XIII. PHYSICAL ACOUSTICS*

Academic and Research Staff

Prof. K. U. Ingard
Prof. L. W. Dean III
Dr. K. W. Gentle

Graduate Students

C. T. Dum
R. H. Katyl
J. L. Macon
W. M. Manheimer
J. A. Ross
H. M. Schulz III
T. B. Smith

RESEARCH OBJECTIVES

Our general objective involves the study of the emission, propagation, and absorption of sound and vibrations in matter. Specific areas of current research in fluids include generation, propagation, and amplification of sound waves in ionized gases and interaction of waves with coherent light beams.

K. U. Ingard

A. DISPERSION RELATION FOR SURFACE WAVES ON A COMPRESSIBLE FLUID

We shall consider the effect of the compressibility of a fluid on the dispersion relation of surface waves, and examine the contribution of the bulk viscosity to the damping coefficient of the waves.

For a lossless compressible fluid, the linearized equations of motion are:

$$\rho_o(\vec{\nabla}\cdot\vec{v}) + \frac{\partial\delta}{\partial t} = 0 \qquad \text{(Continuity)}$$

$$\rho_o\frac{\partial\vec{v}}{\partial t} + \vec{\nabla}p = 0 \qquad \text{(Momentum conservation)}$$

$$p = \frac{\delta}{\rho_o\kappa_s} \qquad \text{(Equation of state)}$$

$$p = -\sigma\left(\frac{\partial^2\zeta}{\partial x^2} + \frac{\partial^2\zeta}{\partial y^2}\right) \qquad \text{(Pressure balance on surface).}$$

In these equations v, δ, and p are the first-order perturbations of the velocity, density, and pressure fields, respectively; σ, the surface tension; κ_s, the adiabatic compressibility; and ζ, the surface displacement. The unperturbed surface has been taken to be the x-y plane, and the fluid exists for $z < 0$.

*This work is supported principally by the U.S. Navy (Office of Naval Research) under Contract Nonr-1841-(42); and in part by the Joint Services Electronics Programs (U.S. Army, U.S. Navy and U.S. Air Force) under Contract DA 36-039-AMC-03200(E).

If we define the velocity potential ϕ by $\vec{v} = -\vec{\nabla}\phi$, these equations reduce to two equations for ϕ:

$$\nabla^2\phi = \rho_o \kappa_s \frac{\partial^2\phi}{\partial t^2} = \frac{1}{c_o^2}\frac{\partial^2\phi}{\partial t^2} \tag{1}$$

$$\rho_o \frac{\partial^2\phi}{\partial t^2} = \sigma\left(\frac{\partial^3\phi}{\partial z\partial x^2} + \frac{\partial^3\phi}{\partial z\partial y^2}\right). \tag{2}$$

By seeking plane wave solutions of the form $Z(z)\, e^{i(Kx-\omega t)}$ we find $Z(z) = e^{\beta z}$, where

$$\beta^2 = K^2 - \left(\frac{\omega}{c_o}\right)^2$$

$$\omega^2 = \frac{\sigma}{\rho_o} K^2 \beta.$$

If β is eliminated between the two equations, one gets

$$\omega^4 + \left(\frac{\sigma K^2}{\rho_o c_o}\right)^2 \omega^2 - \left(\frac{\sigma K^3}{\rho_o}\right)^2 = 0. \tag{3}$$

The solution is

$$\omega^2 = \frac{\sigma K^3}{\rho_o}\left[\sqrt{1 + \left(\frac{\lambda_o}{\Lambda}\right)^2} - \frac{\lambda_o}{\Lambda}\right], \tag{4}$$

where $K = \frac{2\pi}{\Lambda}$, and $\lambda_o = \frac{\pi\sigma}{\rho_o c^2} = \pi\sigma\kappa_s$ is a characteristic wavelength of the substance that is being considered. λ_o is of the order of 10^{-8} cm for most fluids away from the critical point. For wavelengths smaller than λ_o, compressible effects are important. For wavelengths of the order of 10^{-5} cm in water, as would be observed in a light-scattering experiment, the correction to the dispersion relation amounts to approximately 0.1%, and the incompressible approximation is a good one for most purposes.

Note also the relation for β:

$$\beta = K\left[\sqrt{1 + \left(\frac{\lambda_o}{\Lambda}\right)^2} - \frac{\lambda_o}{\Lambda}\right]. \tag{5}$$

In the limit $\Lambda \ll \lambda_o$, we have the reduction

$$\omega^2 \to c_o^2 K^2$$

$\beta \to 2\pi/\lambda_o$.

If the damping coefficient is calculated from the viscous dissipation $D_{ij}\epsilon_{ij}$, one obtains for the attenuation per wavelength $\alpha\Lambda$, where α is the velocity potential damping factor,

$$\alpha\Lambda \approx \sqrt{\frac{\Lambda_o}{\Lambda}} \left\{ \left[1 - \frac{3}{2}\frac{\lambda_o}{\Lambda} - \frac{23}{12}\left(\frac{\lambda_o}{\Lambda}\right)^2 \right] + \frac{4\gamma}{\mu}\left(\frac{\lambda_o}{\Lambda}\right)^2 \right\}$$

with

$$\Lambda_o = \frac{32\pi^3\mu^2}{3\sigma\rho_o} \approx 330\frac{\mu^2}{\sigma\rho_o}.$$

Here, μ and γ are the shear and bulk viscosity coefficients, respectively. We see that to a very good approximation only the shear viscosity contributes to the damping of surface-tension waves.

Observation of the spectrum of light scattered by surface waves would then permit determination of the shear viscosity that is present for strains whose characteristic length is $\sim 10^{-5}$ cm, and whose frequency is less than 100 Mc (that is, the frequency of surface waves at $\sim 6 \cdot 10^{-5}$ cm is almost two orders of magnitude smaller than ordinary sound waves of the same wavelength).

R. H. Katyl, U. Ingard