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Using Double Tagging to measure the Performance of the Same Side Kaon Tagger in Data

LHCb Collaboration

Onuora Awunor, Imperial College London
Ulrik Egede, Imperial College London

Abstract

This note describes a method for the measurement of the wrong-tag fraction of the Same Side Kaon tagger, ω_{SS}, in data using the Double Tagging procedure. The importance of such a measurement is explained and the method of Double Tagging described. This measurement was carried out using samples from reconstructed $B_s \to D_s\pi$ and $B_s \to D_s\mu\nu$ decays. The impact of background in both decay channels on the measurement of ω_{SS} was also studied. A general way of handling tagger correlations is presented. Finally, the Double Tagging measurement and the correlation investigation are performed on the present Monte Carlo data and the results presented. Results show that with $2fb^{-1}$ of data, the Double Tagging procedure can be used to measure the SS Kaon wrong-tag fraction to a statistical accuracy of 3.6% and 1.2% for the $B_s \to D_s\pi$ and $B_s \to D_s\mu\nu$ channels respectively.

1 Introduction

The process of flavour tagging is an important step in achieving the main physics goal of the LHCb experiment which is the measurement of CP asymmetries. Flavour tagging is the process of inferring the initial flavour of the reconstructed B meson in $b\bar{b}$ decays. Any CP asymmetry measurement will require the wrong-tag fraction as an input parameter and the uncertainty on this value will make a significant contribution to the overall systematic error. Therefore, detailed knowledge of the performance of the tagging algorithms will be instrumental to the success of LHCb.

There are two broad categories of tagging algorithms: Same Side (SS) tagging infers the signal B meson production flavour using correlations between the quark fragmentation and the signal B, while Opposite Side (OS) tagging utilises the hadron from the other B meson in the event. SS kaon tagging provides 30% of the total tagging power in hadronic B_s meson decays [1]. The OS tagging performance should be independent of the signal B species, because, by definition, the tagging uses the other B hadron in the event. The SS tagging performance is not independent of the signal B species because it utilises correlations from fragmentation which is dependent on signal B species, B_d or B_s.

The efficiency, ϵ, wrong-tag fraction, ω and dilution, D, are defined by

$$\epsilon = \frac{R+W}{R+W+U}, \quad \omega = \frac{W}{R+W}, \quad D = 1 - 2\omega, \tag{1}$$

where R, W and U refer to the number of right, wrong and untagged events in a given dataset. The amplitude of flavour oscillations is given by the dilution, therefore a mixing analysis can be used to evaluate the performance of the tagging in data. In principle the performance of any OS tagger can be extracted from samples such as inclusive $B \to D^0 \mu X$ and $B \to D^0 \pi X$ which include both $B^+ \to D^0$ (direct measurement) and $B_d \to D^*$ (mixing measurement) transitions while the SS tagging needs to be evaluated for each B species separately.

There are two methods which can be used to measure the wrong-tag fraction in data; by fitting the oscillation amplitude of the CP asymmetry to a flavour specific final state, or by using the Double Tagging method. The former can be used to measure the OS, SS or total wrong-tag fractions but requires excellent vertex resolution while the latter can be used to measure only the SS wrong-tag fraction but is dependent only on high B_s statistics.

This analysis describes a method for using the process of Double Tagging to evaluate the performance of the SS tagging algorithms in data without observing B_s oscillations. This analysis is a continuation of work carried out in [2].

2 Double Tagging

Double Tagging is the process of tagging events using the combination of a same side tagger, t_{SS} and one or more opposite side taggers, t_{OS}. The flavour taggers, t_{OS} and t_{SS}, have wrong-tag fractions, ω_{OS} and ω_{SS} and efficiencies, ϵ_{OS} and ϵ_{SS}. It is assumed that ω_{OS} is known and ω_{SS} is the quantity to be measured. Both tagging algorithms are applied to a sample of signal events of size, N_T. Assuming that t_{OS} and t_{SS} are uncorrelated, the number of double tagged events, N_{DT}, is given by,

$$N_{DT} = \epsilon_{OS} \times \epsilon_{SS} \times N_T. \tag{2}$$

The number of events in which the taggers agree is given by N_{Agree} and the fraction, F, is given by N_{Agree}/N_{DT}. This is equal to the probability that the taggers agree

$$F = P(t^R_{OS})P(t^R_{SS}) + P(t^W_{OS})P(t^W_{SS}), \tag{3}$$

where $P(t^{R(W)}_{OS(SS)})$ is the probability that $t_{OS}(t_{SS})$ is right (wrong). This can be expressed in terms of the wrong-tag fractions

$$F = (1 - \omega_{OS})(1 - \omega_{SS}) + \omega_{OS}\omega_{SS}, \tag{4}$$

and it immediately follows that

$$\omega_{SS} = \frac{1 - \omega_{OS} - F}{1 - 2\omega_{OS}}, \tag{5}$$

with the error, $\sigma(\omega_{SS})$, given by

$$\sigma(\omega_{SS})^2 = \frac{(1-2F)^2}{(1-2\omega_{OS})^4}\sigma(\omega_{OS})^2 + \frac{1}{(1-2\omega_{OS})^2}\sigma(F)^2 \tag{6}$$

$$= \frac{(1-2F)^2}{(1-2\omega_{OS})^4}\sigma(\omega_{OS})^2 + \frac{1}{(1-2\omega_{OS})^2}\frac{F(1-F)}{N_{DT}}, \tag{7}$$

where $\sigma(\omega_{OS})$ is the error on the wrong-tag fraction of t_{OS}.

Equation (7) has two components. The first term is the error due to the knowledge of the tagger t_{OS}, while the second term is a statistical error. As a result there is a limit to the accuracy that ω_{SS} can be calculated which is given by

$$\sigma(\omega_{SS})_{LIMIT} = \frac{(1-2F)}{(1-2\omega_{OS})^2}\sigma(\omega_{OS}). \tag{8}$$

Therefore this measurement requires a balance between optimal understanding of the opposite tagging performance and high statistics.

3 Decay Channels

This analysis evaluates the performance of the Double Tagging algorithm in two decay channels, the $B_s \to D_s\pi$ channel and the $B_s \to D_s\mu\nu$ channels. These channels are used as control channels for the $B_s \to J/\psi\phi$ channel from which time dependent CP asymmetry measurements can be used to extract the phase Φ_s. All MC events which were used for analysis on both channels were generated in the Data Challenge 2004 (DC04) environment and analysed using version v12r18 of the DaVinci physics analysis package and version v6r6 of the Flavour Tagging package.

3.1 Signal

For the $B_s \to D_s\pi$ channel, 92,941 stripped reconstructed $B_s \to D_s\pi$ Monte Carlo (MC) signal events, which are events that have been passed through the Level Zero (L0) trigger and Technical Design Report (TDR) selections were used [3]. Of these, 70,740 passed the Level 1 (L1) trigger. This corresponds to an integrated luminosity of approximately $1.8 fb^{-1}$ of data at LHCb.

For the $B_s \to D_s\mu\nu$ channel, 499,000 $B_s \to D_s\mu\nu X$ cocktail MC signal events, where X is a neutral particle such as π^0 or γ, were generated, simulated and selected as described in [4] with the exception that $\Delta\ln\mathcal{L}_{k\pi} > 0$ to select a $K^\pm$ and $\chi^2 < 10$ to select $D_s\mu$ combinations. Of these events, 4181 pass L0 and 3410 pass L0×L1. This corresponds to an integrated luminosity of approximately $0.007 fb^{-1}$, i.e. 1 LHCb day of data taking.

3.2 Background

In order to estimate the impact of background on signal in both $B_s \to D_s\pi$ channel and the $B_s \to D_s\mu\nu$ channels, a model, shown in Figure (1), was used in which some assumptions were made for simplicity. It was assumed that the combinatorial background follows an exponential in reconstructed D_s mass. The lower side band will contain background from partially reconstructed B decays which cannot affect the signal region. Hence only the upper side band is used for analysis. The model is defined by:

$$N(m) = N_{sig} G(\mu, \sigma) + N_{bkg} exp(-km), \tag{9}$$

where $N(m)$ is the total number of events, $G(\mu, \sigma)$ is a gaussian with mean μ and width σ, and $N_{sig}(N_{bkg})$ are the number of signal (background) events as a function of reconstructed mass. Therefore, N_{sig} and N_{bkg} can be obtained by carrying out a fit on the event sample. This is shown clearly in Figure (1), where the fit was carried out on 553 $B_s \to D_s\mu\nu$ events from an inclusive $b\bar{b}$ event sample.

In order to incorporate this model into the Double Tagging technique, σ(F) in Equation (6) has to be restructured. Since F is given by,

$$F = \frac{N_{Agree}}{N_{DT}} \tag{10}$$

$$= \frac{N_{Agree}}{N_{Agree} + N_{Disagree}}, \tag{11}$$

where N_{DT}, N_{Agree} and $N_{Disagree}$ are the total number of double tagged events and the number of double tagged events for which the OS and SS tagging decisions agree and disagree respectively. Carrying out an error propagation on Equation (11) gives:

$$\sigma(F)^2 = \frac{1}{N_{DT}^2}\left(\sigma(N_{Agree})^2(1-F)^2 + \sigma(N_{Disagree})^2 F^2\right) \ . \tag{12}$$

This is now inserted into Equation (6) to give:

$$\sigma(\omega_{SS})^2 = \frac{(1-2F)^2}{(1-2\omega_{OS})^4}\sigma(\omega_{OS})^2 + \frac{(\sigma(N_{Agree})^2(1-F)^2 + \sigma(N_{Disagree})^2 F^2)}{(1-2\omega_{OS})^2 N_{DT}^2} \tag{13}$$

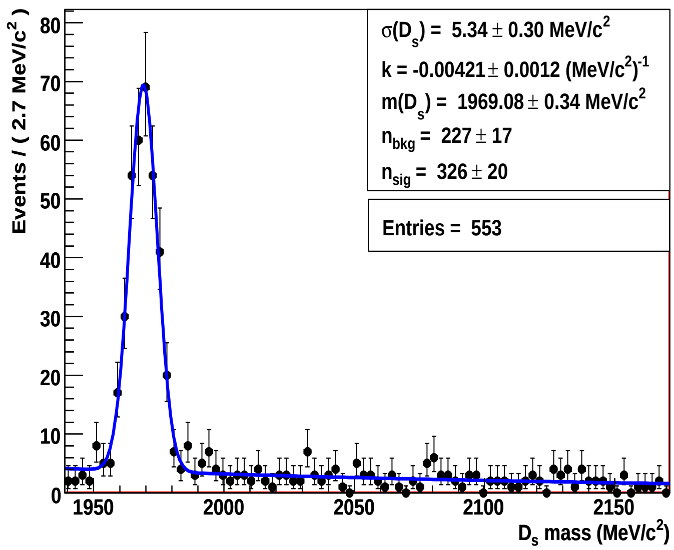

Figure 1: Model used for background analysis. In this example, the fit was performed on the total $B_s \rightarrow D_s\mu\nu$ candidates from the inclusive $b\bar{b}$ event sample.

Note that in order to obtain the appropriate value of F after taking into account background subtraction, the values of N_{Agree} and N_{DT} are obtained from the fit to the signal sample, i.e.:

$$F_{Signal} = \frac{(N_{Agree})\mid_{Signal}}{(N_{Agree} + N_{Disagree})\mid_{Signal}}, \tag{14}$$

which is then substituted into Equation (5) to give the correct value of ω_{SS}. Similarly, the error in $\sigma(\omega_{SS})$ is obtained from Equation (13) where F is taken from Equation (14) and $\sigma(N_{Agree})\mid_{Signal}$ and $\sigma(N_{Disagree})\mid_{Signal}$ are taken from the extended log likelihood fit to the data using the model in Equation (9). Note also that to obtain the above information, the data sample will need to be split into three separate samples of N_{DT}, $N_{Disagree}$ and N_{Agree} events and the fit applied to all three as shown in the example of Figure (1).

With respect to the $B_s \rightarrow D_s\pi$ channel, the absence of any appreciable background data from the inclusive $b\bar{b}$ event sample resulted in $(N_{Agree})\mid_{Signal}$ and $(N_{Disagree})\mid_{Signal}$ being estimated using a toy Monte Carlo generated sample of the reconstructed B_s mass. The sample was generated such that the B/S ratio of 0.05 [5] was satisfied under the mass peak. The fit was performed on a sample of 70,889 events. The results are summarized in Table 1.

With respect to the $B_s \rightarrow D_s\mu\nu$ channel, background analysis was carried out using an inclusive $b\bar{b}$ event sample of 34 million events. The events were analysed under the same conditions described for the signal MC sample with the exception that $Mass(D_s) > 1.940 GeV/c^2$ to remove events around the $D^{\pm}$ mass peak . No triggers were applied. 553 events passed the selection. The fit was not performed on the B_s mass because the absence of the neutrino in the reconstruction of the $B_s \to D_s\mu\nu$ channel results in a spread of the $D_s\mu$ mass peak. The fit was consequently performed on the D_s mass plots of N_{DT}, $N_{Disagree}$ and N_{Agree} events. The result is summarized in Table 1.

This study did not consider possible backgrounds from charm quark production.

$B_s \to D_s\pi$ Channel				
	Signal	Bkg	$\sigma(Mass(B_s))$ MeV/c^2	decay constant from Fit
Candidates	54329 ± 244	16559 ± 147	10.04 ± 0.03	$(-0.2\pm 0.1)\times10^{-4}$
N_{DT}	8461 ± 97	2491 ± 58	10.06 ± 0.09	$(-0.3\pm 0.2)\times10^{-3}$
N_{Agree}	4768 ± 71	1345 ± 40	9.86 ± 0.12	$(-0.1\pm 0.8)\times10^{-4}$
$N_{Disagree}$	3844 ± 64	1057 ± 36	9.76 ± 0.13	$(-0.1\pm 0.9)\times10^{-4}$
$F_{Signal} = 0.554 \pm 0.0055$				

$B_s \to D_s\mu\nu$ Channel						
	Signal		Bkg		$\sigma(Mass(D_s))$ MeV/c^2	decay constant from Fit
	MC	Fit	MC	Fit		
Candidates	286	326 ± 20	267	227 ± 17	5.34 ± 0.3	$(-0.4\pm 0.1)\times10^{-2}$
N_{DT}	46	57.2 ± 8.1	55	43.8 ± 7.3	5.34 ± 0.3	$(-0.2\pm 0.2)\times10^{-2}$
N_{Agree}	28	29.4 ± 5.7	27	25.6 ± 5.4	5.34 ± 0.3	$(-0.1\pm 0.8)\times10^{-2}$
$N_{Disagree}$	20	29.9 ± 6.0	35	25.1 ± 5.6	5.34 ± 0.3	$(-0.3\pm 0.3)\times10^{-2}$
$F_{Signal} = 0.496 \pm 0.0696$						

Table 1: Results from model fit applied to the $B_s \to D_s\pi$ and $B_s \to D_s\mu\nu$ channels respectively. The event sample for the $B_s \to D_s\pi$ channel was generated using a toy Monte Carlo. For the $B_s \to D_s\mu\nu$ channel, the inclusive $b\bar{b}$ event sample was used, and the fit to the N_{DT}, $N_{Disagree}$ and N_{Agree} distributions agreed with the MC to within 2σ. F_{Signal} was computed using Equations (14) and (12), i.e. after background subtraction.

4 Methodology

The use of the Double Tagging algorithm to measure the performance of the SS kaon tagging was carried out by using the OS tagging performance, $\omega_{OS} \pm \sigma(\omega_{OS})$ as input in Equations (5) and (6) to determine the SS wrong-tag fraction, $\omega_{SS(DT)} \pm \sigma(\omega_{SS(DT)})$. This was then compared to the MC value of SS kaon wrong-tag fraction, $\omega_{SS(MC)}$. The Double Tagging algorithm assumes that there is no correlation between OS and SS taggers. A correlations test was carried out to test for this. Finally, a projection was then carried out to determine the effectiveness of this technique taking into consideration background subtraction.

4.1 Opposite side tagging

In order to measure the SS kaon tagging performance, the known tagger, t_{OS}, utilised all the OS taggers available (muon, electron, kaon and vertex). Initially it was assumed the measurement could be done using the muon tagging only but preliminary studies showed that this did not provide adequate statistics. A new tool was written to perform the same combination as the Technical Design Report (TDR) combination [3] but only using the OS taggers. The performance of this combined tagging was measured on $B_s \to D_s\pi$ and $B_s \to D_s\mu\nu$ Monte Carlo signal sample and is summarised in Tables 2 and 3 respectively.

Total number of B candidates = 70740				
Inclusive Performance				
Tagger		ϵD^2 (%)	ϵ (%)	ω (%)
Muon		1.73 ± 0.09	11.36 ± 0.12	30.5 ± 0.5
Electron		0.61 ± 0.06	4.22 ± 0.08	31.0 ± 0.8
OS Kaon		2.23 ± 0.11	30.90 ± 0.17	36.6 ± 0.3
Vertex		1.33 ± 0.08	23.39 ± 0.16	38.1 ± 0.4
SS Kaon		3.23 ± 0.12	30.55 ± 0.17	33.7 ± 0.3
Exclusive Performance (OS Taggers)				
Category		ϵD^2 (%)	ϵ (%)	ω (%)
1	Muon only	1.20 ± 0.08	7.50 ± 0.10	30.0 ± 0.6
2	Electron only	0.44 ± 0.05	2.52 ± 0.06	29.1 ± 1.1
3	Kaon only	1.64 ± 0.09	25.7 ± 0.16	37.4 ± 0.4
4	Muon + Kaon	0.88 ± 0.06	2.27 ± 0.06	18.8 ± 1.0
5	Electron + Kaon	0.27 ± 0.03	0.80 ± 0.03	21.2 ± 1.7
6	Vertex only	0.38 ± 0.05	10.6 ± 0.12	40.5 ± 0.6
TDR Combined Performance (OS Taggers)				
		ϵD^2 (%)	ϵ (%)	ω (%)
		4.81 ± 0.15	49.36 ± 0.19	34.39 ± 0.23

Table 2: The tagging performance for the SS kaon tagger, all the OS taggers (muon, electron, kaon and vertex) and their categories for the $B_s \to D_s\pi$ Monte Carlo signal sample. The tagging performance was evaluated using the TDR combination method described in [3].

4.2 TDR combination

The performance of this algorithm is given in Tables 2 and 3 respectively. Each category is treated as a separate tagger. The motivation lies in the fact that the μ + k and e + k categories have a very low ω that is washed out when taking a "blind" average. Therefore there are six independent measurements, ω^i_{SS}, one for each category. The values are combined via the relations

$$\overline{\omega}_{SS} = \frac{\sum \frac{\omega^i_{SS}}{\sigma(\omega^i_{SS})^2}}{\sum \frac{1}{\sigma(\omega^i_{SS})^2}}, \quad \sigma(\overline{\omega}_{SS})^2 = \frac{1}{\sum \frac{1}{\sigma(\omega^i_{SS})^2}}. \tag{15}$$

Total number of B candidates = 3410				
Inclusive Performance				
Tagger		ϵD^2 (%)	ϵ (%)	ω (%)
Muon		0.79 ± 0.29	7.62 ± 0.45	33.8 ± 2.9
Electron		0.15 ± 0.13	4.46 ± 0.35	40.8 ± 3.9
OS Kaon		1.11 ± 0.36	32.32 ± 0.80	40.7 ± 1.5
Vertex		1.66 ± 0.43	32.05 ± 0.79	38.6 ± 1.5
SS Kaon		4.30 ± 0.67	33.25 ± 0.80	32.0 ± 1.4
Exclusive Performance (OS Taggers)				
Category		ϵD^2 (%)	ϵ (%)	ω (%)
1	Muon only	0.78 ± 0.28	4.66 ± 0.36	29.6 ± 3.6
2	Electron only	0.03 ± 0.05	2.49 ± 0.27	44.7 ± 5.4
3	Kaon only	0.89 ± 0.32	27.62 ± 0.77	40.9 ± 1.6
4	Muon + Kaon	0.22 ± 0.15	1.67 ± 0.22	31.6 ± 6.1
5	Electron + Kaon	0.21 ± 0.14	0.91 ± 0.16	25.8 ± 7.8
6	Vertex only	0.17 ± 0.14	13.31 ± 0.58	44.3 ± 2.3
TDR Combined Performance (OS Taggers)				
		ϵD^2 (%)	ϵ (%)	ω (%)
		2.32 ± 0.50	50.67 ± 0.85	39.29 ± 1.0

Table 3: The tagging performance for the SS kaon tagger, all the OS taggers (muon, electron, kaon and vertex) and their categories for the $B_s \to D_s \mu\nu$ cocktail Monte Carlo signal sample. The tagging performance was evaluated using the TDR combination method described in [3].

4.3 Correlations

The underlying assumption of the Double Tagging method is that the taggers are uncorrelated. This results in Equation (2). If the taggers are correlated, the value for ω_{SS} obtained will be incorrect. In order to achieve the correct value the correlations must be removed which can be done by modifying the OS taggers. This section describes a method for checking the OS and SS taggers for any correlations. This method requires the MC truth information and so can only be performed on MC data. Once the OS tagging is "de-correlated" from the SS tagging in MC data, it is assumed that no correlations exist in the data. In principle, correlations in the data could be checked by using a complex likelihood fit to oscillations.

The method uses the truth information in MC to determine the correct and incorrect tags. A matrix is constructed, called the "Double Tagging truth matrix" that represents the probability of events in which each tagger was correct and incorrect

$$\begin{pmatrix} P(t^R_{OS}t^R_{SS}) & P(t^R_{OS}t^W_{SS}) \\ P(t^W_{OS}t^R_{SS}) & P(t^W_{OS}t^W_{SS}) \end{pmatrix}. \tag{16}$$

In the case of no correlations, this matrix is given by

$$\begin{pmatrix} P(t^R_{OS}t^R_{SS}) & P(t^R_{OS}t^W_{SS}) \\ P(t^W_{OS}t^R_{SS}) & P(t^W_{OS}t^W_{SS}) \end{pmatrix} = \begin{pmatrix} (1-\omega_{OS})(1-\omega_{SS}) & (1-\omega_{OS})\omega_{SS} \\ \omega_{OS}(1-\omega_{SS}) & \omega_{OS}\omega_{SS} \end{pmatrix}. \tag{17}$$

However, if there are correlations present, the following relation is postulated

$$\begin{pmatrix} P(t^R_{OS}t^R_{SS}) & P(t^R_{OS}t^W_{SS}) \\ P(t^W_{OS}t^R_{SS}) & P(t^W_{OS}t^W_{SS}) \end{pmatrix} = \begin{pmatrix} (1-\omega_{OS}+\Delta)(1-\omega_{SS}-\Delta) & (1-\omega_{OS}+\Delta)(\omega_{SS}-\Delta) \\ (\omega_{OS}+\Delta)(1-\omega_{SS}-\Delta) & (\omega_{OS}+\Delta)(\omega_{SS}-\Delta) \end{pmatrix}. \tag{18}$$

The problem of looking for correlations becomes the task of searching for a non-zero Δ. In order to do this, the correlation matrix is determined and then a binned likelihood fit performed to determine Δ. If the observed Double Tagging truth matrix is denoted by O_{ij} and the expected matrix assuming correlations denoted by E_{ij}, the likelihood function is given by

$$L = \prod_{ij} \frac{e^{-E_{ij}} E_{ij}^{O_{ij}}}{O_{ij}!} \times \frac{e^{-N_T} N_T^{O_T}}{O_T!} \times \exp[-\frac{1}{2}\frac{(\omega_{OS}-\omega_{OS}^{MEASURED})^2}{\sigma(\omega_{OS}^{MEASURED})^2}], \tag{19}$$

where $O_T = \Sigma O_{ij}$ and $\omega_{OS}^{MEASURED}$ and $\sigma(\omega_{OS}^{MEASURED})$ are the OS tagging wrong-tag fraction and error. The free parameters in the fit are ω_{SS}, ω_{OS}, Δ and N_T. The first term in Equation (19) is the product of Poisson probabilities for observing O events given an average of E for each element of the matrix. The second term is the overall normalisation and is the Poisson probability for observing O_T given an average of N_T. The final term is a penalty term to include the prior knowledge of ω_{OS} into the likelihood.

The inclusive tagging performance ($\omega_{(OS)i}$ and ω_{SS}) in Tables 2 and 3 were used as input to obtain the Double Tagging truth matrix for the $B_s \to D_s\pi$ and $B_s \to D_s\mu\nu$ channels respectively.

5 Monte Carlo results

This section describes the results of the Double Tagging and correlation investigations on the current DC04 MC signal sample available.

5.1 Double Tagging results

The results from implementing the Double Tagging algorithm over the $B_s \to D_s\pi$ and $B_s \to D_s\mu\nu$ channels are shown in Table 4.

For $B_s \to D_s\pi$ events, the average value of the wrong-tag fraction, $\overline{\omega}_{SS}$, resulting from combining the individual categories using the TDR formulation was observed to be $(30.7 \pm 1.6)\%$, while the actual value measured using the MC truth information was $(33.7 \pm 0.3)\%$. This resulted in an under estimation of ω_{SS} MC by 1.9σ and was largely due to contribution from the under estimation in the OS Kaon category. In order to investigate the under estimation of ω_{SS} MC, the wrong-tag performance was investigated for each tagging category under various trigger conditions. The signal sample was used to investigate the performance under the L0 and L0×L1 trigger conditions, while the *CheatedSelection*, which is a reconstruction algorithm with less stringent preselection requirements that assumes all selected reconstructed events to be true signal, was used to investigate the tagging performance under the No L0,LI trigger condition. The results, which are shown in Table 5, indicate that while the underestimation of $\omega_{SS(MC)}$ appears to be independent of trigger selection, it is least for Muons and greatest for Kaons. There is also a consistent statistical effect for $\sigma(\omega_{SS(DT)})$ between tagging categories which appears to be independent of trigger selection. The scaling disparity of $\sigma(\omega_{SS(DT)})$ observed in the muon category was due to the richer muon content of the stripped decays compared to the CheatedSelection.

With respect to the $B_s \to D_s\mu\nu$ channel, due to poor statistics, all the categories were combined inclusively into a single category and a wrong-tag fraction of $(47.3 \pm 9.5)\%$ was obtained in comparison to the actual value measured using the MC truth information which was $(32.0 \pm 1.4)\%$ resulting in an overestimation of ω_{SS} MC by 1.6σ. The overestimation in this channel was investigated by looking at Double Tagging performances for various trigger conditions using the CheatedSelection to improve statistics. The result, summarised in Table 6, indicates that the overestimation could be due to low statistics in the signal sample.

To summarise, for both the $B_s \to D_s\pi$ and $B_s \to D_s\mu\nu$ channels, $\omega_{SS(DT)}$ was found to be compatible with $\omega_{SS(MC)}$ to within 2σ.

5.2 Correlation results

The measured Double Tagging truth matrices which are shown in Table 7 were used as input to perform a likelihood fit between OS and SS taggers. The results of the likelihood are shown in Table 8. The input $\omega_{OS}^{MEASURED}$ and the error were taken from the inclusive performance section of Tables 2 and 3. It can be seen that for the $B_s \to D_s\pi$ channel, the errors are large but all the values of Δ are consistent with zero except the OS kaon data fit which is non zero at 1.3 sigma.

With respect to the $B_s \to D_s\mu\nu$ channel, the errors are also large but all the values of Δ are consistent with zero. However, there is not enough statistics to draw any significant conclusion.

In order to investigate the presence of a non-zero Δ in the correlation fit between OS and SS kaon taggers observed in the $B_s \to D_s\pi$ channel, $\omega_{SS(MC)i}$ was obtained for the double-tagged data sample corresponding to each tagging category and compared to $\omega_{SS(DT)i}$, where, i refers to a given tagging category. The comparison was carried out for $B_s \to D_s\pi$ events passed through three trigger selections; L0, L1×L1 and No L0,L1.

The signal was used for the L0 and L0×L1 trigger selections, while the CheatedSelection was used for the No L0,L1 trigger selection. A likelihood fit was carried out to test for correlations under the same trigger conditions and the results are shown in Table 9. The results show that the effect of non-zero Δ appears to be trigger independent. Events passed through the No L0,L1 trigger show non-zero Δ in all categories, and only in the OS kaon category for the L0 and L0×L1 trigger selections which both have a smaller data sample indicating that the correlation could be dependent on statistics.

6 Projections

To determine the effectiveness of the Double Tagging technique, a projection was carried out by obtaining the number of double tagged events required to measure $\sigma(\omega_{SS})$ to varying degrees of accuracy. In the projections for the $B_s \to D_s\pi$ ($B_s \to D_s\mu\nu$) channel, the ω_{OS} value of 34.39% (39.29)% from Table 2(3) was used to obtain ω_{SS}. The total number of events were increased, and the number of double tagged events was then calculated using Equation (2) and then used to obtain plots of $\sigma(\omega_{SS})/\omega_{SS}$ illustrated in Figures (2)A,B,C and D respectively. The effect of background subtraction was taken into account by using F_{Signal}, $\sigma(N_{Agree})\mid_{Signal}$ and $\sigma(N_{Disagree})\mid_{Signal}$ from Table 1 to obtain $\sigma(\omega_{SS})$ using Equation (13). The error on F_{Signal}, $\sigma(F)_{Signal}$, was scaled to give $0.0021(2.14 \times 10^{-5})$ for the $B_s \to D_s\pi$ ($B_s \to D_s\mu\nu$) channel at $2fb^{-1}$.

Since it is envisaged that the real measurement of ω_{OS} in data will come from B^+/B_d mixing measurements, the variation of $\sigma(\omega_{OS})$ with luminosity was incorporated into the projection by using MC results from $B^+ \to \overline{D}^0\pi^+$ and $B^+ \to \overline{D}^0\mu^+\nu$ analysis [6]. Figures (2)A and C illustrate the dominance of the statistical component of $\sigma(\omega_{SS})$. Figures (2)B and D show that with $2fb^{-1}$ of data, i.e. one nominal year, ω_{SS} can be measured to an accuracy of 3.57(1.17)% for the $B_s \to D_s\pi(B_s \to D_s\mu\nu)$ channel. The error on this accuracy is negligible due to $\sigma(F)_{Signal}$ being very small.

In carrying out the projections, it was assumed that for the $B_s \to D_s\pi$ channel, the number of events produced in $2fb^{-1}$ is 140,000 using estimates from [5] in which events were passed through L0×L1×HLT triggers. In this study however, the events were only passed through L0×L1. The effect of including the HLT trigger would be a slight decrease in tagging efficiency and therefore a decrease in the total number of double tagged events estimated. However, the estimate of 140,000 events was used for this analysis because this decrease is not expected to be significant. Also the HLT trigger is not expected to significantly affect the wrong-tag performance. With respect to the $B_s \to D_s\mu\nu$ channel, it was assumed that 1.06 million events are produced in $2fb^{-1}$ of data [4]. The error on the estimates for both channels were not taken into account in this analysis.

It is clear from comparing the projections in Figures (2)B and D that the $B_s \rightarrow D_s\mu\nu$ channel offers a greater potential to measure ω_{SS} in data with greater accuracy and with less LHCb run time compared to the $B_s \rightarrow D_s\pi$ channel. In which case, results from implementation of the Double Tagging technique could be used to complement B meson oscillation amplitude fitting studies.

7 Discussion

In deriving the model that was used to implement background subtraction, it was assumed that the background decays exponentially with reconstructed mass. With respect to the $B_s \rightarrow D_s\mu\nu$ channel, with the availability of data from the $b\bar{b}$ inclusive sample, the BackgroundCategory tool [7] was used to quantify the various background contributions. Table 10 shows the breakdown of the signal and main background contributions under the D_s mass distribution.

The signal events were mainly events of type $B_s \rightarrow D_s^*\mu\nu$ (under the D_s mass peak) or $B_s \rightarrow D^{*0}(K\pi X)K\mu\nu$ (outside the D_s mass peak) where the $KK\pi$ is reconstructed as a D_s.

The background event were of three types: The Partially Reconstructed Background were events of type $B \rightarrow DDX$ where a $D^{\pm}$ is reconstructed as a ${D_s}^{\pm}$ due to K/π mis-identification. This background was observed to have a distribution that was fairly flat and therefore can be considered not problematic. The Low Mass Background, which were events of type $B \rightarrow DDX$, where the B meson decays into a D_s and a $D^{\pm}/D^0/D^*$ and the Combinatoric background, which were primarily composed of reconstructed B mesons in which a ${D^{\mp}}_s$ was combined with a random $\mu^{\pm}$, were both observed to have peaking distributions under the D_s mass peak.

The presence of peaking background will introduce a systematic uncertainty on the accuracy with which ω_{SS} is measured. In order to estimate this uncertainty, the sum of the peaking backgrounds (63) was scaled to $2fb^{-1}$. Two conservative assumptions were made: It was assumed that the peaking background in the MC data is estimated with an accuracy set arbitrarily to 10%. It was also assumed that for the Double Tagging of the peaking background, N_{Agree} is equal to $N_{Disagree}$. Therefore, a wrong estimate of the peaking background will lead to background being assumed as signal or vice versa. With these assumptions, the shift in the central value of ω_{SS}, $\Delta(\omega_{SS})$, was seen to give $\frac{\Delta(\omega_{SS})}{\omega_{SS(MC)}}$ = 0.012, i.e. a systematic uncertainty of 1.2%. When combined in quadrature with the statistical uncertainty of 1.17% obtained in Section 6, this results in an upper bound of 1.7% on the accuracy with which ω_{SS} is measured.

With respect to the $(B_s \rightarrow D_s\pi)$ channel, in the absence of any peaking backgrounds, the systematic effect is expected to be very small.

8 Conclusion

This analysis shows that the Double Tagging method can be used to measure the SS tagging performance in data effectively. Projections show that the $B_s \rightarrow D_s\mu\nu$ channel has a much greater accuracy compared to the $B_s \rightarrow D_s\pi$ channel due to its higher yield. In comparison with results from flavour oscillation amplitude fit studies [5] which show that for the $B_s \rightarrow D_s\pi$ channel, the total wrong-tag fraction can be measured to an accuracy of 1.1%, this study shows that the SS wrong-tag fraction, which accounts for 30% of the total tagging power, can be measured to a statistical accuracy of 3.6% and 1.2% for the $B_s \rightarrow D_s\pi$ and $B_s \rightarrow D_s\mu\nu$ channels respectively. These results were obtained with limited statistics, in particular the $B_s \rightarrow D_s\mu\nu$ channel. Thus they will need to be confirmed with more statistics.

The systematic uncertainty for the $B_s \rightarrow D_s\pi$ channel is expected to be very small, while for the $B_s \rightarrow D_s\mu\nu$ channel, it was estimated to be 1.2%, thus giving an upper bound of 1.7% on the accuracy with which the SS wrong-tag fraction can be measured. With respect to the $B_s \rightarrow D_s\pi$ channel, the Double Tagging method provided an uncertainty on ω_{SS} that was ≈ 2 times larger than the flavour oscillation amplitude method, while for the $B_s \rightarrow D_s\mu\nu$ channel, the uncertainty on ω_{SS} was similar in both methods. In addition, the Double Tagging method is not dependent on a detailed understanding of the lifetime resolution model.

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$B_s \to D_s\pi$ Channel				
	Input ω_{OS}	N_{DT}	N_{AGREE}	Output ω_{SS}
Muon	30.0 ± 0.6	1598	902	33.9 ± 3.1
Electron	29.0 ± 1.1	569	327	30.9 ± 5.1
Kaon	37.4 ± 0.4	5768	3207	27.8 ± 2.7
Muon + Kaon	18.9 ± 1.0	486	303	30.2 ± 3.6
Electron + Kaon	21.2 ± 1.7	172	103	32.9 ± 6.6
Vertex	40.5 ± 0.6	2343	1252	32.0 ± 5.5
Final result: $\overline{\omega}_{SS}$ = 30.7 ± 1.6				
$B_s \to D_s\mu\nu$ Channel				
	Input ω_{OS}	N_{DT}	N_{AGREE}	Output ω_{SS}
	39.3 ± 1.0	611	309	47.3 ± 9.5

Table 4: The Double Tagging performance for $B_s \to D_s\pi$ and $B_s \to D_s\mu\nu$ channels respectively using the MC signal samples and assuming that all reconstructed candidates are signal. Results for the $B_s \to D_s\pi$ channel were obtained using the category approach and used as input $\omega_{(OS)i}$ from Table 2. Due to poor statistics, all categories in the $B_s \to D_s\mu\nu$ channel were combined inclusively and used the TDR combined $\omega_{(OS)}$ from Table 3 as input.

Wrong-tag Performance for $B_s \to D_s\pi$ Channel						
Trigger Selection	L0		L0×L1		No L0,L1	
Candidate Events	92,941		70,740		319,970	
	$\omega_{SS(MC)}$	$\omega_{SS(DT)}$	$\omega_{SS(MC)}$	$\omega_{SS(DT)}$	$\omega_{SS(MC)}$	$\omega_{SS(DT)}$
Muon	35.8	33.3 ± 2.8	35.4	33.7 ± 3.1	34.6	33.1 ± 1.6
Electron	35.5	32.6 ± 4.6	34.6	31.0 ± 5.0	35.6	36.8 ± 1.9
Kaon	33.2	26.7 ± 2.4	33.3	27.8 ± 2.6	35.0	30.2 ± 1.1
Vertex	31.7	27.5 ± 5.1	33.0	32.4 ± 5.5	34.6	39.8 ± 2.6

Table 5: Comparing the wrong-tag performance in the $B_s \to D_s\pi$ channel for various categories. $\omega_{SS(MC)i}$ refers to the measured SS wrong-tag fraction from Monte Carlo signal sample in a given category, while $\omega_{SS(DT)i}$ refers to the measured SS wrong-tag fraction obtained using the Double Tagging technique for the same category.

Double Tagging Performance Summary ($B_s \to D_s\mu\nu$)						
	Selection	Trigger	Mass Cuts	Candidates	$\omega_{SS(MC)}$	$\overline{\omega}_{SS(DT)}$
Signal	$\surd$	L0×L1	$\surd$	3410	32.0 ± 1.4	47.3 ± 9.5
Cheated.	×	L0	×	12756	36.2 ± 0.8	37.3 ± 4.9
Cheated.	×	L1	×	5213	34.5 ± 1.2	37.2 ± 6.3
Cheated.	×	L0×L1	$\surd$	4680	34.5 ± 1.3	36.5 ± 6.4

Table 6: Comparison between $\omega_{SS(MC)}$ and $\overline{\omega}_{SS(DT)}$ obtained using the Double Tagging technique for various trigger selections, where Cheated. refers to the *CheatedSelection*.

Observed elements of truth matrix				
	O_{00}	O_{01}	O_{10}	O_{11}
$B_s \to D_s\pi$ Channel				
Muon	1110	569	493	285
Electron	415	226	184	122
OS Kaon	2891	1445	1624	970
Vertex	2147	1075	1303	727
$B_s \to D_s\mu\nu$ Channel				
Muon	35	28	30	9
Electron	20	10	15	9
OS Kaon	145	91	109	50
Vertex	159	83	99	47

Table 7: Measured Double Tagging truth matrices for the $B_s \to D_s\pi$ and $B_s \to D_s\mu\nu$ channels respectively.

Correlation Test Results				
	ω_{SS}	ω_{OS}	Δ	N_T
$B_s \to D_s\pi$ Channel				
Muon	35.7 ± 1.3	30.5 ± 0.5	3.1 ± 2.9	2462 ± 36
Electron	37.7 ± 1.9	31.0 ± 0.8	3.8 ± 4.8	949 ± 23
OS Kaon	35.8 ± 0.9	36.6 ± 0.3	3.2 ± 2.5	6943 ± 63
Vertex	35.1 ± 1.2	38.1 ± 0.4	2.5 ± 3.3	5258 ± 54
$B_s \to D_s\mu\nu$ Channel				
Muon	38.9 ± 5.1	34.3 ± 2.7	9.4 ± 12.1	104 ± 9
Electron	37.7 ± 8.2	41.3 ± 3.6	8.3 ± 20.7	55 ± 7
OS Kaon	35.1 ± 4.5	40.7 ± 1.5	-2.2 ± 13.2	395 ± 15
Vertex	32.3 ± 4.1	38.6 ± 1.4	-3.6 ± 9.7	389 ± 15

Table 8: Correlation test results obtained after likelihood fit to the Double Tagging truth matrices in Table 7 for $B_s \to D_s\pi$ and $B_s \to D_s\mu\nu$ channels respectively.

Correlation Test under various Trigger Selections			
Trigger Selection	L0	L0+L1	No L0,L1
Total Tagged Events	92,941	70,740	319,970
Muon	2.8 ± 2.9	3.1 ± 2.9	3.6 ± 1.5
Electron	1.8 ± 4.8	3.8 ± 4.8	3.3 ± 1.9
OS Kaon	3.1 ± 2.6	3.2 ± 2.5	2.2 ± 1.1
Vertex	-0.1 ± 3.3	2.5± 3.3	1.8 ± 1.4

Table 9: Likelihood fit showing the Δ values for $B_s \to D_s\pi$ events passed through various trigger selections.

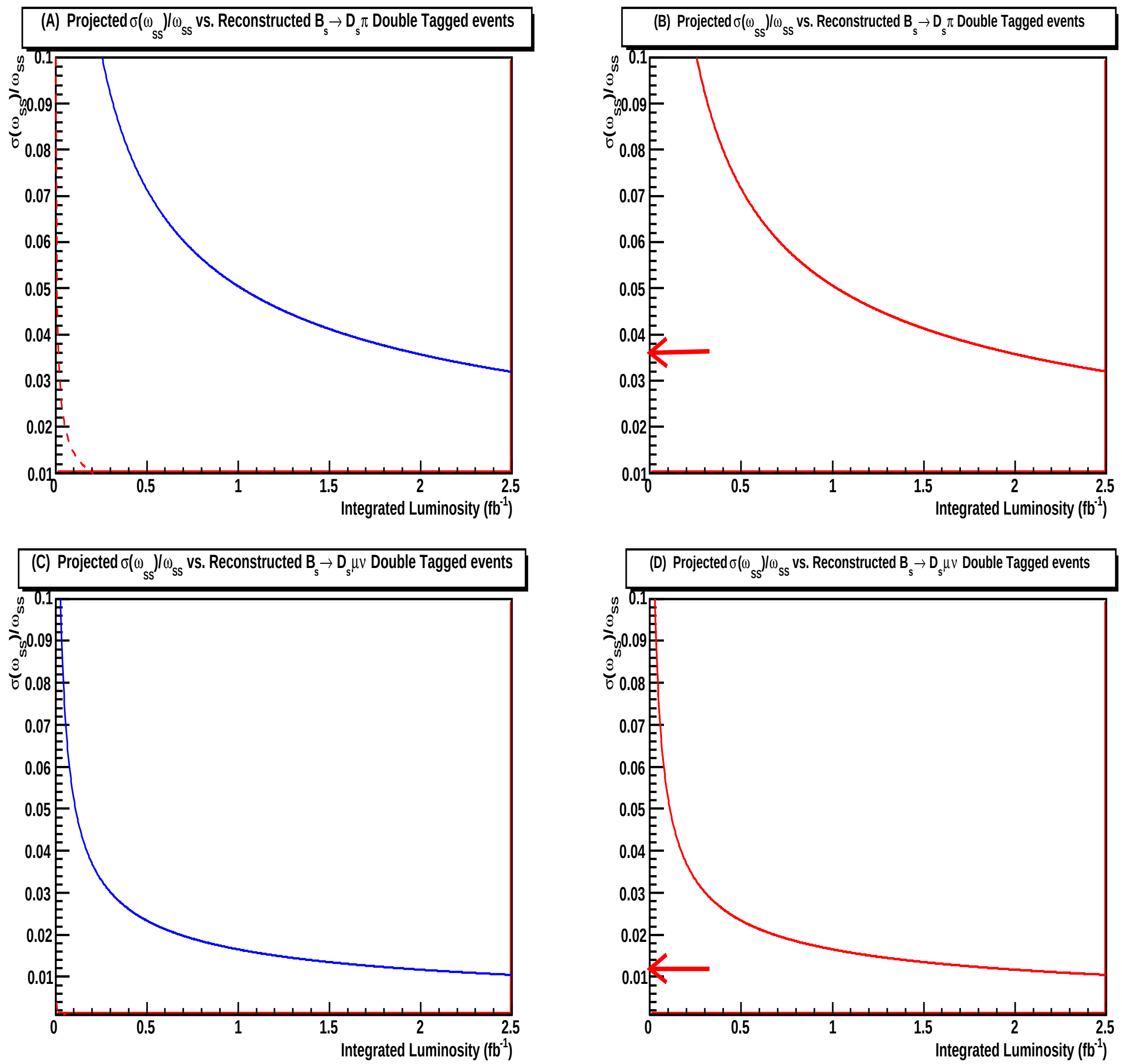

Figure 2: Projections using the Double Tagging technique for the $B_s \to D_s\pi$ and $B_s \to D_s\mu\nu$ channels after background subtraction. Figs. A and C illustrate the effect of the component parts of $\sigma(\omega_{SS})$ in measuring ω_{SS}. $\sigma(\omega_{SS})$ was computed using F_{Signal}, $\sigma(N_{Agree})\mid_{Signal}$ and $\sigma(N_{Disagree})\mid_{Signal}$ from Table 1. The statistical component (in blue) is seen to be clearly dominant, while the error from the knowledge of the OS taggers (in dotted red) is negligible. Figs. B and D show that with $2fb^{-1}$ of data, ω_{SS} is measured to an accuracy of 3.6% and 1.2% respectively for the $B_s \to D_s\pi$ and $B_s \to D_s\mu\nu$ channels.

	A	B
Signal	286	269
Background	267	119
Partially Reconstructed Background	77	34
Low Mass Background	47	35
Combinatorials	56	28

Table 10: Breakdown of Signal and main Backgrounds of selected $B_s \to D_s\mu\nu$ candidate events from the inclusive $b\overline{b}$ event sample using the BackgroundCategory tool [7]. Region A covered the entire D_s mass region, i.e. $Mass(D_s) > 1.9GeV/c^2$ while region B covered a 40MeV mass window under the D_s peak, i.e. $(1.944 < Mass(D_s) < 1.988)GeV/c^2$