PROCEEDINGS OF SCIENCE

Renormalisation of quark bilinears with $\mathrm{N_f} = 2$ Wilson fermions and tree-level improved gauge action

P. Dimopoulos*, **R. Frezzotti**, **G. Herdoiza**, **A. Vladikas**
Dip. di Fisica, Università di Roma Tor Vergata and INFN, Sez. di Tor Vergata, Via della Ricerca Scientifica, I-00133 Roma, Italy
E-mail: {dimopoulos,frezzotti,herdoiza,vladikas}@roma2.infn.it

V. Lubicz, **S. Simula**
Dip. di Fisica, Università di Roma Tre and INFN, Sez. di Roma Tre, Via della Vasca Navale 84, I-00146 Roma, Italy
E-mail: lubicz@fis.uniroma3.it, simula@roma3.infn.it

M. Papinutto
CERN, Physics Department, Theory Division,CH-1211 Geneva 23, Switzerland
E-mail: mauro.papinutto@cern.ch

On behalf of the ETM Collaboration

We present results for the renormalisation constants of bilinear quark operators, using the $N_f = 2$ twisted mass Wilson action at maximal twist (which guarantees automatic $O(a)$ improvement) and the tree-level Symanzik improved gauge action. The scale-independent renormalisation constants are computed with a new method, which makes use of both standard twisted mass and Osterwalder-Seiler fermions. Moreover, the results from an RI-MOM calculation are presented for both scale independent and scale dependent renormalisation constants.

The XXV International Symposium on Lattice Field Theory
July 30-4 August 2007
Regensburg, Germany

*Speaker.

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1. Introduction

We compute quark bilinear renormalisation constants (RCs), based on the ETMC $N_f = 2$ dynamical quark action which consists of a tree–level improved Symanzik gauge action and twisted mass (tm) Wilson fermions at maximal twist [1]. Our results are automatically improved in the spirit of ref. [2]. In section 2 we present a new (non-perturbative) method for the calculation of the scale independent RCs, Z_A and Z_P/Z_S, based on the use of two valence quark actions and a standard calculation of Z_V within the tm valence quark sector. In section 3 we describe the RI-MOM calculation of all RCs (both scale dependent and scale independent ones).

2. Calculation of the scale independent RC

In this section we present a calculation of the scale independent RCs, namely Z_V, Z_A, Z_P/Z_S. The evaluation of Z_V is based on the PCAC Ward identity method (see refs. [3] for details). This calculation leads to very precise results. The computational method for Z_A and Z_P/Z_S is new. It is based on the use of two regularisations for the valence quark actions. One is the standard twisted mass action, while the other is the Osterwalder–Seiler (OS) variant [4]. In the so called physical basis these actions can be compactly written in the form:

$$S_{val} = a^4 \sum_x \bar{\psi}(x)(\gamma\tilde{\nabla} - i\gamma_5\, r\, W_{cr} + \mu_q)\psi(x) \ , \tag{2.1}$$

with $W_{cr} = -\frac{a}{2}\sum_\mu \nabla^*_\mu \nabla_\mu + M_{cr}(r=1)$, $\psi = (u \ \ d)^T$, $r = \mathrm{diag}(r_u \ \ r_d)$ and $\mu_q = \mathrm{diag}(\mu_u \ \ \mu_d)$. The twisted mass case corresponds to $r_u = -r_d = \pm 1$, while the Osterwalder-Seiler case is obtained taking $r_u = r_d = \pm 1$. Sea quarks are regularized in the standard tm framework.

Consider that, for the two different choises of the matrix r, we perform the following two axial transformations of the quark fields, namely $(u,d) = \exp[i(\gamma_5\tau_3\pi/4)](u^{'},d^{'})$ and $(u,d) = \exp[i(\gamma_5\pi/4)](u^{'},d^{'})$, respectively. Each of the actions (2.1) transforms respectively into an action with the Wilson term in the standard form (no γ_5 and no τ_3). This is a rotation into the tm basis at maximal twist. However the tm action has a mass term of the form $i\mu\,\bar{\psi}^{'}\gamma_5\tau_3\psi^{'}$, while the OS one has $i\mu\,\bar{\psi}^{'}\gamma_5\psi^{'}$. Consider, now, an operator O_Γ defined in the physical basis. Under the two axial trasformations this operator transforms into two operators, called $O_{\tilde{\Gamma}}$ and $O_{\tilde{\tilde{\Gamma}}}$, which, in general, are not of the same form. However the respective renormalised matrix elements between given physical states have to be equal up to $O(a^2)$ effects. This is due to the fact that in the continuum limit each of them should coincide, up to $O(a^2)$, with the corresponding matrix element of the unique physical operator, O_Γ. Therefore, if we call $Z_{O_{\tilde{\Gamma}}}$ and $Z_{O_{\tilde{\tilde{\Gamma}}}}$ the respective renormalisation constants for the two operators, we have:

$$Z_{O_{\tilde{\Gamma}}}\langle O_{\tilde{\Gamma}}\rangle^{tm} = Z_{O_{\tilde{\tilde{\Gamma}}}}\langle O_{\tilde{\tilde{\Gamma}}}\rangle^{OS} + O(a^2) \ . \tag{2.2}$$

Renormalisation constants are named, as usual, after the basis in which the Wilson term has its standard form. For maximal twist, the operator renormalization pattern in the physical and twisted bases is shown in Table 1 for both OS and tm formalisms. The primed operators refer to the tm basis while the unprimed ones to the physical basis and we have adopted the notation, $O_\Gamma = \bar{u}\Gamma d$, for both the primed and unprimed case.

OS case	tm case
$(A_R)_{\mu,ud} = Z_A A_{\mu,ud} = Z_A A'_{\mu,ud}$	$(A_R)_{\mu,ud} = Z_V A_{\mu,ud} = -iZ_V V'_{\mu,ud}$
$(V_R)_{\mu,ud} = Z_V V_{\mu,ud} = Z_V V'_{\mu,ud}$	$(V_R)_{\mu,ud} = Z_A V_{\mu,ud} = -iZ_A A'_{\mu,ud}$
$(P_R)_{ud} = Z_S P_{ud} = iZ_S S'_{ud}$	$(P_R)_{ud} = Z_P P_{ud} = Z_P P'_{ud}$

Table 1: Renormalization pattern of the bilinear quark operators for the OS and tm case at maximal twist.

Calculation of Z_P/Z_S: Our method is based on comparing the amplitude $g_\pi =< 0|P|\pi >$, computed both in tm and OS formalisms. We start by considering, in the physical basis, the correlator $C_{PP}(t) \equiv \sum_{\mathbf{x}} < \bar{u}\gamma_5 d(x)\, \bar{d}\gamma_5 u(0) >$, which at large times behaves like $C_{PP}(t) \simeq \frac{|g_\pi|^2}{2m_\pi}[\exp(-m_\pi t) + \exp(-m_\pi(T-t))]$. In the twisted basis, this corresponds to $C_{S'S'}(t)$ in the OS case and $C_{P'P'}(t)$ in the tm one. Based on Table 1, this translates into

$$[g_{\pi^\pm}]^{cont} \;=\; Z_P\,[g'_{\pi^\pm}]^{tm} + O(a^2) \;=\; Z_S\,[g'_\pi]^{OS} + O(a^2)\ , \tag{2.3}$$

from which the ratio Z_P/Z_S is extracted.

Calculation of Z_A: We undertake the calculation of f_π in both OS and tm regularisations. In the tm case we use the Ward identity evaluation of the decay constant: $f_{\pi^\pm}^{tm} = 2\mu_q g_\pi/m_\pi^2$. Note that in this case no renormalisation constant is needed [5]. Thus the pion decay constant can be extracted from the large time asymptotic behaviour of $C_{PP}(t)$ as it is discussed above.

For the OS case we use the correlators C_{PP} and C_{A_0P} (with $r_u = r_d = \pm 1$). The large time asymptotic behaviour of the former correlator has been discussed above, while the latter goes like $C_{A_0P}(t) \simeq \frac{\xi_{A_0P}}{2m_\pi}[\exp(-m_\pi t) - \exp(-m_\pi(T-t))]$. Combining these, we can extract the bare OS estimate of the pion decay constant as $f_\pi^{OS} = \xi_{A_0P}/g_\pi m_\pi$. Since the tm and OS determinations of the (properly normalized) decay constant satisfy the relation

$$[f_{\pi^\pm}]^{cont} \;=\; f_{\pi^\pm}^{tm} + O(a^2) \;=\; Z_A f_\pi^{OS} + O(a^2)\ , \tag{2.4}$$

an estimate of Z_A is readily obtained. Since all computations are performed at finite mass, the final results for Z_A and Z_P/Z_S are finally obtained by extrapolation to the chiral limit. Moreover, maximal twist ensures that cut–off effects are of order $O(a^2)$ ([2],[4]).

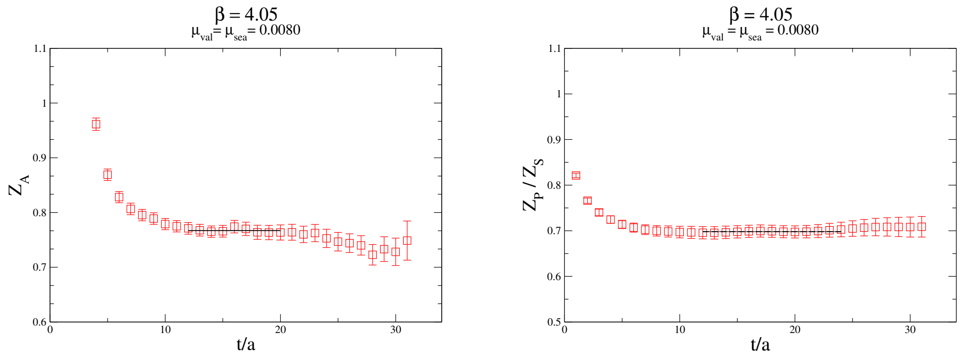
2.1 Results

Our configuration ensembles for $N_f = 2$ sea quarks have been generated at three values of the gauge coupling, $\beta = 3.80,\ 3.90$ and 4.05, corresponding to lattice spacings $a \sim 0.10, 0.09$ and 0.07 fm. We have performed 240 measurements for the two smallest β-values and 150 measurements for the highest one. In order to significantly reduce autocorrelation times, correlators were computed every 20 trajectories (each having trajectory length equal to $\tau = 1/2$). Five sea quark masses have been simulated at $\beta = 3.90$ and four at the other two couplings. The smallest sea quark mass corresponds to a pion of about 300 MeV and the higher one is just above half the strange quark mass. Eight valence quark masses were used at each coupling; the lowest ones are equal to the sea quark masses, whereas the others rise to the region of the strange quark mass. For the inversions in the valence sector we have made use of the stochastic method (one–end trick of ref. [6]) in order

β	3.80	3.90	4.05
Z_A	0.72(2)(1)	0.76(1)(1)	0.76(1)(1)
Z_P/Z_S	0.47(2)(1)	0.61(1)(1)	0.66(1)(1)
Z_V	0.5814(2)(2)	0.6104(2)(3)	0.6451(2)(3)

Table 2: The results for the scale independent RCs for three values of the gauge coupling.

to increase the statistical information. Propagator sources are at randomly located timeslices. This turned out to be an optimal way to reduce the autocorrelation time. Typical plots on the quality of the signal for the RCs (for fixed values of the bare coupling and masses) are shown in Fig. 1.



(a) (b)

Figure 1: Asymptotic behaviour of scale independent normalisation constants

Three methods were implemented in the RC computation. The first consists in calculating the RCs at fixed value of the sea quark mass for a number of valence quark masses and taking the “valence chiral limit"[1]. Subsequently, the RCs were quadratically extrapolated to the sea quark chiral limit [2]. The second method consists in inverting the order of the two chiral limits. The third method is simply the extraction of the RCs from the subset of data satisfying $\mu_{val} = \mu_{sea}$, which allows to reach the chiral limit with one single extrapolation in the quark mass. Our results from all three methods are compatible within one standard deviation. We present preliminary data from the second method, which has fits of better quality, in Table 2. The first error is statistical while the second is systematic coming from the difference between the central values of the various methods. A final analysis will be presented in a forthcoming publication.

[1] First and second degree polynomial fits in μ_{val} have been performed.

[2] A quadratic dependence on $a\mu_{sea}$ is expected from the form of the sea quark determinant, assuming that lattice artifacts on the RCs are not sensitive to spontaneous chiral symmetry breaking. However we have verified that a linear fit in μ_{sea} leads to compatible results.

3. RI-MOM calculation

The RI-MOM method is a non–perturbative, mass independent, renormalisation scheme proposed in ref. [7]. For a detailed presentation of various technical aspects see ref. [8]. In our case the scheme consists in fixing the Landau gauge and computing the momentum space Green function

$$G_\Gamma^{ud}(p,p') = \sum_{x,y} \langle u(x)(\bar{u}\Gamma d)_0 \bar{d}(y)\rangle e^{-ip\cdot x + ip'\cdot y} \ , \tag{3.1}$$

for a general quark bilinear operator $\bar{u}\Gamma d$ (with $\Gamma = A,V,S,P,T$) and the propagator is written as, $S_q = \sum_x \langle q(x)\bar{q}(0)\rangle e^{-ip\cdot x}$ with $q = u,d$.
Then the forward amputated Green function, $\Lambda_\Gamma^{ud} = S_u(p)^{-1} G_\Gamma^{ud}(p,p) S_d(p)^{-1}$, is projected by a suitable projector P_Γ (essentially a properly normalized Dirac matrix). The RCs, Z_Γ and Z_q are obtained by imposing the RI-MOM renormalization conditions

$$Z_\Gamma^{ud}(Z_u Z_d)^{-1/2}\Gamma_\Gamma^{ud}(p)|_{p^2=\mu^2} \equiv Z_\Gamma^{ud}(Z_u Z_d)^{-1/2} Tr[\Lambda_\Gamma^{ud} P_\Gamma]|_{p^2=\mu^2} = 1, \ Z_q \frac{i}{12} Tr\left[\frac{\not{p} S_q(p)^{-1}}{p^2}\right]_{p^2=\mu^2} = 1. \tag{3.2}$$

The computation is done for fixed quark masses. The results are extrapolated to the chiral limit. The renormalisation scale μ has to satisfy the condition: $\Lambda_{QCD} \ll \mu \ll \pi/a$.

The RCs, calculated in the chiral limit in the way described above, are $O(a)$ improved at large momenta [8]. Moreover an analysis based on the symmetries of MtmLQCD and the O(4) symmetry of the underlying continuum theory shows that $\Gamma_\Gamma^{ud}(p)$ and $\Gamma_\Gamma^{du}(p)$ are separately $O(a)$ improved for all momenta. In order to increase the statistical information, we computed the following combinations: $Z_\Gamma = (Z_\Gamma^{ud} + Z_\Gamma^{du})/2$ and $Z_q = (Z_u + Z_d)/2$.

The scale dependent RCs (Z_P, Z_S and Z_T) are obtained at a reference scale $\mu_0 = a^{-1}$, by cancelling the scale dependence μ, at a sufficiently high order in perturbation theory:

$$Z_\Gamma(a\mu_0) = (\ Z_\Gamma(a\mu)/C_\Gamma(\mu)\ )\ C_\Gamma(\mu_0)\ . \tag{3.3}$$

Here $C_\Gamma = \exp\int^{\alpha(\mu)} d\alpha\ [\gamma_\Gamma(\alpha)/\beta(\alpha)]$ and γ_Γ, β are the anomalous dimension of the operator and the beta function respectively. They are known at N^{2}LO for Z_T and N^{3}LO for Z_S and Z_P [9].

It is known that the RI-MOM estimate of Z_P is contaminated by the presence of a Goldstone pole [10]. In the twisted mass theory this problem also arises for Z_S, though $O(a^2)$ suppressed. All these contaminations are removed in the subtracted Green function [11]:

$$\Gamma_{P,S}^{\rm sub}(p^2,\mu_{q_1},\mu_{q_2}) = \frac{\mu_{q_1}\Gamma_{P,S}(p^2,\mu_{q_1}) - \mu_{q_2}\Gamma_{P,S}(p^2,\mu_{q_2})}{\mu_{q_1} - \mu_{q_2}} \tag{3.4}$$

where μ_{q_1}, μ_{q_2} are non–degenerate valence quark masses.

3.1 Results

The simulation parameters are the same as those of section 2.1. The RCs, computed at fixed sea quark mass and several valence quark masses, are first linearly extrapolated to the valence chiral limit. Subsequently, the sea quark chiral limit is obtained by linear extrapolation in μ_{sea}^2. In Fig. 2 we show the effect of the Goldstone boson subtraction for Z_P and Z_S for which the subtracted

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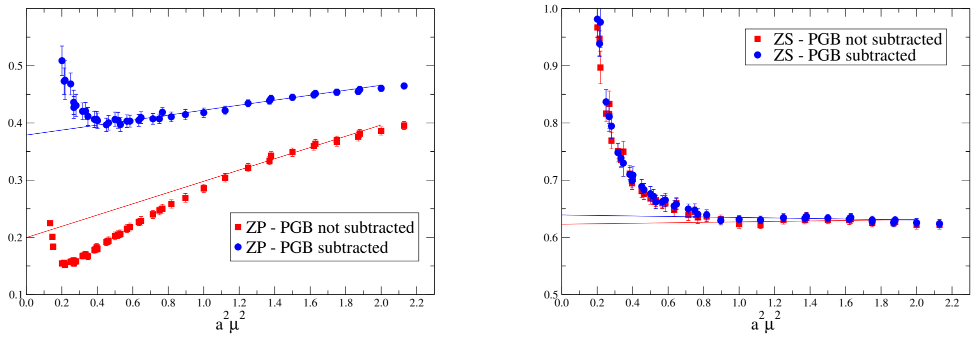

(a) (b)

Figure 2: Goldstone pole subtraction at $\beta = 3.90$; $Z_\Gamma(a\mu_0)$ of Eq. (3.3) is plotted against $(a\mu)^2$.

β	Z_A	Z_P/Z_S	Z_V	Z_P	Z_S	Z_T
3.80	0.75(3)	0.47(3)	0.62(4)	0.30(1)	0.64(2)	0.73(4)
3.90	0.76(2)(1)	0.63(2)(3)	0.65(2)(3)	0.39(1)(2)	0.62(1)(5)	0.75(1)(2)
4.05	0.77(1)	0.65(2)	0.67(1)	0.40(1)	0.61(1)	0.79(1)

Table 3: RI-MOM results for the RCs. Z_P, Z_S and Z_T are calculated at scale $\mu_0 = a^{-1}$ (see Eq. (3.3)). The results at $\beta = 3.80$ and 4.05 are preliminary and the quoted errors are purely statistical in these cases.

Green function of Eq. (3.4) has been used; we see that this has an important effect on Z_P, while Z_S is almost unaffected, as expected. Moreover, we note from Fig. 2 that once the scale evolution has been perturbatively divided out, the scalar RC is indeed scale independent, while the pseudoscalar one is still subject to large discretization effects. These are removed by linear extrapolation, giving a Z_P final estimate as the intercept of the fit.

In Table 3 we show our preliminary results for the RCs; for $\beta = 3.80$ and 4.05 the results correspond to the lighter value of the sea quark mass only. For $\beta = 3.90$ the results come from a full analysis in the valence and the sea sector. The first error is statistical and the second is systematic due to an estimate of the $O(a)$-contribution to the quark propagator which induces an $O(a^2)$ correction in the determination of the RCs. A better estimate of the systematic errors will be available once we finalize the analysis on all the three values of lattice spacing.

A first comparison for $\beta = 3.90$ between the results of Tables 2 and 3 shows that the values of Z_A and Z_p/Z_S are in nice agreement and of comparable statistical accuracy. The corresponding Z_V results, though compatible within the quoted errors, show that the PCAC Ward Identity estimate is statistically more precise[3].

[3] A slightly different determination based on the same WI taken between two one–pion states gives very similar results; for example, for $\beta = 3.90$ it is found, $Z_V = 0.6109(2)$ [12]. Moreover from the Table 1 we find that the value of the ratio $(Z_A/Z_V)^2|_{\beta=4.05}$ is consistent with the one found in [13].

We would like to note that combining the result of Z_P/Z_S from the first method with that of Z_S from RI-MOM, an alternative evaluation of Z_P can be obtained, in which the problem of the pseudoscalar Goldstone boson pole subtraction is avoided[4]. For example, for $\beta = 3.90$ this calculation gives $Z_P = 0.38$ which is compatible, within the errors, with the corresponding value given by the RI-MOM calculation (see Table 3). The results of a precise statistical analysis will be given in a forthcoming publication.

Acknowledgements
We wish to thank G.C. Rossi for fruitful discussions and comments on the manuscript. We also acknowledge C. Michael for helpful discussions. This work was partially supported by the EU Contract MRTN-CT-2006-035482 "FLAVIAnet".

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[4] We note in passing that $Z_P^{-1} = Z_\mu$ is the quark mass renormalisation constant in the twisted mass theory.