Published by Institute of Physics Publishing for SISSA

Received: *November 28, 2007*
Revised: *January 31, 2008*
Accepted: *February 18, 2008*
Published: *February 22, 2008*

Separation of attractors in 1-modulus quantum corrected special geometry

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Abstract: We study the attractor equations for a quantum corrected prepotential $\mathcal{F} = t^3 + i\lambda$, with $\lambda \in \mathbb{R}$,which is the only correction which preserves the axion shift symmetry and modifies the geometry.

By performing computations in the "magnetic" charge configuration, we find evidence for interesting phenomena (absent in the classical limit of vanishing λ). For a certain range of the quantum parameter λ we find a *"separation"* of attractors, i.e. the existence of multiple solutions to the Attractor Equations for fixed supporting charge configuration. Furthermore, we find that, away from the classical limit, a *"transmutation"* of the supersymmetry-preserving features of the attractors takes place when λ reaches a particular critical value.

Keywords: D-branes, Black Holes in String Theory, Supergravity Models.

Contents

1. Introduction

After the discovery of the *Attractor Mechanism* in the mid 90's [1]-[5] in the context of BPS black holes (BHs), recently *extremal BH attractors* have been object of intensive study [6]–[49]. This is mainly due to the (re)discovery of new classes of scalar attractor configurations, which do not saturate the BPS bound and, when considering a supergravity theory, break all supersymmetries at the BH event horizon.

The various classes of BPS and non-BPS attractors are strictly related to the geometry of the scalar manifold. In $\mathcal{N} = 2$, $d = 4$ ungauged supergravity coupled to n_V Abelian vector multiplets, the scalar manifold is endowed with the so-called Special Kähler (SK) geometry (see e.g. [50] and refs. therein), in which the holomorphic prepotential function $\mathcal{F}$ plays a key role. In general, SK geometry admits three classes of extremal BH attractors: $\frac{1}{2}$-BPS (preserving four supersymmetries out of the eight pertaining to asymptotical $\mathcal{N} = 2$, $d = 4$ superPoincaré algebra, and two non-supersymmetric typologies, discriminated by the eventual vanising of the $\mathcal{N} = 2$ central charge function Z: non-BPS $Z \neq 0$ and non-BPS $Z = 0$ (see e.g. [21] for a detailed analysis in the case of symmetric SK geometries).

When considering $\mathcal{N} = 2$, $d = 4$ supergravity coming from a string compactification, the perturbative quantum corrections to the holomorphic prepotential of SK geometries can be polynomials or some non-polynomial (usually polylogarithmic) functions of the moduli (see e.g. [53] and refs. therein). In the case of cubic classical SK geometries, such as the ones obtained in the large volume limit of CY_3-compactifications of Type IIA, the sub-leading nature of the quantum corrections constrains the most general polynomial correction to be at most of degree two in the moduli, with a priori complex coefficients.

In [51] it has been shown that the only polynomial quantum perturbative correction to the prepotential of classical cubic SK geometries which is consistent with the perturbative (continuous) axion-shift symmetry [52] is the constant purely imaginary term ($i = 1, \ldots, n_V$ throughout):

$$\mathcal{F}_{\text{classical}} = d_{ijk} t^i t^j t^k \longrightarrow \mathcal{F}_{quantum-pert.} = d_{ijk} t^i t^j t^k + i\lambda,\ \ \lambda \in \mathbb{R}, \tag{1.1}$$

where d_{ijk} is the real, constant, completely symmetric tensor defining the cubic geometry. Indeed, it can be easily shown that all other polynomial perturbative corrections (quadratic, linear and real constant terms in the moduli) do not modify the classical cubic geometry, since they do not contribute to the Kähler potential at all [51].

Let us start by considering the holomorphic prepotential of a certain class of compactifications of the heterotic $E_8 \times E_8$ superstring over $K_3 \times T^2$, having the form (see e.g. [53] and refs. therein)

$$\mathcal{F}^{\text{heterotic}} = stu - s \sum_{a=4}^{n_V} \left(\widetilde{t}^a\right)^2 + h_{1-loop}\left(t, u, \widetilde{t}\right) + f_{non-pert.}\left(e^{-2\pi s}, t, u, \widetilde{t}\right). \tag{1.2}$$

Since in this case the dilaton s belong to a vector multiplet, there exist (T-symmetric) quantum perturbative string-loop corrections and non-perturbative corrections, as well. The tree-level, classical term $stu - s\sum_{a=4}^{n_V}\left(\widetilde{t}^a\right)^2$ is the prepotential of the generic cubic sequence $\frac{\mathrm{SU}(1,1)}{U(1)} \otimes \frac{SO(2+n,2)}{SO(2+n)\otimes SO(2)}$ ($n_V = n+3$) of homogeneous symmetric SK manifolds (see e.g. [51], and [21] and refs. therein), in the symplectic gauge exhibiting the largest possible amount of explicit symmetry $SO\left(n+1,1\right)$. Due to non-renormalization theorems, all the quantum perturbative string-loop corrections are encoded in the 1-loop contribution h_{1-loop}, which contains a constant term, a purely cubic polynomial term and a polylogarithmic part (see e.g. [53] and refs. therein). Finally, $f_{non-pert.}$ encodes the non-perturbative corrections, exponentially suppressed in the limit $s \to \infty$.

By exploiting the Type IIA/heterotic duality and correspondingly by suitably identifying the relevant moduli fields, the heterotic prepotential (1.2) becomes structurally identical to the one arising from Type IIA compactifications over Calabi-Yau threefolds (CY_3s). In such a case, the prepotential of the resulting low-energy $\mathcal{N} = 2$, $d = 4$ supergravity is purely of classical origin. Indeed, there are only Kähler structure moduli, and the dilaton s belongs to an hypermultiplet; thus, there are no string-loop corrections, and all corrections to the large-volume limit cubic prepotential come from the world-sheet sigma-model [54]. As shown in [55 − 57], there are no 1-, 2- and 3-loop contributions. Moreover, the non-perturbative, world-sheet instanton corrections destroy continuous axion-shift symmetry, by making it discrete [52].

By disregarding such non-perturbative world-sheet instanton corrections, the Type IIA prepotential can be written as follows ($n_V = h_{1,1}$) [56 − 58, 53]:

$$\mathcal{F}^{IIA} = \frac{1}{3!}\mathcal{C}_{ijk} t^i t^j t^k + \mathcal{W}_{0i} t^i - i\frac{\chi\zeta\left(3\right)}{16\pi^3}. \tag{1.3}$$

The $\mathcal{C}_{ijk}$ are the real classical intersection numbers, determining the large volume limit cubic SK geometry. The perturbative contributions from 2-dimensional CFT on the world-sheet are encoded in a linear and in a constant term.

The linear term is determined by the $\mathcal{W}_{0i} = \frac{1}{4!}c_2 \cdot J_i = \frac{1}{4!}\int_{CY_3} c_2 \wedge J_i$, which are the real expansion coefficients of the second Chern class c_2 of CY_3 with respect to the basis J_i^* of the cohomology group $H^4(CY_3, \mathbb{R})$, dual to the basis of the (1,1)-forms J_i of the cohomology $H^2(CY_3, \mathbb{R})$. It has been shown that the linear term $\mathcal{W}_{0i}t^i$ can be reabsorbed by a suitable symplectic transformation of the period vector, and thus in the heterotic picture it has just the effect of a constant shift in $\mathtt{Im}\, s$ ([59]; see also [53]).

From the general analysis of [51], the constant term is the only relevant one. It is determined by the Euler character[1] χ of CY_3 (ζ is the Riemann zeta-function), and it has a 4-loop origin in the non-linear sigma-model [54 – 57].

It is worth pointing out here that CY_3-compactifications of Type IIB do not admit a large volume limit; moreover, the Attractor eqs. only depend on the complex structure moduli (which are the scalars of the $\mathcal{N} = 2$ vector multiplets). The solutions to $\mathcal{N} = 2$, $d = 4$ Attractor eqs. for the resulting SK geometries were studied in [26] for the particular class of Fermat CY_3s with $n_V = 1$, and in [24] for a particular CY_3 with $n_V = 2$.

Aim of the present paper is to study the solutions to the $\mathcal{N} = 2$, $d = 4$ Attractor eqs. in a dyonic background in the simplest case of perturbative quantum corrected cubic SK geometry, namely in the 1-modulus SK geometry described (in a suitable special symplectic coordinate) by the holomorphic Kähler gauge-invariant prepotential $\mathcal{F} = t^3 + i\lambda$, with $\lambda \in \mathbb{R}$ (see eq. (3.1) below). By doing so, we will extend the BPS analysis of [53].

The plan of the paper is as follows.

In section 2 we shortly review the classical so-called t^3 model, focussing on the attractors supported by the "magnetic" BH charge configuration, in which the charge vector $Q \equiv \left(q_0, q_1, p^0, p^1\right) = \left(q_0, 0, 0, p^1\right)$. Thence, in section 3 we will study the solutions to $\mathcal{N} = 2$, $d = 4$ Attractor eqs. for the $t^3 + i\lambda$ model, dividing our analysis in the classical BPS (Subsection 3.1) and non-BPS (Subsection 3.2) charge domains. Despite the simplicity of the correction added to the classical prepotential $\mathcal{F} = t^3$ (see eq. (2.1) below), we will find evidence for interesting phenomena, such as the *"separation"* and the *"transmutation" of extremal BH attractors*. Finally, in Subsection 3.3 we briefly consider the so-called $D0 - D6$ BH charge configuration $Q = \left(q_0, 0, p^0, 0\right)$, pointing out that it does not support admissible BPS attractors, both at the classical and quantum level. An appendix, containing some technical details, concludes the paper.

2. t^3 model

The classical so-called t^3 model is a 1-modulus model based on a cubic holomorphic pre-

[1]It is worth remarking that for typical CY_3s $|\chi| \leqslant 10^3$, and thus $\frac{\chi\zeta(3)}{16\pi^3}$ is of order 1. Notice that $\chi = 0$ for *self-mirror* CY_3s, and thus such a constant term vanishes. Moreover, for some particular *self-mirror* models, such as the so-called FHSV one [60], also the non-perturbative, world-sheet instanton corrections vanish; thus, in such models, up to suitable symplectic transformations of the period vector, the classical cubic prepotential does not receive any perturbative and non-perturbative correction.

potential which (in the local special symplectic coordinate t) reads

$$\mathcal{F}(t) = t^3, \tag{2.1}$$

constrained by the condition

$$\mathtt{Im}\, t < 0. \tag{2.2}$$

The corresponding manifold is the rank-1 symmetric special Kähler (SK) space $\frac{SU(1,1)}{U(1)}$. This is an isolated case in the classification of the symmetric SK manifolds (see e.g. [51], and [21] and refs. therein). Furthermore, it is worth pointing out that $\frac{SU(1,1)}{U(1)}$ can be endowed with a Kähler gauge-invariant quadratic prepotential $\mathcal{F} = \frac{i}{4}\left(t^2 - 1\right)$, as well; in such a case, it corresponds to the $n = 0$ element of the sequence of symmetric irreducible quadratic SK manifolds $\frac{\mathrm{SU}(1,1+n)}{\mathrm{U}(1)\otimes\mathrm{SU}(1+n)}$ ($n = n_V - 1$, see e.g. [51], and [21] and refs. therein), with geometric properties completely different from the cubic case (see e.g. the discussion in [29]). The reason for such a difference lies in the fact that the four charges q_0, q_1, p^0, p^1 sit in the spin $\frac{3}{2}$ real representation of $SU\,(1,1)$ in the former case, and in the spin $\frac{1}{2}$ complex representation of $SU\,(1,1)$ in the latter case.

At the present time, the t^3 model is the only model whose $\mathcal{N} = 2$, $d = 4$ Attractor eqs. have been solved for a completely generic BH charge configuration $\left(q_0, q_1, p^0, p^1\right)$. Its $\frac{1}{2}$-BPS solutions were known after [61], whereas in [10] its non-BPS solutions with non-vanishing $\mathcal{N} = 2$ central charge Z were determined for $q_1 = 0$. Recently, in [32] such a result was extended to the general case $q_1 \neq 0$.

Since such a model does not admit non-BPS $Z = 0$ attractors, in the following "non-BPS" will be understood for "non-BPS $Z \neq 0$". Furthermore, we will always consider the simple case corresponding to the so-called "magnetic" charge configuration, in which the only non-vanishing BH charges are q_0 and p^1 (in a stringy interpretation, this corresponds to consider only $D0$ and $D4$ branes).

In the remaining of this section we will report the main known results about t^3 model in "magnetic" BH charge configuration, in order to make the comparison with the quantum-perturbed case easier.

The Kähler potential and metric function are given by

$$K = -ln\left[-8(\mathtt{Im}\, t)^3\right], \qquad g \equiv \partial_t\overline{\partial}_{\overline{t}}K = \frac{3}{4(\mathtt{Im}\, t)^2}; \tag{2.3}$$

thus the condition (2.2) is nothing but the reality condition for K. Notice that $g > 0$ for $\mathtt{Im}\, t \neq 0$, and in particular in the lower half of the Argand-Gauss plane $\mathbb{C}$ determined by (2.2). The superpotential, its covariant derivative and the effective black hole (BH) potential are respectively given by

$$\begin{aligned} &W = q_0 - 3p^1t^2, \qquad D_tW = -6p^1t + \frac{3i}{2\mathtt{Im}\, t}W; \\ &V_{BH} = -\frac{3\left(\left(\mathtt{Im}\, t\right)^2 p^1\right)^2 + 12\left(p^1\mathtt{Im}\, t\,\mathtt{Re}\, t\right)^2 + \left(q_0 - 3p^1\left(\mathtt{Re}\, t\right)^2\right)^2}{2\mathtt{Im}\, t^3}. \end{aligned} \tag{2.4}$$

The BPS and non-BPS solutions to the Attractor eq.

$$\frac{\partial V_{BH}}{\partial t} = 0 \tag{2.5}$$

are always stable, and they are supported by BH charges satisfying $p^1 q_0 > 0$ and $p^1 q_0 < 0$, respectively[2] [61, 10, 32]. They are both axion-free:

$$\mathtt{Re}\, t|_{cr.} = 0, \qquad \mathtt{Im}\, t|_{cr.} = \sqrt{\left|\frac{q_0}{p^1}\right|}, \tag{2.6}$$

and the corresponding BH entropy reads

$$S_{BH} = \pi V_{BH,cr.} = 2\pi \sqrt{\left|q_0 \left(p^1\right)^3\right|}. \tag{2.7}$$

3. $t^3 + i\lambda$ model

From the considerations made in the Introduction, it follows that the most general form of 1-modulus cubic geometry with polynomial quantum perturbative corrections consistent with axion-shift symmetry is based on the holomorphic Kähler gauge invariant prepotential function

$$\mathcal{F}(t) = t^3 + i\lambda, \ \lambda \in \mathbb{R}. \tag{3.1}$$

Let us start by noticing that the SK manifold based on the prepotential (3.1) is no more symmetric nor homogeneous. Its Kähler potential and metric function have the following form:

$$K = -ln\left[-4\lambda - 8\left(\mathtt{Im}\, t\right)^3\right], \qquad g = 3\frac{\left(\left(\mathtt{Im}\, t\right)^3 - \lambda\right)\mathtt{Im}\, t}{\left(\lambda + 2(\mathtt{Im}\, t)^3\right)^2}. \tag{3.2}$$

In the "magnetic" BH charge configuration, the superpotential, its covariant derivative and the effective BH potential respectively reads

$$W = q_0 - 3p^1 t^2, \quad D_t W = -6p^1 t + \frac{3i\left(\mathtt{Im}\, t\right)^2 W}{\lambda + 2\left(\mathtt{Im}\, t\right)^3}, \qquad V_{BH} = e^K\left[W\bar{W} + g^{-1} D_t W \overline{D_t W}\right]. \tag{3.3}$$

When switching $\lambda \neq 0$, the reality condition (2.2) on the Kähler potential gets modified as follows:

$$\mathtt{Im}\, t < -\sqrt[3]{\frac{\lambda}{2}}. \tag{3.4}$$

Furthermore, the positivity of the metric function yields another restriction on allowed values of $\mathtt{Im}\, t$:

$$\left(\left(\mathtt{Im}\, t\right)^3 - \lambda\right)\mathtt{Im}\, t > 0. \tag{3.5}$$

Thus, $\mathtt{Im}\, t$ belong to the shaded regions represented in figure 1. If $\lambda > 0$, the allowed region consists of a single interval, otherwise it is the union of two non-overlapping intervals:

$$\begin{aligned} \lambda > 0: \qquad & \mathtt{Im}\, y \in \left(-\infty, -\sqrt[3]{\tfrac{\lambda}{2}}\right) \\ \lambda < 0: \qquad & \mathtt{Im}\, y \in \left(-\infty, \sqrt[3]{\lambda}\right) \cup \left(0, -\sqrt[3]{\tfrac{\lambda}{2}}\right). \end{aligned}$$

[2]It is easy to recognize $p^1 q_0 > 0$ and $p^1 q_0 < 0$ respectively as the "magnetic" branches of the 2 homogeneous symmetric BH charge BPS and non-BPS $Z \neq 0$ orbits of the symplectic vector representation space of the U-duality group $SU(1,1)$ of the t^3 model (see appendix II of [21]).

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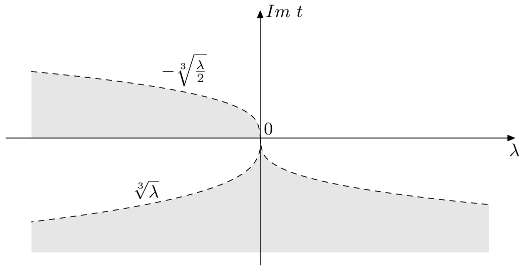

Figure 1: Admissible regions for $\mathtt{Im}\,t$.

It is instructive to consider the λ-dependence of the Ricci scalar curvature R. Whereas in the classical case $R = -\frac{2}{3}$, in the considered quantum case R is not constant nor necessarily negative anymore:

$$R = \frac{\lambda^4 + 32\lambda^3 (\mathtt{Im}\,t)^3 - 48\lambda^2 (\mathtt{Im}\,t)^6 + 104\lambda (\mathtt{Im}\,t)^9 - 8 (\mathtt{Im}\,t)^{12}}{12 (\mathtt{Im}\,t)^3 (-\lambda + (\mathtt{Im}\,t)^3)^3}. \tag{3.6}$$

For $\lambda > 0$ the $\mathtt{Im}\,t \to \infty$ limit gives $R = -2/3$; on the other hand, R decreases approaching the boundary value $\mathtt{Im}\,t = -\sqrt[3]{\lambda/2}$, where it equals -2.

For $\lambda < 0$, R increases starting from its value at $\mathtt{Im}\,t \to -\infty$, and it diverges in the limit $\mathtt{Im}\,t \to \sqrt[3]{\lambda}^{-}$. In the other interval $\left(0, -\sqrt[3]{\frac{\lambda}{2}}\right)$, R diverges in the limit $\mathtt{Im}\,t \to 0^+$, whereas it reaches its minimal value -2 at the boundary $\mathtt{Im}\,t = -\sqrt[3]{\frac{\lambda}{2}}$.

Concerning the effective BH potential, it is a real function of one complex variable t, and it contains three parameters: the charges q_1 and p^0 and the quantum parameter λ. It enjoys the following properties with respect to reflection of its arguments:

$$V_{BH}(-t; p^1, q_0, -\lambda) = -V_{BH}(t; p^1, q_0, \lambda), \qquad V_{BH}(t; -p^1, -q_0, \lambda) = V_{BH}(t; p^1, q_0, \lambda). \tag{3.7}$$

In order to decrease the number of independent parameters and simplify the analysis, in the following treatment we will redefine t and λ in such a way to factorize the dependence of V_{BH} on the charges.

The second reflection property shows that V_{BH} is somehow sensitive to the sign of $p^1 q_0$. This fact, in light of the form of the "magnetic" supporting BH charge orbits in the classical case, leads us to divide the treatment of the quantum model in two parts, respectively corresponding to:

1. $p^1 q_0 > 0$ and refer to this range of charges as *classical BPS charge domain* (or shortly *BPS domain*; for $\lambda = 0$ it supports only BPS attractors);

2. $p^1 q_0 < 0$ and refer to this range of charges as *classical non-BPS charge domain* (or shortly *non-BPS domain*; for $\lambda = 0$ it supports only non-BPS attractors).

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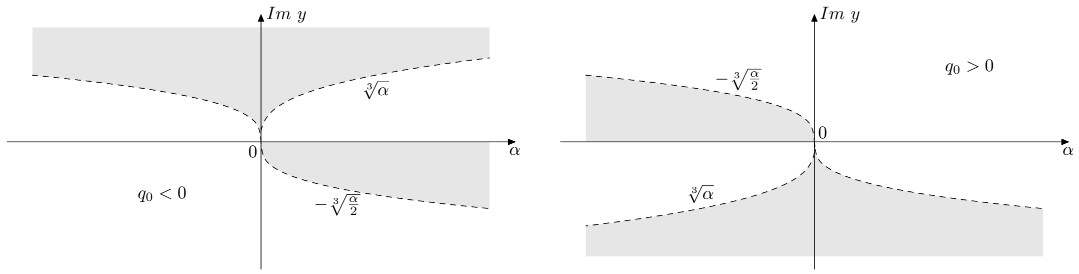

Figure 2: Domains of positivity of g and e^K.

3.1 BPS domain

Let us switch to a new coordinate y and to a new quantum parameter α:

$$t \equiv y p^1 \sqrt{\frac{q_0}{(p^1)^3}}, \qquad \lambda \equiv \alpha q_0 \sqrt{\frac{q_0}{(p^1)^3}}, \tag{3.8}$$

in terms of which the dependence of all the considered quantities on the charges is factorized:[3]

$$\begin{aligned}
& W = q_0\left(1 - 3y^2\right), \qquad e^{-K} = -4q_0\sqrt{\frac{q_0}{(p^1)^3}}\left(\alpha + 2\mathtt{Im}\,y^3\right); \\
& g = \frac{3p^1}{q_0}\frac{\left((\mathtt{Im}\,y)^3 - \alpha\right)\mathtt{Im}\,y}{\left(2(\mathtt{Im}\,y)^3 + \alpha\right)^2}, \qquad V_{BH} = v(y, \bar{y}, \alpha)\frac{q_0}{\sqrt{\frac{q_0}{(p^1)^3}}}; \\
& v(y, \bar{y}, \alpha) \equiv \frac{1}{4\mathtt{Im}\,y\left(\alpha^2 + \alpha\mathtt{Im}\,y^3 - 2\mathtt{Im}\,y^6\right)}\Big[12\alpha^2\left(\mathtt{Im}\,y^2 + \mathtt{Re}\,y^2\right) \\
& \qquad + 4\mathtt{Im}\,y^4\left(3\mathtt{Im}\,y^4 + 12\mathtt{Im}\,y^2\mathtt{Re}\,y^2 + \left(1 - 3\mathtt{Re}\,y^2\right)^2\right) \\
& \qquad + \alpha\mathtt{Im}\,y\left(3\mathtt{Im}\,y^4 - \left(1 - 3\mathtt{Re}\,y^2\right)^2 - 6\mathtt{Im}\,y^2\left(3 + \mathtt{Re}\,y^2\right)\right)\Big].
\end{aligned} \tag{3.9}$$

As stated above, the topology of the allowed regions of $\mathtt{Im}\,t$ depends on the sign of λ, as given by figure 1. In terms of y and α defined in the BPS domain by eq. (3.8), the corresponding allowed regions are represented in figure 2.

For example, if one considers $q_0 < 0$ then for $\alpha > 0$ there are two disconnected allowed intervals for $\mathtt{Im}\,y$:

$$\mathtt{Im}\,y \in \left(-\infty, \sqrt[3]{\alpha}\right) \bigcup \left(0, -\sqrt[3]{\frac{\alpha}{2}}\right), \tag{3.10}$$

while for $\alpha < 0$ it can range in only one interval:

$$\mathtt{Im}\,y \in \left(-\sqrt[3]{\frac{\alpha}{2}}, \infty\right). \tag{3.11}$$

In the classical case there is a one-to-one correspondence between the symplectic vector of BH charges inserted as input in the Attractor eq. (2.5) and the solutions to this equation. Thus, for example in the BPS domain $p^0 q_1 > 0$, when inserting two arbitrary values for

[3] For brevity's sake, we write $\mathtt{Im}\,y^n$ instead of $(\mathtt{Im}\,y)^n$. The same holds for $\mathtt{Re}\,y$.

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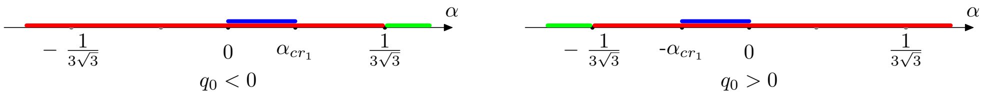


Figure 3: Ranges of the quantum parameter α supporting minima of V_{BH}, for the cases $q_0 < 0$ and $q_0 > 0$. The red line corresponds to BPS minima, the green one to axion-free non-BPS minima, and the blue one to non-BPS minima with non-vanishing axion.

p^0 and q_1 of the same sign, in the classical case there exists one and only one solution (in particular, of BPS type) to the classical Attractor eq. (2.5).

This is no more true in the considered quantum case: in the BPS domain $p^0 q_1 > 0$, when inserting two arbitrary values for p^0 and q_1 of the same sign, now there exist two stable critical points[4] of V_{BH}, one BPS and the other non-BPS. It is worth remarking that in the considered BPS domain the non-BPS critical points of V_{BH} do not admit a classical limit $\alpha \to 0$.

Furthermore, while in the classical case both BPS and non-BPS critical points of V_{BH} supported by a "magnetic" BH charge configuration are axion-free, in the quantum case also non-BPS critical points of V_{BH} with non-vanishing axion may arise out (and they can also be stable). As in the classical case, also in the considered quantum case non-BPS critical points of V_{BH} with $Z = 0$ do not exist at all. A pictorial view of the more complicated situation typical of the quantum case is given by figure 3. As mentioned in the appendix, in both domains (BPS and non-BPS) of the quantum case the BPS solutions are known analytically, whereas the non-BPS solutions can be investigated only numerically.

Axion-free stable non-BPS critical points arise only for $\alpha > \frac{1}{3\sqrt{3}}$ (in the case $q_0 < 0$) and for $\alpha < -\frac{1}{3\sqrt{3}}$ (in the case $q_0 > 0$). On the other hand, non-BPS critical points with non-vanishing axion field exists only for $0 < \alpha < \alpha_{cr_1}$ (in the case $q_0 < 0$) and for $-\alpha_{cr_1} < \alpha < 0$ (in the case $q_0 > 0$), where the value of the α_{cr_1} is known just numerically: $\alpha_{cr_1} \in (0.030101, 0.030102)$.

By looking at figure 3, one can realize at a glance that two new phenomena, absent in the classical case, arise in the quantum case. The first one, which we will name *"separation" of attractors*, occurs when the value of the quantum parameter is fixed in a certain range. The second one, which we will call *"transmutation" of attractors*, occurs when the value of the quantum parameter is varied, and becomes greater (or smaller) of a certain critical value.

Concerning the *"separation"*, from figure 3 one sees that for $|\alpha| < \alpha_{cr_1}$ two different stable critical points of V_{BH} exist for the same BH charge configuration: one of them is BPS (whose classical limit is the well known BPS solution (2.6)), the other one is non-BPS with $\mathtt{Re}\, y \neq 0$ (which does not have a classical limit). Such a *"separation"* of the solutions of the Attractor eq. (2.5) can ultimately be traced back to the presence of two disconnected regions in the domain of allowed values for $\mathtt{Im}\, y$ (see above treatment), which determines two disconnected regions of asymptotical ($r \to \infty$) values for $\mathtt{Im}\, y$ (also called *"area codes"*

[4] Also unstable non-BPS critical points of V_{BH} may exist; however, we will not deal with them, because they do not determine *attractors* in strict sense.

or *"basins of attraction"* in literature [62 – 64, 11, 39]). The dynamical radial evolution of the modulus t is completely deterministic, and determines one unique solution for an arbitrary but fixed "magnetic" BH charge configuration given as input to the Attractor eq. (2.5), *but only inside each "area code"*. Thus, the core of the Attractor Mechanism is preserved in the considered framework of dyonic, extremal, static, spherically symmetric and asymptotically flat BHs in $\mathcal{N} = 2$, $d = 4$ supergravity. In other words, the Attractor Mechanism gets two-fold split due to the presence of two disconnected regions in the allowed values of the modulus t, separated by a region where the metric function g is negative. Such a result allows one to conjecturally argue that in presence of m disconnected regions allowed in the moduli space, in general there exist m solutions to the Attractor eqs. for an arbitrary but fixed supporting BH charge configuration.[5] A similar phenomenon was observed some years ago in $\mathcal{N} = 2$, $d = 5$ supergravity in [62, 63]; the disconnectedness of the moduli space preserves the validity of the Attractor Mechanism inside *each* allowed connected region, and therefore the results of existence and uniqueness of the solutions to the Attractor eqs. obtained in [65] are still valid, but inside *each "area code"*.

Concerning the *"transmutation"*, let us take $q_0 < 0$ (this does not imply any loss of generality). Thence, from figure 3 one sees that varying the quantum parameter α across the critical value $\frac{1}{3\sqrt{3}}$ the BPS stable critical point *"transmutes"* into the non-BPS critical point (or *vice versa*).

By recalling that α depends on (a dimensionless ratio of) q_0 and p^1, it is easy to realize that the occurrence of such a phenomenon can be traced back to the fact that the supporting "magnetic" (branches of the) BH charge orbits still exist in the considered quantum case, but, as the quantum moduli space itself, they are not symmetric nor homogeneous manifolds any more.

As mentioned above, since the BPS critical points can be computed analytically, one can calculate the α-dependent expression of the BPS BH entropy in the BPS domain to be

$$S_{BH} = \pm\frac{\pi}{4}\frac{\left(1+3\mathtt{Im}\,y^2\right)^2}{\alpha+2\mathtt{Im}\,y^3}, \quad \text{with } \mathtt{Im}\,y \text{ satisfying} \quad \mathtt{Im}\,y^3 - \mathtt{Im}\,y + 2\alpha = 0, \tag{3.12}$$

where the solution of the cubic eq. must be chosen inside the allowed region(s) of the moduli space, and the $\pm$ branches of S_{BH} must be chosen in order to obtain $S_{BH} > 0$. In the classical limit $\alpha \to 0$, one recovers the well known value $S_{BH} = 2\pi\sqrt{q^0(p_1)^3}$ given by eq. (2.7).

Let us now analyze the evolution V_{BH} with respect to the quantum parameter α. Choosing $q_0 < 0$ without loss of generality, we consider $\alpha > 0$, because, due to the presence of the *"separation" of attractors* described above, it is the most interesting case.

The classical limit of the function v defined in eq. (3.9) has the form of a "scoop", with a minimum at the point $\mathtt{Re}\,y = 0$, $\mathtt{Im}\,y = 1$ (see eq. (2.6) and figure 4).

[5] In the context of $n_V = 1$ SK geometries, the phenomenon of *"separation"* of the solutions of the Attractor eq. (2.5) can be observed also for more general geometries, which do not corrispond to cubic geometries in the large modulus limit.

An example is given by the 1-modulus SK geometry based on the holomorphic prepotential $\mathcal{F} = t^4$ (we thank Mario Trigiante for discussions on this issue).

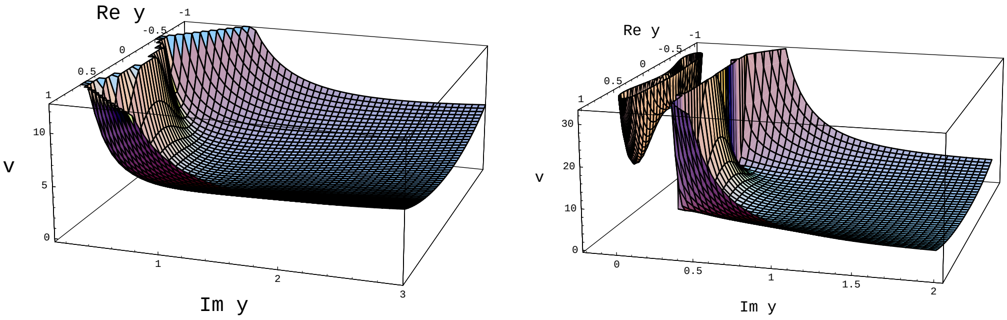


Figure 4: Plots of V_{BH} for the classical case ($\alpha = 0$, left) and for $\alpha = 1/100$ (right).

Let us now switch the (considered class of) quantum corrections on, and slightly increase the value of α from 0 to 0.01. As one can see from figure 3, $\alpha = 0.01$ is in the range supporting the *"separation" of attractors*, i.e. the coexistence of the axion-free BPS attractors with the non-BPS ones with non-vanishing axion. Such stable critical points of V_{BH} respectively have coordinates

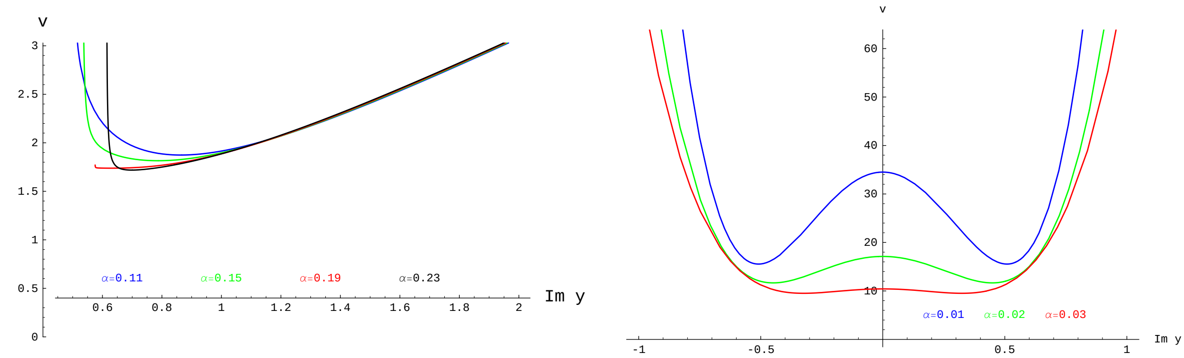
$$\begin{array}{llll} BPS: & \mathtt{Re}\, y = 0, & \mathtt{Im}\, y \approx 0.99, & V_{BH,BPS} \approx 1.99\sqrt{q_0(p^1)^3}; \\ non-BPS: & \mathtt{Re}\, y \approx \pm 0.51, & \mathtt{Im}\, y \approx -0.08, & V_{BH,non-BPS} \approx 15.54\sqrt{q_0(p^1)^3}. \end{array} \tag{3.13}$$

As one can see from the plot of V_{BH}, in contrast to the classical case there appears a new chump of the form of camel humps, which contains a stable non-BPS critical point (not existing in the classical limit). The width of such a disconnected branch of V_{BH} is $\sqrt[3]{\alpha/2}$, and thus it vanishes in the classical limit. The region of separation between the two branches of V_{BH} has width $\sqrt[3]{\alpha}$, and in such a region $g < 0$.

As yielded by eq. (3.13), for $\alpha = 0.01$ (and actually in the whole range of α supporting the *"separation" of attractors*) it holds that $V_{BH,BPS} < V_{BH,non-BPS}$. It is worth remarking that this is the opposite of what happens in globally supersymmetric field theories with multiple local minima, where the non-supersymmetric stable critical points of the superpotential are energetically favored with respect to their supersymmetry-preserving counterparts (see e.g. [66–68] and refs. therein).

Increasing the value of α the flatness of the humps increases (on the figure 5 are presented sections of hump chump by planes $\mathtt{Re}\, y = const$ passing through minima), and at the critical value α_{cr_1} the humps disappear. For $\alpha > \alpha_{cr_1}$ the non-BPS branch of V_{BH} in the BPS domain does not contain critical points of V_{BH} any more.

Let us now analyze the evolution of the BPS branch of V_{BH} when increasing α. When $\alpha \to \frac{1}{3\sqrt{3}}^{-}$, the BPS minimum point gets closer and closer to the boundary of the region where $g < 0$ (see figure 2), until it sits exactly on the curve where the metric function vanishes, becoming a non-admissible critical point of V_{BH} (red curve on the figure 5). Differently from what happens in the non-BPS branch when α gets bigger than α_{cr_1}, in the BPS branch when α gets bigger than the critical value $\frac{1}{3\sqrt{3}}$ the critical point does not disappear, but rather it changes its supersymmetry-preserving features: from BPS it *"transmutes"* into a non-BPS one, preserving its axion-free character. Such a phenomenon



Sections $\mathtt{Re}\, y = 0$ of the scoop branch (admitting a classical limit)

Sections $\mathtt{Im}\, y = const$ of the hump branch (vanishing in the classical limit)

Figure 5: Different sections of the two branches of V_{BH}.

of *"transmutation" of attractors* can be seen by looking at the $q_0 < 0$ branch of figure 3: when α increases and it passes through $\frac{1}{3\sqrt{3}}$ the BPS minimum transforms into a non-BPS minimum.

Finally, it is worth plotting in figure 6 the α -dependence of the BPS and non-BPS entropies (e.g. for $q_0 < 0$, without any loss of generality). Consistently with figure 3, the red line corresponds to BPS minima, the blue line to non-BPS minima with non-vanishing axion, and the green line to axion-free non-BPS minima. As it is seen, as previously mentioned in all the range of α supporting coexistence of minima it holds $S_{BH,BPS} < S_{BH,non-BPS}$.

Let us also notice that the definitions (3.8) are dimensionless in charges, and thus all considered phenomena hold for any magnitude of BH charges q_0 and p^1.

3.2 Non-BPS domain

Let us switch to a new special coordinate y and to a new quantum parameter α (notice the different definition with respect to the corresponding quantities in the BPS domain defined by eq. (3.8)):

$$t \equiv y p^1 \sqrt{-\frac{q_0}{(p^1)^3}}, \qquad \lambda \equiv \alpha q_0 \sqrt{-\frac{q_0}{(p^1)^3}}, \tag{3.14}$$

in terms of which the dependence of all the considered quantities on the charges is factorized:

$$W = q_0 \left(1 + 3y^2\right), \qquad e^{-K} = -4q_0 \sqrt{-\frac{q_0}{(p^1)^3}} \left(\alpha - 2\mathtt{Im}\, y^3\right);$$

$$g = -\frac{3p^1}{q_0} \frac{\left(\mathtt{Im}\, y^3 + \alpha\right) \mathtt{Im}\, y}{\left(2\mathtt{Im}\, y^3 - \alpha\right)^2}, \qquad V_{BH} = v(y, \bar{y}, \alpha) \frac{q_0}{\sqrt{-\frac{q_0}{(p^1)^3}}};$$

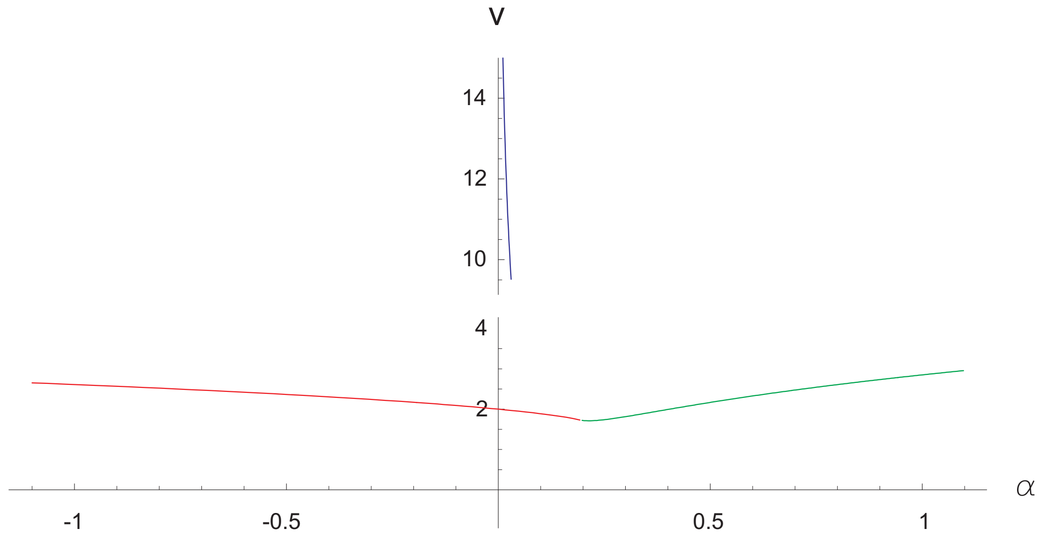


Figure 6: α-dependence of the BH entropy S_{BH} for $q_0 < 0$.

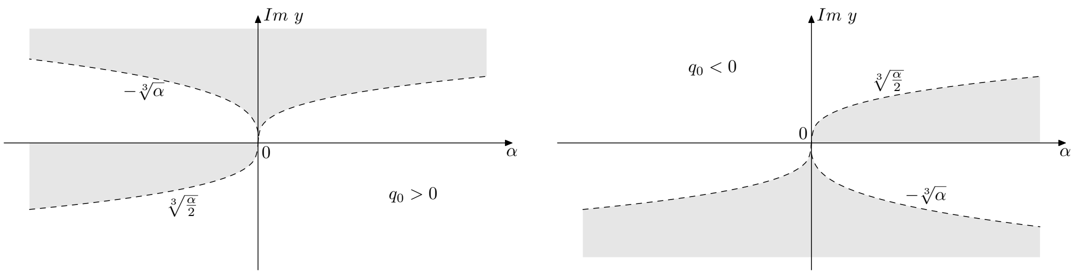


Figure 7: Domains of positivity of g and e^K.

$$
\begin{aligned}
v(y,\bar{y},\alpha) \equiv -\frac{1}{4\mathtt{Im}\,y\left(\alpha^2-\alpha\mathtt{Im}\,y^3-2\mathtt{Im}\,y^6\right)} \Big[& 12\alpha^2\left(\mathtt{Im}\,y^2+\mathtt{Re}\,y^2\right) \\
& +4\mathtt{Im}\,y^4\left(3\mathtt{Im}\,y^4+12\mathtt{Im}\,y^2\mathtt{Re}\,y^2+\left(1+3\mathtt{Re}\,y^2\right)^2\right) \\
& +\alpha\mathtt{Im}\,y\left(-3\mathtt{Im}\,y^4+\left(1+3\mathtt{Re}\,y^2\right)^2+6\mathtt{Im}\,y^2\left(-3+\mathtt{Re}\,y^2\right)\right)\Big]. \quad (3.15)
\end{aligned}
$$

Notice that $v(y,\bar{y},\alpha)$ defined in the non-BPS domain by eq. (3.15) is different from its counterpart defined in the BPS domain by eq. (3.9). As pointed out above, the topology of the allowed regions of $\mathtt{Im}\,t$ depends on the sign of λ, as given by figure 1. In terms of y and α defined in the non-BPS domain by eq. (3.14), the corresponding allowed regions are given by figure 7.

As in the BPS domain, also in the non-BPS domain the phenomena of *"separation" and "transmutation" of attractors* arise, but in a slightly different way. A pictorial view of the ranges of α supporting the various typologies of minima of V_{BH} is given by figure 8.

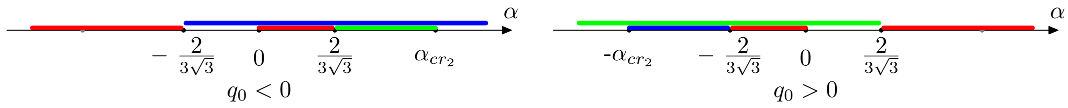


Figure 8: Ranges of the quantum parameter α supporting minima of V_{BH}, for the cases $q_0 < 0$ and $q_0 > 0$. The red line corresponds to BPS minima, and the green and blue lines to the two different kinds of axion-free non-BPS minima.

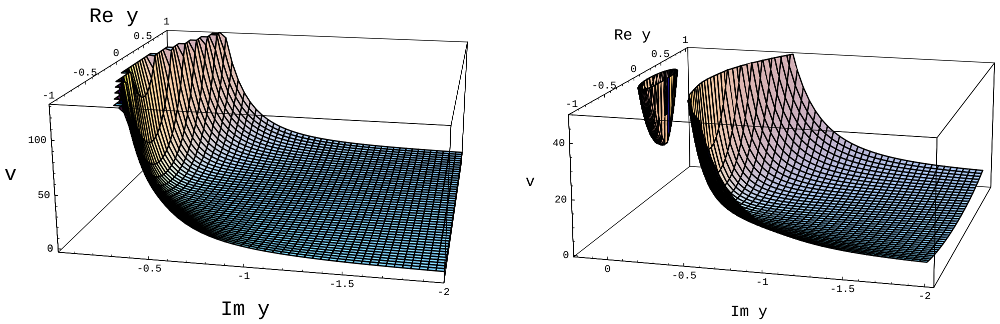


Figure 9: Plots of V_{BH} in the classical limit ($\alpha = 0$, left) and for $\alpha = 1/100$ (right).

Differently from what happens in the BPS domain, in the non-BPS domain no non-BPS minima with non-vanishing axion exist at all, and there are two distinct typologies of axion-free non-BPS minima.

Concerning the BPS minima, for $q_0 < 0$ they exist in the range $0 < \alpha < \frac{2}{3\sqrt{3}}$, whereas for $q_0 > 0$ they exist for $\alpha > \frac{2}{3\sqrt{3}}$. The same holds for $\alpha < 0$, but with opposite sign of q_0.

As mentioned, in the considered non-BPS domain there are two kinds of non-BPS minima, both axion-free (see figure 8). The corresponding relevant critical value of the quantum parameter is $\alpha_{cr_2} \approx 0.934$.

As mentioned above, since the BPS critical points can be computed analytically, one can calculate the α-dependent expression of the BPS BH entropy in the non-BPS domain to be

$$S = \pm\frac{\pi}{4}\frac{\left(1 - 3\mathtt{Im}\,y^2\right)^2}{\alpha - 2\mathtt{Im}\,y^3}, \quad \text{with } \mathtt{Im}\,y \text{ satisfying} \quad \mathtt{Im}\,y^3 + \mathtt{Im}\,y - 2\alpha = 0, \tag{3.16}$$

where the solution of the cubic equation must be chosen inside the allowed region(s) of the moduli space, and the $\pm$ branches of S_{BH} must be chosen in order to obtain $S_{BH} > 0$. As it has to be in the non-BPS domain, in the classical limit the cubic equation has no admissible solutions.

Let us now analyze the evolution V_{BH} with respect to the quantum parameter α, choosing $q_0 < 0$ without loss of generality.

The classical limit of the function v defined in eq. (3.15) has the form of a "scoop", with a minimum at the point $\mathtt{Re}\,y = 0$, $\mathtt{Im}\,y = 1$ (see eq. (2.6) and figure 9).

As done in the BPS domain, let us now slightly increase the value of α from 0 to 0.01. As one can see from figure 9, $\alpha = 0.01$ is in the range supporting the *"separation" of attractors*, in this case corresponding to the coexistence of a BPS attractors with a non-BPS

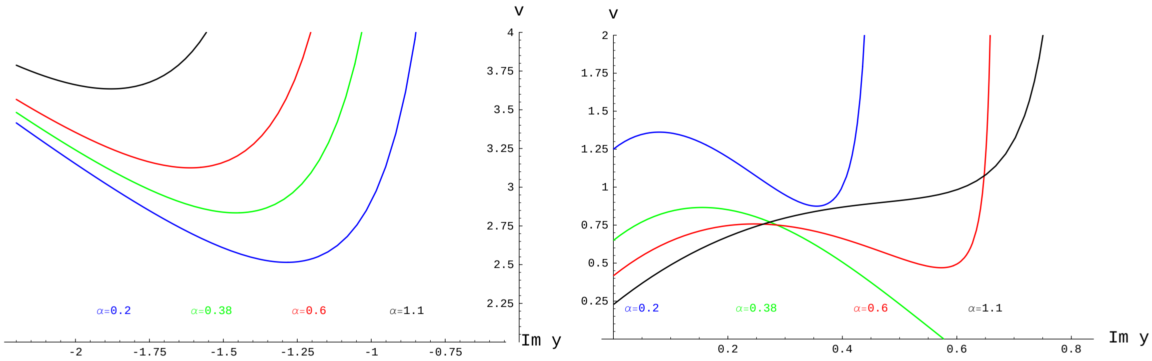


Figure 10: The sections $\mathtt{Re}\, y = 0$ of the two branches of V_{BH}: scoop (admitting a classical limit, left) and hump (vanishing in the classical limit, right).

one, both axion-free. Such stable critical points of V_{BH} respectively have coordinates

$$\begin{array}{llll} non-BPS: & \mathtt{Re}\, y = 0, & \mathtt{Im}\, y \approx -1.02, & V_{BH,non-BPS} \approx 2.03\sqrt{-q_0(p^1)^3}; \\ BPS: & \mathtt{Re}\, y = 0, & \mathtt{Im}\, y \approx 0.02, & V_{BH,BPS} \approx 24.98\sqrt{-q_0(p^1)^3}. \end{array} \tag{3.17}$$

As one can see from the plot of V_{BH} and analogously to what happens in the BPS domain, in contrast to the classical case there appears a new chump, which contains a stable BPS critical point (not existing in the classical limit). The width of such a disconnected branch of V_{BH} (which vanishes in the classical limit) and its separation from the other branch admitting a consistent classical limit are both proportional to $\sqrt[3]{\alpha}$. As in the BPS domain, such two branches of V_{BH} are separated by a region in which $g < 0$.

As yielded by eq. (3.17), for $\alpha = 0.01$ it holds that $V_{BH,non-BPS} < V_{BH,BPS}$ (see discussion below).

However in the non-BPS domain, once again differently from the BPS domain, in the range $0 < \alpha < \alpha_{cr_2}$ supporting the *"separation" of attractors* one can observe also the phenomenon of the *"transmutation"* of the supersymmetry-preserving features of the such minima of V_{BH}. This can be seen by increasing α. As evident by looking at figure 10, the scoop chump of V_{BH} (i.e. the branch of V_{BH} containing an axion-free non-BPS minimum and admitting a classical limit), does not undergo any qualitative change when increasing α. On the other hand, the hump chump of V_{BH} (i.e. the one vanishing for $\alpha \to 0$) exhibits a *"transmutation"* of the minimum: for $0 < \alpha < \frac{2}{3\sqrt{3}}$ it constains a BPS minimum. For $\alpha = \frac{2}{3\sqrt{3}}$ such a critical point of V_{BH} sits on the boundary (see figure 7) where e^K diverges, so it is not an allowed critical point of V_{BH}. For $\frac{2}{3\sqrt{3}} < \alpha < \alpha_{cr_2}$ the minimum is still present, it is not BPS any more, but rather it is non-BPS (and axion-free). Thence, for $\alpha > \alpha_{cr_2}$ the hump chump of V_{BH} does not contain any critical point.

Also the scoop chump of V_{BH} exhibits a *"transmutation"* of the minimum, but for negative values of α. Indeed, when α gets smaller than $-\frac{2}{3\sqrt{3}}$ the axion-free non-BPS minimum *"transmutes"* into a BPS one (axion-free, as all the admissible BPS critical points found in our analysis).

The shape of V_{BH} for various values of α is plotted in figure 10. One might see that for small values of α (blue line) the hump chump contains a BPS minimum. At the

JHEP02(2008)088

critical value $\alpha = \frac{2}{3\sqrt{3}}$ (green line) the minimum disappears and reappears, but non-BPS, when $\alpha > \frac{2}{3\sqrt{3}}$ (red line). For $\alpha > \alpha_{cr_2}$, no critical points exist at all (black line).

The α-dependence of the BH entropy is shown in figure 11. Consistently with the colors used in figure 8, the red plot corresponds to BPS BH entropy, whereas the blue and green plots to non-BPS BH entropy.

As done in the BPS domain, it is worth pointing out that the definitions (3.14) are dimensionless in charges, and thus all considered phenomena hold for any magnitude of BH charges q_0 and p^1.

Thus, the non-BPS domain exhibits various differences with respect to the BPS domain, which can be summarized as follows:

(i) all non-BPS minima in such a domain are axion-free;

(ii) the *"transmutation"* of the supersymmetry-preserving features of the minimum of V_{BH} happens not only in the branch of V_{BH} admitting a classical limit, but also in the one vanishing in such a limit;

(iii) the *"transmutation"* inside the branch of V_{BH} vanishing in the classical limit actually determines a *"transmutation"* in the region of *"separation"* of attractors: for $0 < \alpha < \frac{2}{3\sqrt{3}}$ a BPS and a non-BPS minimum coexist; when α gets bigger than $\frac{2}{3\sqrt{3}}$ the BPS minimum *"transmutes"* into a non-BPS one, who then coexists with the pre-existing non-BPS (axion-free) minimum until α reaches the critical value α_{cr_2};

(iv) as evident by looking at figure 11, in the range $0 < \alpha < \frac{2}{3\sqrt{3}}$ supporting the coexistence of a BPS and a non-BPS minimum, the sign of $S_{BH,non-BPS} - S_{BH,BPS}$ (for fixed BH charges p^0 and q_1 satisfying $p^0 q_1 < 0$) changes. Indeed, there exists a particular value $\widehat{\alpha} \approx 0.1 \in \left(0, \frac{2}{3\sqrt{3}}\right)$ for which remarkably $S_{BH,non-BPS} = S_{BH,BPS}$. This means that for $\alpha = \widehat{\alpha}$ two *different* purely charge-dependent attractor configurations of the scalar t exist (one preserving one half of the 8 supersymmetries pertaining to the asymptotical $\mathcal{N} = 2$, $d = 4$ superPoincaré algebra and the other not preserving any of them) such that they determine the *same* BH entropy. For $0 < \alpha < \widehat{\alpha}$ it holds $S_{BH,non-BPS} < S_{BH,BPS}$, whereas for $\widehat{\alpha} < \alpha < \frac{2}{3\sqrt{3}}$ it holds $S_{BH,BPS} < S_{BH,non-BPS}$;

(v) for $\frac{2}{3\sqrt{3}} < \alpha < \alpha_{cr_2}$ it holds $\widetilde{S}_{BH,non-BPS} < S_{BH,non-BPS}$, where $\widetilde{S}_{BH,non-BPS}$ is the BH entropy determined by the non-BPS minimum originated from the BPS one by *"transmutation"* (see the green and blue plots in figure 11);

(vi) for $\alpha > \alpha_{cr_2}$, the branch of V_{BH} vanishing in the classical limit actually still contains critical points of V_{BH}, but they are not admissible and/or they are not stable.

It is worth pointing out that all the obtained results are expressed in terms of coordinates and quantum parameters *dimensionless* in the BH charges (i.e. depending on *ratios* of BH charges, as given by eqs. (3.8) and (3.14)). Thus, *apriori* they hold for every range of values of the BH charges. Actually, we disregard the actual quantization of the electric

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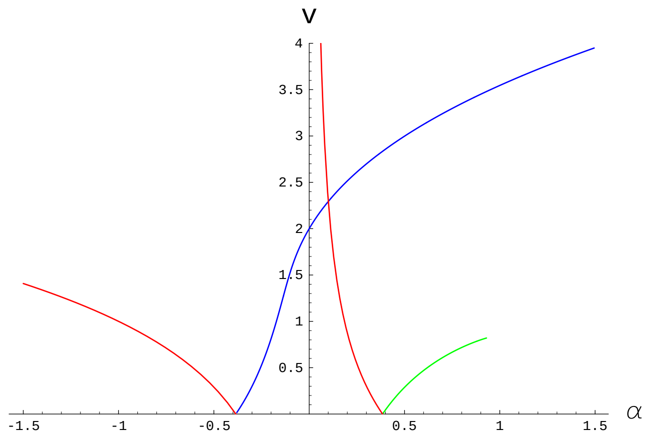

Figure 11: α-dependence of the BH entropy S_{BH} for $q_0 < 0$

and magnetic charges, considering $\mathbb{R}^4$, rather than the 4-dimensional charge lattice $\widehat{\Gamma}$, to be the space spanned by the BH charge vector Q. In other words, all our analysis and the obtained results hold in the semiclassical regime of large, continuous, real BH charges.

3.3 The $D0 - D6$ BH charge configuration

From the above analysis of the extremal BH attractors supported by the "magnetic" charge configuration $Q = \left(q_0, 0, 0, p^1\right)$ in the $t^3 + i\lambda$ model, one might be lead to argue that starting from a BH charge configuration supporting only one class of attractors at the classical level, at the quantum level one obtains that such a charge configuration generally supports more than one class of attractors (at least for a certain range of the quantum parameter λ).

This is actually not true, and a counterexample is provided by the BH charge configuration $Q = \left(q_0, 0, p^0, 0\right)$, usually named Kaluza-Klein BH (in M-theory language) [69] or $D0 - D6$ system (in Type IIA Calabi-Yau compactifications in the language of superstring theory) [10, 34], for which the superpotential acquires the form

$$W = q_0 - 2i\lambda p^0 + p^0 t^3. \tag{3.18}$$

It is easy to check that at the classical level such a charge configuration only supports non-BPS attractors [10, 34, 40]. Differently from what happens for the "magnetic" charge configuration, this holds also at the quantum level, $\forall \lambda \in \mathbb{R}$; in other words, the $t^3 + i\lambda$ model does not have admissible BPS critical points of V_{BH} supported by the $D0 - D6$ charge configuration. This can be easily realized by looking at the real and imaginary parts of the BPS Attractor Equation for such a charge configuration, respectively of the following form (having a smooth classical limit):

$$\begin{aligned}
\frac{(\mathtt{Im}\, t^3 - \lambda)(\mathtt{Re}\, t^2 + \mathtt{Im}\, t^2)}{\lambda + 2\mathtt{Im}\, t^3} &= 0; \\
\frac{6\mathtt{Im}\, t(2\lambda p^0 \mathtt{Re}\, t + p^0 \mathtt{Im}\, t^3 \mathtt{Re}\, t + \mathtt{Im}\, t(q_0 + p^0 \mathtt{Re}\, t^3))}{\lambda + 2\mathtt{Im}\, t^3} &= 0.
\end{aligned} \tag{3.19}$$

By recalling the expression of the metric function g given by eq. (3.2), it is clear that all BPS critical points of V_{BH} supported by the $D0-D6$ charge configuration are points where $g=0$, and therefore they are not admissible.

Also in view of the analysis of [53], such a result can be directly extended to the axion-free BPS critical points of V_{BH} in a λ-*corrected* SK cubic geometry, based on the Kähler gauge-invariant holomorphic prepotential (in a suitable basis of special symplectic coordinates $t^i \equiv \varsigma^i - i\omega^i$, $i = 1, \ldots, n_V$)

$$\mathcal{F} = \frac{1}{3!} d_{ijk} t^i t^j t^k + i\lambda, \tag{3.20}$$

which is nothing but the n_V moduli generalization of eq. (3.1). The λ-*corrected* real metric g_{ij} can be easily computed from eq. (3.20) to be

$$g_{ij} = -\frac{3}{2}\left[\frac{d_{ij}}{d-3\lambda} - \frac{3}{2}\frac{d_i d_j}{(d-3\lambda)^2}\right], \tag{3.21}$$

where $d_{ij} \equiv d_{ijk}\omega^k$, $d_i \equiv d_{ijk}\omega^j\omega^k$ and $d \equiv d_{ijk}\omega^i\omega^j\omega^k$. Notice that the limit $\lambda \to 0$ consistently yields the expression of the real metric of a SK cubic geometry, given by eq. (2.4) of [40]. In [53] the axion-free BPS critical points of V_{BH} in the SK geometry based on $\mathcal{F}$ given by eq. (3.20) have been computed to be

$$t^i_{BPS,axion-free} = \omega^i_{BPS,axion-free} = i\frac{p^i}{\mathfrak{c}}, \tag{3.22}$$

where the charge-dependent quantity $\mathfrak{c}$ satisfies the third order algebraic equation

$$2\lambda\mathfrak{c}^3 - q_0\mathfrak{c}^2 + \frac{1}{3!} d_{ijk} p^i p^j p^k = 0. \tag{3.23}$$

It is then clear that the $D0-D6$ BH charge configuration does not support axion-free BPS critical points of V_{BH}, because for $p^i = 0\ \forall i = 1, \ldots, n_V$ eqs. (3.21) and (3.22) yield $g_{ij,BPS,axion-free} = 0$.

Notice that the non-admissibility of the axion-free BPS critical points of V_{BH} in the $D0-D6$ BH charge configuration would not be evident by only looking at the BPS BH entropy determined by axion-free solutions, which reads [6]

$$S_{BH,BPS,axion-free} = -2\pi\left(\mathfrak{c} + \frac{(p^0)^2}{\mathfrak{c}}\right)\left(q_0 - \frac{3}{2}\lambda\mathfrak{c}\right), \tag{3.24}$$

whose limit (independent on n_V) in the $D0-D6$ BH charge configuration is finite (and constrains $\lambda \in \mathbb{R}^-_0$):

$$S_{BH,BPS,axion-free,D0-D6} = -\frac{\pi}{2}\left(\frac{(q_0)^2}{2\lambda} + 2\lambda\left(p^0\right)^2\right). \tag{3.25}$$

[6] Such an expression corrects a misprint in eq. (3.35) of [53].

Acknowledgments

The work of S.B., S.F. and A.S. has been supported in part by the European Community Human Potential Program under contract MRTN-CT-2004-005104 *"Constituents, fundamental forces and symmetries of the universe"*.

The work of S.F. has been supported in part by European Community Human Potential Program under contract MRTN-CT-2004-503369 *"The quest for unification: Theory Confronts Experiments"*, in association with INFN Frascati National Laboratories and by D.O.E. grant DE-FG03-91ER40662, Task C.

The work of A.M. has been supported by a Junior Grant of the *"Enrico Fermi"* Center, Rome, in association with INFN Frascati National Laboratories.

A. Attractor equations

The complex BPS Attractor eq. for the quantum 1-modulus model based on the prepotential $\mathcal{F} = t^3 + i\lambda$ has the form

$$\frac{t^4p^1 - 3p^1\bar{t}^2t^2 + 2p^1\bar{t}^3t - q_0\bar{t}^2 - q_0t^2 + 2q_0\bar{t}\,t + 8i\lambda p^1 t}{(t-\bar{t})^3 - 8i\lambda} = 0. \tag{A.1}$$

By taking its real and imaginary parts, one obtains the following two real eqs.:

$$\frac{\mathtt{Re}\,t(\mathtt{Im}\,t^3 - \lambda)}{2\mathtt{Im}\,t^3 + \lambda} = 0; \qquad \frac{\mathtt{Im}\,t(\mathtt{Im}\,t^3p^1 + 3\mathtt{Im}\,t\mathtt{Re}\,t^2p^1 + 2\lambda p^1 - \mathtt{Im}\,tq_0)}{2\mathtt{Im}\,t^3 + \lambda} = 0, \tag{A.2}$$

whose solution reads

$$\mathtt{Re}\,t = 0, \qquad \mathtt{Im}\,t^3p^1 - \mathtt{Im}\,tq_0 + 2\lambda p^1 = 0. \tag{A.3}$$

Obviously, the number of real roots of the obtained algebraic cubic eq. depends on the relations between its coefficients.

On the other hand, the real part of the complex non-BPS Attractor eq. for the model based on $\mathcal{F} = t^3 + i\lambda$ reads

$$\frac{\mathtt{Re}\,t[8\mathtt{Im}\,t^6p^1 - \mathtt{Im}\,t^3\lambda p^1 + 2\lambda^2p^1 + 4\mathtt{Im}\,t^4(3\mathtt{Re}\,t^2p^1 - q_0) + \mathtt{Im}\,t\lambda(-3\mathtt{Re}\,t^2p^1 + q_0)]}{4\mathtt{Im}\,t^7 - 2\mathtt{Im}\,t^4\lambda - 2\mathtt{Im}\,t\lambda^2} = 0; \tag{A.4}$$

its imaginary part is rather cumbersome, and it is not worth presenting it here.

The above eqs. get modified when passing to the special coordinate and quantum parameter suitably defined in the BPS and non-BPS domains (see definitions (3.8) and (3.14), respectively). In the following treatment we will analyze them case-by-case.

A.1 BPS domain

A.1.1 BPS attractor equations

In the BPS domain the BPS Attractor Equations (AEs) (A.2) acquire the form

$$\frac{\left((\mathtt{Im}\,y)^3 - \alpha\right)\mathtt{Re}\,y}{\alpha + 2(\mathtt{Im}\,y)^3} = 0, \qquad \frac{\mathtt{Im}\,y\left(2\alpha + \mathtt{Im}\,y\left(-1 + (\mathtt{Im}\,y)^2 + 3(\mathtt{Re}\,y)^2\right)\right)}{\alpha + 2(\mathtt{Im}\,y)^3} = 0. \tag{A.5}$$

JHEP02(2008)088

From eq. (3.9) it follows that, in order to have a regular solution, the real part of the modulus y has to vanish, while its imaginary part has to satisfy a cubic eq.:

$$\mathtt{Re}\, y = 0, \qquad (\mathtt{Im}\, y)^3 - \mathtt{Im}\, y + 2\alpha = 0. \tag{A.6}$$

Depending on α, the obtained cubic eq. has up to three real roots. If $\alpha^2 < 1/27$ there are three real roots but, due to the consistency conditions on the metric and Kähler potential, only one of them turns out to determine an admissible critical point of V_{BH}. On the other hand, for $\alpha^2 > 1/27$ the situation is described by figure 3. Notice that, as it has to be in the BPS domain, in the classical limit $\alpha \to 0$ one reobtains the classical solutions given by eq. (2.6).

A.1.2 Non-BPS attractor equations

In the BPS domain the real part (A.4) of the non-BPS AEs acquire the form

$$\frac{\mathtt{Re}\, y}{\mathtt{Im}\, y}\, \frac{2\alpha^2 - \alpha \mathtt{Im}\, y(-1 + \mathtt{Im}\, y^2 + 3\mathtt{Re}\, y^2) + 4\mathtt{Im}\, y^4(-1 + 2\mathtt{Im}\, y^2 + 3\mathtt{Re}\, y^2)}{(\alpha - \mathtt{Im}\, y^3)(\alpha + 2\mathtt{Im}\, y^3)} = 0. \tag{A.7}$$

Once again, the imaginary part is cumbersome, and it is not worth writing it here. Eq. (A.7) can be solved with respect to $\mathtt{Re}\, y$, obtaining two solutions:

$$\mathtt{Re}\, y = 0; \qquad \text{or} \qquad \mathtt{Re}\, y^2 = -\frac{1}{3}\frac{2\alpha^2 + \alpha \mathtt{Im}\, y - \alpha \mathtt{Im}\, y^3 - 4\mathtt{Im}\, y^4 + 8\mathtt{Im}\, y^6}{4\mathtt{Im}\, y^4 - \alpha \mathtt{Im}\, y}. \tag{A.8}$$

We are now going to substitute each of these solutions in the imaginary part of the non-BPS AEs.

Axion-free non-BPS solutions. In this case the imaginary part of the non-BPS AEs acquires form

$$(\mathtt{Im}\, y^3 - \mathtt{Im}\, y + 2\alpha)[2\alpha^3 - 5\alpha^2 \mathtt{Im}\, y - 3\alpha^2 \mathtt{Im}\, y^3 + 4\alpha \mathtt{Im}\, y^4 + 36\alpha \mathtt{Im}\, y^6 - 8\mathtt{Im}\, y^7 - 8\mathtt{Im}\, y^9] = 0. \tag{A.9}$$

It is worth remarking that the cubic term $\mathtt{Im}\, y^3 - \mathtt{Im}\, y + 2\alpha$ which gets factorized determines the BPS solution (A.6), and thus here it has to be disregarded. The other factor in eq. (A.9) is the one yielding the non-BPS solutions. As it has to be since we are considering the BPS domain, in the classical limit $\alpha \to 0$ this factor reduces to a polynomial of the form $x^9 + x^7$, which has no admissible solutions.

Since the solution of an algebraic eq. of ninth order cannot be found analytically, one can analyze it numerically, and determine the number of real solutions depending on the value of α. It turns out that α has two critical values: one of them is $\frac{1}{3\sqrt{3}}$, while the other only numerically, $\widetilde{\alpha}_{cr_1} \in (1.1010864, 1.1010865)$. For $0 < \alpha < \frac{1}{3\sqrt{3}}$ and $\alpha > \widetilde{\alpha}_{cr_1}$, only one real root exists, while for $\frac{1}{3\sqrt{3}} < \alpha < \widetilde{\alpha}_{cr_1}$ there are tree real roots. By choosing $q_0 < 0$ (without any loss of generality) and requiring the stability of the solutions and the belonging to the allowed regions of the moduli space given by figure 2, one gets that in the BPS domain stable and admissible non-BPS axion-free critical points of V_{BH} exist for $\alpha > \frac{1}{3\sqrt{3}}$ (see the green line in the plot on the left of figure 3).

Non-axion-free non-BPS solutions. The other possibility is to substitute the more complicated expression for $\mathtt{Re}\,y$ given by eq. (A.8) into the imaginary part of the non-BPS AEs. By doing so, the imaginary part of the non-BPS AEs reduces to an algebraic eq. of fifteenth order:

$$\begin{aligned}2\alpha^5+\alpha^4\mathtt{Im}\,y-13\alpha^4\mathtt{Im}\,y^3+8\alpha^3\mathtt{Im}\,y^4-76\alpha^3\mathtt{Im}\,y^6-120\alpha^2\mathtt{Im}\,y^7-\\-92\alpha^2\mathtt{Im}\,y^9+320\alpha\mathtt{Im}\,y^{10}-32\alpha\mathtt{Im}\,y^{12}-128\mathtt{Im}\,y^{13}-32\mathtt{Im}\,y^{15}=0,\end{aligned}\tag{A.10}$$

which clearly cannot be solved analytically. As it has to be since we are considering the BPS domain, in the classical limit $\alpha\to 0$ eq. (A.10) reduces to a polynomial of the form $x^{15}+x^{13}$, which has no admissible solutions.

By inspecting the number of real roots of such an eq. with respect to the value of α, one obtains two critical values of α. One of them is known just numerically: $\alpha_{cr_1}\in(0.030101, 0.030102)$, while the other one is $\frac{2}{3\sqrt{3}}$. When $0<\alpha<\alpha_{cr_1}$, five real roots exist but only one of them is admissible, whereas for $\alpha_{cr_1}<\alpha<\frac{2}{3\sqrt{3}}$ and $\alpha>\frac{2}{3\sqrt{3}}$ three and one real root exist respectively, but none of them is admissible. Thus (by choosing $q_0<0$, once again without loss of generality), in the BPS domain stable and admissible non-BPS critical points of V_{BH} with non-vanishing axion exist for $0<\alpha<\alpha_{cr_1}$ (see the blue line in the plot on the left of figure 3).

A.2 Non-BPS domain

A.2.1 BPS attractor equations

In the non-BPS domain the BPS AEs (A.2) acquire the form

$$\frac{\left(\mathtt{Im}\,y^3+\alpha\right)\mathtt{Re}\,y}{\alpha-2\mathtt{Im}\,y^3}=0,\qquad \frac{\mathtt{Im}\,y\left(-2\alpha+\mathtt{Im}\,y\left(1+\mathtt{Im}\,y^2+3\mathtt{Re}\,y^2\right)\right)}{\alpha-2\mathtt{Im}\,y^3}=0.\tag{A.11}$$

From eq. (3.15) it follows that, in order to have a regular solution, the real part of the modulus y has to vanish, while its imaginary part has to satisfy a cubic eq.:

$$\mathtt{Re}\,y=0,\qquad \mathtt{Im}\,y^3+\mathtt{Im}\,y-2\alpha=0.\tag{A.12}$$

Such a cubic eq. has only one real root $\forall\alpha\in\mathbb{R}$:

$$\mathtt{Im}\,y=\sqrt[3]{\alpha-\sqrt{\alpha^2+\frac{1}{27}}}+\sqrt[3]{\alpha+\sqrt{\alpha^2+\frac{1}{27}}}.\tag{A.13}$$

Even if such a solution exists $\forall\alpha$, the admissible BPS minima exist for $q_0<0$ in the range $0<\alpha<\frac{2}{3\sqrt{3}}$, and for $q_0>0$ in the range $\alpha>\frac{2}{3\sqrt{3}}$. The same holds for $\alpha<0$, but with opposite sign of q_0 (see the red lines in figure 8). Notice that, as it has to be in the non-BPS domain, in the classical limit $\alpha\to 0$ the cubic eq. has not admissible solutions.

A.2.2 Non-BPS attractor equations

In the non-BPS domain the real part (A.4) of the non-BPS AEs acquire the form

$$\frac{\mathtt{Re}\,y}{\mathtt{Im}\,y}\,\frac{2\alpha^2+\alpha\mathtt{Im}\,y(1+\mathtt{Im}\,y^2+3\mathtt{Re}\,y^2)+4\mathtt{Im}\,y^4(1+2\mathtt{Im}\,y^2+3\mathtt{Re}\,y^2)}{(\alpha+\mathtt{Im}\,y^3)(\alpha-2\mathtt{Im}\,y^3)}=0.\tag{A.14}$$

JHEP02(2008)088

Once again, the imaginary part is cumbersome, and it is not worth writing it here. Eq. (A.14) can be solved with respect to $\mathtt{Re}\,y$, obtaining two solutions:

$$\mathtt{Re}\,y=0; \qquad \text{either} \qquad \mathtt{Re}\,y^2=-\frac{1}{3}\frac{2\alpha^2+\alpha\mathtt{Im}\,y+\alpha\mathtt{Im}\,y^3+4\mathtt{Im}\,y^4+8\mathtt{Im}\,y^6}{4\mathtt{Im}\,y^4+\alpha\mathtt{Im}\,y}. \tag{A.15}$$

Notice that the non-axion-free branch of solutions given by the second expression of eq. (A.15) is not consistent with the classical admissible and stable non-BPS solutions, which are axion-free (see eq. (2.6); on the other hand, as stated below, admissible and stable non-axion-free non-BPS solutions do not exist in the considered quantum case, too).

We are now going to substitute each of these solutions in the imaginary part of the non-BPS AEs.

Axion-free non-BPS solutions. In this case the imaginary part of the non-BPS AEs acquires form

$$(\mathtt{Im}\,y^3+\mathtt{Im}\,y-2\alpha)[2\alpha^3-5\alpha^2\mathtt{Im}\,y+3\alpha^2\mathtt{Im}\,y^3-4\alpha\mathtt{Im}\,y^4+36\alpha\mathtt{Im}\,y^6-8\mathtt{Im}\,y^7+8\mathtt{Im}\,y^9]=0. \tag{A.16}$$

It is worth remarking that the cubic term $\mathtt{Im}\,y^3+\mathtt{Im}\,y-2\alpha$ which gets factorized determines the BPS solution (A.13), and thus here it has to be disregarded. The other factor in eq. (A.16) is the one yielding the axion-free non-BPS solutions, and, as it has to be in the non-BPS domain, in the classical limit $\alpha\to 0$ it gives the classical solutions (see eq. (2.6)).

Since the solution of an algebraic eq. of ninth order cannot be found analytically, as done in the BPS domain one can analyze it numerically, and determine the number of real solutions depending on the value of α. It turns out that α has two relevant critical values, one is $\frac{2}{3\sqrt{3}}$ and the other is known only numerically: $\alpha_{cr_2}\approx 0.934$. For $\alpha>\alpha_{cr_2}$ and $\alpha<0$, only one real root exists, while for $0<\alpha<\alpha_{cr_2}$ there are tree real roots. By choosing $q_0<0$ (without any loss of generality) and requiring the stability of the solutions and the belonging to the allowed regions of the moduli space given by figure 7, one gets that in the non-BPS domain stable and admissible non-BPS axion-free critical points of V_{BH} exist for $\alpha>-\frac{2}{3\sqrt{3}}$ and that they double in the range $\frac{2}{3\sqrt{3}}<\alpha<\alpha_{cr_2}$ (see the blue and green lines in the plot on the left of figure 8).

Non-axion-free non-BPS solutions. The other possibility is to substitute the more complicated expression for $\mathtt{Re}\,y$ given by eq. (A.15) into the imaginary part of the non-BPS AEs. By doing so, as in the BPS domain the imaginary part of the non-BPS AEs reduces to an algebraic eq. of fifteenth order:

$$\begin{aligned}&2\alpha^5+\alpha^4\mathtt{Im}\,y+13\alpha^4\mathtt{Im}\,y^3-8\alpha^3\mathtt{Im}\,y^4-76\alpha^3\mathtt{Im}\,y^6-120\alpha^2\mathtt{Im}\,y^7-\\&\quad+92\alpha^2\mathtt{Im}\,y^9-320\alpha\mathtt{Im}\,y^{10}-32\alpha\mathtt{Im}\,y^{12}-128\mathtt{Im}\,y^{13}+32\mathtt{Im}\,y^{15}=0,\end{aligned} \tag{A.17}$$

which cannot be solved analytically. By performing a numerical inspection of the number of real roots depending on the values of α, and imposing the conditions of stability and belonging to the allowed regions of the moduli space given by figure 7, in this case one finds that there are no admissible solutions. In other words, in the non-BPS domain admissible and stable non-BPS critical points of V_{BH} with non-vanishing axion do not exist $\forall\alpha\in\mathbb{R}$.

JHEP02(2008)088

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