

PHYSICAL MECHANISMS CAUSING NONLINEAR MICROWAVE LOSSES IN HIGH- T_c SUPERCONDUCTORS

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We present a short overview of the mechanisms of microwave losses in high- T_c superconductors with special attention to high-power losses. An impedance plane analysis is used as a tool for quantitative comparison of the experimental data to the models. We discuss several models of nonlinear microwave performance of high- T_c superconductors, including coupled-grain and rf-critical state models, and estimate their characteristic time scales.

Keywords: Superconductivity; Radiofrequency; Cavities; Surface impedance; Thin films

1. INTRODUCTION

The use of superconductors in microwave technology has considerably increased with the advent of high- T_c materials which are going to be used mostly in thin film applications such as transmission lines, resonators, filters, and special elements based on Josephson junctions. Several overviews feature microwave properties and possible applications of high- T_c superconductors,^{1–3} their nonlinear microwave performance,^{4–6} and their microwave properties in a dc magnetic field.⁷ These works point on several key problems:

- (1) The surface resistance of high- T_c superconductors is not low enough compared to conventional superconductors.

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- (2) Nonuniform current distribution in planar microwave circuits, namely, strong current concentration at the edges of a superconducting strip.
- (3) Sensitivity of microwave properties to temperature even at low temperatures.
- (4) Nonlinearity which appears as Q -degradation at high microwave power, intermodulation, and harmonic generation.

While the achievement of low surface resistance is an important goal but at present not a bottleneck; current concentration at the edges may be avoided by the choice of microwave components that have more uniform current distribution, such as disk resonators;⁸ the nonlinearity turns out to be a bottleneck in applications of high- T_c superconductors in passive microwave devices. Therefore, the study of nonlinear performance of high- T_c superconducting films draws considerable theoretical and experimental attention. The important task which the researchers are presently faced with is a choice of a proper model to account for their particular experimental results rather than development of new models. In this study I will concentrate on the impedance plane analysis as a tool for comparison of the experimental data on the nonlinear performance of superconductors to the models. I will also discuss several mechanisms of nonlinearity with an emphasis on their characteristic time scales.

2. PHENOMENOLOGY OF NONLINEAR MICROWAVE PERFORMANCE OF SUPERCONDUCTORS

2.1. General Description

Typical dependence of the surface resistance of a superconducting film on microwave current may be separated into four regimes: linear (at small currents), weakly nonlinear, strongly nonlinear, and breakdown (at highest currents). Most researchers agree that the surface resistance of high- T_c superconducting films in the linear regime is extrinsic and is determined by the defects such as weak links. The linear regime is conveniently described by the coupled-grain model. In the weakly nonlinear regime the surface resistance gradually increases, and this increase is usually quadratic in current. The weakly nonlinear regime is

also believed to arise from the presence of defects such as weak links at grain boundaries. This regime is described by the extended coupled-grain model which takes into account nonlinear inductance of the weak links. In best films this region is absent. Above some threshold current, the surface resistance increases more rapidly. This regime of strong nonlinearity is usually ascribed to vortex generation (either Josephson or Abrikosov) by intense microwave magnetic field. At very high microwave current the breakdown occurs, i.e., at a certain value of the microwave current the surface resistance increases abruptly. This breakdown is believed to arise from heating and formation of normal-state domains.

2.2. Correlation to Material Properties

There were many attempts to find empirical correlation between microwave performance of superconducting films and their material properties. On the one hand, clear correlation was demonstrated between the *linear* surface resistance and: (i) penetration length;¹ (ii) mosaic spread in the a - b -plane;⁹ (iii) sensitivity of the surface resistance to the dc magnetic field.^{10,11} On the other hand, correlation between *nonlinear* performance and material properties was not established unambiguously. In particular, Ma *et al.*¹² demonstrate that while there is a clear correlation between high power performance and material properties (such as penetration depth and normal-state conductivity) for YBCO films fabricated by the same deposition technique, there is no such correlation for the films fabricated by different deposition techniques. Even more puzzling is the absence of clear correlation between linear and nonlinear performance of high- T_c superconducting thin films. Those films that have the lowest surface resistance in the nonlinear regime do not necessarily have the lowest surface resistance in the linear regime.^{6,13} In other words, the films that are not optimal with respect to their low-power performance, may demonstrate the best high-power performance. This feature prevents screening the films on the basis of their linear surface resistance.

2.3. Classification

Hein *et al.*^{1,6} proposed to classify nonlinear performance of high- T_c superconducting films according to the functional dependence of the

surface resistance on microwave current (i.e., linear, power-law, breakdown) and according to the value of the crossover field that marks the onset of nonlinearity. While this classification is very useful for comparison of different samples, it is not the optimal one for the purpose of modeling. Since the microwave current is known with the accuracy of 20–50% and the fit to the dependence of $R_S(J_{rf})$ requires several fitting parameters, the fit of the experimental dependence $R_S(J_{rf})$ to the model is often successful but not persuasive enough.

A classification based on impedance plane analysis turns out to be more useful for comparison to the models. While the impedance plane analysis is not very efficient for comparing different samples, it has several advantages for the quantitative comparison to the models since the fitting to the model requires only few (if any!) fitting parameters.

3. IMPEDANCE PLANE ANALYSIS (R_S vs X_S)

3.1. Definition

This analysis consists of plotting variation of surface resistance δR_S versus variation of surface reactance δX_S at varying microwave power and analyzing the resulting plots.^{14,15} Very often dependence of δR_S on δX_S is close to a straight line which may be characterized by the dimensionless slope r . Different mechanisms of nonlinearity are characterized by different values of r . A very useful feature of δR_S vs δX_S plot is that it allows the comparison of the R_S vs X_S dependence in the nonlinear regime (at varying microwave current) to similar plots in the linear regime (at varying temperature, dc magnetic field, etc). This provides a quantitative basis for comparison of linear and nonlinear microwave properties of the *same* sample.

The impedance plane analysis is closely related to the Cole–Cole plot which is widely used in studies of linear dielectric response of materials. The Cole–Cole plot is a parametric representation of the lossy, resistive part of the dielectric susceptibility on its real, reactive part at varying frequency (typical form of such plot in the impedance plane is a semi-circle). Another closely related plot is a Smith chart (parametric representation of the complex impedance of a rf/microwave network in the complex plane with the frequency as an implicit parameter) which is a very important tool in microwave engineering. An impedance

plane analysis (with either frequency or probe–sample separation as an implicit parameter) is also widely used in eddy current nondestructive testing for identification of various defects.

3.2. Justification

Why the dependence of R_S on X_S upon variation of almost any parameter (excluding frequency) is so close to a linear one? A possible explanation is as follows. If the surface impedance is an analytical function of some parameter x , i.e. $Z = Z(x)$, we can use Taylor expansion of $Z(x + \delta x)$. For the real and imaginary parts of Z we find

$$\delta \operatorname{Im}(Z) = \frac{\partial \operatorname{Im}(Z)}{\partial x} \delta x + \frac{\partial^2 \operatorname{Im}(Z)}{\partial x^2} (\delta x)^2 + \dots \quad (1a)$$

$$\delta \operatorname{Re}(Z) = \frac{\partial \operatorname{Re}(Z)}{\partial x} \delta x + \frac{\partial^2 \operatorname{Re}(Z)}{\partial x^2} (\delta x)^2 + \dots \quad (1b)$$

Since the surface impedance Z is a generalized susceptibility, its real and imaginary parts are *linearly* related through the Kramers–Kronig relations:

$$\operatorname{Im}(Z) = -\frac{2\omega}{\pi} \int_0^\infty \frac{\operatorname{Re}(Z)}{y^2 - \omega^2} dy. \quad (2)$$

This linear relation holds also for the derivatives of $\operatorname{Im}(Z)$ and $\operatorname{Re}(Z)$ with respect to any parameter x (excluding frequency, since it appears in Eq. (2) in explicit form). Hence, if the leading term in the Taylor expansion of $R_S = \operatorname{Re}(Z)$ is of the order n , the leading term in the Taylor expansion of the $X_S = -\operatorname{Im}(Z)$ is of the *same* order n . Leaving only leading terms in Eq. (1) we find

$$\delta R_S(x) \approx \left[\operatorname{Re} \left(\frac{\partial^n Z}{\partial x^n} \right) \right] (\delta x)^n, \quad \delta X_S(x) \approx - \left[\operatorname{Im} \left(\frac{\partial^n Z}{\partial x^n} \right) \right] (\delta x)^n. \quad (3)$$

Therefore, small variations of surface resistance and surface reactance upon small variation of any parameter (such as J_{dc} , H_{dc} , T , but not ω)

are linearly related and the dimensionless ratio of the two variations is:

$$r = \frac{\delta R_S}{\delta X_S} = - \frac{\operatorname{Re}(\partial^n Z / \partial x^n)}{\operatorname{Im}(\partial^n Z / \partial x^n)}. \quad (4)$$

Strictly speaking, the above analysis is not applicable to the dependence of surface impedance on J_{rf} because the derivation of the Kramers–Kronig relations assumes linear relation between the force and the response.¹⁶ Nevertheless, the experimental data very often demonstrate linear dependence between variations of the real and imaginary parts of the surface impedance of superconductors upon varying J_{rf} . This linear dependence is not specific for nonlinear electrodynamic properties, it is well known for nonlinear elastic properties of materials.¹⁷ May be, the linear or quasilinear dependence of the real part of the generalized susceptibility on its imaginary part upon varying force can be justified through the generalization of the Kramers–Kronig analysis for nonlinear and hysteretic phenomena.

3.3. Survey of Experimental Data

Very often experimental dependence of R_S vs X_S upon varying microwave current is a straight line which is characterized by dimensionless slope $r = dR_S/dX_S$. (Note, that in some works the inverse value, namely, dX_S/dR_S is defined as r .) If the measurements are done for several orders of magnitude of current variation, then different mechanisms may be responsible for nonlinearity at low and at high currents. Since each mechanism has its own r -value, the switching of the mechanisms of nonlinearity is clearly seen in the impedance plane as a change in slope of R_S vs X_S dependence.¹⁸

Table I lists experimental values of r for superconducting films. Here r_{rf} characterizes dependence on microwave current J_{rf} (nonlinear regime), while r_H and r_T characterize dependence on the static magnetic field and on the temperature (in the linear regime). In what follows we use the data for r -value for identification of the mechanism of nonlinearity in each particular case. We observe that the typical values for r -parameter are: $r_T \sim 0.01$, $r_H \sim 0.2-0.3$, $r_{rf} \sim 1$ (at the onset of nonlinearity r_{rf} is usually smaller). In the strongly nonlinear regime r_{rf} depends to some extent on frequency, temperature and dc magnetic

TABLE I Nonlinear properties of superconductors

<i>Material</i>	r_{rf}	r_H	r_T at, 4.2 K	$T(K)$	$f(GHz)$	<i>Reference</i>
Nb	0.3		0.01	4.2	3.4	Andreone <i>et al.</i> ¹⁹
Nb	0.8–1.5 ^a		0.01	4.2–8	1.6–5.4	Golosovsky <i>et al.</i> ¹⁴
Nb ₃ Sn	0.4	0.2	0.01	4.2	1.4	Andreone <i>et al.</i> ²⁰
BSCCO	0.8		0.1	4.2	2.2	Andreone <i>et al.</i> ²¹
TBCCO	1			4.2	18	Portis <i>et al.</i> ²²
GdBCO	1	0.1		21	5.5	Gallop <i>et al.</i> ¹⁰
YBCO	0.6		0.05	4.2	2.2	Andreone <i>et al.</i> ¹⁹
YBCO	1			4.2–76	10	Findikoglu <i>et al.</i> ⁹
YBCO	0.8–2 ^a	0.2		23–60	5.5	Tsindlekht <i>et al.</i> ²³
YBCO	0.5 ^b			20–80	1.5–16	Herd <i>et al.</i> ²⁴
	1 ^c					
YBCO	0.7–0.8			15	8	Porch <i>et al.</i> ¹³
YBCO	0.25 ^b			77	1.5–7.7	Nguyen <i>et al.</i> ¹⁸
	0.5 ^c					Halbritter ¹⁵
YBCO ^c	2	0.7		77		Hein <i>et al.</i> ²⁵
YBCO	0.7 ^d			4–15	1.7	Belk <i>et al.</i> ²⁶

^aEach sample has a definite value of r but this value varies from sample to sample. ^bLow currents. ^cHigh currents. ^dIn magnetic field of 2 T. ^eGranular sample.

field, although these dependences are very weak. The parameter r_{rf} does not vary considerably from sample to sample and is almost the same for high- and low- T_c superconductors.

The model of developed nonlinearity should account for all these features. As we will see, only a few models are able to do it. In what follows we briefly describe different models of nonlinearity in superconductors paying special attention to the r -value that they predict.

4. INTRINSIC NONLINEARITY OF SUPERCONDUCTORS

Intrinsic electrodynamics of superconductors in terms of the two-fluid model is given by

$$\sigma = \sigma_1 - i\sigma_2, \quad \sigma_1 = \frac{(1-K)n_0e^2\tau}{m}, \quad \sigma_2 = \frac{Kn_0e^2}{m\omega}, \quad (5)$$

where σ is the complex conductivity, τ is the scattering time, ω is the microwave frequency, n_0 is the normal-state carrier density, K is the fraction of condensate, and $(1-K)$ is the fraction of quasiparticles.

The surface impedance is

$$Z_S = R_S - iX_S = \sqrt{\frac{i\mu_0}{\omega\sigma}}, \quad R_S \cong \frac{\sigma_1}{2\sigma_2} \left(\frac{\omega\mu_0}{\sigma_2}\right)^{1/2}, \quad X_S = \left(\frac{\omega\mu_0}{\sigma_2}\right)^{1/2}. \quad (6)$$

The density of the superconducting condensate decreases at high velocities (pair-breaking). Since the microwave current is directly related to velocity, the surface impedance of an ideal superconductor depends on current. This dependence is cast in parameter $K = K(J)$. For isotropic s -wave superconductors this dependence was treated by Parmenter²⁷ in terms of the Ginzburg–Landau model. He finds that:

$$\delta R_S \propto J^2, \quad \delta X_S \propto J^2. \quad (7)$$

The microwave nonlinearity in d -wave superconductors was studied theoretically by Dahm and Scalapino²⁸ who find:

$$\delta R_S \propto J, \quad \delta X_S \propto J \text{ (low-temperatures)} \quad (V_S k_F \gg T), \quad (8a)$$

$$\delta R_S \propto J^2, \quad \delta X_S \propto J^2 \text{ (high-temperatures)} \quad (V_S k_F \ll T). \quad (8b)$$

Here V_S is the velocity of the condensate, k_F is the Fermi-vector.

The r -parameter for intrinsic nonlinearity may be estimated as follows. If we assume that the only current-dependent term in Eqs. (5) and (6) is $K = K(J)$, the r -parameter may be estimated by excluding K from Eq. (5). The expression for r_{rf} becomes especially simple in the low-temperature limit at which $K \sim 1$, namely,

$$r \approx \omega\tau \ll 1. \quad (9)$$

Since Eqs. (6) and (7) may be used for d -wave superconductors as well, we expect that Eq. (9) is also valid for d -wave superconductors in the low-temperature limit. [The above analysis assumes that the dependence on microwave current arises from pair-breaking and is accounted for by the parameter K . It is not clear to which extent this is true for high- T_c superconductors. Indeed, since the scattering time in these materials is strongly temperature-dependent, it may be also current-dependent (for example, if there is strong quasiparticle–quasiparticle scattering). This requires further theoretical analysis.]

We conclude that if the nonlinearity is dominated by intrinsic mechanism, the r -parameter should be small and strongly frequency-dependent. This differs from what is usually observed in microwave experiments (Table I). It means that other mechanisms mask the intrinsic nonlinearity. However, there is a better chance to observe intrinsic nonlinearity at higher frequencies. In the following we will show that the most probable mechanism of strong nonlinear behavior in high- T_c superconducting films is the vortex penetration. Since this process requires finite time (for example, to nucleate the vortex), the vortex penetration should be negligible at sufficiently high frequencies. Therefore, high frequencies are more favorable for the observation of intrinsic nonlinearity. Indeed, Orenstein *et al.*²⁹ has recently observed intrinsic quadratic dependence given by Eq. (8b) in the THz transmission experiments on BSCCO films. To the best of my knowledge, intrinsic linear dependence of the surface impedance on current, predicted by Eq. (8a), has not been observed yet for high- T_c superconductors.

5. COUPLED-GRAIN MODEL

This model treats a superconducting sample as a network of Josephson junctions extending along grain boundaries.^{30,31} Although there are many modifications of the coupled-grain model which differ in representation of the equivalent circuit of the sample containing Josephson junctions (parallel, series, transmission line, etc.), in all of these modifications the nonlinearity arises from a nonlinear inductance of a Josephson junction. The R_S vs X_S plot as predicted by this model has a very special form which has been observed for the granular films^{25,32} and for a single Josephson junction.¹ In particular, at low currents the coupled-grain model yields a very small r -parameter with strong dependence on temperature and on dc magnetic field.¹⁴ However, in good films the experimentally observed r -parameter is close to unity and almost does not depend on temperature and on dc magnetic field. Therefore, while the simple coupled-grain model describes fairly well the microwave nonlinearity in granular films and the onset of nonlinearity in some epitaxial films, it fails to describe strong nonlinearity in good films.

Inability of the coupled-grain model to yield $r \sim 1$ has been recently overcome by taking into account distribution of junction properties. The first step in this direction was done by Bonin and Safa³³ who assumed an ensemble of Josephson junctions with wide distribution of critical currents. Independently, Herd *et al.*²⁴ took into account distribution of critical currents I_c of the junctions and the distribution of their $I_c R_n$ products as well. More than this, while the Hylton–Beasley coupled-grain model^{30,31} deals with junction behavior only for $I < I_c$, the model of Herd *et al.* accounts for junction behavior at $I > I_c$ as well. As a result, this model²⁴ yields the R_S vs X_S plot which at small currents (at the onset of nonlinearity) is close to that predicted by a simple coupled-grain model for a single junction,³⁰ while at higher currents it approaches a straight line with the slope $r \sim 1$. This model²⁴ accounts quite well for experimental results. However, it makes an important assumption that the distribution of $I_c R_n$ products is very wide, in other words, the junctions with high critical currents (and small Josephson penetration length) are required.⁶ Although this requirement seems very stringent, the experiments in high magnetic field indeed indicate on the presence of such junctions³⁴ in YBCO.

Of course, the model of Herd *et al.* is limited by a threshold current above which the vortices (Josephson or Abrikosov) should appear.

6. NONLINEARITY ARISING FROM ABRIKOSOV VORTICES

In the sample free of defects the nonlinearity should eventually arise from the appearance of vortices. Let us discuss Abrikosov vortices. There are several ways through which introduction of Abrikosov vortices might affect the nonlinear microwave properties. This may be analyzed using an equation of motion of a single vortex:

$$\eta \mathbf{V} + \alpha [\hat{\mathbf{n}} \times \mathbf{V}] + k_p \mathbf{x} = \Phi_0 [\hat{\mathbf{n}} \times \mathbf{J}] + \mathbf{F}_T(U). \quad (10)$$

Here V is the vortex velocity, n is the direction of the vortex, η is viscosity, α is the Hall coefficient, k_p is the pinning constant, J is a current density, F_T is a stochastic thermal force, and U is the pinning potential.³⁵ Any of these terms may be a source of nonlinearity.

Viscosity η may be nonlinear due to Larkin–Ovchinnikov instability³⁶

$$\eta = \frac{\eta_0}{1 + (V/V^*)^2}. \quad (11)$$

Equation (11) states that above some critical velocity V^* the viscosity decreases. Such instability was observed in dc experiments with YBCO films^{36,37} at vortex velocities of the order of 1 km/s. However, the maximum vortex velocity in microwave experiments with high- T_c superconductors is far smaller, $V \sim 10$ m/s, so this instability was not observed yet at high frequencies.

Hall coefficient α may depend on vortex displacement. Indeed, since α depends on the interaction with impurities (Kopnin–Kravtsov force³⁵) it might be different for vortex displacements much smaller and much higher than the distance between impurities. One can argue that the Hall coefficient is usually very small and its effect on vortex dynamics is negligible. Although this is true with respect to conventional superconductors, it is not so with respect to high- T_c superconductors since these materials are in the superclean limit at low temperatures^{7,35,38} (superclean limit indicates on appreciable Hall coefficient).

Pinning constant k_p may depend on rf-magnetic field through nonparabolicity of the pinning potential. However, since the typical vortex displacement in microwave experiments is $\sim 1 \text{ \AA}$ which is smaller than the effective range of the pinning potential $\sim 20 \text{ \AA}$,²⁶ this type of nonlinearity seems to be negligible.

Stochastic thermal force $F_T(U)$ is the source of the flux creep. The activation energy of the flux creep is strongly current-dependent^{35,39,40} and this is a dominant source of nonlinearity at lower frequencies, i.e. below 100 MHz. Since characteristic time for each individual act of flux hopping is rather big, $\tau_{\text{hop}} \sim 10^{-8}$ s,⁷ the flux creep is not effective in the microwave range (the microwave period is less than 10^{-9} s). In high magnetic fields, the effects of flux hopping still can be observed at microwave frequencies.²⁶ It occurs due to wide distribution of the pinning energies which allows to a small part of vortex segments to be very loosely pinned.

Nonlinearity due to proximity to the vortex phase transition^{35,40} also seems to be less pronounced at microwave frequencies. Indeed, Wu *et al.*⁴¹ observe that the phase transition in the vortex state in YBCO

is barely visible above 1 GHz. This group also shows that the vortex response at microwave frequencies has a mean-field behavior and is almost linear. Observations of a different group, Wu *et al.*⁴² indicate that the vortex melting (as detected through disappearance of the shear modulus of the vortex lattice) is barely seen above 50 MHz.

Vortex generation by the microwave magnetic field is one of the dominant source of nonlinearity in superconductors in the microwave range as we will advocate below.

7. RF-CRITICAL STATE MODEL

This model quantitatively accounts for the vortex generation by the microwave magnetic field. It was developed by Sridhar⁴³ and then by McDonald *et al.*⁴⁴ who extended the Bean model to account for microwave nonlinearity in high- T_c superconducting films. The rf-critical state model assumes a superconductor carrying an rf current and the vortex generation by the magnetic field of this current. McDonald *et al.* calculated the inductive part of the impedance and extended the Bean model to a thin strip geometry. The surface resistance and surface reactance were found to depend on dimensionless parameter J_{rf}/J_c where $J_c = J_c(H, T)$ is the critical current, while the dependence of R_S on X_S is almost linear with the slope $r = \delta R_S / \delta X_S$ depending only on the sample geometry. In particular, $r = 0.42$ for an ellipse and $r = 0.67$ for a thin strip. Experimental r -values (Table I) are rather close to those predicted by the model. The rf-critical state model predicts that while the R_S and X_S depend on temperature and on dc magnetic field (through the critical current), their ratio r is almost field- and temperature-independent. This prediction is also in good agreement with experimental results. A weak temperature dependence of the r -parameter observed in some experiments may be attributed to the change of effective geometry with temperature (since the ratio between the penetration depth and film thickness varies with temperature, then the thin strip geometry may effectively change to ellipse geometry).

The rf-critical model as described in Refs. [43,44] totally neglects surface barriers and assumes that the threshold field for vortex penetration is zero ($H_{c1} = 0$). Nguyen *et al.*¹⁸ took into account the finite H_{c1} .

They find that the finite penetration field modifies dependences of R_S on current. Introduction of a finite penetration field also provides a basis for modeling nonlinear microwave performance of superconducting films in a dc magnetic field. Experiments demonstrates that the application of the dc magnetic field shifts the onset of nonlinearity towards lower currents,^{23,45,46} although the plot of R_S vs X_S almost does not change. This can be accounted by the rf-critical state model since the surface barriers are strongly decreased in the presence of a static magnetic field.

The rf-critical state model seems to describe all major features of the microwave nonlinearity in high- T_c superconductors. It is important to note that this model is very general and does not assume any specific vortex properties. It may apply for Abrikosov and for Josephson vortices as well. However, the rf-critical state model as presented in Refs. [18,43,44] is static, i.e., it assumes that the critical state adiabatically follows the field.^{9,43} This assumption requires a special analysis.

8. TIME SCALES INVOLVED IN THE CRITICAL-STATE MODEL

In order to build a critical state, the vortices should be nucleated at the edge of the film and they should propagate inside the film.

8.1. Vortex Nucleation Time

This time is not understood well. A generally accepted estimate for bulk Nb is 10^{-6} s.⁴⁷ Samoilova⁴ shows from theoretical considerations that the vortex nucleation time scales with the inelastic scattering time. She estimates vortex nucleation time for NbN at 4.2 K as 3×10^{-10} s and for YBCO at 77 K as 10^{-12} s. Using numerical solution of the time-dependent Ginzburg–Landau equations, Aranson *et al.*⁴⁸ have studied dynamics of normal-superconducting transition in superconducting strips under the action of strong dc current and estimated vortex nucleation time. However, the results of Ref. [48] are cast in so dimensionless a form that comparison to experiments requires a very considerable effort.

Dynamics of the normal-superconducting transition in current-carrying superconducting strips turns out even more complicated since different mechanisms can compete, namely, vortex formation vs phase-slip center formation. While it is generally believed that the phase-slip centers are formed in narrow films (film width is smaller than coherence length) while the vortices form in wider films (film width exceeds coherence length), numerical simulations and experiments with current pulses demonstrate that the phase-slip centers may form in wide films as well.⁴⁹

To the best of my knowledge the experimental measurements of the vortex nucleation time in high- T_c superconductors are almost absent. The nucleation time may be measured by passing a narrow current pulse through a superconducting film and observing corresponding voltage pulse. The delay between the two allows to estimate the nucleation time. In this manner Maneval *et al.*⁴⁹ find for YBCO films at 4.2 K the delay time varying from 10 to 400 ns depending on the value of the current.

8.2. Vortex Propagation Time

Critical state develops on the length scale which we roughly estimate as

$$L_{\text{pinning}} \cong \frac{H_{\text{rf}}}{\mu_0 J_c}. \quad (12)$$

Here, H_{rf} is the microwave magnetic field. To cover this distance vortex needs some time. Neglecting all forces in Eq. (10) except the viscous and the Lorentz forces, we find vortex velocity $V = \Phi_0 J_{\text{rf}} / \eta$. Using relation $H_{\text{rf}} = \mu_0 J_{\text{rf}} \lambda$ we find

$$\tau_{\text{cr.state}} \geq \frac{\eta \lambda}{\Phi_0 J_c}. \quad (13)$$

Assuming $J_c = 10^7 \text{ A/cm}^2$, $\eta = 10^{-6} \text{ MKS units}$ and $\lambda = 150 \text{ nm}$ (realistic parameters for YBCO at 4.2 K) we find $\tau_{\text{cr.state}} \approx 10^{-9} \text{ s}$. It means that at microwave frequencies (microwave period is less than 10^{-9} s) and at 4.2 K there is not enough time to build a critical-state based on Abrikosov vortices. At higher temperatures the situation may change since η , λ and J_c are temperature-dependent (although Eq. (13) shows that these temperature dependences are partially canceled). It is

tempting to compare the $\tau_{\text{cr.state}}$ to the viscoelastic vortex relaxation time $\tau_{\text{ve}} \approx \eta/k_p$ (which is inverse of depinning frequency⁷). Since $\Phi_0 J_c = k_p r_p$ where r_p is the radius of the pinning potential $\sim 20 \text{ \AA}$, we find $\tau_{\text{cr.state}} \approx \tau_{\text{ve}} \lambda / r_p$. While for high- T_c superconductors $\tau_{\text{ve}} \sim 10^{-11} \text{ s}$, the ratio $\lambda / r_p \sim 100$. Therefore, the time it takes to build a critical-state model is considerably longer than the viscoelastic vortex relaxation time.

In conclusion, the rf-critical state accounts quite well for nonlinear microwave losses in high- T_c superconducting films. The main difficulty is that there is not enough time to build a critical state using Abrikosov vortices at microwave frequencies. However, there is enough time to build a critical state at microwave frequencies using Josephson vortices or so-called Abrikosov–Josephson vortices, since they have much lower viscosity.

9. ABRIKOSOV–JOSEPHSON VORTICES

Abrikosov–Josephson (AJ) vortex has been discussed theoretically in the works of Halbritter⁵⁰ and Gurevich.⁵¹ According to Gurevich,⁵¹ this vortex appears at grain boundaries with high critical current J_b which satisfies the following inequality: $J_c < J_b < J_d$. Here J_b is a critical current through this grain boundary, J_d is a depairing current and J_c is the critical current in the bulk. Gurevich names these grain boundaries “hidden weak links”. The viscosity of AJ vortices which move along such “hidden weak links” has an intermediate value between that for Abrikosov and Josephson vortices (for YBCO at 4.2 K $\eta_A \approx 10^{-6}$ MKS units and $\eta_J \approx 10^{-10}$ MKS units⁷). Hence, AJ vortices require less time to organize themselves into a critical-state, hence they are very probable candidates to account for high-power microwave performance of YBCO.

The idea of “hidden weak links” with high critical current and small Josephson penetration length is very appealing for the explanation of microwave nonlinearity in high- T_c superconductors. Indeed, Hein *et al.*⁶ point out that postulating such grain boundaries is mandatory for explanation of the microwave nonlinearity in high- T_c superconductors in terms of intrinsic granularity. The presence of “hidden weak links” explains (i) the absence of correlation between linear and

nonlinear performance of superconducting films; (ii) dependence of the microwave surface impedance on the orientation of a dc magnetic field;⁵² (iii) reduced threshold rf field for the onset of nonlinearity (in comparison to H_{c1} for Abrikosov vortices); (iv) the absence of nonlinearity until breakdown in best films.

The notion of “hidden weak links” allows merging the model of microwave nonlinearity proposed by Herd *et al.*²⁴ (ensemble of junctions with wide distribution of critical currents) with the rf-critical state model.⁴⁴ Indeed, at the lowest microwave current the nonlinearity is determined by the nonlinear inductance of grain-boundary junctions, whereas the contribution from the junctions with the smallest critical current is the most important. Upon increasing microwave current the junctions with higher critical current come into play. Simultaneously, Josephson and Abrikosov–Josephson vortices start to enter the junctions with smaller critical current.

10. BREAKDOWN

At very high microwave power the surface resistance and reactance abruptly increase, in other words, breakdown occurs. The breakdown is due to the fact that the whole film or part of it undergoes transition into the normal state. Heating and heat transfer to the substrate are shown to play an important role here.^{53–55} In distinction to nonlinearity at lower currents which seems to be distributed across the film, there are several indications that the breakdown is triggered by local defects^{14,56} and usually occurs locally, in one point.⁵⁷ This indicates the importance of studying local rf/mw properties of superconducting samples and devices. It has been done by mapping physical properties of high- T_c superconducting films using scanning probes and methods including thermal imaging,⁵⁶ Raman microscopy,⁵⁸ critical current mapping,^{59,60} local penetration depth (through mutual inductance),^{61,62} scanning SQUID microscopy,^{63,64} laser microscopy,⁶⁵ e-beam microscopy.⁶⁶ Such studies reveal noticeable inhomogeneity in material properties of high- T_c superconducting films. The most relevant to microwave applications is microwave near-field imaging using scanning probe,^{67–71} that are being intensively developed nowadays. These probes directly map surface resistance of

the superconducting films. Hopefully, the development of the microwave near-field probes will very soon have considerable impact on the study of nonlinearity in superconductors.

11. CONCLUSIONS

- (1) Nonlinear performance of high- T_c superconductors is a major problem for their microwave applications. Impedance plane analysis is a valuable tool to uncover the mechanism of nonlinearity in each particular case.
- (2) Two complementary models account for nonlinearity at intermediate power levels in high- T_c superconducting films:
 - (i) Extended coupled-grain model which assumes intrinsic granularity and grain boundaries acting as weak links having wide distribution of $I_c R_n$ products.
 - (ii) rf-critical state model based on Abrikosov–Josephson vortices. Both models postulate weak links with high critical currents.
- (3) It is necessary to go beyond static rf-critical state model and to take into account relevant time scales such as vortex nucleation time and propagation time. An experimental measurement of the vortex nucleation time is required.

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