VI. GEOPHYSICS

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RESEARCH OBJECTIVES

This group will be concerned with a variety of problems of geophysical interest, including laboratory investigations of the properties of matter and radiation under extreme conditions. The initial program includes two principal projects.

1. Planetary Radiation Belts

Theoretical and experimental studies will be undertaken to extend our knowledge of the interaction of charged particles in a magnetic field, especially to include conditions under which electric and magnetic forces are of comparable magnitude. These studies should have a bearing on scattering, recombination, and radiation processes in very low density plasmas.

Laboratory studies are planned for the investigation of the diffusion of charged particles in the fields of magnetic dipoles.

F. Bitter

2. Scattering of Light in the Earth's Atmosphere

Studies of the Earth's atmosphere by scattering of laser radiation will be pursued. An important part of this program will involve studying the dust layers existing at an altitude of 20 km and above. We are planning to take the apparatus to Sweden during next summer to observe noctilucent clouds.

The possibility of using Raman scattering in order to obtain profiles of density and temperature for various species in the lower atmosphere will be investigated.

The scattering cross section in the vicinity of an absorption line will be investigated experimentally with a tunable laser.

G. Fiocco

A. TRANSITION FROM PREDOMINANTLY ELECTRIC TO PREDOMINANTLY MAGNETIC BINDING IN THE HYDROGEN ATOM IN A MAGNETIC FIELD

Attempts to obtain complete solutions for the hydrogen atom in a magnetic field thus far have been only partially successful, and will not be reported now. The following simple semiclassical calculation of the effect of combined magnetic and electric forces acting on electrons in circular orbits in a hydrogen atom with the nucleus at rest indicates the kind of phenomena to be expected. For this case, for the forces, we have

$$\frac{mv^2}{r} = \frac{ke}{r^2} + evB, \tag{1}$$

and for the canonical angular momentum

$$p = n\hbar = mr^2\left(\dot{\phi} - \frac{eB}{2m}\right). \tag{2}$$

Upon eliminating the velocity, we obtain an expression for the radius

$$n^2 = \frac{r}{a} + \frac{r^4}{b^4}, \tag{3}$$

where a is the first Bohr orbit

$$a = \frac{\hbar^2}{kem} = 0.53 \times 10^{-10} \text{ meter},$$

and b is the lowest cyclotron orbit radius

$$b = \frac{3.62 \times 10^{-8} \text{ meter}}{\sqrt{B}}.$$

This case is illustrated in Fig. VI-1. For small values of r and B, the radius is independent of B. This is in accordance with the Larmor theorem, which states that, to a first approximation, magnetic fields do not distort atomic configurations, but merely produce the Larmor precession. At high fields and in large orbits in which

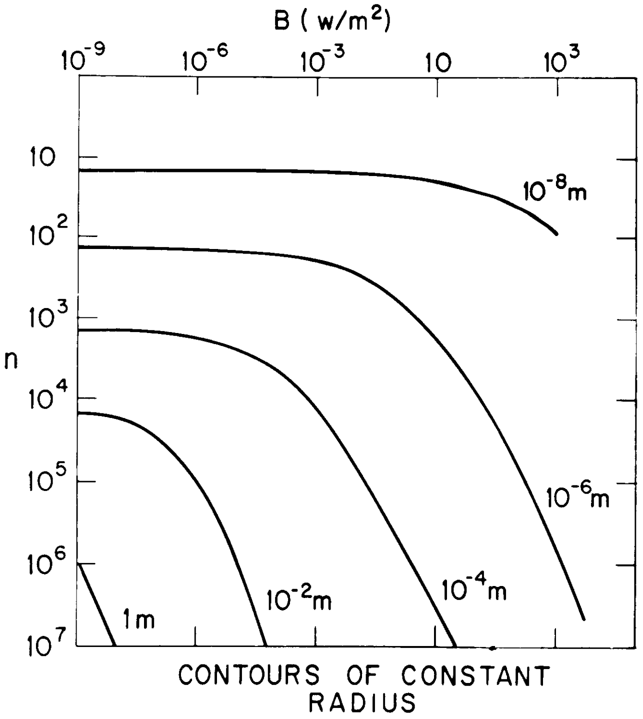

Fig. VI-1. Radius of circular Bohr orbits with magnetic quantum number m = ±n plotted as a function of n and B.

the electrostatic forces are negligible, the orbits approach the cyclotron orbits of free electrons.

The total energy of the system,

$$E = \frac{1}{2} mv^2 - \frac{ke}{r}, \tag{4}$$

may be put into the form

$$E = \pm\mu_B B \left[n \pm \sqrt{n^2 - \frac{r}{a}} \right] - \frac{1}{2} \frac{ke}{r}. \tag{5}$$

For small orbits in weak magnetic fields $n^2 = \frac{r}{a}$, and the energy is simply the zero-field energy $\pm n \mu_B B$. For large fields $\frac{r}{a} \ll n^2$, and the energy becomes

$$E = 2 n \mu_B B$$

when the magnetic moment is antiparallel (diamagnetic), or zero when it is parallel (paramagnetic) to the applied field. In the diamagnetic case, the energy levels approach the cyclotron or Landau levels.

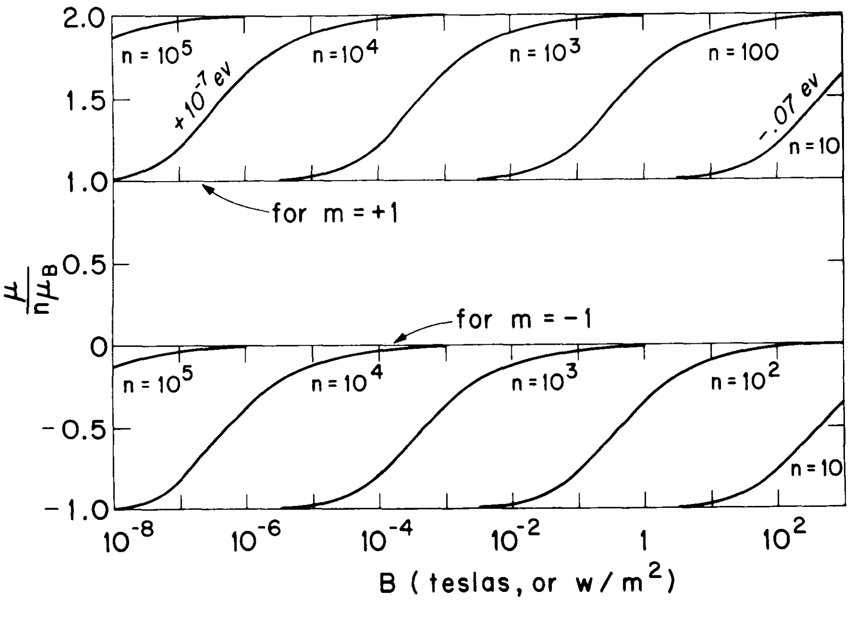

Fig. VI-2. Magnetic moment of quantized circular orbits in arbitrary magnetic fields.

The magnetic moment of the atom in these states is plotted in Fig. VI-2 in units normalized to "n" Bohr magnetons. The density of energy levels, normalized to the corresponding density of Landau levels, is plotted in Fig. VI-3. Note that the usually accepted density of states of an atom in zero magnetic field, which rises to infinity as negative

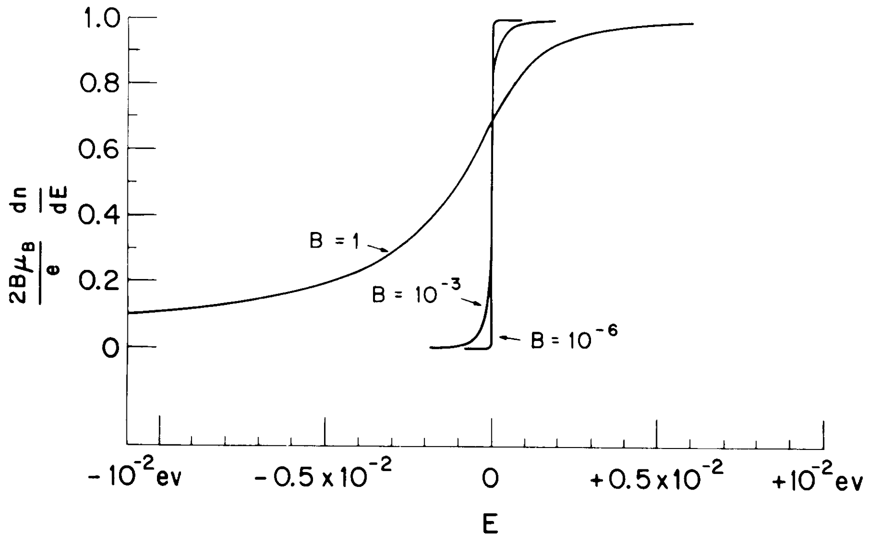

Fig. VI-3. Density of atomic-energy levels for circular diamagnetic orbits in the vicinity of the ionization potential.

energies approach the ionization potential and is zero for positive energies, is a "singular" or physically unachievable condition. In arbitrarily weak fields these particular states have a very low density for negative energies, and a constant density equal to the density of Landau levels for free electrons for positive energies.

This work was partly carried out at the Computation Center of the Massachusetts Institute of Technology.

F. Bitter

B. AN INTERPRETATION OF SOME OPTICAL RADAR RESULTS*

Recent optical radar observations of echoes from the upper atmosphere have been reported elsewhere by Fiocco and Smullin.[1] The results of several successive nights of observations indicated, among other features, weak sporadic echoes at altitudes between 110 km and 140 km; between 100 km and 110 km a noticeable reduction of the returned signal was observed. The interpretation of the results reported here has been developed in collaboration with Dr. Giusseppe Colombo, of the Smithsonian Institution Astrophysical Observatory, Cambridge, Massachusetts, and the University of Padua, Italy.

The results that are of present interest are shown in Fig. VI-4. The diagram gives the observed radar cross section per unit volume, averaged over 10-km range intervals

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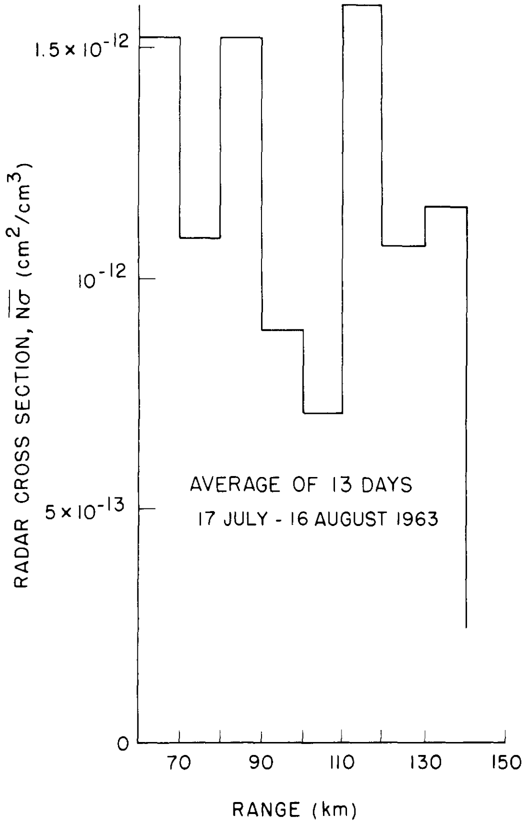

Fig. VI-4. Average of observed radar cross sections per unit volume versus range.

and over 13 nights during which observations were possible, from 17 July to 16 August 1963. The radar's wavelength is 0.694 micron.

We are concerned at present with an interpretation of the echoes obtained at heights above 100 km, that is, with the presence of maxima in the radar cross section above 100 km, and of minima between 100 km and 110 km. The diagram results from averaging the successive observations of each single night (several hundred pulses), and then averaging again the results of the 13 successive nights, giving to each night the same weight. A few remarks have to be made with regard to the accuracy of these determinations. Some measure of accuracy is provided by the amplitude of the rms background fluctuations, which is equivalent to 1.5×10^{-13} $(Z/100)^2$ cm^2/cm^3 (Z = height in km). Because of slow recovery and nonlinearity of the photodetector, the sensitivity of the receiver decreases with decreasing height. The effect is noticeable at altitudes below 90 km; the cross sections, as given in Fig. VI-4, are less accurate at this range. No correction has been introduced, since the present discussion is limited to the results of observations at higher levels. We note, however, that the echoes observed between 80 km and 90 km, at the mesopause, are indicative of cross sections much smaller than those that would be expected in noctilucent clouds,[2] and are in agreement qualitatively with the measurements of Mikirov.[3] The appearance of these echoes could be related to the same process that is responsible for the formation of the noctilucent clouds.

It seems reasonable to assume that these echoes at higher altitudes are produced by small particles, meteoroids. A knowledge of their shape and of the ratio of the size to the wavelength is therefore essential in order to establish their individual cross sections.

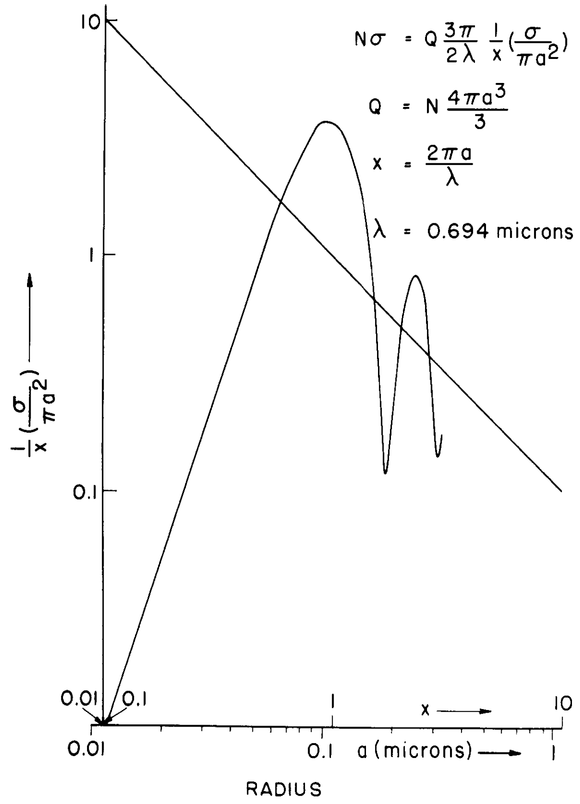

Fig. VI-5.

Radar cross section $N\sigma$ of N totally reflecting spheres of equal radius a and constant volume Q.

For a particle much larger than the wavelength the radar cross section, σ, is proportional to the geometrical cross section and independent of wavelength; for dimensions much smaller than the wavelength λ, the scattering cross section decreases with λ^{-4} dependence. In the intermediate region of sizes, large fluctuations occur. Values of the ratio $\sigma/\pi a^2$ for totally reflecting spheres are given by Van de Hulst[4] as a function of $x = 2\pi a/\lambda$. Suppose that a given volume Q is fragmented into N totally reflecting spheres of radius a, the density remaining constant. The total radar cross section of the N spheres, taken as independent scatterers, is

$$N\sigma = Q\frac{3\pi}{2\lambda}\frac{1}{x}\left(\frac{\sigma}{\pi a^2}\right). \tag{1}$$

A plot of $\frac{1}{x}\frac{\sigma}{\pi a^2}$ is shown in Fig. VI-5. As the wavelength of the radar is 0.694 micron, a reduction in size corresponds to a large increase in total cross section until an optimum size ($x \approx 1$, $a \approx 0.11$ micron) is reached beyond which the cross section decreases very rapidly. Thus one may conceive that on its flight through the atmosphere a meteoroid undergoes progressive fragmentation in such a way that the collective radar cross section of the fragments increases until an average optimum size is reached and that further breakup, at lower heights, is responsible for a successive reduction of the observed cross section.

In the real case the particles will not behave as totally reflecting scatterers; the choice of spherical shape is unfavorable because it minimizes the geometrical cross section for a given mass and is probably far from reality, since published photographs of "fluffy" particles by Hemenway and Soberman[5] indicate very complex shapes. For these filamentary shapes the radar cross section is much larger than the geometrical cross section.

Since we are interested in obtaining from the measure of the collective radar cross section an order-of-magnitude estimate of mass density and influx, but do not have any measurement of individual radar cross sections, we shall assume that at some stage in the descent the meteoroid breaks up into fragments of optimum size for observation at the radar wavelength. We shall also assume that when the optimum size is reached, the average radar cross section σ is 10 times larger than the average geometrical cross section A.

Witt, Hemenway, and Soberman[2] have collected particles from the mesopause. They found a large number of submicron particles of a size that would not be expected to scatter efficiently at wavelength $\lambda = 0.69$ micron, and that cannot exist in elliptical orbit in the solar system, because of radiation pressure.

Some conditions of a purely mechanical character, which we have formulated but will not present here, indicate that physically the following process is possible: A meteoroid, with a radius of 10 microns or more, a density between 0.1 and 0.5,[1] and entering velocity of 30 km/sec, will begin to fragment in sizes of 0.1-0.5 micron at a height of 120-140 km (which is above the height Z previously defined) if it has a crushing strength somewhat higher than 10 dynes/cm^2; these fragments of density close to one will keep breaking up in flight downward into smaller crystals, typically of 0.02-0.05 micron, at an altitude of 110-120 km, having reached a speed of approximately 10 km/sec, if they have a crushing strength of the order of 100 dynes/cm^2. The size distribution should be variable with height, and the atmosphere would be working as a "filter."

On the bases of the radar observations and of the present model of fragmentation, an attempt can be made to estimate the influx on Earth of meteoric material that is undergoing fragmentation and is of a size suitable for observation.

From the size population found by Hemenway and Soberman it can be established that most of the total mass is obtained in sizes smaller than 0.1-micron radius. According to our model, all of this material is produced by fragmentation. Thus an estimate of the flux based on these particles of larger size that are observable by optical radar at higher altitudes accounts for most of the mass obtained at lower levels. Some of the larger particles may well enter at lower speed in such a way that their contribution to the total flux will be small.

Since the experiments have been performed in a period of high meteoric activity and at night, a seasonal and diurnal rate factor affects the measurements. For the sake of obtaining an order-of-magnitude estimate, we take the yearly average of the observed difference in cross section between the maximum above 110 km and the minimum between 100 km and 110 km to be of the order of 10^{-13} cm^2/cm^3. For particles of dimensions close to optimum, we shall assume an average radar cross section of the order of 7×10^{-9} cm^2, an individual mass of the order of 2.5×10^{-14} gm, a density of 1.5, with an average radial velocity of 5 km/sec. A total influx of the order of 6×10^4 tons a day on Earth is thus obtained. It is unnecessary to emphasize the great uncertainty attached to this estimate, in view of the scarcity of experimental data and the variety of assumptions. The number obtained is, however, in agreement with other published results. For instance, it is in fair agreement, within an order of magnitude, with evaluations obtained from measurements of meteoric impact on satellites by Dubin and McCracken.[6] The hypothesis of fragmentation may account for the relatively low influxes obtained on the basis of Volz and Goody's twilight experiments.[7,8]

G. Fiocco

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