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SIMULATION OF SPACE CHARGE AND INTEGER RESONANCE CROSSING IN STORAGE RINGS AND ITS APPLICATION TO HEAVY ION FUSION

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Space charge and its effect on dipole error resonances of circular accelerators is investigated by means of computer simulation. The problem is applied to storage rsp. compression rings in heavy ion fusion, where extreme space charge conditions are expected to occur. A major point is that crossing of the single particle (incoherent) tune through an integer is of no effect on dipole error response, but only crossing of the coherent tune, which depends on image charge effects. It is found that even for strong space charge tune depression and large nonlinear tune spread, as typical for heavy ion fusion applications, space charge does not wash out the beneficial effect of phase memory, hence the beam always responds coherently on dipole errors. This coherent behaviour is conserved even in cases where the nonlinear spread of the incoherent tune extends over several integers. Thus only crossing of the coherent tune through an integer has to be avoided. We suggest a qualitative criterion for conservation of coherence, which is analogous to the Landau damping criterion. We also discuss conservation of emittance and the formation of beam halo due to strong space charge in an ideal lattice.

1 INTRODUCTION

In high-current synchrotrons or in bunch compression rings the question may arise whether crossing of the space charge shifted single particle tune over one or several integers is possible. Such a crossing may be desirable during final bunch compression in heavy ion fusion storage rings, or, alternatively, during beam accumulation by non-Liouvillean stacking, which has been studied in different schemes of heavy ion fusion recently.[1,2] A similar situation arises in recirculators based on the induction accelerator principle.[3] For these applications such a crossing occurs during typically ten or several tens of revolutions. Beams with such strong space charge effects as considered in these schemes are closer to the conditions of high-current beam transport in straight beam lines rather than usual storage ring or synchrotron conditions. Due to the challenging large space charge tune shift envisaged for heavy ion fusion we focus on this application. The basic findings of our study should, however, be of interest for all circular machines with large incoherent tune shift.

A principal feature of circular accelerators is the repeated action of magnet errors. Off resonance the effect of errors cancels periodically and the beam remains on its equilibrium orbit with invariant emittance even for a large number of revolutions. The most dangerous resonance is the integer resonance associated with dipole field errors. In the presence of space charge it is necessary, for a correct treatment of integer resonance crossing, to distinguish between incoherent and coherent betatron tunes. According to analytical estimates[4] crossing of the coherent betatron tune, which is responsible for the dipole motion of the beam, is expected to pose alignment requirements, which are very difficult to satisfy in practice.

The real situation of a beam is somewhat more complicated by at least two additional effects:

1) the need to consider a relatively large spread in the single-particle tune due to non-uniform space charge,

2) Changes in both the incoherent and the coherent tunes due to step-wise increasing current during stacking, or during bunch compression.

There is some concern that both effects might destroy the phase memory of particles when passing a certain error element repeatedly after each revolution. This would have the effect that errors add up in a random fashion, which could be controlled only by extremely small alignment errors.

We have studied this problem by computer simulation with a self-consistent calculation of the (transverse) space charge force by means of a Poisson solver. The initial phase space distribution is assumed to be of the'waterbag' type, which results in a close to parabolic self-consistent density profile at low current. The self-consistent tracking shows non-negligible emittance growth in the absence of alignment errors for currents, which give a significant tune depression. This can be explained by transport instabilities of intense beams in AG focusing systems[5] and avoided, except for some halo effect, by choosing an appropriate phase advance per focusing cell.

In section 2 we summarize the standard expressions for incoherent and coherent tunes, which are needed to interpret the simulation results. These results are presented in a way largely independent of a particular ring or ion. In section 3 we study the effect of space charge in an ideal lattice, in section 4 the effect of errors and in section 5 the situation in a large ring.

2 BASIC BEAM AND ACCELERATOR PARAMETERS

2.1 Parameters for Heavy Ion Fusion Applications

In the following we present a typical set of parameters envisaged for inertial fusion with heavy ions[2]. Basic parameters are summarized in Table 1. Here it is assumed that the total intensity is focused on two small convertors (diameter 2.7 mm) inside the target cavity. The beam kinetic energy is converted into radiation, which drives the pellet implosion and ignition. The advantage of this conversion ('indirect drive') is the smoothing of the radiation field due to multiple reflections in the target hohlraum. The small size of the convertors is required in order to obtain the temperature of about 300 eV necessary for effective conversion into radiation. This determines the small emittance of the beams, which is responsible for the large incoherent tune shift considered in storage and bunch compression rings.

TABLE 1: Parameters for Heavy Ion Fusion Drivers

Ion (typical)	Bi^{+1} stripped to Bi^{+2}
Kinetic energy	10 GeV
Total energy	5 MJ ($N = 3.1 * 10^{15}$)
Final pulse duration	20 ns (full width)
Final momentum spread	$\Delta p/p = 3.0 * 10^{-4}$
Emittance at target	6π mm mrad
Spot diameter	2.7 mm
Ion range	0.3 g/cm^2
Specific Power	10^{16} W/g (peak)

In the scenario of Ref. [2] it is assumed that the total intensity is provided by a linac and stored in typically 20 storage rings in a large number of short bunches. After filling of all rings a non-Liouvillean method of stacking these bunches in compression rings is considered by means of photo-ionization from 1+ to 2+. Ideally it appears possible to stack - typically - ten bunches from the storage ring into the phase space volume of a single bunch in the compression ring. This process takes ten revolutions in the compression ring during which the intensity increases in equal steps.

A representative number considered here is to combine 480 bunches per storage ring into 48 bunches per compression ring by photo-ionization stacking. With the parameters of Table 1 and a bunch length of 45 nsecs we obtain a particle current of 1.75 A at the bunch center. After ionization the current is doubled and increased to 35 A after ten-fold stacking. A final r.f. bunch rotation performs the compression to 20 nsec as required at the target.

2.2 *Tune shifts as function of space charge*

Results can be represented in terms of tune shifts induced by space charge and rates of change of tune shifts due to varying current. As usual we distinguish between:

2.2.1 Incoherent tune shift It is defined as frequency shift of single particles caused by the space charge of a stationary beam. For a uniform beam it is given by

$$\Delta Q_{inc} = \frac{r_p Z^2 N}{2\pi\epsilon\beta^2\gamma^3 A B_f} \tag{1}$$

with r_p the classical proton radius (1.53×10^{-18}), Z charge state of ion with atomic number A, N number of ions per bunch, $\beta = v/c$, γ relativistic mass factor, ϵ emittance (area/π of a uniformly filled ellipse), and B_f bunching factor. ϵ is roughly given by a^2Q/R, with Q the machine tune, R the machine radius and a the average beam radius.

2.2.2 Coherent tune shift If the beam oscillates as a whole, the frequency of its center of mass is not affected by the direct space charge, but only by image charges. The resulting shift is given by Ref. [4]:

$$\Delta Q_{coh} = \Delta Q_{inc} \frac{a^2}{g^2} \tag{2}$$

with g the vacuum chamber radius. Here we have ignored a geometry factor, which is close to unity for a circular chamber. It is noted that ΔQ_{coh} can, in principle, be made small by choosing a large vacuum chamber.

2.2.3 Tune spread For a non-uniform beam density profile there is a nonlinear dependence of the restoring force on the amplitude, which leads to anharmonic betatron oscillations. Assuming a parabolic density profile (consistent with the initial waterbag phase space distribution of the simulation) one can readily calculate the difference between the (incoherent) tune shift for maximum amplitude oscillations to that of small amplitude oscillations, ΔQ_{min}, which in turn can be related to the incoherent tune shift ΔQ_{inc} of a uniform beam of the same rms size as a reference. This nonlinear tune spread is calculated as

$$\Delta Q_{spread} = \frac{3}{8} \Delta Q_{min} = \frac{1}{2} \Delta Q_{inc} \tag{3}$$

2.3 Ring Lattice and Dipole Errors

2.3.1 Lattice We have employed a simple doublet focusing with 64 unit cells of 10 m length and phase advance $\sigma_0 \approx 60°$, hence a circumference of 640 m. The quadrupole magnets have 1 m length and a gradient of 25 T/m. The bending requires two dipoles per cell with 2.85 m length and 1.8 T.

As reference ion we have chosen Bi^{2+} at 10 GeV, which corresponds to a magnetic rigidity of $B\rho$=105 T-m.

2.3.2 Dipole errors We have assumed, for simplicity, a single dipole error element per cell, i.e. 64 random value errors per circumference. For the rms value of the error we have chosen a standard value of $\Delta(Bl) = 1.5 \times 10^{-3}$ T-m, which results in an angle error of $\delta x' = \Delta(Bl)/(B\rho) \approx 0.14 \times 10^{-4}$. This can be due to an error in the field of one of the dipoles of 5×10^{-4} T (relative error 3×10^{-4}) or an alignment error of 60μm for one of the quadrupoles. The resulting displacement after a 90° rotation in betatron phase space is obtained by means of the lattice beta function:

$$\delta x = \beta \delta x' \tag{4}$$

We note that the assumed standard error per cell results in an average random angle deviation after passing of n cells given by

$$\delta x' \propto \sqrt{n} \tag{5}$$

provided that errors add completely at random.

In our case this leads to an average angle deviation after one revolution of $\delta x' \approx 10^{-4}$. The corresponding displacement using a beta function of 17 m in the middle of a focusing quadrupole is thus obtained as $\delta x \approx 1.7$ mm.

3 EMITTANCE GROWTH IN IDEAL LATTICE (NO FIELD ERRORS)

3.1 Step-wise Increasing Current

In order to simulate non-Liouvillean stacking (or in an analogous way a bunch compression scheme) we make the assumption that the current of a sample of beam is increased every turn in equal steps to achieve the maximum current after 10 turns. What matters here is not the absolute value of the current or emittance, but the incoherent tune shift and the total tune. Applying this to the example of the previous section we note that a current of 3.5 A for the first turn is equivalent to a shift $\Delta Q_{inc} = 0.53$.

Given a ring with 64 cells this is equivalent to a change of the phase advance per cell by 3°. It should be noted that close to the maximum current the decrease of the incoherent tune is somewhat less than linear. According to Ref. [5] in such an ideal transport lattice the excitation of space charge instabilities depends on the zero current phase advance per cell, σ_0, and its shift due to space charge.

Results are compared in Fig. 1 for three different values of the zero current phase advance. As a measure for the emittance growth we use both ϵ_{90} and ϵ_{100} as emittances (area/π) of 90% rsp. 100% of the particles.

(a) $\sigma_0 = 100°$: A strong growth of both ϵ_{90} and ϵ_{100} is found to start after 2-3 revolutions leading to rapid phase space dilution by a factor of about 4 in both planes.

(b): $\sigma_0 = 75°$: ϵ_{90} grows by less than 5%, whereas ϵ_{100} grows by about 300% during 10 revolutions.

(c) $\sigma_0 = 60°$: The growth of ϵ_{90} is almost identical to case (b), but ϵ_{100} grows by about 40% only.

The origin of emittance growth can be identified more easily by using phase space projections into the emittance planes for the different cases. In Fig. 2 we show the results after completion of the 10^{th} revolution. It is noted that for case (a) the emittance growth stems from an excitation of a multipole oscillation in phase space with fourfold symmetry. The excitation of such fourth-order modes (besides second-order, i.e. envelope oscillations) is discussed in Ref. [5]. The fourth-order mode gets out of resonance with the periodic focusing for $\sigma_0 \leq 90°$. For cases (b) and (c) we find indeed that the rms emittance is nearly constant, but there is a halo effect, which is much less pronounced for case (c) with $\sigma_0 = 60°$. We thus choose a phase advance near to 60° as 'safe' with respect to high space charge. Note that $\sigma_0 = 60°$ is equivalent to $Q_0 = 10.667$ in our model storage ring.

We have also compared this result for a step-wise current increase by 0.35 A over 100 revolutions and $\sigma_0 = 60°$ giving practically the same amount of emittance growth. This shows that the emittance growth observed here seems not to be related to the size and number of current steps, but rather to the total increase of current.

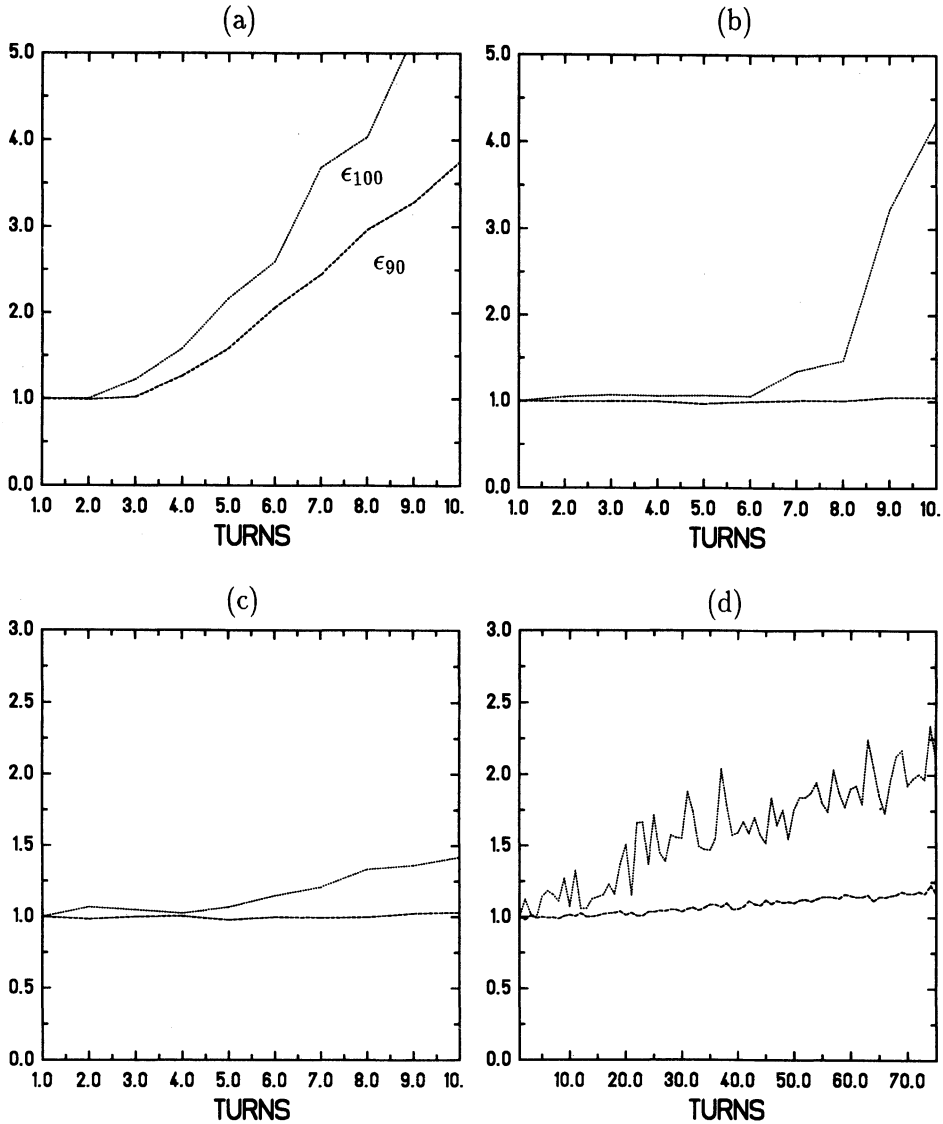

FIGURE 1: Emittance growth factors for step-wise increasing current during 10 turns and $\sigma_0 = 100°, 75°, 60°$ (a,b,c); for constant maximum current over 75 revolutions (d).

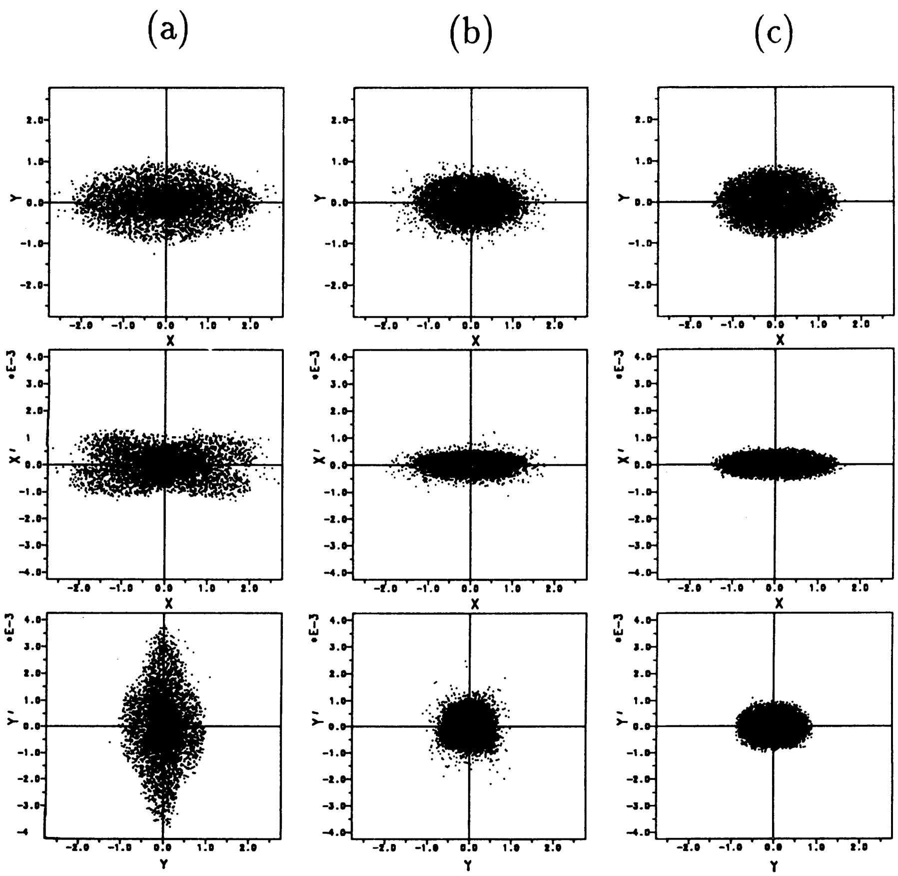

FIGURE 2: Final stage of simulation: projections into real space (x-y) and phase space (x-x', y-y') for examples a,b,c of previous figure.

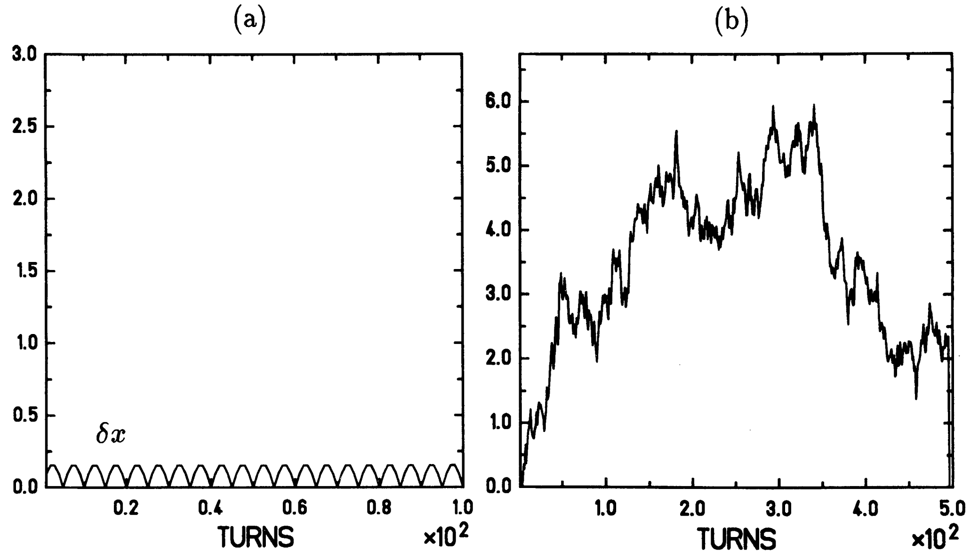

FIGURE 3: Beam displacement (cm) due to errors for $Q_0 = 10.8$ (a) and for fully random errors (b).

3.2 Constant Current

For comparison we have also explored the emittance growth if the full current in the above example is kept constant over a large number of turns. We have applied rms matching at start and no simultaneous matching of the density profile. This causes some coherent oscillation, which might also contribute slightly to emittance growth as a result of transformation of space charge field energy into thermal energy, i.e. emittance (see Ref. [6]). The result is shown in Fig. 1d for over 75 revolutions, which is equivalent to 4800 cells ($\sigma_0 = 57.2°$). It is noted that there is a slow increase of ϵ_{90} by about 30% and an increase of ϵ_{100} by about 100%, which indicates a halo effect. It cannot be excluded, however, that the slow increase of ϵ_{90} is also caused by noise due to a limited number of simulation particles.

4 EFFECT OF FIELD ERRORS

In this section we introduce a standard random set of field errors, which is identically repeated after each turn in the model storage ring. For comparison we have an option where we redefine errors completely at random after each revolution, which simulates extinction of memory. The average field error strength is kept constant in all cases.

4.1 Zero Current

We first consider the effect of random errors distributed over the circumference of our 64 cells model storage ring. We choose $\sigma_0 = 60.75°$ corresponding to a tune $Q_0 = 10.8$.

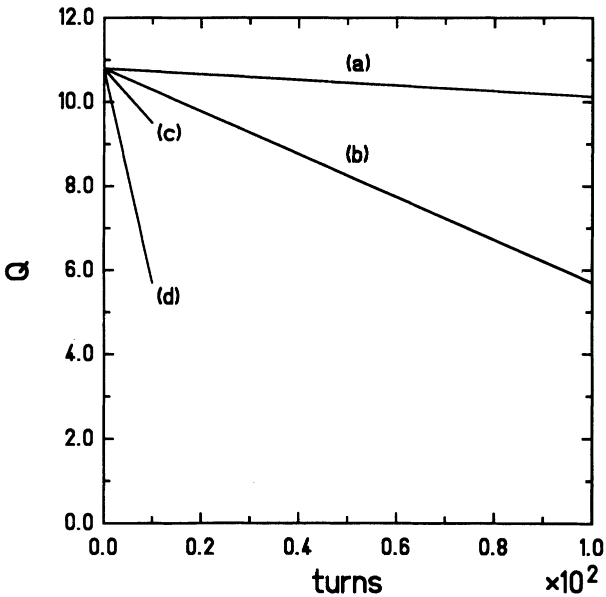

FIGURE 4: Step-wise decreased Q_0 at different rates

In Fig. 3 we plot the displacement of the beam center at a fixed position of the ring. As expected we find in Fig. 3a a cancellation of the displacement to zero after every 5 turns due to the difference 0.2 of the tune from the next integer. This is the usual situation in storage rings, where the tune is constant or nearly constant.

For illustration we show in Fig. 3b the corresponding result if the errors used in a) are (artificially) re-defined every revolution at random, for a total of 500 revolutions. The displacement is the result of adding up a large number of random numbers leading to large fluctuations. Obviously the result depends on the particular statistical set. Eq. 5 suggests an average displacement, after 500 revolutions, of $\delta x' \approx 2.2 \times 10^{-3}$, or $\delta x \approx 38$ mm. Note that the rms value of the error is the same in cases a) and b).

In order to illustrate the effect of a rapid change of tunes on the resonance crossing we have allowed for a variation of Q_0 at four different rates (see Fig. 4). The result is shown in Fig. 5a-d, where we have reduced Q_0 in 100 steps (100 turns) from 10.8 to 10.13 (a) and from 10.8 to 5.7 (b); and in 10 steps (10 turns) from 10.8 to 9.5 (c) as well as from 10.8 to 5.7 (d). In case a) the change results in a variation of the period of displacement, i.e. the coherent betatron oscillation wavelength. In case b) the beam center jumps during crossings of the integers 10, 9, 8, 7 and 6, with the largest effect occurring at $Q_0 = 8$ (as a feature of the particular random set chosen here).

By carrying out the variation of Q_0 much faster the resonance response is reduced. In case c) we still observe the effect of crossing $Q_0 = 10$; whereas d) shows no pronounced resonance response due to the large tune jumps of about half an integer per turn.

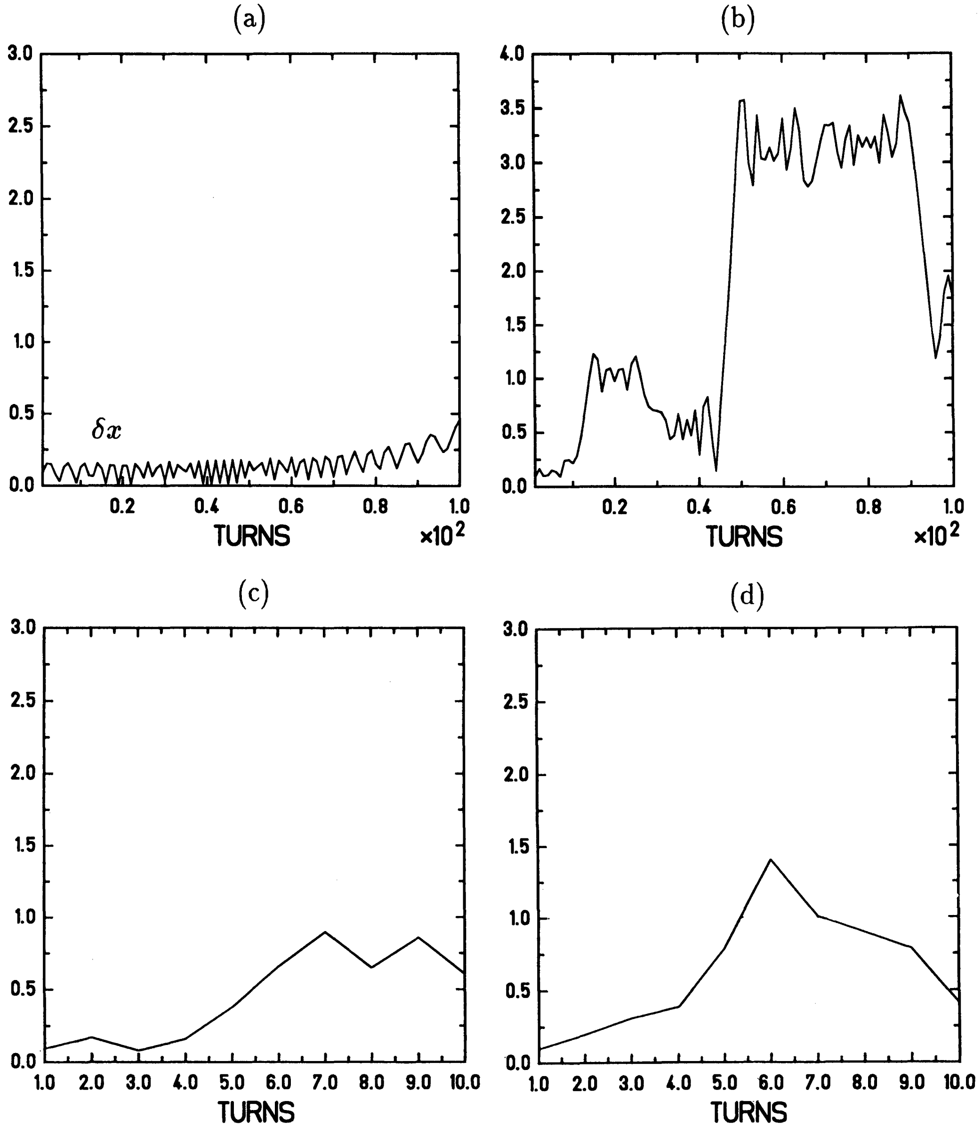

FIGURE 5: Beam displacement (cm) for step-wise reduced Q_0 (no space charge) at different rates according to Fig. 4.

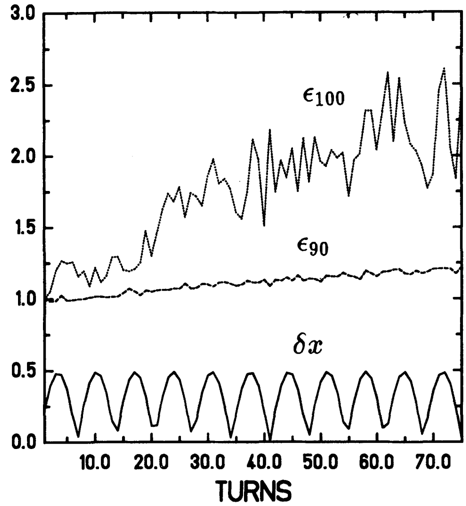

FIGURE 6: Coherent beam displacement (cm) and emittance growth factors in the presence of field errors and space charge for $Q_0 = 10.17$ and beam of Fig. 1 ($g = 3.16$; $\Delta Q_{inc} = 4.2$)

4.2 *Effect of Space Charge*

The essential point of the calculations in this section is to keep Q_0 fixed and study resonance crossing due to a varying current density.

As a first example we take the beam of Fig. 1d with a constant current of 35 A and $Q_0 = 10.17$. The corresponding incoherent tune shift is found as $\Delta Q_{inc} = 4.2$, i.e. shifted by four integers (note that Eq. 1 is strictly valid only for small tune shifts). From Eq. 2 we expect a coherent image charge tune shift $\Delta Q_{coh} = 0.35$ for the pipe radius $g = 3.16$ in this example. This is in good agreement with the observed period of oscillation, which is inferred from Fig. 6 as 9.85, i.e. 11 periods over 75 revolutions. It is noted that there is a gradual increase of ϵ_{90} and a halo formation indicated by the enhanced increase of ϵ_{100}. The increase is, however, practically identical with Fig. 1d, hence not related to the dipole errors.

The important conclusion is that the space charge dominated beam oscillates in a *coherent* fashion defined by the coherent tune shift. This coherence is not affected by the large intrinsic (nonlinear) spread of incoherent tunes, which in our example extends over two integers, centered around the incoherent tune of about 6, according to Eq. 3. The absence of any effect of the incoherent tunes crossing resonances is reflected by the emittance behaviour, which is the same with the dipole errors off and on.

4.3 *Stepwise Varying Current*

Next we assume a current increment of 3.5 A per turn over 10 turns for the same beam as above in a smaller vacuum pipe of $g = 2.16$, which doubles the coherent tune shift compared with the above example. In Fig. 7 we indicate the calculated variation of single

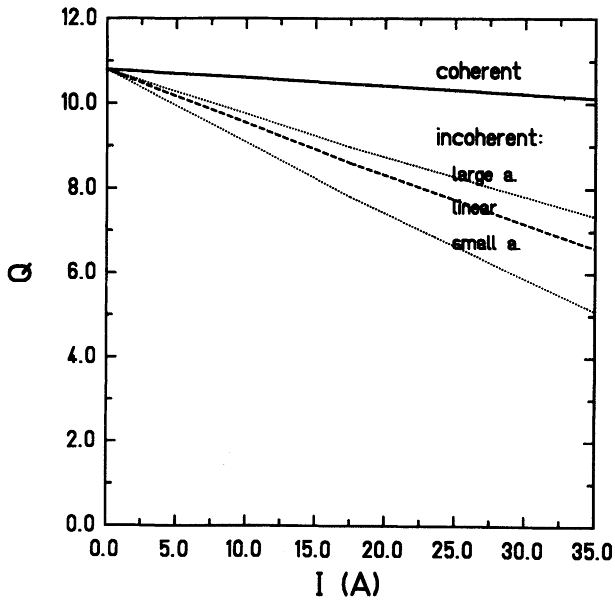

FIGURE 7: Calculated tunes as function of current ($g = 2.16$)

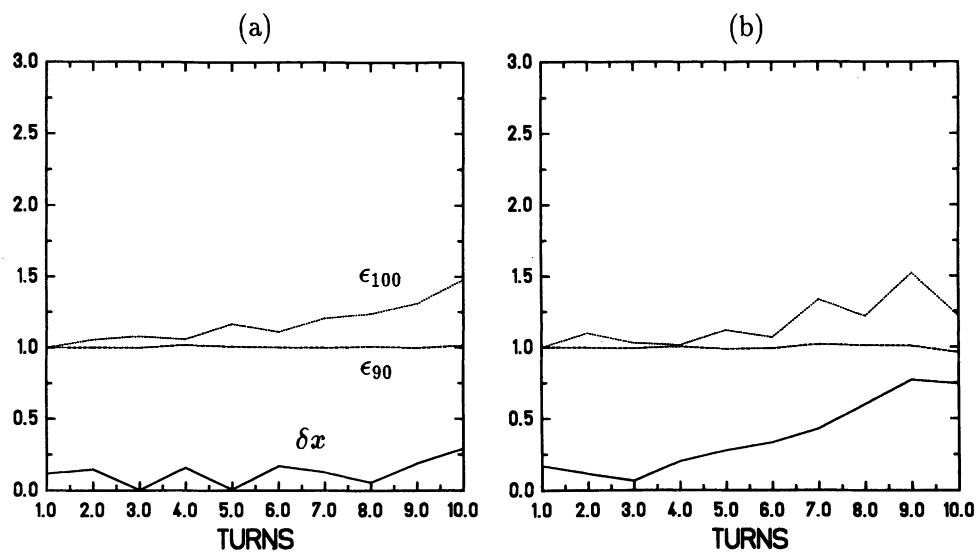

FIGURE 8: Coherent beam displacement (cm) and emittance growth factors with step-wise increasing current ($g = 2.16$); (a) $Q_0 = 10.8$; (b) $Q_0 = 10.5$

particle and coherent tunes. The calculated coherent image tune shift at full current is 0.67. In Fig. 8a we show the result for $Q_0 = 10.8$ and in Fig. 8b for $Q_0 = 10.5$. In the first case Q_{coh} remains above the integer 10 (as shown in Fig. 7), but approaching the integer leads to a slight increase of the amplitude. The integer 10 is crossed in the second case with a final $Q_{coh} = 9.83$ (calculated).

The latter case shows the characteristic displacement due to the crossing of Q_{coh} through 10. The crossing of the single particle tunes through as much as 5 integers is of no effect. Every turn corresponds to a reduction of the coherent tune by 0.067. We conclude from the results that such a change of tune still conserves the memory of the phase of the error response during previous turns. In other words, there is no random error summation; instead we have error cancellation away from the resonance and linear summation of errors at the resonance. This behaviour is consistent with Fig. 5b, where the crossing of $Q_0 = 10$ was at nearly the same rate.

5 ERROR RESPONSE IN A LARGE RING

In this section we study a large ring, where the shift of Q_{coh} due to increasing space charge goes over a few integers and the tune spread extends over a large number of integers. The most unfavourable situation would be that errors add up randomly from turn to turn if the phase memory was completely extinguished.[4]

We have carried out a computer simulation in a ring with ten times the circumference of the previous model storage ring, i.e. 640 cells, and $Q_0 = 104.5$. After each turn in the large ring the current is increased by 0.8 A. This corresponds to a change of the incoherent tune shift of $\Delta Q_{inc} \approx 1.12$. Hence, a total of 12 revolutions leads to 9.6 A and $\Delta Q_{inc} \approx 13.5$. The calculated shift of betatron tunes is shown in Fig. 9. Results are shown in Fig. 10a,b using 1000 simulation particles. Note that a) corresponds to a calculation through 7680 cells and b) through 6400 cells, where Poisson's equation has been solved 7 times per focusing cell. Beam displacement and emittance growth factors are plotted every 64 cells.

It is noted (Fig. 10a) that the displacement can be fully described as resonance crossing $Q_{coh} = 104$ between turn 4 and 8, which is consistent with the expected tune behaviour according to Fig. 9. The larger oscillations (by a factor of 3) compared with Fig. 8 can be explained by the expected increase of the average displacement (per turn) with the square root of the circumference, i.e. the number of random error elements. ϵ_{90} is practically constant, which confirms that single particle resonance (incoherent tune) effects are absent, in spite of the crossing of the incoherent tunes through many integers. The development of a slight halo is not related to the presence of errors as already discussed above.

In Fig. 10b we show the result for constant current. The constancy of the maximum displacement reflects that the phase memory is perfectly maintained from revolution to revolution, even though the spread of single particle tunes is as big as 7. Hence, although a fraction of the particles sit exactly on machine integer resonances, there is no real effect, since these particles see the sum of the dipole error force *and* the electric force due to the coherent motion. The slight halo effect is again not related to the errors.

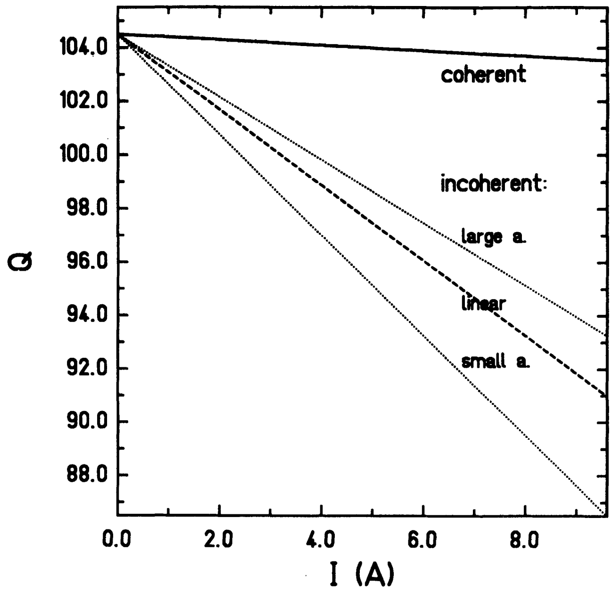

FIGURE 9: Calculated tunes as function of current for large ring ($Q_0 = 104.5$; g= 3.06)

The difference between coherent error response and random error response can be appreciated by comparing the displacement in Fig. 10b with that in Fig. 3. It is to be noted that one turn of the large ring is equivalent to 10 turns in the smaller ring of Fig. 3 as we have assumed the same density and rms strength of errors.

6 DISCUSSION

6.1 Error Tolerance

With dipole errors close to technically feasible values the displacement even in the small ring is of the order of 1 cm for the assumed speed of crossing. The amount of growth follows basically the single particle integer resonance crossing, except for the value of the tune, which is determined by image charges.

A displacement, which is as large as the beam size itself is not tolerable. Hence, a practical requirement is that crossing of the *coherent* tune through an integer must be avoided under all circumstances. For large currents this leads to the requirement of a sufficiently large vacuum chamber, which may be in conflict with the demands to focusing and bending magnets.

Incoherent tune shifts, on the other hand, may cross integers without an effect on displacement or emittance. This applies to dipole errors and crossing speeds of several turns per unit tune shift. At much smaller crossing rates gradient errors may come into play, which has not been studied here.

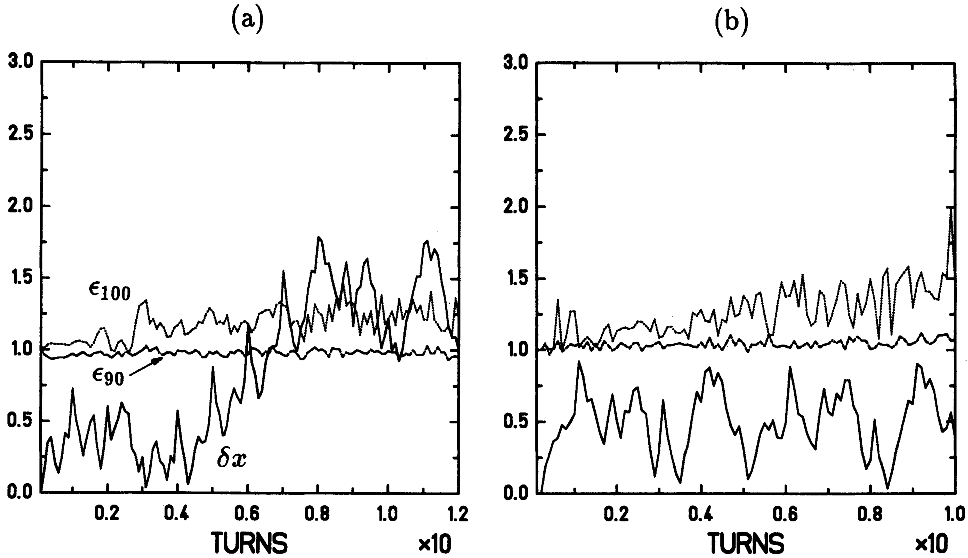

FIGURE 10: Coherent beam displacement (cm) and emittance growth factors in large ring with $Q_0 = 104.5$; (a) step-wise increasing current up to 9.6 A; (b) constant current of 9.6 A.

6.2 *Analogy with Landau damping*

It can be concluded from the examples that the coherence of the dipole error response is not affected by a large spread of single particle tunes due to non-linear space charge. Single particle tunes can be shifted over integer machine (dipole) resonances with no effect, since they are out of resonance with the total force, which is the sum of the forces from the dipole error and the electric field of the coherent motion. Since the spread of tunes is much larger here than the shift of the coherent tune this seems - at first glance - opposed to the usual Landau damping behaviour, where a coherent oscillation is damped if the spread is comparable (or larger than) the shift.

There is, however, an analogy with Landau damping, if one defines as 'shift' the difference between the average single particle tune and the coherent tune. This shift is (see Figs. 7,9) then practically equal to the average single particle tune shift in view of Eq. 2. Note that this argument applies to non-relativistic beams, where space charge is the dominant effect. Hence the coherence of dipole error response can be explained as a direct consequence of the smallness of the nonlinear tune spread compared with the incoherent tune shift. Due to this coherence crossing of resonance of the dipole error type is only important at points where the *coherent* tune becomes an integer. Avoidance of such a coherent crossing is thus a necessary criterion unless extremely small error tolerances can be realized.

We have also found that independent from whether dipole errors are present or not, conservation of beam quality is dependent on the strength of space charge effects and on the actual distance the beam is moving. There is an analogy, under conditions where the tune is strongly depressed by space charge, with linear transport systems: 'structure resonances' can be avoided by choosing a phase advance per cell near or below 60°; on the other hand development of some kind of halo in phase space (involving only a few percent of the total intensity) seems to be a common feature of long systems in general. This requires further study in the future.

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