ENERGY RECOVERY LINACS

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Abstract

Energy Recovery Linacs (ERL) is a technique that makes use of the fact that linear accelerators can provide electron beam qualities which in some respects are superior to storage rings, without having to pay the price of unacceptable power consumption. The limitations to this statement, what the ERL is, and its advantages and disadvantages are discussed.

1 PATH OF DEVELOPMENT

Over the last few years there has been a running discussion on what will become the fourth-generation light source. At the moment there is no definite answer. New storage rings are being constructed but will they be fourth-generation sources? Free electron lasers have unique capabilities which no other source can beat. Energy Recovery Linacs (ERLs) come in between, having most of the good qualities of a storage ring and many of the good qualities of an FEL. So, will the ERL be the fourth-generation light source? We do not know. We do not even know how to define a fourth-generation light source. What special quality defines the fourth-generation source? Coherence, pulse length, diffraction-limited, etc.

The first-generation synchrotron light sources operated parasitically on high-energy physics machines. No real optimization was done to improve the generation of light. The second-generation sources were built to generate synchrotron radiation from bending magnets, and the third-generation sources were optimized for generating synchrotron radiation in undulators. This is where we are today.

Soleil in Paris and Diamond in Oxford are improving on almost all details of the radiation from current sources, but is that enough to call them fourth-generation sources? I do not think so and prefer to call them three-and-a-half-generation sources.

The FEL is able to generate coherent, powerful, tunable, diffraction-limited radiation in truly femtosecond pulses. This perhaps qualifies them technically as fourth-generation sources, but will they be available for everyone? A few large facilities will be built, each one of them providing a handful of parallel beamlines. A small number of key experiments which can not be performed elsewhere will be performed here, but the majority of science will be done on other sources.

Will the ERL become the fourth-generation source? Many aspects tell us yes, but there are also question marks. We shall see.

2 WHAT IS AN ERL

In a storage ring, electrons emitting light come back turn after turn, we can say that they are re-used. This has certain disadvantages as the electron beam characteristics are defined by an equilibrium that is found after a large number of turns.

In the ERL it is not the electrons that are re-used, but only their energy. Thus the electron emitting light at this moment, is not the same one as during the last turn. This is favourable as the equilibrium electron beam has not been created and a 'one-time-use' beam can be made with a smaller emittance both transversally and longitudinally.



Fig. 1: Step one of an ERL – acceleration

In the first step of an ERL, electrons from the injector are accelerated (Fig. 1). The electron beam is then conducted to the experimental area where the synchrotron radiation is extracted.



Fig. 2: Step two of an ERL – deceleration

In the second stage of the ERL, the electron beam is directed back to the accelerating structure but with a phase change of 180 degrees. Thus the electrons are decelerated instead of accelerated, and after the deceleration they are extracted at low energy and dumped (Fig. 2).



Fig. 3: Alternative layout of an ERL – acceleration and deceleration over several turns

An alternative way of operating the ERL is to run acceleration over several turns, using the same accelerating structure more than once (Fig. 3). In the final (outermost) turn the generation of synchrotron radiation takes place and the electrons arrive in the subsequent turns in the decelerating phase. Thus passing the same orbits in reverse order until they are slowed down to the injection energy and can be dumped.

The energy recovery in this process takes place in the accelerating structure. The energy taken from the electron beam in the decelerating phase is 'stored' in the accelerating structure and can be

used to accelerate a following electron bunch. When put into operation it is not necessary to supply energy for the electron beam anymore, but only the losses in the system. These losses are resistive losses in the accelerating structure and losses of already accelerated particles, which then can not give their energy back.

The power loss in a linac is given by

$$P_{\text{wall}} = \frac{\hat{E} L}{Z_{\text{s}}}$$

where E is the field strength, L the length of the structure, and Z_s the shunt impedance. The ability to store energy in the linac is given by

$$W = \frac{Q P_{\text{wall}}}{\omega}$$

where W is the stored energy, Q the quality factor and ω the frequency.

The build up, or decay, of fields in a linac is given by

$$E = E_0 \left(1 - e^{-\frac{\omega t}{Q}} \right) \; .$$

The fields in a cavity in an ERL are sketched in Fig. 4. For a standard normal-conducting (NC) linac the $Q = 1 \times 10^4$ and for the TESLA superconducting linac the $Q = 3 \times 10^9$ (unloaded Q-values). The loading of a cavity goes exponentially towards the maximum value. When a beam is accelerated in the cavity, the fields are decreased towards a new equilibrium. Finally the accelerated beam returns to further load the cavity and the fields increase towards a higher equilibrium. If we increase the Q-value we achieve much smaller wall losses for a given stored energy. The decay and change over time will also be slower and less sensitive. On the other hand the 'memory' of the cavity will be much longer.



Fig. 4: Fields in a cavity. a) filling an empty cavity, b) an accelerated beam loads the cavity, c) a decelerated beam fills the cavity.

3 BRILLIANCE, EMITTANCE, DIFFRACTION LIMIT, AND PULSE LENGTH

With these machines the aim is of course to produce radiation with the best quality possible. What quality actually means can be fairly arbitrary, but when looking at your experiment you are normally interested in how many 'good' photons you get on your sample. That would mean the number of photons at a certain wavelength, with a certain polarization, within a time window. This can be given as flux (normally: photons/(second * 0.1% bandwidth * A electron beam). Unfortunately, the flux for one machine will not tell you much when comparing the flux with the one you will get with another machine.

3.1 Brilliance

The ability to control your photon beam and focus it on your sample is given by the flux divided by the phase space of the radiation, this is called brilliance (or brightness) (flux / $mm^2 \times mrad^2$). Brilliance is a unit possible to compare between different sources.

$$Brilliance = \frac{Flux}{4\pi^2 \Sigma_x \Sigma_{x'} \Sigma_y \Sigma_{y'}}$$

where Σ is the width of the distribution, *x* defines the horizontal plane, and *y* the vertical. Σ_x refers to size and $\Sigma_{x'}$ to angle.

The Σ contains one contribution from the electron beam (the source) and one from the radiation field, giving

$$\Sigma_{x} = \sqrt{\sigma_{x}^{2} + \sigma_{r}^{2}} \quad \Sigma_{x'} = \sqrt{\sigma_{x'}^{2} + \sigma_{r'}^{2}}$$

for the horizontal direction, and similar for the vertical direction.

The electron beam quality is given by the electron beam emittance, defined by

$$\mathcal{E} = \sigma_{x} \sigma_{x'}$$

where ε is the emittance of the electron beam.

Regarding the photon beam the emittance is given by

$$\varepsilon = \sigma_r \sigma_{r'} = \frac{\lambda}{2\pi}$$

Exactly what you will find in the denominator depends on which reference you read, if you look at fields or powers etc. What is quite clear is that the emittance of the photon beam is a fundamental of the wavelength and we can not do much about it.

3.2 Diffraction limit

We can tailor the electron beam emittance with the optics of the storage ring or the emittance of our linac source. If we can make the σ_x and $\sigma_{x'}$ smaller than the σ_r and $\sigma_{r'}$ then it is no use going any further because we can not gain anything more in brilliance. In this case we call our accelerator 'diffraction-limited'.

From Fig. 5 we can see that if we want to generate diffraction-limited radiation at 1 keV we need an emittance a bit below 1 nm rad. This is lower than what the thind-generation storage rings can make today, but a few of the rings under construction and design can do it. The reason is that the emittance of a storage ring is given by the equilibrium emittance which is the fight between excitation through emission of radiation and damping.



Fig. 5: The diffraction emittance for different photon energies

3.3 Pulse length

The pulse length in a storage ring can be reduced by playing with the momentum compaction, but there is a price to pay: lifetime and current of the electron beam. When the electron bunch length is reduced, the electron density is increased and thus scattering and collisions within the electron beam increase. This will reduce the lifetime of the beam and the capability to cope with high currents. In a single-pass machine the lifetime is of almost no importance, and thus the electron beam density can be increased without having to pay (at least not in this respect).

4 WHAT CAN AN ERL GIVE US

The linac is a machine that regarding brilliance and bunch length can in principle produce a better beam than a storage ring. The ordinary linac though has a power consumption that makes it almost impossible to compete on any scale with a storage ring regarding average radiation. The exception is the free electron laser (FEL) which can extract very much more energy from the electron beam. Let us now look at the capabilities of the ERL compared in particular with the storage ring.

4.1 Emittance and brilliance

The emittance of an electron storage ring scales as

$$\mathcal{E}_{\rm h} \propto \frac{\gamma^2}{M^3}$$

where γ is the energy and *M* the periodicity of the storage ring. Thus when going to higher electron energies the emittance increases. The counteraction is to design a lattice with a high periodicity, requiring many strong-focusing elements. The best storage rings existing or under design today are approaching 1 nm rad horizontal emittance. The vertical emittance though is given by the coupling in the storage ring. A typical value has been 10% but newer rings are performing better mainly because improved alignment procedures. The value is closer to 1% today. This would imply a vertical emittance of 10 pm rad.

In a linac, the normalized emittance (= $\epsilon\gamma$) can in principle be conserved from the electron gun and down the accelerator. A good gun can produce a normalized emittance of 1 µm rad. The actual emittance for a 5 GeV (γ = 10000) linac can then be ϵ = 100 pm rad in both the horizontal and vertical direction. The linac, or ERL, thus has a smaller horizontal emittance than a storage ring but a bigger vertical emittance. The brilliance can be enhanced either by increasing the flux or by decreasing the emittance until the diffraction limit is reached. Development is ongoing but the ERL will not be significantly better than the storage rings as regards emittance or brilliance.

4.2 Bunch length

The ERL bunch length can be achieved in a number of ways. As the equilibrium situation from a storage ring will not have time to develop, the bunch length can be tailored at the electron gun and maintained through the system. There is also the possibility to compress the bunch at certain locations around the machine.

The electron gun normally consists of an RF-gun structure with a photocathode onto which a laser beam is focused. A typical RF structure operates around 3 GHz and such a gun creates a bunch of a few ps length if left free-running. If a short laser pulse is used, a shorter window can be created in the femtosecond range. Lasers providing 100 fs pulses are available, but not trivial to construct.

Compression of an electron bunch is done by letting the bunch pass an accelerating structure slightly off-crest. The head and the tail will thus receive slightly different energies and will follow paths with different lengths in a dispersive section. The tail can catch up with the head and thus compress the bunch. The result is a bunch that in principle can reach below 100 fs, which is far below what can be achieved in a storage ring. The price is paid in increased energy spread of the bunch.

An obstacle though is that a short bunch will generate coherent synchrotron radiation (see below).

4.3 Energy savings

The name Energy Recovery Linac cries out for an investigation of the energy savings that are possible. The first important concept is to actually recover the already accelerated beam. In the existing ERL systems, as at Jefferson Lab, efficiencies up to 99.97% have been achieved. One can thus assume energy recovery works, there are savings to be made.

An important fact is that a 100 mA 5 GeV beam that is used only once means a power of 500 MW. It is impossible to spend this amount in any realistic project, and thus ways of reducing the consumption have to be made. The easiest way is to reduce the average current, but then the average brilliance falls equally fast. Another way is, of course, to recover the energy.

To make the most efficient energy recovery system, one normally considers the superconducting linac technology as superior. In the LBL LUX proposal [1] the two alternatives of superconducting or normal-conducting linacs have been investigated (Table 1). (Four passes through a 600 MeV linac generating 2.4 GeV and a repetition rate of 10 kHz.)

	NC	SC
RF-power peak	240 MW	0.288 MW
RF-power average	6 MW	0.288 MW
Cooling power	0	~3.5 MW
Total	6 MW	3.8 MW

Table 1: Comparison of power consumption: normal-conducting v. superconducting linac for the LBL LUX

It is obvious that for peak power, the normal-conducting linac can never compete with the superconducting alternative. On the other hand, we are most interested in average power in this case. To get a full picture we also have to add the cooling power of the superconducting system. The final figure is still better for the superconducting technology, but only by a factor of two. This has to be compared with the added complexity of such a system. If the repetition rate could be increased to 100 kHz the SC would be outstanding.

Let us compare an ERL with a storage ring (Table 2). The Cornell ERL-2 [2] will operate at 5 GeV with a current of 100 mA. This is comparable to the ESRF (at 5 GeV and 200 mA).

	Cornell	ESRF
RF-power peak	1.1 MW	2.6 MW
RF-power average	1.1 MW	2.6 MW
Cooling power	16.4 MW	0
Total	17.5 MW	2.6 MW

Table 2: Comparison of RF power consumption ERL vs. storage ring

The cooling power of the superconducting ERL consumes more than the storage ring RF power, and thus there is no gain. On the other hand, the electron beam power is 500 MW for the Cornell machine, and thus the ERL is a solution if the unique beam of a linac source is desired.

One can argue whether these comparisons are correct, but what is obvious is that the ERL does not always save energy.

4.4 Radiation savings

By continuously dumping a high-current beam there is a large potential of generating radiation. A storage ring slowly loses its current over several hours and the total number of electrons is fairly low, thus the 'dumped' power is very low. In a single-pass machine the power might get very large.

A 100 mA beam which is dumped at 5 GeV would mean dumping 500 MW. This is of course impossible on account of energy considerations, but also the generated radiation would be a severe problem. Decelerating the beam down to below 10 MeV reduces the dump power to 1 MW. What is also important is that by going below 10 MeV with an electron beam means that one comes below the neutron production threshold and thus it is easier to shield the radiation dump. For a single-pass machine, like an ERL, energy recovery is important to reduce the radiation hazards.

A storage ring like the ESRF (200 mA, 5 GeV, 24 hours lifetime) generates 30 mW in 'dump' power.

5 ACCELERATOR PHYSICS ISSUES, LIMITATIONS

The ERL is built with either one recirculation or with several passages through the linac systems. In one passage there are a number of key issues to take into account (Fig. 6).



Fig. 6: Beam issues in a one-recirculation ERL

5.1 Coherent Synchrotron Radiation (CSR)

A short electron bunch can radiate synchrotron radiation coherently at wavelengths that are longer than the length of the bunch. In this case all electrons radiate in phase. This can be used to generate powerful infrared radiation, but the radiation emitted from the back end of the bunch can also overtake the head of the bunch which then will be influenced by the electromagnetic field. This can cause both a change of energy and a displacement, and thus the energy spread and the emittance can be destroyed. The effect is most prominent for short bunches at low energies. Thus a sub picosecond bunch just after the gun is very sensitive, and so are the bunches after hard compression later in the system. The effect can be reduced by having larger radii in the bending magnets and, in some cases, by small diameters on the vacuum vessel that shields the CSR.

5.2 Phase slip

It is important to find a proper balance between the accelerated and decelerated beam to match the energy taken and deposited. A low-energy beam will slip in phase relative to the linac phase differently than a high-energy beam. Choosing a low injection and extraction energy makes the problem worse, though this is the choice that is good for the efficiency of the recovery process.

5.3 Multi-energy beam focusing

As beams with different energies pass through the same focusing elements, special care has to be taken in the design. Proper focusing of the lower energies, which is the normal way, will give too-weak focusing for higher energy beams. The contrary would mean over-focusing of the low-energy beams and is less favourable.

The focusing of multi-energy beams is even more difficult as there is a larger span in the different energies and they must pass the same elements going in and out from the accelerating structure. In addition, there is also the problem of dividing and merging the beams in the arcs of the recirculation.

5.4 RF focusing

The linac itself focuses the electron beam both horizontally and vertically. This focusing is dependent on the beam energy and energy gain.

5.5 Beam loss

A loss of the beam will first generate heating-up and activation of the components where it is lost. Another effect is that the balance of the fields in the linac system will not be correct. A larger power has to be supplied by the klystron to the linac. The balance between the different fields can be seen in Fig. 6. If the decelerated particles suddenly become fewer, a new equilibrium point will be found and thus the energy in the process will change.

The loss of particles can be due either to the design of the system, where the aperture is not large enough, or to instabilities. What is more of a problem are 'halo particles'. These are particles in the beam but very far out from the centre, normally disregarded when discussing the emittance of the beam.

5.6 Beam break-up (BBU)

If the beam does not pass in the centre of the accelerating cavity it will induce fields of other modes in the cavity. The reason for not passing in the middle could be either misalignments of the accelerator, or misplacements of the beam itself. The induced fields can oscillate in the cavities if they fit a cavity mode. Such modes other than the main mode are called higher-order modes (HOM). A subsequent electron bunch, or the same bunch after one turn, will sense these fields and instabilities can be induced (Fig. 7).



Fig. 7: Beam break-up (BBU)

The BBU effect can be treated by a good damping of the unwanted modes in the cavity and a good alignment of the accelerator. In the existing and earlier ERLs this effect has been a limiting factor on the achieved maximum current. The MUSL ERL in Illinois [3] never became stable and at Jefferson Lab IR-demo [4] the current limit is around 10 mA.

In a multiturn machine the complexity of the BBU is increased as there will be a larger number of turns and sensitivity for longer range modes in the cavity.

5.7 Challenges

There are several challenges for the future development of ERLs. The first is the construction of continuous electron guns. The gun not only has to operate CW, but to make use of the capabilities of an ERL the emittance has to be very small. To achieve the small emittance, the most common route is to use a photocathode RF gun. A laser is focused onto the cathode surface extracting the wanted electrons. The cathode sits inside an RF cell which gives the initial acceleration immediately. This kind of device can produce very small emittances. Problems lie mainly in the areas of high-repetition-rate lasers and on making the RF structures superconducting, and maintaining them cold while shining into them with a laser. Alternative solutions would be thermionic guns, where the cathode is heated to continuously emit electrons. The emittance of such guns though is poorer.

Controlling HOMs in superconducting cavities is a key topic to be able to reach high average currents.

The optics of recovery lines and especially the systems where multi-energy beams are envisioned is a true challenge. The splitting and joining of beams also poses significant problems. These problems have to be solved and effects diluting the emittance, like CSR, have to be taken into account.

6 THE CURRENT SITUATION

In 1965 Mauro Tigner published the first ideas for an ERL [5]. It took a number of years until the ideas were put into realisation at the University of Illinois in the MUSL accelerator in 1977. Unfortunately, stability was not attained and we had to wait another 10 years until T. Smith in 1986 operated the SCA at Stanford in energy recovery mode [6] (Fig. 8).



Fig. 8: The SCA at Stanford 1986 [6]

Following the Stanford development the S-DALINAC [7] in Darmstadt, Germany, came into operation in 1990. It is built from a 10 MeV injector and three passes through a 40 MeV superconducting linac, making a maximum energy of 130 MeV.

The most intense development has been done at Jefferson lab [4] in the US. In 1995 the CEBAF [4] came into operation. It is a continuous 6 GeV accelerator for nuclear physics. Today there are plans to upgrade this machine up to 12 GeV.

At Jefferson lab there is also an infrared FEL that has been in operation since 1999 and is now being upgraded to a 160 MeV 10 mA machine [4]. A comparable machine to the Jlab FEL, but at lower energy, is the JAERI-ERL and FEL in Japan [8]. Looking into the future there are a number of laboratories with intense activities and development. One of the most thrilling concepts was presented by G. Kulipanov et al. in Novosibirsk [9], the MARS (Multipass Accelerator Recuperator Source) (Fig. 9).



Fig. 9: The MARS concept [9]

To overcome some of the problems of accelerating low- and high-energy beams in the same structures and being able to optimize the optics, the system is divided into two accelerating linacs.

There are three further proposals for synchrotron radiation sources in the US.

The LUX at LBL is a 3 GeV racetrack-shaped machine [10] (Fig. 10). In the first stage it will not operate in energy recovery mode; this option is being kept for a second step. This is because of the problems of stability at higher currents. The main objective of the project is to create short femtosecond pulses of radiation.



Fig. 10: Layout of the LBL LUX proposal [1]

Another proposal is the joint Jefferson lab–Cornell ERL [11]. The first step consists of a 100 MeV ring-like machine operating with one pass. A second step would be to build the full-scale machine which is a 5 GeV ERL for synchrotron radiation.

At Brookhaven on Long Island the NSLS is proposing the PERL project which is a 2.7 GeV recirculated linac for synchrotron radiation [12].

In Europe we have the 4 GLS in Daresbury UK [13]. This is the 'accelerator physicists dream', a machine that can do almost anything. One part of the project is a 600 MeV superconducting linac capable of running in energy recovery mode.

At the university of Erlangen in Germany there is an ambitious proposal for a 3.5 GeV energy recovery linac for synchrotron radiation [14], the ERLSYN.

Finally, there is, of course, also a Japanese proposal from KEK for a 2.5–5 GeV energy recovery linac [15].

	Operation	Туре	Type Energy		Purpose
MUSL	1977	unstable		multiturn	
SCA	1986	ERL	50 MeV	ring	SR+FEL
S-Dalinac	1990	ERL	130 MeV	multiturn	SR+FEL
CEBAF	1995	ERL	6-12 GeV	multiturn	nuclear
Jlab FEL	1999	ERL	40–160 MeV	ring	FEL
JAERI-ERL	199?	ERL	17 MeV	/leV ring	
MARS	idea	ERL	6 GeV	2×multiturn	SR
LUX	prop	linac	3 GeV	multiturn	SR
Cornell-ERL	prop	ERL	100 MeV-5 GeV	ring	SR
PERL	prop	ERL	2.7 GeV	ring	SR
4 GLS	prop	ERL	600 MeV	ring	SR+FEL
ERLSYN	prop	ERL	3.5 GeV	ring	SR
KEK-ERL	prop	ERL	2.5–5 GeV	ring	SR

Table 3: Overview of ERL machines

7 SUMMARY

Energy recovery linacs have been operated in lower energy applications for several years. Experiments proving the principle and achieving very high recovery rates have been done. Today, several proposals for new large projects are being put forward around the world, but especially in the US.

The ERLs beat storage rings in horizontal emittance and short (fs) pulses. They can not compete with the FELs in peak power or coherence (if not the ERL is used as an FEL driver). The vertical emittance of a modern storage ring is better than both the ERL and the FEL, and if this is properly used in the beam line design, the diffraction limit is reached and there is no need for smaller beams. Thus the main advantage of the ERL is to provide fs pulses to many users on undulator beam lines.

A superconducting ERL will be able to operate almost in CW mode (several KHz to MHz), thus leaving warm systems behind. They will also be an important way of reducing the radiation hazards of linac systems. The SC ERL will also be able to efficiently drive a CW FEL.

On the other hand, no direct energy savings are made. Some applications will only be possible by energy recovery, but comparing the final radiation to power consumption will show that energy savings is a relative term.

Many problems still remain to be solved and worked on. The instabilities limiting higher currents and the higher order modes (HOM) in the linac structures are key areas.

The question of the fourth-generation light source still has to be solved. As the situation looks today, we can expect a number of different sources providing different capabilities. A few of them will most probably be Energy Recovery Linacs.

	Diffraction limit	Coherence	fs pulses	Multi user	Brilliance, average	Brilliance, peak	Rep. rate
Storage	-hor, +vert	_	—	+	0	—	+
ring							
FEL	0	+	+	0	0	+	-
ERL	0	_	+	+	0	_	0

Table 4: Pros and cons for some machines

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