

LATTICE METHODS FOR QUANTUM CHROMODYNAMICS

Thomas DeGrand

University of Colorado, USA

Carleton DeTar

University of Utah, USA

 **World Scientific**

NEW JERSEY • LONDON • SINGAPORE • BEIJING • SHANGHAI • HONG KONG • TAIPEI • CHENNAI

Contents

<i>Preface</i>	vii
1. Introduction	1
2. Continuum QCD and its phenomenology	5
2.1 The Lagrangian and QCD at short distance	6
2.2 The nonrelativistic quark model	7
2.2.1 Explanations and puzzles	7
2.2.2 Quark model interpolating operators	9
2.2.3 Glueballs, hybrids, and exotics	10
2.2.4 Flavor and glueball mixing example: the anomaly . .	11
2.2.5 Quark model hadron masses	11
2.3 Heavy quark systems	13
2.3.1 Quarkonium	13
2.3.2 Heavy quark symmetries	14
2.4 Chiral symmetry and chiral symmetry breaking	15
2.4.1 Chiral symmetry	15
2.4.2 Linear sigma model	16
2.4.3 Nonlinear sigma model	18
2.4.4 Nonlinear effective chiral Lagrangian for QCD	20
2.5 A technical aside: Ward identities	21
2.6 The axial anomaly and instantons	24
2.6.1 Nonconservation of the flavor-singlet axial current . .	24
2.6.2 Witten-Veneziano formula	25
2.6.3 Suppression of the topological susceptibility	26
2.6.4 QCD vacuum	27

2.7	The large N_c limit	27
3.	Path integration	33
3.1	Lattice Schwinger model	33
3.1.1	Free fermions in one dimension	33
3.1.2	Species doubling	35
3.2	Hamiltonian with gauge fields	36
3.3	Feynman path integral	38
3.3.1	Pure gauge theory	39
3.3.2	Generalization to $SU(3)$ pure gauge theory	43
3.3.3	Static quark potential	45
3.3.3.1	Polyakov loops and potential energy of point charges	45
3.3.3.2	Wilson loops and point charge potential	46
3.4	Free fermions	48
3.4.1	Grassmann calculus	48
3.4.2	Grassmann path integral for a two-level system	50
3.4.3	Fermion propagator	52
3.4.4	Path integral with particles and antiparticles	54
3.4.5	Generalization to higher dimensions	56
3.5	The interacting theory	57
3.5.1	Functional integral representation for the lattice Schwinger model	57
4.	Renormalization and the renormalization group	61
4.1	Blocking transformations	61
4.2	Renormalization group equations	71
4.2.1	Renormalization in scalar field theory	72
4.3	Renormalization group equations for the scalar field	77
4.4	Effective field theories	79
5.	Yang-Mills theory on the lattice	87
5.1	Gauge invariance on the lattice	87
5.2	Yang-Mills actions	89
5.3	Gauge fixing	90
5.3.1	Maximal tree gauge	90
5.3.2	Landau and Coulomb gauge	91
5.4	Strong coupling	92

5.4.1	Wilson loop and confinement	93
5.4.2	Glueball mass	96
5.4.3	Polyakov loop	98
6.	Fermions on the lattice	101
6.1	Naive fermions	101
6.2	Wilson-type fermions	106
6.2.1	Twisted-mass fermions	113
6.3	Staggered fermions	115
6.4	Lattice fermions with exact chiral symmetry	122
6.5	Exact chiral symmetry from five dimensions	127
6.5.1	Five dimensions in the continuum	127
6.5.2	Five dimensions on the lattice	130
6.6	Heavy quarks	134
6.6.1	Heavy quark effective theory	135
6.6.2	Nonrelativistic QCD	137
6.6.3	Heavy, relativistic quarks	141
7.	Numerical methods for bosons	143
7.1	Importance sampling	143
7.1.1	Monte Carlo methods	144
7.1.1.1	Metropolis <i>et al.</i> method	145
7.1.1.2	Heat bath method	145
7.1.2	Molecular dynamics method	146
7.1.3	Refreshed molecular dynamics	147
7.1.4	Hybrid Monte Carlo	148
7.1.5	Leapfrog algorithm and improvements	148
7.2	Special methods for the Yang-Mills action	150
7.2.1	Heat bath	150
7.2.2	Overrelaxed updates	151
7.2.3	Molecular dynamics	152
8.	Numerical methods for fermions	155
8.1	Taming the fermion determinant: the Φ algorithm	155
8.2	Taming the fermion determinant: the R algorithm	159
8.3	The fourth root approximation	160
8.4	An exact algorithm for the fourth root: rational hybrid Monte Carlo	162

8.5	Refinements	163
8.5.1	Sexton-Weingarten scheme	163
8.5.2	Hasenbusch method	163
8.5.3	Schwarz alternating method	164
8.6	Special considerations for overlap fermions	165
8.7	Monte Carlo methods for fermions	170
8.7.1	Multiboson method	171
8.7.2	Ratio of determinants	171
8.8	Conjugate gradient and its relatives	173
8.8.1	Even-odd preconditioning	174
8.8.2	The conjugate gradient algorithm	175
8.8.3	Biconjugate gradient	178
8.8.4	Stabilized biconjugate gradient	179
8.8.5	Shifted solvers	180
8.8.6	Computing fermion eigenmodes	181
9.	Data analysis for lattice simulations	183
9.1	Correlations in simulation time	184
9.2	Correlations among observables	186
9.2.1	Correlated least chi square	186
9.2.2	Truncating the correlation matrix	187
9.2.3	Jackknife and bootstrap methods	189
9.3	Fitting strategies	191
9.3.1	Fitting range	191
9.3.2	Signal to noise ratio	192
9.3.3	Interpolating operator	192
9.3.4	Asymmetric lattice	193
9.3.5	Bayesian methods	194
10.	Designing lattice actions	197
10.1	Motivation	197
10.2	Symanzik improvement	199
10.2.1	Gauge action improvement	200
10.2.2	Fermion action improvement	201
10.2.3	Nonperturbative improvement	202
10.3	Tadpole improvement	204
10.4	Renormalization-group inspired improvement	209
10.5	“Fat link” actions	211

11. Spectroscopy	215
11.1 Computing propagators and correlation functions	215
11.2 Sewing propagators together	219
11.2.1 Mesonic correlators – Wilson fermion formalism . . .	219
11.2.2 Baryonic correlators – Wilson fermion formalism . . .	222
11.2.3 Mesonic correlators – staggered fermion formalism . . .	224
11.2.3.1 Flavor nonsinglet mesons	225
11.2.3.2 Flavor singlet mesons	229
11.2.4 Baryonic correlators – staggered fermion formalism . . .	229
11.3 Glueballs	232
11.4 The string tension	233
12. Lattice perturbation theory	235
12.1 Motivation	235
12.2 Technology	235
12.2.1 Free fermion and gluon propagators	236
12.2.2 Quark-gluon vertices	238
12.2.3 Fat links	239
12.2.4 Quark self energy and tadpole	241
12.2.5 Vertex graph	242
12.2.6 Ultraviolet divergences	243
12.2.7 Automation	243
12.3 The scale of the coupling constant	244
13. Operators with anomalous dimension	249
13.1 Perturbative techniques for operator matching	250
13.2 Nonperturbative techniques for operator matching	253
13.2.1 Methods for approximations to conserved currents . . .	253
13.2.2 Regularization-independent scheme	254
13.2.3 Schrödinger functional methods	258
14. Chiral symmetry and lattice simulations	261
14.1 Minimal introduction to chiral perturbation theory	261
14.2 Quenching, partial quenching, and unquenching	265
14.2.1 The eta prime correlator	266
14.2.2 Chiral Lagrangian with partial or complete quenching	268
14.3 Chiral perturbation theory for staggered fermions	270
14.3.1 Staggered chiral perturbation theory	270

14.3.2	Rooted staggered chiral perturbation theory	273
14.4	Computing topological charge	275
15.	Finite volume effects	279
15.1	Finite volume effects in chiral perturbation theory	279
15.2	The ϵ -regime	282
15.2.1	Banks-Casher formula	282
15.2.2	Chiral Lagrangian in the epsilon regime	283
15.2.3	Random matrix theory	284
15.2.4	Further applications	286
15.3	Finite volume, more generally	286
15.3.1	Single particle states	286
15.3.2	Two particle states	288
15.3.2.1	Scattering lengths from the two-particle ground state	289
15.3.2.2	Decay widths from level repulsion	290
15.4	Miscellaneous comments	293
16.	Testing the standard model with lattice calculations	295
16.1	Overview	295
16.2	Strong renormalization of weak operators	295
16.2.1	Effective Hamiltonian	295
16.2.2	An example: $c \rightarrow s\bar{d}u$	297
16.2.3	Lattice <i>vs</i> continuum renormalization of the effective Hamiltonian	300
16.2.4	Mixing with operators of higher and lower dimension	301
16.3	Lattice discrete symmetries	303
16.4	Some simple examples	306
16.4.1	Leptonic decay constants of mesons	306
16.4.2	Leptonic decay constants in the heavy quark limit	307
16.4.3	Electromagnetic widths of vector mesons	308
16.4.4	Form factors	308
16.4.5	Meson B parameters	310
16.4.6	Other purely hadronic weak interactions	314
16.5	Evading a no-go theorem	314
17.	QCD at high temperature and density	317
17.1	Simulating high temperature	317

17.2	Introducing a chemical potential	318
17.3	High quark mass limit and chiral limit	318
17.4	Locating and characterizing the phase transition	320
17.5	Simulating in a nearby ensemble	320
17.6	Dimensional reduction and nonperturbative behavior	322
17.7	Miscellaneous observables	323
17.7.1	Quark number susceptibilities	323
17.7.2	Quarkonium potential	323
17.7.3	Equation of state	323
17.8	Nonzero density	325
17.9	Spectral functions and maximum entropy	326
	<i>Bibliography</i>	329
	<i>Index</i>	341