cancellation conditions are

$$\text{Tr}\, \gamma_{\theta,7_3} - \text{Tr}\, \gamma_{\theta,7_1} - \text{Tr}\, \gamma_{\theta,7_2} + 3\text{Tr}\, \gamma_{\theta,3} = 0 \tag{A.10}$$

These equations are equivalent to the non-abelian anomaly cancellation conditions.

B. A Z_7 compact example

All compact orientifold examples up to now were based on the Z_3 orientifold. One can see that there are instanton induced superpotentials in other Z_N examples. Although it has no phenomenological interest, let us briefly mention the case of the compact Z_7 orientifold which is the next simplest compact orientifold with only O3-planes. The twist θ is generated by $v = \frac{1}{7}(1, 2, -3)$. The twisted tadpole cancellation condition implies $\mathrm{Tr}\,\gamma_\theta = 32\cos\frac{\pi}{7}\cos\frac{2\pi}{7}\cos\frac{3\pi}{7} = 4$. Then locating all D3-branes at the origin with CP matrix

$$\gamma_\theta = \mathrm{diag}\,(\delta \mathrm{I}_4, \delta^2 \mathrm{I}_4, \bar{\delta}^3 \mathrm{I}_4, \mathrm{I}_4, \bar{\delta}\mathrm{I}_4, \bar{\delta}^2\mathrm{I}_4, \delta^3\mathrm{I}_4, \mathrm{I}_4) \tag{B.1}$$

twisted tadpoles cancel. Here $\delta = \mathrm{e}^{2i\pi/7}$ and $\bar{\delta} = \delta^*$. The gauge group is $\mathrm{U}(4)^3 \times \mathrm{SO}(8)$ and the matter spectrum is given by

$$(\underline{4, 1, 1, 8_v}) + (\underline{\overline{4}, \overline{4}, 1, 1}) + (\underline{6, 1, 1, 1}) \tag{B.2}$$

$$+(\overline{4}, 4, 1, 1) + (1, \overline{4}, 4, 1) + (4, 1, \overline{4}, 1) \tag{B.3}$$

where the underlining means one has to add permutations. Now there are $E3^r\ O(1)$ instantons transverse to the r-th plane. They give rise to fermionic zero modes η_a^r which transform like

$$\eta^1 = (4, 1, 1, 1) \ ; \ \eta^2 = (1, 4, 1, 1) \ ; \ \eta^3 = (1, 1, 4, 1, 1) \tag{B.4}$$

respectively for $r = 1, 2, 3$ labeling each complex plane. The main difference here with the case of the $\mathbb{Z}_3$ orientifold is that each instanton $E3^r$ has zero modes transforming non-trivially under a different gauge group. Each zero mode couples to a different antisymmetric 6-plet according to eq. (2.3). These are couplings of the form $\eta_a^r 6_r^{ab} \eta_b^r$. Then mass terms of the form

$$\sum_{r=1}^{3} e^{-S_{E3^r}} \ \epsilon^{abcd} \ 6_{ab}^r \ 6_{cd}^r \tag{B.5}$$

are generated by the instanton. Indeed one can check the $\mathrm{U}(1)^3$ gauge invariance of this operator.

C. E3-brane instantons in general toric singularities

In this appendix we provide the basic tools to generalize the analysis in the main text to the computation of non-compact E3-brane instanton effects for systems of D3-branes at general toric singularities. For simplicity (and due to limitations in the available tools) we restrict to configurations with no D7-branes.

Many properties of the geometry of toric singularities, as well as of the gauge theories on D3-branes located at them, can be studied using the so-called brane tilings or dimer diagrams, see e.g. [46–48]. The structure of the gauge theory is encoded in a tiling of a 2-torus by a graph, with faces corresponding to gauge factors, edges to chiral multiplets in bi-fundamental representations (under the gauge factors of the two faces separated by

the edge), and nodes to superpotential couplings (among the chiral multiplets associated to the edges ending on the node). Referring to these papers for details, let us simply say that the corresponding gauge theories have a product gauge group $\prod U(n_i)$ and a set of bifundamental chiral multiplets $\Phi_{ij}^{a_{ij}}$ in the $(\square_i, \overline{\square}_j)$, with the index a_{ij} distinguishing possible multiplets in the same representation (in what follows we omit this index a_{ij} for clarity). We focus on systems of D3-branes at non-orientifold toric singularities, but the generalization of our discussion below to systems of D3-branes at orientifolded toric singularities can be treated similarly, using the techniques in [19].

We would like to consider possible instanton effects in this kind of configuration. Instantons corresponding to euclidean branes wrapped on the collapsed cycles at the singularity (the analog of E(−1)-brane instantons in the orbifolds in the main text), can be efficiently described using the dimer diagrams, and instanton effects for non-gauge instantons (namely euclidean branes associated to a face / gauge factor not occupied by the 4d spacefilling branes) have been described (upon the introduction of orientifold actions) in [19]. For this reason, and also to keep with the main line in this paper, we rather consider instantons arising from E3-branes wrapped on non-compact 4-cycles passing through the singularity.

The general problem of describing possible non-compact holomorphic 4-cycles in general toric singularities was addressed in appendix B of [49]. The motivation there was to wrap D7-branes on them to introduce flavors for the D3-brane gauge theory, but the results can be applied to the description of the wrapped E3-brane instantons, and of the coupling of their zero modes to the D3-brane fields. The main result from the analysis is as follows. For each bi-fundamental Φ_{ij} in the D3-brane gauge theory, there exist a non-compact supersymmetric 4-cycle passing through the singularity, such that an E3-brane instanton wrapping it leads to charged fermion zero modes α_i, β_j, charged in the $\square_i$ and $\overline{\square}_j$ of the $U(n_i)$ and $U(n_j)$ D3-brane gauge factors, and having couplings

$$\Delta S_{E3} = \alpha_i \, \Phi_{ij} \, \beta_j \tag{C.1}$$

with the D3-brane chiral multiplets. When the D3-brane gauge theory has several multiplets in the same gauge representation (labeled by $a_{ij} = 1, \ldots, K_{ij}$), they all correspond to E3-branes on the same 4-cycle, but carrying different world-volume gauge bundles (distinguished by a $\mathbb{Z}_{K_{ij}}$ Wilson line at infinity). This correspondence between 4d bi-fundamentals and non-supersymmetric 4-cycles follows (as discussed in [49]) from the AdS/CFT correspondence between di-baryonic operators and 3-cycles on the 5d horizon of the gravity dual of the D3-brane gauge theory. Namely the dibaryons are constructed from chiral multiplets by antisymmetrization of indices, and the 3-cycles are the bases of 4d cones describing the non-compact 4-cycles.

It is easy to obtain the 4d effective vertex generated by one such D3-brane instanton. Let us focus on the case $n_i = n_j$, otherwise the charged fermion zero modes are unpaired, leading to a vanishing contribution (or rather, requiring the presence of additional intersections of the E3-brane instanton with other branes in a globally consistent example, see

footnote 8). The integration over the charged fermion zero modes leads to a superpotential

$$W \simeq e^{-T} \det \Phi_{ij} \tag{C.2}$$

where T denotes the modulus associated to the 4-cycle in an eventual global embedding of the local configuration.

It is easy to recover the results for orbifold singularities in this language. Similarly it is possible to construct explicit models of D3-branes at such more general toric singularities, and to describe the possible effects of E3-brane instantons in the field theory. We refrain from this more extensive discussion, leaving it for future work.

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