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Absolute Luminosity from Machine Parameters

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Abstract

The expected rates for proton proton collisions in the LHC are rather high. Monitoring can be based on several detector components and different physics channels can be used together and should allow for a good accuracy in the relative luminosity determination. The accuracy in the absolute luminosity determination may soon be limited by the uncertainty in the knowledge of the proton proton cross section at the LHC energy.

Here we discuss the possibility to determine the absolute luminosity in the LHC from machine parameters, which does not require the knowledge of particle cross sections.

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1 Introduction

The event or collision rate $\dot{N}$ for a process of cross section σ produced by a machine running with luminosity $\mathcal{L}$ is

$$\dot{N} = \mathcal{L}\,\sigma \ . \tag{1}$$

If the cross section for a process is known, then we can use this relation to determine the luminosity from the observed event rate. In $\mathrm{e}^+\mathrm{e}^-$ colliders, the theoretically well known $\mathrm{e}^+\mathrm{e}^- \rightarrow \mathrm{e}^+\mathrm{e}^-$ scattering or Bhabha process is often used for this purpose.

For hadron colliders the situation is more complex. There is no corresponding process with a well-known cross section that can be used in a direct way. There are electromagnetic processes like muon pair production via two photon exchange that can be calculated to better than 1% but the rates are extremely low and the experimental acceptance and efficiency is difficult to estimate accurately. For proton-proton collisions at 14 TeV c.m.s energy, the rate of W and Z production is high and those processes are potentially suitable for luminosity determination if the cross section could be calculated with precision. However at the moment the uncertainty in the calculations are in the 5-10% range.

Traditionally the luminosity at hadron colliders is determined via elastic scattering of protons at small angles. An extrapolation to zero scattering angles in combination with a measurement of the total inelastic rate can be used to determine the luminosity via the optical theorem. This approach is taken by TOTEM [1] and ATLAS [2] using detectors housed in Roman Pots. The method has the potential to be accurate to a couple of percent but requires quite demanding beam conditions during special high beta runs in the LHC.

In this report we consider the complementary possibility to determine the absolute luminosity in the LHC directly from the machine parameters. The basic idea is to measure the absolute luminosity under much simplified and careful controlled conditions and calibrate any relative luminosity monitor of the machine or of the experiments under such optimal conditions.

Luminosity is a general concept. The luminosity for colliding beams can be directly obtained from geometry and numbers of particles flowing per time unit, see e.g. [3].

We will first illustrate a simple case and introduce generalisations later.

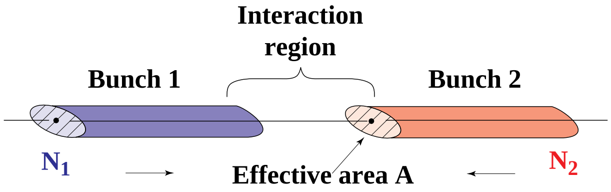

Figure 1: Luminosity from particles flux and geometry.

We start considering two bunches of N_1 and N_2 particles of equal beam sizes colliding head-on in an interaction region. For bunches crossing with the frequency f the luminosity is given as

$$\mathcal{L} = \frac{N_1\,N_2\,f}{A_{\text{eff}}} \ . \tag{2}$$

A_{eff} is the *effective transverse area* in which the collisions take place. For a uniform transverse particle distribution, A_{eff} would be directly equal to the transverse beam cross section. More generally, the effective area can be calculated from the overlap integral of the two transverse beam distributions $g_1(x,y)$, $g_2(x,y)$ according to

$$\frac{1}{A_{\text{eff}}} = \int g_1(x,y)\, g_2(x,y)\; dx\, dy\ . \tag{3}$$

For equal Gaussian beams

$$g_1 = g_2 = \frac{1}{2\pi\sigma_x\sigma_y}\ \exp\left[-\frac{x^2}{2\sigma_x^2} - \frac{y^2}{2\sigma_y^2}\right] \tag{4}$$

we obtain for head-on collisions

$$A_{\text{eff}} = 4\pi\,\sigma_x\sigma_y\ . \tag{5}$$

so that

$$\mathcal{L} = \frac{N_1\,N_2\,f}{4\pi\sigma_x\sigma_y} \qquad \text{which becomes} \qquad \mathcal{L} = \frac{N_1\,N_2\,f}{4\pi\sigma_r^2} \tag{6}$$

in case of round beams where $\sigma_x = \sigma_y = \sigma_r$.

The revolution frequency in a collider is accurately known. The number of particles circulating will be continuously measured with beam current transformers to roughly 10^{-2} accuracy and maybe better under certain conditions [4]. However, as discussed below in Sect. 3.3, we also have to make sure that there is no significant unknown component of particles outside the nominal bunches and thus the determination of number of particles contributing to the luminosity is non trivial. Still the dominant uncertainty in the prediction of the absolute luminosity from machine parameters is expected to come from the knowledge of the effective beam sizes.

Safe operation of the LHC requires a rather good knowledge of the optics and beam sizes and we expect that this should already allow a determination of the luminosity from machine parameters to about $20-30$ percent.

We believe that a much better accuracy can be reached if an extra effort is made. In the following text we will describe how this could be done and which methods and work would be involved.

2 Beam parameters

Table 1 summarises relevant LHC design beam parameters at 7 TeV.

The normalised emittance is $\epsilon_N = 3.75\,\mu\text{m}$. The horizontal and vertical emittances and the β functions and beams sizes σ^* at the interaction points are by design equal. The beam-beam parameter ξ which for round beams and constant normalised emittance ϵ_N only depends on the bunch population N_p

$$\xi = \frac{r_c\,N_p}{4\pi\,\epsilon_N} \tag{7}$$

is also given; r_c is the classical particle (here proton) radius.

The first line with numerical values in Table 1 was chosen to be what we expect would be typical for an early luminosity calibration run, i.e. a moderate bunch intensity of about 4×10^{10} protons and the initial $\beta^* = 11\,\text{m}$. Even with only a single bunch, counting rates would already be sufficient to get below 1% statistical accuracy within a minute.

Table 1: Single bunch luminosities. Event rates are given for $\sigma = 10\,\mathrm{mb}$ as roughly expected for the relative luminosity monitor Bran. The ratio of $\dot{N}$ and the revolution frequency f_{rev} is also given as a measure of the expected pile-up in the Bran. The numbers for $\beta^* = 0.55\,\mathrm{m}$ include the effect of the crossing angle.

β^* m	σ^* μm	N_p	$\mathcal{L}$ $\mathrm{cm}^{-2}\mathrm{s}^{-1}$	$\dot{N} = \mathcal{L}\,\sigma$ Hz	$\frac{\dot{N}}{f_{\mathrm{rev}}}$	ξ
11	74.36	4×10^{10}	2.59×10^{28}	259	0.023	0.001 30
2	31.71	1.15×10^{11}	1.18×10^{30}	11773	1.047	0.003 74
0.55	16.63	1.15×10^{11}	3.54×10^{30}	35400	3.15	0.003 74

We do not think that it is essential to restrict luminosity calibration runs to single bunch operation. For operation with $43 - 156$ bunches, the bunch spacing is large and does not require any crossing angle. Parasitic beam-beam effects will be negligible and all bunches of a beam travel on average on the same orbit. The luminosity calibration can be done by properly summing up over all bunch pairs which cross in a given point [5, 6]. By symmetry, the same bunches collide in the LHC points 1 and 5.

3 Systematic uncertainties

We know of a number of effects which have an impact on the luminosity. We discuss here how these effects could be minimised or measured and corrected for with good accuracy.

3.1 Crossing angle

For high luminosity operation with many (> 156) bunches, a crossing angle will be required to avoid parasitic collisions. This will reduce the luminosity by a factor

$$F_c = \sqrt{1 + \left(\frac{\theta_c \sigma_z}{2\sigma^*}\right)^2} \tag{8}$$

where θ_c is the full crossing angle between the two beams, σ_z the bunch length and σ^* the transverse r.m.s beam size at the interaction point.

Table 2: Luminosity reduction factor F_c for $\theta_c = 0.285\,\mathrm{mrad}$ and $\sigma_z = 7.55\,\mathrm{cm}$.

β^* m	σ^* μm	F_c
11	74.36	1.010
2	31.71	1.056
0.55	16.63	1.191

Table 2 shows numerical values for the LHC design parameters at 7 TeV. While the reduction is still nearly negligible with about 1% at a β^* of 11 m it becomes rather significant with about 21 % reduction at 0.55 m. We believe that the absolute luminosity calibration can be done such, that the uncertainty due to the luminosity reduction by the crossing angle will be negligible. For this, initial luminosity calibration runs would be best performed without crossing angle at $\beta^* = 2\,\mathrm{m}$ or larger which is planned anyway in the LHC commissioning.

3.2 Beams not colliding head-on

There is a loss in luminosity if the beams are not colliding head-on. For Gaussian beams, the remaining luminosity fraction is [3, 7]

$$\frac{\mathcal{L}}{\mathcal{L}_0} = \exp\left[-\left(\frac{\delta x}{2\sigma_x}\right)^2 - \left(\frac{\delta y}{2\sigma_y}\right)^2\right] . \tag{9}$$

$\delta x, \delta y$ is the horizontal and vertical separation between the two beams and σ_x, σ_y the r.m.s beam sizes. Numerical values are listed in Table 3. Using separation scans, we expect to be able to obtain less than $0.1\,\sigma$ separation, such that the uncertainty from this source would be negligible.

Table 3: Remaining luminosity fraction for 0 to 2 σ separation, for Gaussian beams.

δx σ_x	δy σ_y	$\mathcal{L}/\mathcal{L}_0$
0	0	1.0000
0.1	0	0.9975
0.2	0	0.9901
0.3	0	0.9778
0.4	0	0.9608
0.5	0	0.9394
0.5	0.5	0.8825
1	0	0.7788
1	1	0.6065
2	0	0.3679
2	2	0.1353

3.3 Bunch shape

We have seen that the luminosity depends on the overlap integral of the two transverse distribution functions. The luminosity is mainly produced by the core of the distribution. The LHC is equipped with profile monitors which allow to measure the transverse beam shapes. Additional information on the transverse distributions is obtained from the separation scans. We expect that the uncertainty will mainly depend on our knowledge of the transverse distributions at large amplitudes. Basically, particles at large amplitudes would be fully counted in the intensity determination but only contribute marginally to the luminosity. For a detailed discussion with analytic expressions and numerical estimates see [8]. The LHC is equipped with wire scanners with extra electronics for an enhanced sensitivity to measure tails. At the moderate intensity proposed for the absolute luminosity determination, it should also be possible to detect and eliminate tails with collimator scans.

It has also been proposed to improve the knowledge on the beams sizes and shapes using beam-gas interactions [9].

Another potential uncertainty could come from the longitudinal charge distribution. Un-bunched particles or extra bunches in one beam for example would be counted in the intensity as measured with a DC-BCT (direct current beam transformer) but would not contribute to the luminosity. This will be observable with several instruments : the gap monitor, comparison of fast and DC beam current transformers, rf-pickups and to some extend using beam loss monitors. The transverse damper can be used to eliminate such unwanted beam components.

3.4 Hour glass effect

The β functions and beam sizes have a minimum at the interaction point. For collisions of long bunches, the luminosity decrease because of the increase in beam sizes around the interaction point. This effect is known as hour glass effect. It is significant if the β function at the IP and the bunch length are comparable, that is where the ratio $r = \beta^*/\sigma_z$ is of order one or less.

Rather general expressions for the hour glass effect which require numerical integration for their evaluation can be found in [3, 10]. Without crossing angle and in the case of round beams, it is possible to write the luminosity reduction $H(r)$ by the hour glass effect in closed form as

$$H(r) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{e^{-s^2}}{1 + s^2/r^2}\, ds = \sqrt{\pi}\, r\, e^{r^2}\, \mathrm{Erfc}(r) \tag{10}$$

using the complementary error function $\mathrm{Erfc}(z) = \frac{2}{\sqrt{\pi}} \int_z^{\infty} e^{-t^2}\, dt$. Numerical values are given in Table 4.

Table 4: Luminosity reduction factor $H(r)$ by the hour glass effect, for the nominal LHC bunch length at 7 TeV of $\sigma_z = 7.55\,\mathrm{cm}$ without crossing angle.

β^*	$r = \beta^*/\sigma_z$	$H(r)$
2 m	26.5	0.9993
0.55 m	7.28	0.9908

We conclude that this effect is negligible for luminosity calibration done at a β^* of 2 m or larger for zero crossing angle.

4 Separation Scans

A direct and potentially very precise method to measure the overlap distribution of the two colliding beams are separations scans. They were frequently used during the first years of LEP operation [11, 12].

Separation scans were pioneered in the ISR by Van der Meer [13] and allowed an absolute calibration in luminosity at the 1% level [14].

The LHC will operate with round beams. Separation scans will have to be performed in both the vertical and horizontal direction, see Fig. 2.

Beam-beam effects have been studied for the LHC using detailed simulations [15]. For the intensities proposed for luminosity calibration, emittance blow-up and lifetime

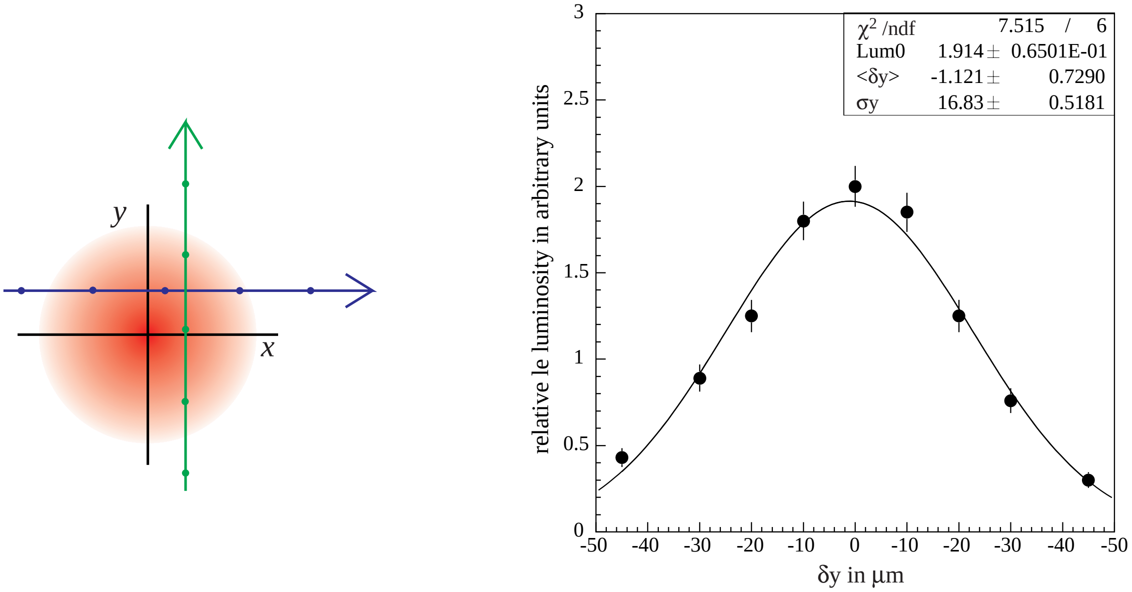

Figure 2: Schematic view of the steps involved in an orthogonal separation scan proposed for the LHC (left) and a possible result in one direction (based on early LEP data) shown on the right.

reduction are not expected to be critical. The simulations show, that separation scans for the LHC should still be possible even for nominal intensities. Blow-up and lifetime would be worse for 0.2 - 0.3 σ separation. The result would be mainly a slow diffusion of particles resulting in a 2 h lifetime and some blow-up depending on how long one would stay at this separation.

5 BPM precision and bump calibration

The knowledge of the length scale in δ_x, δ_y is required for the measurement of the absolute luminosity. In the ISR, this was achieved using precision scrapers [16, 17].

For the LHC, we propose to use a combination of several methods. Optics, orbit correctors and beam position pickups can be intercalibrated using an orbit response matrix measurements and analysis [18, 19]. In addition, wire scanners, collimators and roman pot detectors can be used to check an calibrated position and length scales.

The LHC will be equipped with over 1000 beam position monitors [20]. Of particular importance for the separation measurements will be the warm directional strip line couplers (BPMSW) installed next to Q1 towards the interaction points. These monitors will provide a direct measurement of the beam separation. For operation without crossing angle and small separation, the accuracy is expected to be of order $10\,\mu\mathrm{m}$ with an uncertainty in the zero position of about $50\,\mu\mathrm{m}$. The uncertainty in the zero position could be eliminated for operation with large bunch spacings as relevant here using additional button pickups next to the BPMSW with identical readout electronics for both beams [21]. We believe that this would also be very useful to obtain collisions and efficiently optimise luminosity in early operation and that this would be needed anyway to reach the required accuracy to measure and minimise the residual crossing angle in high-β ATLAS and TOTEM operation [22, 23].

6 Alternative methods

The beam-beam parameters ξ will be over an order of magnitude smaller in the LHC compared to LEP. We do not expect, that beam-beam deflection scans which were routinely used in LEP2 will be practical in the LHC.

An alternative method we expect to be practical and yield additional information in the LHC is to optimise luminosity and minimise the separation between the two colliding beams by measuring the beam-beam transfer function. This method was successfully used in the ISR [24, 25] and HERA [26, 27]. Small coherent beam oscillations excited in one beam are observed on the other beam.

7 Conclusion

We have looked into the possibility to calibrate the absolute LHC luminosity from machine parameters and think that a precision of a few per cent should be reachable. In addition, we think that this should be possible with the LHC instruments and procedures which already exist or which are foreseen anyway. Optimal running conditions would be moderate bunch intensities, large bunch spacings, no crossing angle and $\beta^* = 2\,\mathrm{m}$ or larger as in fact already planned for the LHC commissioning.

We plan to have a PhD student to work on this subject, in close collaboration with the commissioning and operations teams.

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