# COMMUNICATION SCIENCES

# AND

# ENGINEERING

Prof. Y. W. Lee
Prof. A. G. Bose
Prof. M. Schetzen
Prof. H. L. Van Trees
Prof. V. R. Algazi
R. Alter
A. B. Baggeroer
R. F. Bauer
C. J. Boardman
J. D. Bruce
A. M. Bush
M. Chessman
L. D. Collins
G. Crimi
B. A. Delagi
T. A. Froeschle
F. E. Glave
S. Greenblatt
T. G. Kincaid
D. C. Kurtz
D. E. Nelsen
A. V. Oppenheim
M. O. Pace
R. B. Parente
M. A. Rich
P. M. Rego
M. J. Schaffer
J. E. Shindall
D. W. Steele
C. I. M. Uchoa

## A. SYSTEMS OF THE CLASS NONLINEAR NO-MEMORY FOLLOWED BY LINEAR WITH MEMORY

In this report, we present some results concerning the class of nonlinear systems of the form shown schematically in Fig. XI-1. As shown, the nonlinear system can be divided into three sections that are connected in tandem: a nonlinear no-memory system whose transfer characteristics, $N_m[x(t)]$, form a complete set of functions; a

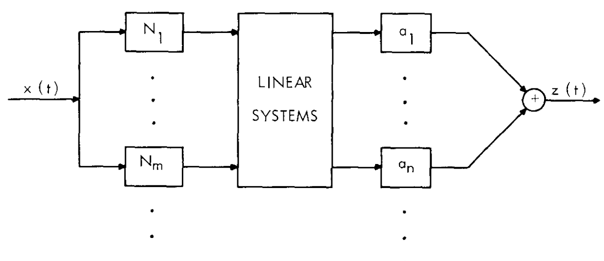


Fig. XI-1.

multiple-input, multiple-output linear section whose impulse responses, $h_{ij}(t)$, form a complete matrix of linear operators; and a linear section consisting of amplifiers whose outputs are then summed. This class of nonlinear systems is not as general as that described by Wiener[1]; it is a subset that we shall describe in this report.

We first observe that, by the use of superposition in linear systems, the system of Fig. XI-1 can be reduced to the form shown schematically in Fig. XI-2. For this system, the set of impulse responses, $h_n(t)$, form a complete set. Thus the nonlinear systems of the class that we are discussing are each of the form shown in Fig. XI-3, in which $k_m(t) = \sum_n a_{mn}h_n(t)$. In order to analyze these systems, we shall approximate

* This work was supported in part by the National Science Foundation (Grant G-16526), the National Institutes of Health (Grant MH-04737-03), and in part by the National Aeronautics and Space Administration (Grant NsG-496).

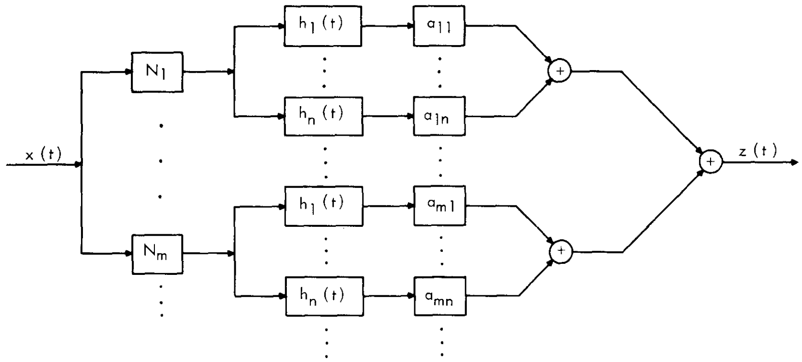


Fig. XI-2.

the complete set of linear operators by the set of delayed impulses, $u(t-t_n)$. Such a set is obtained by taps on a delay line. Any linear system can be approximated arbitrarily closely by this set. That is, the set of delayed impulses is complete in the limit as the delay between impulses approaches zero so that the set becomes dense on the t axis.

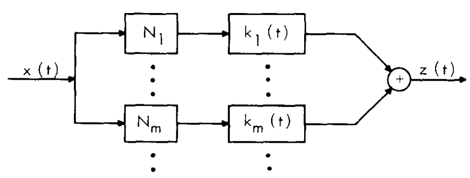


Fig. XI-3.

The system of Fig. XI-2 can thus be approximated arbitrarily closely by the delayed impulses as shown in Fig. XI-4. The output, z(t), of this system is

$$z(t) = \sum_m \sum_n a_{mn} N_m[x(t-t_n)],$$

which is identical with the output of the nonlinear system shown in Fig. XI-5. Thus both systems are equivalent.

Each of the nonlinear systems shown in Fig. XI-5 are of the form shown in Fig. XI-6 in which $M_n[x(t)] = \sum_m a_{mn} N_m[x(t)]$. Thus, as the delay between impulses approaches zero, the class of systems of the form shown in Fig. XI-6 is equivalent to that of the form shown in Fig. XI-3. From Fig. XI-6, the nonlinear systems can be described generally as those whose output is the sum of a different nonlinear no-memory function of x(t) for each instant of time in the past; there are no cross products between the input

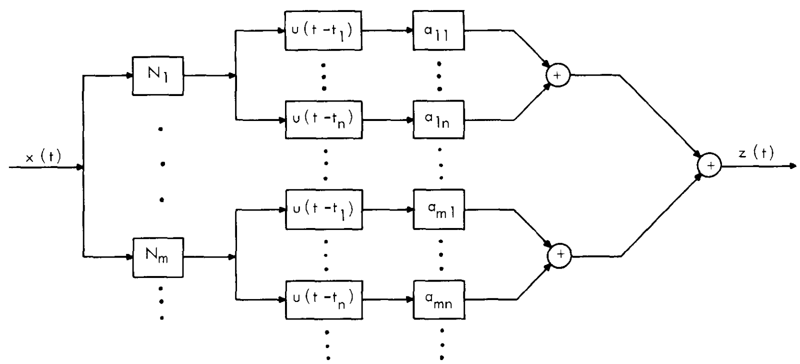


Fig. XI-4.

at different instants of time. Since the set of nonlinear no-memory operators, $\{N_m\}$, is complete, the system of Fig. XI-2 is complete in the sense that any set of nonlinear no-memory functions of x(t) for each instant of time in the past can be realized. In this connection we should note that the system of Fig. XI-6 is not equivalent to that of

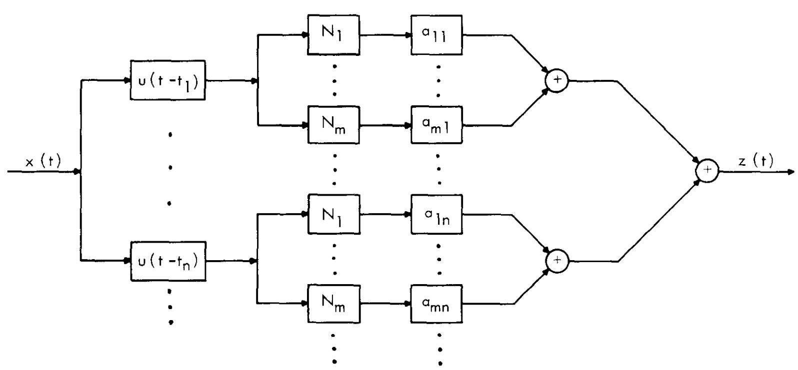


Fig. XI-5.

Fig. XI-7, in which the impulse responses, $h_n(t)$, are not delayed impulses. This is easily seen, since the output of each of the nonlinear no-memory systems will contain cross products between the input at different instants of time. The system of Fig. XI-7 is a subset of those described by Wiener which contain no cross products between the

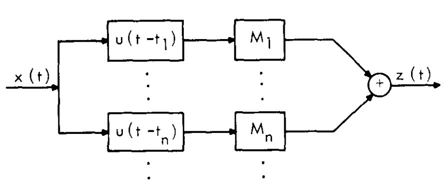


Fig. XI-6.

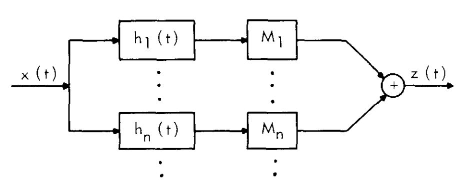


Fig. XI-7.

outputs of the linear systems. We observe from our discussion that the class of nonlinear systems of the form shown in Fig. XI-7 is dependent upon the set of linear systems used even though the set of impulse responses forms a complete set. Only if the cross products between the outputs of the linear systems are included, so that the class becomes that described by Wiener, will it be independent of whatever complete set of linear systems is used.

M. Schetzen

References

1. N. Wiener, Nonlinear Problems in Random Theory (The Technology Press of Massachusetts Institute of Technology, Cambridge, Mass., and John Wiley and Sons, Inc., New York, 1949).