VIII. NOISE IN ELECTRON DEVICES[*]

Academic and Research Staff

Prof. H. A. Haus
Prof. P. Penfield, Jr.
Prof. R. P. Rafuse

Graduate Students

J. L. Doane   H. J. Pauwels
R. L. Guldi   V. K. Prabhu

RESEARCH OBJECTIVES

Our research is concentrated in three areas: noise in optical masers, noise in frequency multipliers, and noise in parametric amplifiers.

The experimental study of noise in the laser oscillator has established the validity of the semiclassical van der Pol oscillator model in the regime near threshold. More rigorous quantum-theoretical studies have established the range of validity of this model and brought out effects that are not present in the semiclassical theory. Studies are under way to check experimentally the predictions of the quantum-theoretical analysis. The potentialities of the photoelectron counting method in studying amplitude fluctuations of the incident light is the subject of further study.

Another problem that has been undertaken is the basic formulation of the circuit theory of noise in nonlinear systems that are strongly driven by periodic signals. This theory should find application in the noise theory of frequency multipliers and degenerate parametric amplifiers, both of which are also under active investigation.

H. A. Haus, P. Penfield, Jr., R. P. Rafuse

A. PHOTOELECTRON STATISTICS PRODUCED BY A LASER OPERATING BELOW THE THRESHOLD OF OSCILLATION

This report deals with the theory and experimental verification of the probability distribution of photoelectron counts produced by a narrow-band Gaussian light source. In the experiments described here photoelectron counts were taken within both shorter and longer time intervals than the inverse bandwidth of the light.

When narrow-band Gaussian light is incident upon a photomultiplier the probability distribution of the photoelectrons emitted within a time interval that is short compared with the inverse bandwidth of the light should follow the Bose-Einstein distribution.[1-5] Until the advent of the laser, it was not possible to make counting measurements within time intervals as short as this. A laser operating slightly below threshold may emit narrow-band light with a Gaussian amplitude distribution centered about one frequency, provided one axial mode is sufficiently near the threshold of oscillation that its gain is high enough compared with the other modes to make it dominant. By "Gaussian

amplitude distribution" of the light, we mean that the light is describable in Glauber's $P(\alpha)$ representation[3,4] in which each mode has a $P(\alpha)$ function of the form

$$P(\alpha) = (1/\pi\bar{n}) \exp(-|\alpha|^2/\bar{n}).$$

In our experiments we used a helium-neon gas laser operating at 6328 Å with internal mirrors of hemispherical geometry and reflectivities greater than 99%.

By monitoring the spectrum of the photomultiplier current produced by the light, we adjusted the laser to operate slightly below threshold with a light bandwidth of approximately 380 cps. In an experimental environment the vibrations of the mirror mounts introduce frequency fluctuations that cause deviations of the light from an ideal narrowband Gaussian waveform. The frequency fluctuations are not detected directly in a photomultiplier homodyne detector and, with proper care, the conversion of the frequency fluctuations into amplitude fluctuations can be made negligible.[6] It is legitimate, therefore, to treat the waveform emitted from the laser as an essentially ideal narrowband Gaussian waveform. The power output of the laser was stabilized[7] against long-term drifts by a feedback network with a time constant of the order of 2×10^{-2} sec. In order to reduce background, the light was passed through a polarizer that was adjusted for maximum transmission. The photoelectrons emitted in the photomultiplier were counted in a high-speed counter with a double-pulse resolution time of 14×10^{-9} sec. Figure VIII-1 shows data for counting intervals of 10^{-5}, 10^{-4}, and 10^{-3} seconds, respectively. The number of photoelectrons observed in successive time intervals were recorded on tape and then processed by a computer. Approximately 13,000 samples were taken for each of the three counting intervals shown in Fig. VIII-1. If the laser output indeed consisted of a pure narrow-band Gaussian light centered at a single frequency, the probability of observing exactly n photoelectrons within a time interval that is short compared with the inverse bandwidth would follow the familiar Bose-Einstein Distribution Law.

$$P(n) = (1+\bar{n})^{-1} (1+1/\bar{n})^{-n}, \tag{1}$$

where $\bar{n}$ is the average count. On the semi-log plot of Fig. VIII-1a, this should appear as a straight line. One can see from the figure that the experimental points lie on or about a straight line down to counts of approximately 4. The deviation from the theoretical curve at lower counts is due to admixture of other modes at other frequencies. If we assume that these other modes are broadband and therefore effectively produce a Poisson probability distribution of their own, a probability distribution slightly different from the Bose-Einstein distribution can be calculated:

$$P(n) = (1/n!) (1+\bar{n}_m)^{-1} (1+1/\bar{n}_m)^{-n} \exp(\bar{n}_b/\bar{n}_m) \int_{\bar{n}_b(1+1/\bar{n}_m)}^{\infty} \exp(-u^n \, du), \tag{2}$$
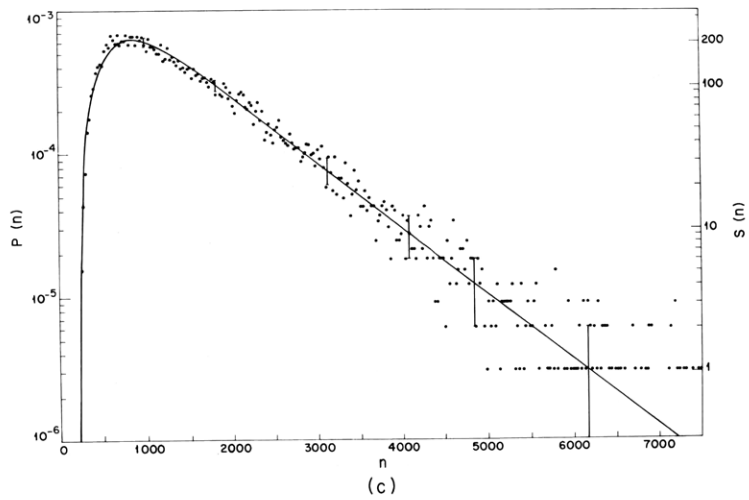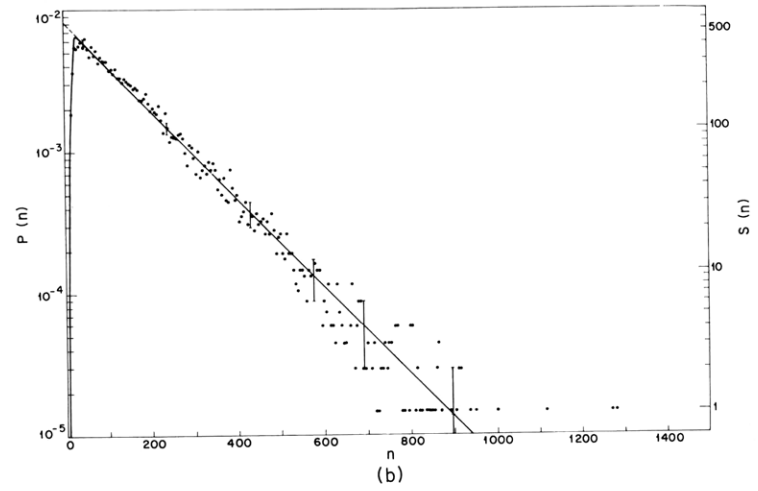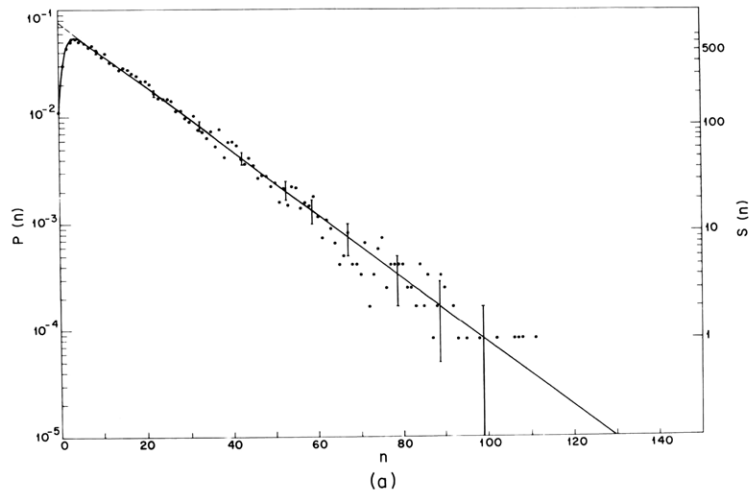
Fig. VIII-1.

Probability distribution and number of observed samples vs photoelectron count.

(a) Counting interval $T = 10^{-5}$ sec; 12,251 samples.

(b) Counting interval $T = 10^{-4}$ sec; 13,619 samples.

(c) Counting interval $T = 10^{-3}$ sec; 13,093 samples.

where $\bar{n}_m$ and $\bar{n}_b$ are the average photoelectron counts resulting from the narrow-band mode and the broadband background, respectively. Because the dominant mode is linearly polarized, the background can be estimated from the degree of polarization of the outcoming light. Since the background is generally unpolarized, it has twice the intensity of the light polarized perpendicular to the polarization of the dominant mode. By measuring the ratio of polarized to unpolarized light, we found that the average total photoelectron count within 10^{-5} second was 15.8, of which the dominant mode and the background counts were $\bar{n}_m = 14$ and $\bar{n}_b = 1.8$, respectively. Introducing these values into Eq. 2 results in the theoretical curve shown as a solid line in Fig. VIII-1a. The scattering of the experimental points, because of the finite number of samples taken, should show a mean-square deviation proportional to the square root of the number of samples taken. The predicted spread is indicated by vertical bars in the figures. The vertical scale S(n) on the right side of Fig. VIII-1 indicates the number of samples observed for any given photoelectron count, n. In order to simplify the plotting of data and to reduce the scatter, the experimental points shown in Fig. VIII-1b and VIII-1c are based on the averages of 5 and 25 adjacent data points, respectively. Figure VIII-1b shows the results obtained for a counting interval of 10^{-4} sec. The average values of photoelectron counts used in Eq. 2 to compute the theoretical curve indicated by a solid line in Fig. VIII-1b were $\bar{n}_m = 140$ and $\bar{n}_b = 19$. Figure VIII-1c shows the corresponding observations for a counting interval $T = 10^{-3}$ sec, which is longer than the inverse bandwidth (in radians) of the light (approximately 4.2×10^{-4} second for Fig. VIII-1). For counting intervals long compared with the inverse bandwidth, an asymptotic expression developed by Glauber[8] is applicable:

$$P(n) = (\bar{n}/\sqrt{2\pi\mu})\,(n)^{-3/2}\,\exp[-(\sqrt{n}-\bar{n}/\sqrt{n})^2/2\mu], \qquad (3)$$

where $\mu = w/\gamma$, γ is the linewidth, w is the average rate of photoelectron counts, and $\bar{n}$ is the average photoelectron count within the counting time interval, T. The inverse light bandwidth is determined from the spectral measurement and all other parameters in this equation are obtained from the photoelectron counting experiment. Inserting these values into Eq. 3, we obtain the theoretical curve shown as a solid line in Fig. VIII-1c. The shape of the probability distribution is not sensitive to small admixtures of broadband modes when the time interval becomes comparable to the inverse bandwidth. Therefore, the equation developed by Glauber need not be corrected for the background modes. In the present example, the counting interval is 2.4 times longer than the inverse bandwidth, yet the fit of experimental points to Glauber's equation is satisfactory.

Data for a counting interval of $T = 10^{-6}$ sec were also taken; in this case the effect of the background modes was even less apparent than in Fig. VIII-1a, insofar as only the $n = 0$ point deviated visibly from the extrapolated Bose-Einstein distribution. This point

fell at 84% (a factor of $e^{-\overline{n}_b}$) of the straight-line extrapolation.

We believe that this experiment is the first observation of a (nearly) Bose-Einstein probability distribution of photoelectrons. Also, the results shown in Fig. VIII-1c constitute the first experimental test of Eq. 3.

A determination of the probability distribution of photoelectron counts has been reported by Johnson, McLean, and Pike,[9] using an incoherent light source and, therefore, counting intervals much longer than the inverse bandwidth. Their work provided the initial impetus for the measurements reported here.

This work has been carried out jointly with C. Freed of Lincoln Laboratory, M.I.T. Many helpful discussions with R. J. Glauber, P. L. Kelley, R. H. Kingston, and E. R. Pike are gratefully acknowledged; P. Trent's computer programming is especially appreciated.

H. A. Haus

References

1. L. Mandel, Proc. Phys. Soc. (London) 72, 1037 (1959).

2. E. Wolf, Proceedings of the Symposium on Optical Masers, Polytechnic Institute of Brooklyn (Polytechnic Press, New York, 1963), Vol. 13, p. 29.

3. R. J. Glauber, Phys. Rev. 130, 2529 (1963).

4. R. J. Glauber, Phys. Rev. 131, 2766 (1963).

5. P. L. Kelley and W. H. Kleiner, Phys. Rev. 136, A316 (1964).

6. C. Freed and H. A. Haus, Appl. Phys. Letters 6, 85 (1965).

7. C. Freed and H. A. Haus, "Photocurrent Spectrum and Photoelectron Counts Produced by a Gas Laser," Phys. Rev. (to be published).

8. R. J. Glauber, Physics of Quantum Electronics, P. L. Kelley, B. Lax, and P. E. Tannenwald (eds.) (McGraw-Hill, New York, in press).

9. F. A. Johnson, T. P. McLean, and E. R. Pike, Physics of Quantum Electronics, P. L. Kelley, B. Lax, P. E. Tannenwald (eds.), op. cit.