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A. MODEL OF LARYNX ACTIVITY DURING PHONATION

We have developed a quasi-static model of larynx operation during the production of voiced sounds, using as an empirical basis the results of the experiments of van den Berg[1] on DC flow through a cast model of the larynx. This model has been simulated by using the time-sharing facilities of Project MAC, and the simulation has produced results that appear to be qualitatively similar to observed larynx activity.

1. Structure

The larynx is represented in simplified form in Fig. XIV-1, and it is for this pipe and duct representation that mathematical relations describing the operation have been developed. The region between the vocal folds, called the glottis, is represented by the rectangular duct connecting the trachea, or wind pipe, on the left with the vocal tract on the right. In opening and closing, the vocal folds increase and decrease the diameter, $2x$, of the glottis. Through the application of basic fluid-flow equations, the pressures at cross sections a, b, c, d, and e in Fig. XIV-1b can be related to each other and to the volume velocity of flow through the larynx.

2. D-C Flow

The mathematical relationships describing fluid flow in the larynx are based on the assumptions that, except at junctions between pipes or ducts of different cross-sections, the flow is parallel to the walls of the pipe and the pressure is uniform across the pipe. Two equations are needed: one to describe the relationship between pressure and flow at junctions between pipes, and the other to relate pressure and velocity in a uniform tube when there are viscous effects. Because both of these equations will relate pressure drop across an element of the system to current flow through that element, they may be thought of as defining fluid-flow resistances. The behavior at the junction can be described by Bernoulli's equation with a correction factor. The change in particle velocity necessitated by the change in tube area causes a change in the kinetic energy of the flowing air, the energy being proportional to the particle velocity squared:

$$\Delta KE \propto (\Delta v)^2, \tag{1a}$$

where ΔKE is the change of kinetic energy across the junction, and Δv is the change of particle velocity across the junction. Second, to keep the total energy in the system

TYPICAL VALUES

$2x_{max}$ = 0.1 cm

th = 0.32 cm

ℓ = 1.8 cm

D = 1.0 cm

(a)
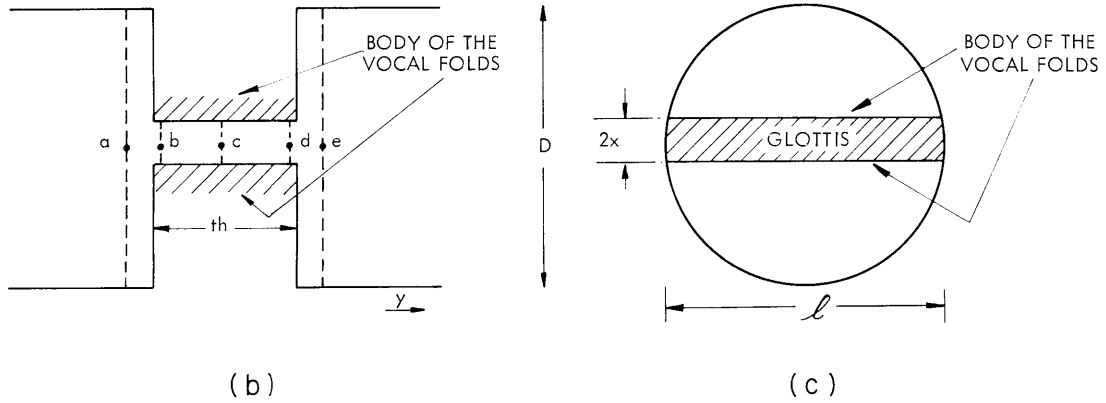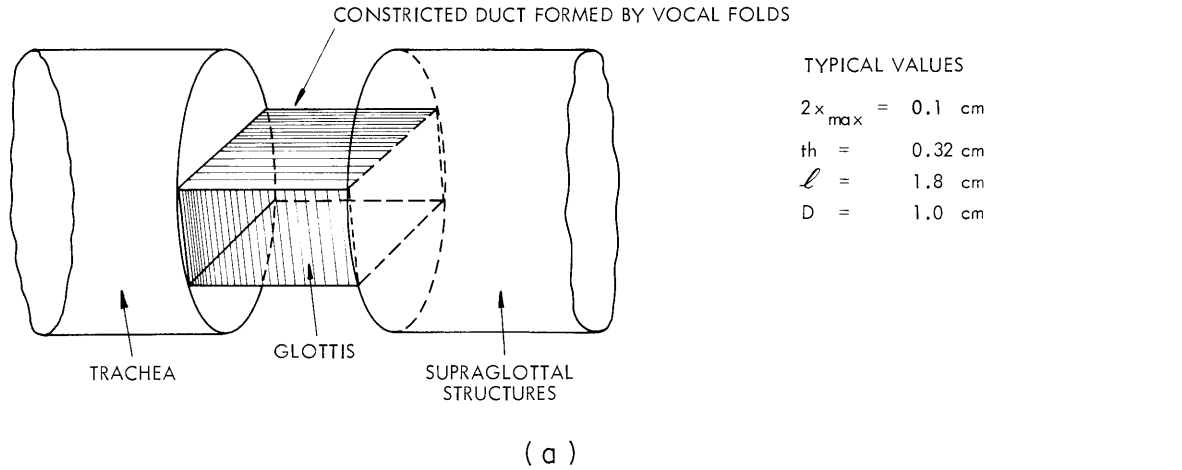
(b)   (c)

Fig. XIV-1.   Simplified larynx structure. (a) Fluid-flow model of the larynx.
(b) Frontal section through vocal folds. (c) Transverse section
through vocal folds.

constant, the potential energy, which is proportional to the pressure, must change by an equal amount. Third, the particle velocity

$$\Delta p \propto \Delta PE = \Delta KE, \tag{1b}$$

where Δp is the change of pressure across the junction, and ΔPE is the change of potential energy across the junction. The particle velocity in the tube, for uniform flow, is the volume velocity divided by the area:

$$\Delta v^2 = \left(\frac{A_2 - A_1}{A_1 A_2}\right)^2 u^2 \longrightarrow \left(\frac{\pm 1}{A_s}\right)^2 u^2,$$ (1c)

where

u = volume velocity

A_1, A_2 = areas on either side of the junction

$A_s = \min (A_1, A_2)$.

One can make the necessary approximation for deriving the rightmost term of relation (1) because, as may be seen from the typical dimensions given in Fig. XIV-1, the area of the glottis is much smaller than the areas of the adjacent tubes. This glottal
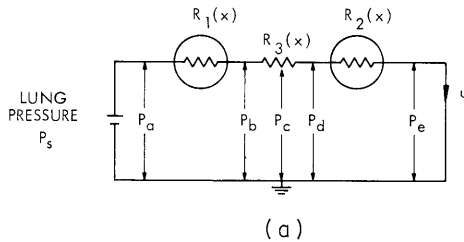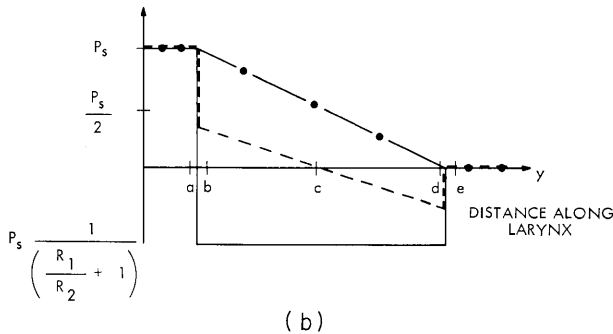

(a)

Fig. XIV-2.

(a) Electrical analog of DC flow.
(b) Pressure vs distance along the larynx.

(b)

area is directly proportional to x in Fig. XIV-1c. The sign of the last term of Eq. 1c will be positive for flow from a larger to a smaller tube, and negative if the situation is reversed.

Combination of Eqs. 1a, 1b, and 1c gives what may be considered a two-terminal Δp-u relationship relating the pressure drop across the junction to the flow through the junction:

$$\Delta p = R(x) \cdot u^2,$$ (2)

where R(x) may be considered a second-order, nonlinear resistor.

Because the effect of viscosity may also be considered to be resistive, it is convenient to introduce the electrical analog of Fig. XIV-2a. In the analog current, u represents the volume velocity (cm^3/sec) of air flow through the larynx, and voltage represents pressure. When measured relative to the datum node, the voltage represents a pressure measured at a point in the tube relative to atmospheric pressure; and the voltage measured between other points in the circuit represents the difference in pressures of these two points. The circuit is specifically used to relate the pressures on either side of each junction and half-way through the larynx to each other.

The junctions are modeled by resistors $R_1(x)$ and $R_2(x)$ which are of the form described in (2). The values are computed by inserting the proper constants of proportionality in Eqs. 1a and 1b:

$$R_1(x) = (k_1 R_{bb})/x^2 \tag{3}$$

$$R_2(x) = (k_2 R_{bb})/x^2, \tag{4}$$

where

$$R_{bb} = \frac{\zeta_o}{8\ell^2} = 4.40 \times 10^{-5} \tag{5}$$

with ζ_o = density of air = 1.14×10^{-3} and ℓ = length of the glottis = 1.8. (Numerical results are calculated from the typical values given in Fig. XIV-1.)

The magnitudes of k_1 and k_2 must be found empirically. They indicate the effects of the particle velocity profile across the glottis, the vena contracta effect when tube size decreases suddenly (spatially) along the direction of flow, and turbulent loss effects when tube size increases suddenly along the direction of flow.[2]

From measurements on a model, van den Berg,[1] found

$$k_1 = 1.37 \tag{6}$$

$$k_2 = -0.5. \tag{7}$$

Because the actual junction between the trachea and the glottis is quite smooth, there should be no vena contracta, with the result that k_1 may be considered to be determined solely by the velocity profile. The measured k_1 lies between the calculated value for uniform flow ($k_1 = 1.0$) and for parabolic flow ($k_1 = 1.54$). The difference in magnitude between k_1 and k_2 is indicative of the turbulence at the glottis-vocal tract junction. This quantity cannot be predicted from present knowledge of the glottis.

The effect of viscous drag in the narrow glottis may be modeled by a linear resistance. Under the assumption that the glottis is a section, ℓ units long, of an infinitely

long channel, the resistance $R_3(x)$ is

$$R_3(x) = R_v/x^3, \tag{8}$$

where

$$R_v = \frac{3\mu th}{2\ell} = 4.96 \times 10^{-5} \tag{9}$$

with μ = viscosity of air = 1.86×10^{-4}, and th = thickness of glottis = 0.32. (Numerical results are calculated from the typical values given in Fig. XIV-1.) Van den Berg's[1] measurements agree closely with this prediction. Viscous flow theory also predicts that the velocity profile is parabolic which, in turn, would give $k_1 = 1.6$ in Eq. 3. The discrepancy between measured and calculated values can be resolved by assuming that the actual flow is nearly uniform with turbulent boundary layer, but that the resistance is nearly the same as for nonturbulent, parabolic flow.

Using the electrical analog of Fig. XIV-2a, one may investigate the distribution of pressure along the larynx for various values of x. When x is very small, the viscous resistance dominates, and the pressure distribution for the simplified larynx structure is shown by the dot-dash line of Fig. XIV-2b. Note that the pressure at point C, which might be considered an average intraglottal pressure, is positive. At the other extreme, for very large values of x, the Bernoulli resistances dominate, giving the solid line in Fig. XIV-2b. For in-between values of x, both terms are present as shown by the dashed line of Fig. XIV-2b. For this last line, the value of x was chosen so that the intraglottal pressure was zero. The changing sign of the pressure in the glottis provides the possibility of cyclic operation of the vocal folds. To investigate this further, the steady-state model of air flow must be made into a quasi-static one, and a mechanical model of the vocal cords must be presented.

3. Quasi-Static and Mechanical Model

Included in the quasi-static model are the effects of the mass and compressibility of the air in the larynx and the rest of the vocal organs. In the glottis itself, the major effect is the inertia of the air mass flowing through. This includes the effect of the mass of air in the glottis at any one instant plus the radiation reactance into both the trachea and the vocal tract. All have been represented by inductances I_{11} and I_{12} in Fig. XIV-3b. The effects of the compressibility of the air in the glottis and the apparent lateral flow of air caused by the variation in the x dimension of the glottis have been considered unimportant and are neglected. The first effect has been neglected because the volume of the glottis is small, and the second because the rate of volume change of the glottis is small compared with the rate of volume flow through the larynx.

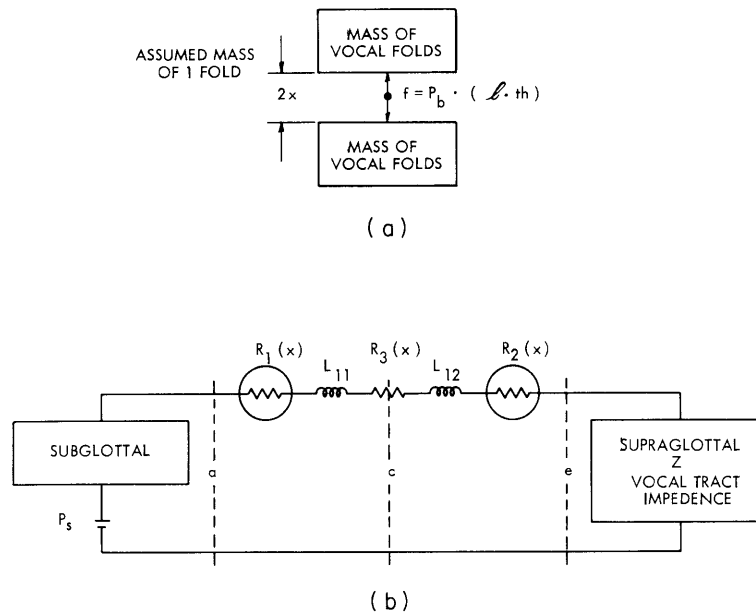Both the subglottal and supraglottal systems can be represented by impedances, as

Fig. XIV-3. Quasi-static larynx model. (a) Mechanical
system. (b) Flow system.

shown in Fig. XIV-3b. For the initial studies both of these impedances have been set
to zero.

The vocal folds themselves are modeled as a spring-mass system driven by the force
exerted by the intraglottal pressure on the surfaces of the vocal folds which are parallel
to the direction of air flow through the glottis. In the initial studies presented here, the
spring constant has been assumed to be zero, giving the resulting mechanical system
shown in Fig. XIV-3a. The mass is calculated by assuming that only the portion of each
vocal fold within 0.32 cm of the glottis takes part in the oscillation. Use of the typical
dimensions of Fig. XIV-1 and assumption of a vocal-cord density of 1 gm/cc gives a
constant mass of 0.018 gram for each vocal fold. This situation obtains during low-
pitch, chest-register voicing, when the tension of the muscles in the larynx is assumed
to be at minimum. This model is also assumed to be quasi-statically valid. This per-
mits the flow and mechanical models to be simulated separately and sequentially if the
interval between each iteration is kept small.

4. Results

Computer simulations of the model have been performed by using both the non-
inductive model of Fig. XIV-2a and the inductive model of Fig. XIV-2b with the subglottal
and supraglottal impedances set to zero. The results are presented as plots of x, u,
and p_c against time in Figs. XIV-4, XIV-5, and XIV-6, respectively. The time scale
is represented in terms of samples which in the simulation were at time intervals of
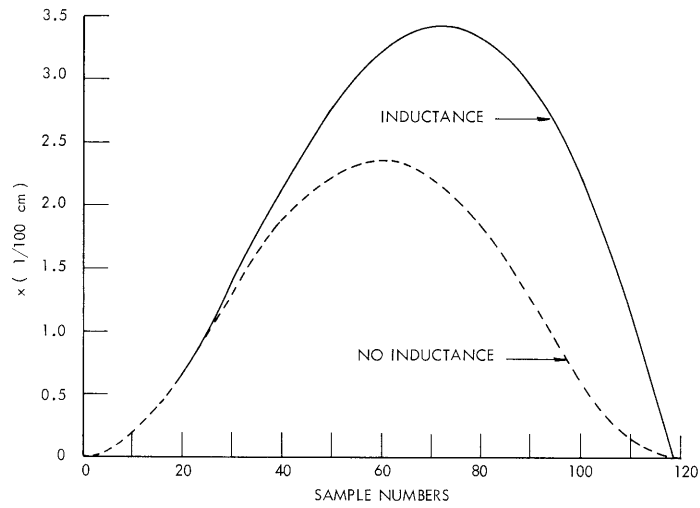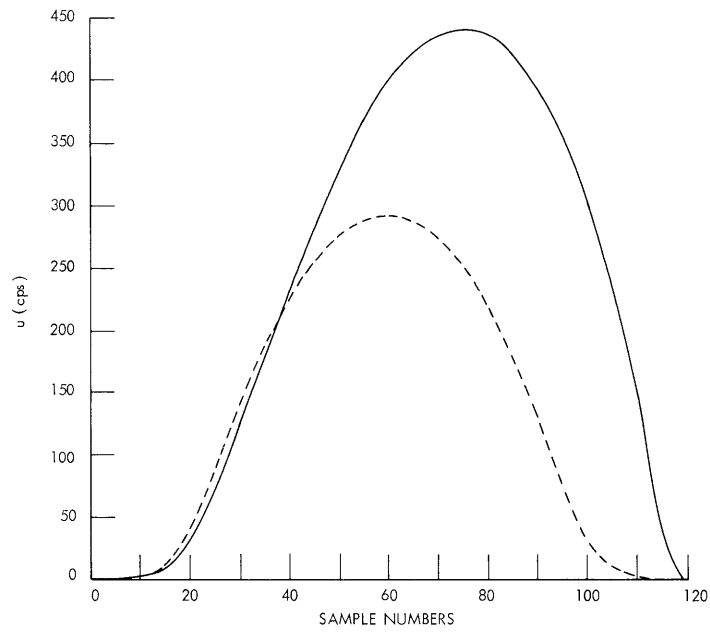
Fig. XIV-4. Diameter of the glottis during open cycle.

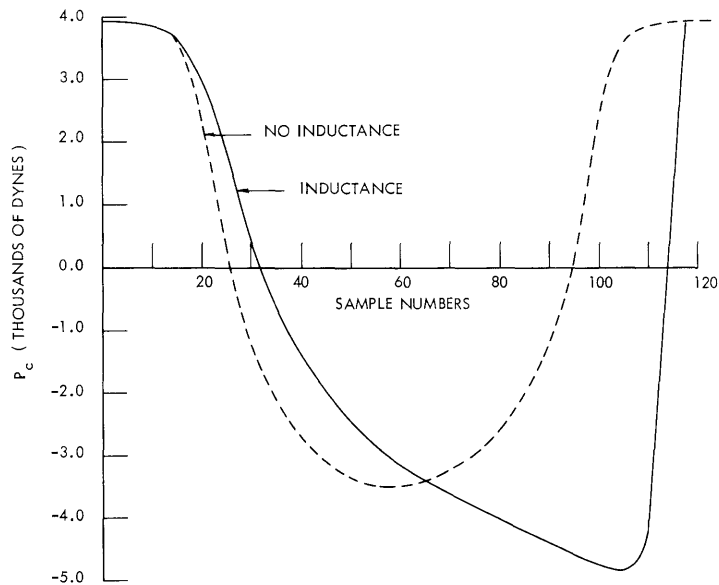Fig. XIV-5. Air flow through the glottis during open cycle.

Fig. XIV-6. Air pressure in the glottis during open cycle.

0.5×10^{-4} second. The open-time of the larynx of 5.9×10^{-3} msec is in the correct range for a normal male speaking with a low pitch. Note that the model does not enable one to predict the closed time of the pitch period and, therefore, duration of the entire pitch period. The symmetry of the noninductive waveforms is a natural consequence of the model. Adding the inductance produces waveforms similar to those found by applying inverse-filtering techniques to natural speech.[3]

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References

1. J. W. van den Berg, J. T. Zantema, and P. Doornenbal, Jr., "On the Air Resistance and the Bernoulli Effect of the Human Larynx," J. Acoust. Soc. Am. 29, 626-631 (May 1957).

2. V. L. Streeter, Fluid Mechanics (McGraw-Hill Book Company, New York, 1951).

3. Miss J. L. Miller, Bell Telephone Laboratories, Inc. (Private communication, 1965).