VIII. PHYSICAL ACOUSTICS*

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A. SOUND WAVE GENERATION AND AMPLIFICATION IN PLASMAS

In a gas discharge, the electron temperature is generally much greater than the neutral-gas temperature; consequently, energy is transferred to the neutral particles through electron-neutral collisions. This energy transfer implies that any sound waves in the neutral fluid will be nonadiabatic and the pressure must be considered as a function of both density and entropy. Since energy is being delivered to the neutrals, we may expect to observe this energy either in the steady increase of the neutral temperature, or in the generation of growing waves in the neutral fluid. It is this generation of sound waves which is of interest here.

If the linearized continuity and momentum equations for a nondissipative gas

$$\frac{\partial \rho}{\partial t} + \rho \nabla \cdot \bar{u} = 0 \tag{1}$$

$$\rho \frac{\partial \bar{u}}{\partial t} + \nabla p = 0 \tag{2}$$

are combined by subtracting the time derivative of Eq. 1 from the divergence of Eq. 2, we obtain

$$\frac{\partial^2 \rho}{\partial t^2} = \nabla^2 p, \tag{3}$$

where ρ is the mass density, and p the pressure of the neutral fluid. The density is related to the pressure and entropy, S, through the equation of state. If we assume a perfect gas, then

$$d\rho = (1/c^2)\, dp - (\rho/C_p)\, dS, \tag{4}$$

where c is the sound velocity, and C_p is the specific heat at constant pressure. Combining Eqs. 3 and 4 gives

$$\frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} - \nabla^2 p + \frac{\partial p}{\partial t}\frac{\partial}{\partial t}\left(\frac{1}{c^2}\right) = \frac{\partial}{\partial t}\left[\frac{\rho}{C_p}\frac{\partial S}{\partial t}\right]. \tag{5}$$

*This work was supported principally by the U. S. Navy (Office of Naval Research) under Contract Nonr-1841(42).

Using the fact that $\rho T_n \, dS/dt = H$, the rate of heat transfer to the neutral fluid, we obtain

$$\frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} - \nabla^2 p + \frac{\partial}{\partial t}\left(\frac{1}{c^2}\right)\frac{\partial p}{\partial t} = (\gamma-1)\,\frac{\partial}{\partial t}\left(\frac{H}{c^2}\right), \tag{6}$$

where γ is the ratio of specific heats. If we neglect the variations in sound velocity, Eq. 6 becomes

$$\frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} - \nabla^2 p = \frac{\gamma - 1}{c^2}\frac{\partial H}{\partial t}. \tag{7}$$

It is now necessary to derive an expression for H in the case of energy transfer from electrons to neutrals through elastic collisions. This expression is found by integrating the product of the energy transferred per collision and the probability of collision over all possible collisions. Thus

$$H = \int d^3 v_e \int_0^{\pi} \sin\theta \, d\theta \, \frac{m_e v_e^2}{2}\,\frac{2m_e}{m_n}\,(1-\cos\theta)\, v_e \sigma(v_e, \theta)\, f(\overline{v}_e)\, N_n, \tag{8}$$

where θ is the angle of scattering, σ is the differential cross section, $f(\overline{v}_e)$ is the electron distribution function, and N_n is the neutral density. The integrals in Eq. 8 depend on the energy and angular dependence of the cross section, but for our purposes, we shall assume an effective electron-neutral collision frequency, and set

$$H = kT_e(2m_e/m_n)\, N_e \nu_e. \tag{9}$$

The collision frequency, ν_e, is linearly proportional to the neutral density, and also varies with the electron temperature, T_e. From Eqs. 7 and 9, it can be seen that there will be a source term in the acoustic wave equation if the electron density, the electron temperature, or the collision frequency varies with time.

In the afterglow of a gas discharge, the electron temperature decays exponentially and provides a source term in Eq. 7. Experimental studies[1] performed in cryogenic afterglow plasmas have shown that the transverse acoustic modes of the discharge tube are excited, and the relative amplitude of oscillation increases with decreasing neutral temperature. These observations are in agreement with results derived from Eqs. 7 and 9 for an exponentially decreasing electron temperature.

If we consider variations in the collision frequency caused by variations in the neutral density (or pressure), then

$$\frac{\partial \nu_e}{\partial t} = \frac{\nu_e}{p_o}\frac{\partial p}{\partial t} = \frac{\nu_e}{m_n N_n c^2}\frac{\partial p}{\partial t}. \tag{10}$$

Inserting this into Eqs. 7 and 9 gives the homogeneous equation

$$\frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} - \nabla^2 p = \frac{2\beta}{c^2}\frac{\partial p}{\partial t}, \tag{11}$$

where

$$\beta = \frac{\gamma - 1}{2}\frac{T_e}{T_n}\frac{m_e}{m_n}\frac{N_e}{N_n}\nu_e. \tag{12}$$

By substituting a plane wave, $p = P \exp(ikx - i\omega t)$, in Eq. 11, we obtain the dispersion relation

$$\omega = i\beta \pm \sqrt{c^2k^2 - \beta^2} \tag{13}$$

which represents growing waves. Similarly, if we consider ω to be real, then the waves will grow in space. Equation 11 has also been studied for waves in cylindrical and spherical discharge tubes, and in both cases, yields self-excited sound waves. In these problems, the fact that the electron density is not uniform leads to coupling between the various acoustic modes, but this does not alter the fact that the waves are self-excited.

In an experimental study of ionic sound waves, Alexeff and Neidigh[2] also observed ordinary sound waves in a spherical discharge tube. They found that these ordinary sound waves did not occur at low pressures, but as the pressure increased (above ~1 mm), sound waves were spontaneously generated. From Eq. 12, we see that the growth rate, β, is proportional to pressure for constant degree of ionization. Dissipative effects in the bulk of the gas and at the boundaries, resulting from heat conduction and viscosity, decrease with increasing pressure. Thus as the pressure increases, the growth mechanism will dominate the dissipative mechanisms as observed experimentally.

In the experiments mentioned above, these sound waves have been detected through their effect on the electron fluid. The waves in the neutral fluid tend to produce waves in the ion fluid through ion-neutral collisions. These ion waves then tend to produce electron waves through the electric fields that they establish. It is these electron waves that may be detected by microwave interactions or with probes.

As seen in Eq. 9, any electron waves that are in phase with the sound waves, will increase the source term in Eq. 7. If we assume that the electron waves have the same relative amplitude as the neutral waves

$$n_e/N_e = n_n/N_n,$$

then the growth rate, β, has twice the value given in Eq. 12.

The important factors in maximizing the growth rate for these acoustic waves are: decreasing the neutral temperature (as has been done by Berlande, Goldan, and Goldstein), and increasing the pressure and degree of ionization (as has been done by Alexeff and Neidigh). If the features of both these experiments were combined, it should be possible to detect the amplification of a propagating sound wave fairly easily.

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References

1. J. Berlande, P. D. Goldan, and L. Goldstein, Appl. Phys. Letters 5, 51-2 (1964).
2. I. Alexeff and R. V. Neidigh, Phys. Rev. 129, 516-27 (1963).

B. HYPERSONIC RELAXATION EFFECTS AND BRILLOUIN SCATTERING

In a previous report,[1] some preliminary results of hypersonic velocity and absorption measurements in acetic acid were presented. These measurements were made by observing the frequency shifts and broadening in light from an He^3-Ne laser scattered by the liquid's thermal phonons. The sound propagation characteristics are inferred from the scattered light through the equations

$$\nu_s = \pm 2\nu_o \eta \frac{v_s}{c} \sin \theta/2$$

$$\delta\nu_s = \frac{a_s v_s}{2\pi},$$

where ν_o and ν_s are frequencies of incident light and hypersound; θ is the scattering angle; a_s is the sound absorption coefficient; η is the liquid's refractive index, and c is the velocity of light in vacuo; and $\delta\nu_s$ is the broadening in frequency of the scattered light.

In the previous report[1] the possibility of resonant acoustic damping in acetic acid, suggested by small oscillations observed in the velocity with frequency and by some roughly corresponding peaks in the total scattered linewidth, was expressed. Since these initial measurements were taken, sufficient improvement has been made in the accuracy of the experiment to rule out any acoustic resonance in acetic acid between 1 kMc and 6 kMc of sufficient strength to cause a velocity change $\Delta v_s/v_s > 0.1\%$. The previously observed peaks in linewidth were attributable to fluctuations in the maser power.

As Fig. VIII-1 shows, we have now observed a new relaxation in acetic acid. Other liquids that were studied either showed no dispersion from the ultrasonic (~1 Mc) to the hypersonic (~6 kMc) ranges or exhibited relaxations. Table VIII-1 summarizes the data for these liquids. Those liquids exhibiting an increase in sound velocity from the

ultrasonic to hypersonic range, evidence acoustic relaxation processes that are active in the kilomegacycle frequency range. Thus far, no resonances have been observed in the damping of hypersonic waves.

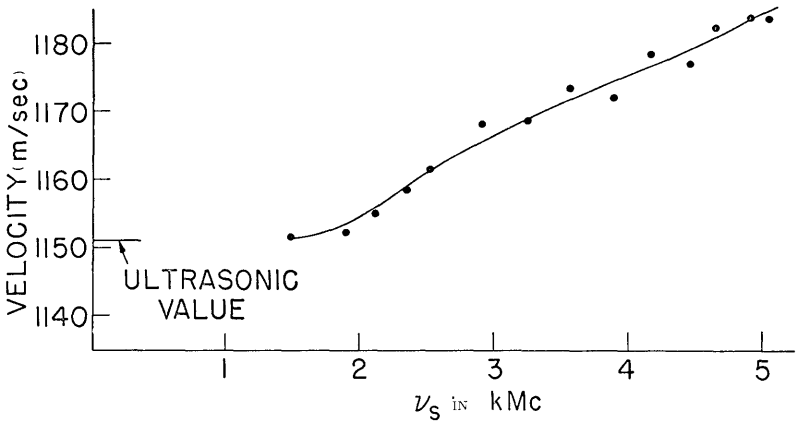

Fig. VIII-1. Hypersonic velocity in acetic acid values normalized to T = 21.6°C by using $\frac{\Delta\nu_s}{\Delta T}$ = -4.8 m/sec/°C.

Previous work in the hypersonic range has suggested a possible resonance in H_2O, attributable to reported decreases in the sound velocity with increasing frequency.[2] These results, however, were reported only for single hypersonic frequencies. Therefore, we measured sound velocity in H_2O at a variety of frequencies (1-7 kMc) and temperatures (10°-50°C). We found no departure from the 5-Mc values reported by Greenspan.[3]

The high accuracy (<0.1%) of our velocity measurements for maximum scattering angle (θ=180°) was achieved largely through a greatly improved signal-to-noise ratio resulting from the arrangement shown in Fig. VIII-2. This arrangement increases the intensity of the light scattered into the receiving system by a factor of approximately 100, as compared with that viewed in the variable angle arrangement.[4] Also, a new He^3-Ne laser, capable of a continuous output of 65 mW at 6328 Å, was constructed and used for these experiments. A much improved Fabry-Perot interferometer, capable of resolving 150 Mc, was also used.

Because of the different line shapes of the maser, the Fabry-Perot interferometer, and the phonon broadening, the extraction of $\delta\nu_s$ from the total observed scattered linewidth is cumbersome. A direct subtraction of the instrumental linewidth (maser plus Fabry-Perot) from the total linewidth is presented in Table VIII-1. Such subtraction is only valid when all line shapes involved are Lorentzian. In the present situation this procedure gives phonon broadenings 10-30% too small. A numerical procedure to correct

Table VIII-1. Comparison of ultrasonic and hypersonic velocity data in liquids with and without relaxation effects.

Liquid	v_o (~1 Mc)[a]	v_s (θ=180°)	T °C	ν_s (θ=180°)	$(\delta\nu_s)_{sub}$	$(\delta\nu_s)_{ext}$
Acetic Acid	1144 (m/sec)	1180 (m/sec)	22.0°	5.11 (kMc)	370 (Mc)	550 (Mc)
Benzene	1324	1501	21.9°	7.10	245	8,400
Toluene	1324	1376	21.8°	6.49	480	735
Nitrobenzene[b]	1462	1558	22.5°	7.62	670	1,200
CCl_4	920	1046	22.5°	4.82	430	1,285
CS_2	1158	1250	21.5°	6.40	55	43,900
H_2O[c]	1488.6	1488.0	22.0°	6.26	200	220
Acetone	1190	1190	21.6°	5.08	175	155
Ethyl Ether	1000	999	21.4°	4.28	180	–

[a] K. F. Herzfeld and T. A. Litovitz, Absorption and Dispersion of Ultrasonic Waves (Academic Press, New York, 1959), pp. 506, 419, 357. The temperatures of the ultrasonic measurements were not specified beyond identifying them as "room temperature." $(\delta\nu_s)_{ext}$ is the linewidth predicted by ultrasonic absorption data, when scaled up as ν_s^2 to kMc frequencies.

[b] J. R. Pellam and K. K. Galt, J. Chem. Phys. 14, 608, 1946.

[c] See M. Greenspan and T. Tschiegg, J. Res. Natl. Bur. Std. 59, 249 (1957).

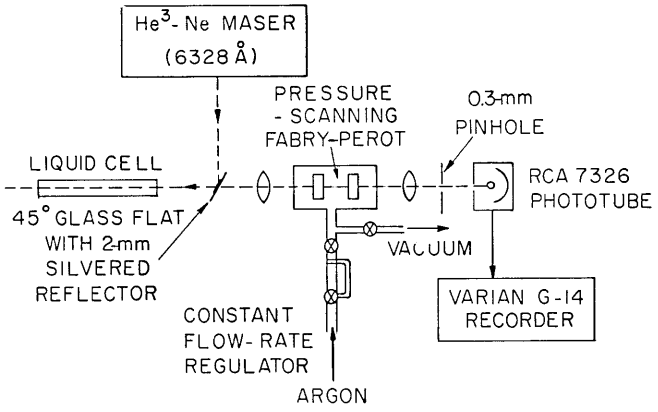

Fig. VIII-2. Experimental arrangement for observation of Brillouin scattering at 180°. The maser is linearly polarized perpendicular to the scattering area.

this error is now being worked out. The present results on linewidths are only semi-quantitative.

These experiments were done in collaboration with R. Y. Chiao of the Optical Maser Group,* M. I. T., which is directed by Professor C. H. Townes and Professor A. Javan.

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3. M. Greenspan and C. Tschiegg, J. Res. Natl. Bur. Stds. (U. S.) 59, 249 (1957).
4. P. A. Fleury V, op. cit., see Fig. XII-1.

*The research of this group is supported in part by the National Aeronautics and Space Administration.