COUPLED SUPERCONDUCTING CAVITIES FOR THE DETECTION OF WEAK FORCES

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The detection of weak forces acting on macroscopic bodies often entails the measurement of extremely small displacements of these bodies or their boundaries. This is the case in the attempts to detect gravitational waves or in the search for possible new long range interactions. In 1978, Pegoraro *et al.*[3] suggested that superconducting coupled cavities could be used as a sensitive detector of gravitational effects, through the direct coupling of the gravitational wave with the electromagnetic field stored inside the cavities. The detector sensitivity is linearly dependent on the quality factor of the cavities and on the square root of the stored energy; for this reason the cavities are made out of superconducting material. In 1984, Reece *et al.*[5] constructed such a detector in the 10 GHz frequency range. They achieved a sensitivity to fractional deformations of the order $\delta x/x = 4 \cdot 10^{-18}$, with a quality factor $Q = 2 \cdot 10^8$ and a stored energy $U = 2 \cdot 10^{-4}$ J. Our proposal is to repeat the Reece, Rainer and Melissinos (RRM) experiment, pushing the cavity quality factor into the 10^{10} range and the stored energy into the 1 J range, in order to increase the detector sensitivity by a factor of 10^3, thus reaching a sensitivity to fractional deformations of the order of 10^{-21}. If these results could be obtained, this detector would be an interesting candidate for the detection of gravitational waves or in the search of long range interactions of weak intensity.

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1 INTRODUCTION

The detection of weak forces acting on macroscopic bodies often entails the measurement of extremely small displacements of these bodies or their boundaries. This is the case in the attempts to detect gravitational waves[1] or in the search for possible new long range interactions.[2]

In 1978, Pegoraro *et al.*[3] suggested that superconducting coupled cavities could be used as a sensitive detector of gravitational effects, through the direct coupling of the gravitational wave with the electromagnetic field stored inside the cavities.[3]

This system is an example of a *parametric converter*, i.e. of a device which converts energy from a reference frequency to a signal at a different frequency as a consequence of the time variation of a parameter of the system. An interesting aspect of the proposed detector is that it will respond to gravitational perturbations through their direct coupling to the electromagnetic field.[3] From this point of view the time dependent gravitational potential $h^{\mu\nu}$ modulates the energy stored in the cavity and one need not consider the motion of the cavity walls. The detector sensitivity is linearly dependent on the quality factor of the cavities; for this reason the cavities are made out of superconducting material.

The proposed detector consists of two cylindrical rf superconducting cavities coupled together. A system made of two coupled resonators shows a frequency spectrum with two energy levels (or resonant frequencies) for each energy level of the single resonator. Just to make a simple mechanical analogue this system is roughly equivalent to a system of two pendulums coupled together by a spring, as shown in Figure 1; the two oscillating modes of the
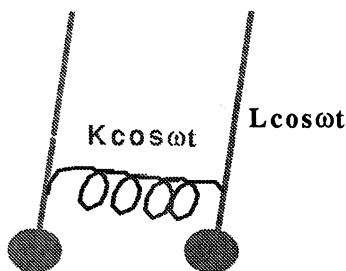

FIGURE 1   Simple mechanical analogue of the parametric converter.

system being the symmetric and antisymmetric modes of the mechanical system.

If we store some energy in the lower energy mode, the interaction of the gravitational wave with the electromagnetic field stored inside the cavities transfers some energy from the lower to the higher mode when the frequency difference of the two modes is equal to the wave frequency.[4]

For a gravitational wave of amplitude h, the energy transfer ΔU is proportional to $h^2 Q^2 U$, where Q is the electromagnetic quality factor and U is the electromagnetic energy stored in the lower energy mode. Using the former relation we find the following equation which gives the minimum measurable amplitude in terms of cavity properties:

$$h_{\text{min}} \propto \frac{1}{Q} \sqrt{\frac{\Delta U_{\text{min}}}{U}}, \qquad (1)$$

where ΔU_{min} is the minimum measurable energy transfer. From Eq. (1) it is clear that to obtain the best detector sensitivity a cavity with large quality factor and large stored energy is needed.

2 STATE OF THE ART

A parametric converter of the type proposed in Ref. [3] has been built and used as a harmonic mechanical motion transducer in 1984 by Reece, Rainer and Melissinos (RRM).[5] In this experiment two cylindrical niobium cavities have been built (see Figure 2), driven in the TE_{011} mode at the frequency $f_0 \approx 10\,\text{GHz}$, with a frequency split of the two resonant modes $\Delta f \approx 1\,\text{MHz}$. The quality factor of the two resonant cavities was $Q = 2 \cdot 10^8$ and the stored energy in the lower mode $U \approx 2 \cdot 10^{-4}\,\text{J}$.

In order to measure the sensitivity limit of the detector, one of its walls was excited by a harmonic perturbation (by a piezoelectric) at a frequency equal to the mode splitting. The results showed a sensitivity to relative deformations $\delta x / x \approx 10^{-18}$, which corresponds to a wall absolute displacement $\delta x \approx 10^{-17}\,\text{cm}$.
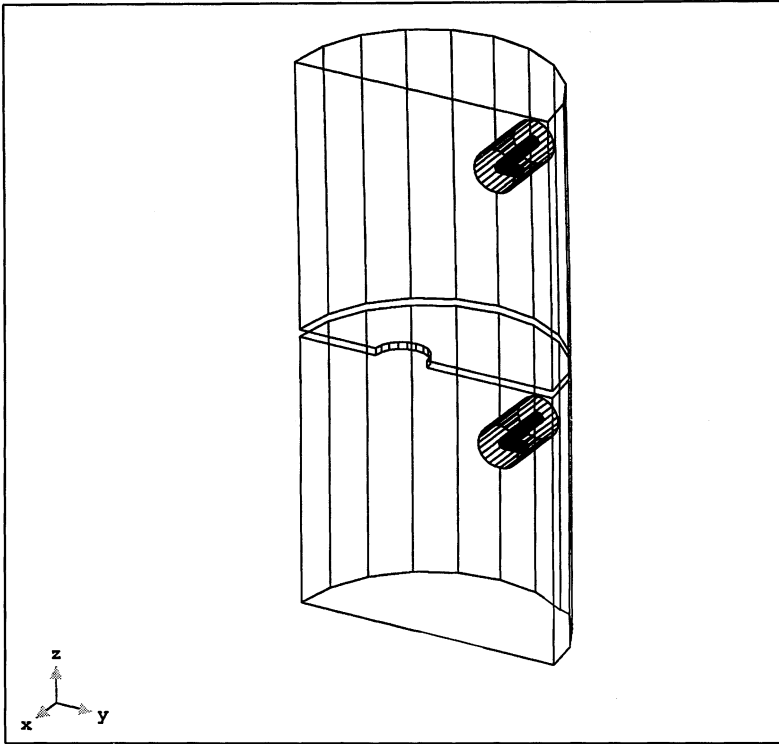
FIGURE 2   Schematic view of two coupled cylindrical cavities.

3   PROPOSAL

These results, even if of great interest, do not allow the use of the device for the detection of gravitational waves or weak forces. To reach this goal the detector sensitivity should be increased at least by three orders of magnitude, approaching $\delta x/x \approx 10^{-21}$ limit, similar to the expected sensitivity of the *Weber-like* detectors.

We think that keeping the conceptual design of the RRM experiment unaltered, several aspects of it can be improved.

At a first stage one can choose to work at a somewhat lower fre-quency taking advantage of the frequency dependence of the surface resistance of superconductors. We remind that the unloaded cavity

quality factor is given by the relation:

$$Q_0 = \frac{\Gamma}{R_s}, \tag{2}$$

where Γ is a geometrical factor which for a cylindrical cavity with its diameter equal to its length, in the TE_{011} mode, is equal to 780 Ω and R_s is the material surface resistance. The surface resistance itself is made up of two contributions:

$$R_s = R_{BCS} + R_{res}. \tag{3}$$

Since the BCS surface resistance for temperatures lower than half of the superconductor critical temperature follows approximately a temperature and frequency dependence given by

$$R_s \propto \frac{\omega^{1.7}}{T} \exp\left[-\frac{\Delta(T)}{kT}\right], \tag{4}$$

one can gain something choosing to work at lower frequencies. For example the theoretical surface resistance of niobium at 10 GHz and 1.5 K is $2.5 \cdot 10^{-8}\,\Omega$ which gives a quality factor $Q_0 = 3 \cdot 10^{10}$. From Eq. (4) one can guess that at 3 GHz the BCS surface resistance should be $R_{BCS} = 3 \cdot 10^{-9}\,\Omega$ and the theoretical quality factor $Q_0 = 2 \cdot 10^{11}$.

Obviously one has to consider the residual contribution to the surface resistance, which is in general the dominant one. To lower the residual resistance contribution, niobium surface treatments are critical. In the last decade however a great deal of progress has been made in this field due to several large scale applications of rf superconductivity which pushed the development of the technology of surface preparations and obtained very interesting results. A recent experimental result obtained at CERN, showed that it was possible to store in a mono-cell copper cavity, sputter coated with a niobium film, operated in the TE_{011} mode at 966 MHz, an energy exceeding 10 J, at $Q \approx 10^{10}$.[6] The maximum surface magnetic field amplitude obtained in this cavity was 38 mT, and the BCS and residual surface resistances at 4.2 K were 192 and 47 nΩ respectively.

From the experimental set-up point of view, the major difference between our proposal and the RRM experiment will be in the signal detection arrangement. While they used the signal *reflected* from the

cavity either to phase lock the signal generator to the symmetric cavity frequency, or to detect the converted signal at the antisymmetric frequency, we plan to use the signal *transmitted* through the cavity to do these jobs. With this arrangement we should improve system selectivity, or, in other words, we should have a lower contribution of the symmetric mode at the higher, antisymmetric, frequency, lowering the overall noise level in the system and increasing sensitivity.

A further improvement in system sensitivity should be obtained by choosing to work with spherical cavities, as shown in Figure 3, instead of cylindrical resonators. With spherical cavities one should gain on two sides:

- on a first side the geometrical factor of a spherical cavity is higher than the geometrical factor of a cylindrical one; this means that for equal surface characteristics (equal R_s) the quality factor of a spherical cavity will be higher;
- field configuration is more favourable in a spherical cavity and should be much easier to build the coupled cavity system with the coupling hole in the higher field region.

Nevertheless we plan not to build a spherical resonator until a full set of tests have been made on the cylindrical one.

On the electronics side a large amount of microwave devices (low noise amplifiers, filters, etc.) have been developed, mainly for applications in telecommunication networks. These devices should allow to reduce the contribution of electronics to the noise in the experiment by at least one order of magnitude. Just to make a comparison, while RRM used an rf low noise amplifier with a noise figure $N_f = 4.7$ dB, nowadays low noise amplifiers with N_f as low as 0.8 dB are commercially avaliable. A further gain could be obtained using a cold preamplifier. In this way the minimum detectable energy is lowered by the ratio of the equivalent temperatures of the preamplifiers, which is, in the ideal case, the ratio between room temperature and the operating temperature of the electronics.

4 CONCLUSIONS

We plan to develop our work in a two-year period, starting with the characterization of a cavity prototype, made of copper, in the first half
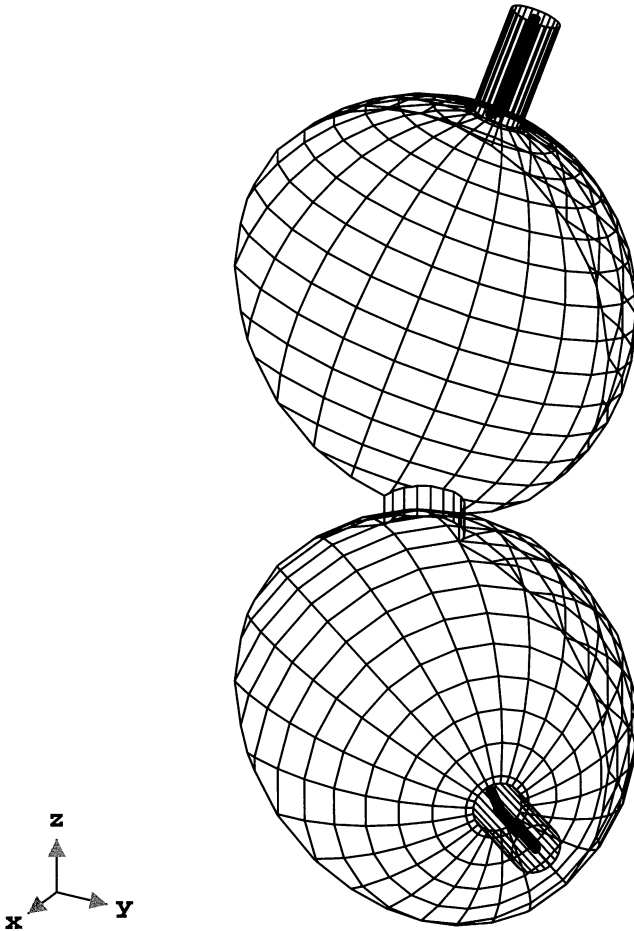
FIGURE 3   Schematic view of two coupled spherical cavities.

of 1998, to begin final system optimization and characterization in the second half of 1999.

Based on the considerations made in the previous sections our proposal is to repeat the RRM experiment increasing the detector sensitivity by at least three orders of magnitude, reaching the $\delta x/x \approx 10^{-21}$ limit. If these results could be obtained, this detector would be an interesting candidate for the detection of gravitational waves or in the search of long range interactions of weak intensity.

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[4] If we consider the interaction between a gravitational wave and the electromagnetic field stored inside a resonant cavity, the system parameter which drives the transition between the energy levels is the permeability tensor ε, whose components are functions of the metric and, for a monochromatic gravitational wave, are sinusoidal functions of time. Furthermore it must be taken into account that in order to interact with the gravitational wave the electromagnetic field must have a quadrupole moment; this result can be achieved choosing with care the geometry of the coupled cavities system.

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