Particle Accelerators, 1992, Vols. 37–38, pp. 409–416
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MUON CATALYZED FUSION AT HIGH DENSITY

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(Received 3 December 1990)

We consider muon catalyzed fusion (MuCF) in the environment of a heavy ion inertial fusion (HIIF) facility and show that it is an interesting fusion option. We present MuCF reaction rates of dt fusion in inertially confined (IC) high density matter.

1 OVERVIEW

The natural existence of a *heavy* electron, the muon, bridges the enormous energy gap between the atomic and nuclear domains and facilitates spontaneous nuclear fusion reactions of hydrogen isotopes[1–5]. This process is termed muon catalyzed fusion, or MuCF. At the origin of the diverse effects is the muonic hydrogen atom, a small neutral object, much like a neutron, capable of entering into chains of complex resonant reactions. Because of the interconnection of atomic, molecular and nuclear phenomena, the chain of processes into which a single muon engages in a target consisting of a mixture of hydrogen isotopes, is very complex[6–8]. The rate at which a muon accomplishes each fusion is referred to as the *cycling rate* λ_c. The achievable yield of fusions per muon, Y, is obtained from the ratio of λ_c to the muon loss rate of λ_l. The latter contains, aside from the rate of natural muon decay $\lambda_0 = 4.552 \cdot 10^5\,\mathrm{s}^{-1}$, also the rate at which the muon catalyst is poisoned. This is the product of the cycling rate λ_c and the probability of muon poisoning per cycle $W_s = \omega_s^{dt} + \delta W_s$, of which the main cycle sticking to the α-particle fusion product[5], ω_s^{dt}, is the dominant contribution:

$$Y = \frac{\lambda_c}{\lambda_l} = \frac{1}{\lambda_o/\lambda_c + \omega_s^{dt} + \delta W_s} < \frac{1}{W_s} < \frac{1}{\omega_s^{dt}}. \tag{1}$$

Typical *best* values today are[6] $Y \approx 150$, $\lambda_c \simeq 10^8\,\mathrm{s}^{-1}$. Contrary to popular belief, it is not the speed of the nuclear reaction occurring in hydrogen muo-molecular ions in conventional MuCF which imposes the limit on the number of fusions possible per muon. *Muon sticking and the muon cycling rate are the key limiting factors in conventional MuCF.* Muon sticking depends on the initial stage of the fusing nuclei, as well as on the (temperature) environment in which the fusion occurs: if sticking occurs the produced muonic ion $\alpha\mu^+$ has 3.5 MeV energy and a significant chance of breaking apart again, a point which we will further address below. It is obviously essential to be in an environment in which the cycling rate is much greater than the natural decay rate, which as we shall see implies high densities of hydrogen. We further note that in MuCF exothermic fusion reactions can occur between all combinations of hydrogen isotopes. If, however, thousands of fusions are desired per muon, only the cases of dd, and in particular the 100-times-faster dt fusion reaction are of interest. A further advantage of dt over the dd fusion is that half of all dd reactions produce ^{3}He, which due to the smaller Q value has only 0.82 MeV kinetic energy. This makes the sticking probability in dd fusion much greater and as dd fusion will always occur at some rate in a D–T mixture, it ought to be studied as a first step in an IF environment.

Every d–t fusion releases $Q = 17.6$ MeV, and hence the maximal direct energy yield per muon $Y_E = Q \cdot Y$ is presently 2.6 GeV. In the present effort we will consider the possibility that muon catalyzed fusion can lead to a much greater fusion yield per muon in a high-density (inertial confinement) environment[9,10]. In part our renewed interest in IC-MuCF is motivated by the observation that the first step in any MuCF process is the production of muons[11–14], and the recognition that a totally negligible portion of the beam energy is used up in this process. We note that even if $Y = 5{,}000$ were possible (see below), it leads to the power production per muon:

$$P_\mu = Q \cdot Y \cdot \lambda_0,$$

which is 0.007 W_{thermal}. Thus a power plant with the power $P_{\text{thermal}} = 2{,}000$ MW and based in a significant way on MuCF would require at any time $N_\mu = P_{\text{thermal}}/P_\mu = 2.4 \cdot 10^{11}$ muons in the reactor, or an average flux $F_\mu = N_\mu \lambda_0 = 1.1 \times 10^{17}\ \text{s}^{-1}$, that is nearly 20 (particle) mA. The practical path to such high-intensity muon beams employs hadronic interactions at GeV energies, in which muons emerge as decay products of pions. The qualititative result is that per π^- produced we need about 5 GeV beam energy, to be compared to $Y_E = 90\ \text{GeV}_{\text{thermal}}$ (for simplicity we assume that each produced π^- can be turned into usable muons; the precise fraction depends on the method of muon production). Thus a beam capable of producing a muon flux of desired intensity must carry about 6% of the thermal power of the reactor (assuming $Y = 5{,}000$). This would imply, in the example discussed above, a 120-MW_{beam} beam of 2–10 GeV/A ions.

Muon-catalyzed fusion in a high-density environment was first suggested by Tan[9] and it was subsequently[10] severely criticized. These objections and our current position are as follows:

1. Energy cost for muon production—there has been much progress in the past 15 years, and we consider the parameters to be well established[11–14] and to not exclude the application of MuCF to inertial fusion.

2. Stopping distance of muons in a $T > 10$ keV electron plasma being too long (due to reduced stopping power at sufficiently high temperature). Due to the much-enhanced direct fusion rate presented in the next section, the required temperature is 1,000 times lower than considered originally[9,10]. Temperature must be chosen wisely such that the stopping distance for muons is sufficiently short, while the regeneration of muons is enhanced[15–18] (see below).

3 Rate of formation of muonic hydrogen atoms being too slow and fusion reactions being too slow up to very high temperatures. A detailed study[15,16] suggests that muonic hydrogen formation is not a bottleneck. We have recently explored[20–23] nuclear fusion reactions catalyzed by muons which occur without the need for the resonant formation of the intermediate muo-molecular state. These reactions are furthermore also most likely accompanied by a small intrinsic sticking and hence would be ideally suited for MuCF in high-density tepid plasmas; see Section 2.

4 ICF-confinement time is thought to be much shorter than the required 5 μs—this is a question related to the required densities and temperature. In the next section we will show that temperatures even of the order of 1–100 eV are sufficient, as we found new direct fusion mechanisms. Hence the emphasis shifts to the identification of the conditions of density and temperature, target size and its geometry, for which confinement time of 5 μs can be achieved. This question will require much further consideration.

The main steps of the MuCF-dt cycle occurring in a D-T mixture are summarized below, adapted to the situation likely to occur in an inertially confined high density plasma target—in order to simplify the situation, we will consider a completely ionized target. Rates given are in most cases normalized to the (atomic) density of liquid nitrogen (LHD) $\rho_0 = 4.25\ 10^{22}\ \mathrm{cm}^{-3}$, which is the convenient density scale. In converting to the plasma process one must adjust diverse Auger processes to reflect the greatly reduced electron density at the site of the muonic system: normally the electron density inside an electron atom is:

$$|\Psi(0)|^2 = \frac{1}{64\pi a_0^3} = 50\rho_0.$$

We find that, at densities well above 50 LHD and $T > 30$ eV, the Auger processes proceed at a rate greater than computed for conventional MuCF atomic processes.

1. Muons are stopped within 10^{-10} (ρ_0/ρ) s in a hydrogen target.

2. Muons are captured into atomic orbits by Auger processes, which usually takes less time than the stopping, with an estimated value 10^{-11} $(70\ \rho_0/\rho)$ s or shorter due to more complex processes[16]. Since muons are bound with an energy 207 times greater than electrons, they can form atomic structures at temperatures up to several keV.

3. Muon cascades down by (external) Auger processes induced by two-body collisions to the muo-atomic L-shell within 10^{-11} (ρ_0/ρ) s. The final transition to the ground state takes less than the radiative rate, which is just about 10^{-11} s, as it is likely that this transition will also be dominated by collision processes, much like the transfer processes. We note that should the muon be captured initially by a deuteron, transfer processes to the heavier isotope compete with the cascade processes in $d\mu$. These transfer processes will be greatly enhanced as compared to conventional MuCF due to three body collisions involving the muo-atom and two hydrogen ions. The relative population is given by the Boltzmann factor $e^{-\Delta E/T}$, where $\Delta E = 48$ eV is the energy between the 1s-states in muo-deuterium and muo-tritium.

4. The de-excited muo-atom collides with another hydrogen ion and undergoes direct fusion reaction at a rate exceeding $10^9\ \mathrm{sec}^{-1}$ at $\rho = 1{,}000\rho_0$. We will compute this crucial rate in the next Section. In this respect the IC-MuCF system differs completely from conventional MuCF which proceeds via a complex chain of molecular resonance processes.

5. If the muon has been captured by (becomes stuck to) the helium produced in fusion, it can be regenerated in collisions.

With reference to the last point we note that the sticking probability in the Born-Oppenheimer approximation[5] is $\omega_s^0 \sim 1.2\%$. More sophisticated three-body non-adiabatic wave fusions lead to a somewhat smaller value of $\omega_s^0 = 0.89\%$; if the fusion reaction occurs from the (11) state, sticking would be much reduced[24]. Our belief is that sticking in direct fusion reactions would be similarly reduced. We have discussed this point and its possible experimental manifestation previously[22,23].

Once the muon sticks to the α particle, it is not entirely lost from the cycle of reactions: at the initial velocity of about $v_{\alpha\mu} = 5.82\alpha c$ for dt fusion it carries about 86 keV kinetic energy, which is significantly greater than the energy needed, 11 keV, to strip it from the α particle. Even more importantly, it takes many atomic collisions before the $\alpha\mu^+$-ion loses its energy, ca. 3.5 MeV. In order to relate the initial sticking ω_s^0 to the final sticking ω_s after regeneration, it is necessary to consider muon stripping processes in competition with the rate of energy loss of the $(\alpha\mu)^+$-ion in the hydrogen medium:

$$\frac{dE}{dt} = -\rho v S(v), \tag{2}$$

$$\frac{d\omega_s}{dt} \simeq -\sigma_{\mathrm{str}}(v)\rho v \omega_s. \tag{3}$$

The stripping cross section $\sigma_{\mathrm{str}}(v)$ is the sum of ionization and transfer cross sections. The time required to bring the $(\alpha\mu)^+$-ion to rest in liquid hydrogen is of the order of $t_{\mathrm{stop}} \approx 4 \times 10^{-11}$ at LHD, so muon stripping, if it occurs, does not have a significant impact on the cycling rate of the muon. We find:

$$\omega_s(E_f) = \omega_s^0 \exp\left(-\int_{E_f}^{E_0} \frac{\sigma_{\mathrm{str}}(E)}{S(E)}\, dE\right), \tag{4}$$

where E_0 is the initial and $E_f \simeq 0$ the final energy of the $(\alpha\mu)^+$-ion. Sticking is reduced by about 30% in liquid hydrogen. However, we can see that this effect is exponentially increasing with decreasing stopping power and at sufficiently high electron degeneracy, i.e., at small S, we find ω_s reduced by as much as a factor of 3×10^{-5} [22].

2 DIRECT REACTIONS

We have studied several direct reaction mechanisms that may occur in IC-MuCF environments[20]. The most immediately obvious is in-flight fusion, in which the

Coulomb barrier between the d and t is substantially screened, permitting fusion at low temperatures[9]. At energies below a few keV, tunneling through the barrier is essentially energy-independent and the fusion cross-section consequently changes like $1/v$. The resulting fusion rate at LHD, which we have computed using an R-matrix parameterization of the dt nuclear interaction and which is in substantial agreement with results obtained using optical potentials, is then approximately[21]:

$$\lambda_{if} = 1 \times 10^5 (\rho/\rho_0)\,\mathrm{s}^{-1} \qquad (0 < T < 100\,\mathrm{eV}).$$

This rate scales with density, so at 10^3 LHD we can therefore expect something less than 100 fusions per muon.

A second direct reaction channel which we have studied recently relies upon the *below*-threshold amplitudes of the $dt\mu$-$\alpha n\mu$ continuum[20,22,23]. We observe that in addition to the above d + t threshold continuum, which is usually most strongly coupled to the α + n continuum far above the Coulomb barrier where tunneling is easy, there also exist a d + t continuum *below* the d + t threshold. In the S-wave channel the dt wavefunction of energy E relative to the d + t threshold has the form[20]:

$$\psi_{dt}(R) = \frac{\eta(E)}{R} e^{-\kappa R} Y_{00}(\hat{R}) \qquad (R \to \infty), \tag{5}$$

where $\kappa = \sqrt{-2\mu_{d,t}E}$ is the relative 'imaginary' momentum of the d and t at energy $E < 0$, and R is the d–t separation. η is a numerical factor that contains the tunneling amplitude and the nuclear interaction strength. Usually, as $E \to 0^-$, which is when ψ_{dt} can be expected to have any appreciable size, η is vanishingly small due to suppression of the coupling by the Coulomb barrier. In the presence of the muon however, the coupling between channels is greatly enhanced due to the lowering of the barrier. When the d$t\mu$-αnμ continuum is plane-wave normalized in the αn channel, we then find $\eta = 0.196$. Thus, in the d$t\mu$ system, there exists a large and long-ranged leakage of the 'fused' αn continuum into the dt channel. dt states can fuse by making transitions to this below-threshold continuum. Typically what we have in mind is the transition of a d + tμ continuum wave to the below-threshold continuum, following some interaction that permits the transition. The contribution transitions are those with initial and final energies very close to the d + tμ threshold, with sizes of order Å, in which case many particles are within the reaction region and many-body reactions become important.

The process we have found to be most favorable at high temperatures is the scattering of tμ off d^+, placing the tμ off mass shell with respect to a second deuteron and enabling the transition to the below-threshold state. Since this is a *three-body* reaction, the fusion rate scales as ρ^2. The details of this calculation are given in Refs. 22, 23. This reaction exhibits a mild but important temperature dependence, favoring low temperatures. The temperature here refers specifically to the temperature of the d^+ ions and the tμ, and in fact most of the fusions come from the *low energy* part of the thermal distribution. Any deviation from a Maxwellian distribution would have an important impact on the fusion rate. This phenomenon of an increasing fusion rate with decreasing temperature is due to the increasing integrated strength of the below-threshold wavefunction wave in the dt channel as the energy approaches

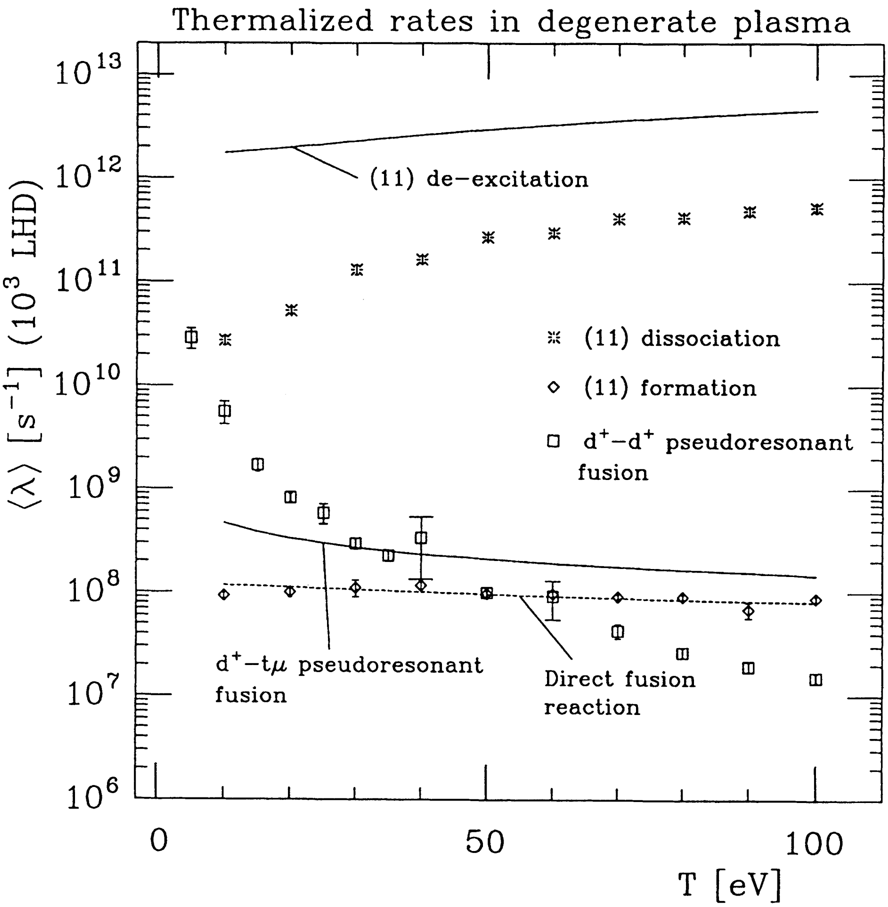

FIGURE 1 IC-MuCF rates at density $\rho = 10^3$ LHD. The direct reaction rate scales with ρ, and the pseudoresonant rates with ρ^2. The Auger processes scale in a more complicated way due to the degeneracy of the electron plasma.

the d + tμ threshold, in which case it is the low-energy part of the thermal spectrum that makes the dominant contribution to the rate.

We find that another three-body direct fusion reaction could also be important; this involves the initial scattering of two d^+ ions before one ion fuses with the tμ. In this case, the scattering matrix element is strongly energy-dependent, although the dominant contribution comes from the region of the thermal spectrum in which the momentum of the scattering d^+ ions is comparable to the reciprocal of the screening length. In consequence, the fusion rate is more temperature-dependent than for the d^+-tμ pre-scattering case. The fusion rate for a screening length of 0.2 Å is given in Figure 1, along with the two other direct reaction channels discussed above.

Other direct reactions to be still considered include in particular the formation of deeply bound muo-molecules in three-body collisions, as well as the rates of their dissociation, to be compared to fusion rates which for J = 0 angular momentum states are 10^{11} s^{-1}. These have been computed in Ref. 22, and are summarized in Figure 1. The most important molecular processes are the Auger molecular formation, dissociation, and de-excitation of the dtμ $(J, v) = (1, 1)$ molecular state. It should be noted that the de-excitation rate exceeds the dissociation rate, so molecular formation is a viable fusion channel. However, molecular formation is generally slower than the competing pseudoresonant rates.

3 DISCUSSION

We have shown that one cannot dismiss IC-MuCF without a thorough study. It seems that after a period of false starts we have now identified a viable path of high density and (relatively) low temperature targets in which thousands of fusions seem possible. Comparing the fusion rates from Section 2 with the natural decay rate λ_0, we see that more than 1,000 fusions and perhaps as many as 5,000 fusions per muon is possible. Assuming that the direct fusion rate is the limiting factor we find that already, in present theoretical models, more than 1000 fusions are possible. However, we have not yet evaluated all direct fusion mechanisms. On one hand this will increase the cycling rate, on another it may increase muon sticking. The unavoidable occasional dd fusions will also contribute more to sticking due to the reduced helium velocity, as was discussed. A very important effect may be the enhancement of muon regeneration by reduced stopping power. But how much regeneration do we need? That of course depends on the initial sticking in direct fusion reactions, which we presently do not know. Discussion of fusion rates in Section 2 suggests that we may not increase temperature without reducing the cycling rate too much—but on the other hand we saw that increasing electron temperature reduces sticking. Also, T influences the confinement time of the target. Thus, there is an optimum temperature for each target density which maximizes the IC-MuCF fusion yield.

We would like to mention here that in principle we must expect that the electron temperature will be higher than the nuclear temperature. The temperature entering the direct fusion rates in Section 2 was the nuclear temperature, which as we have seen we wish to be small. Thus, compounding all the scientific complexity of IC-MuCF, we see that we win if the ion and electrons do not reach thermal equilibrium! This of course is in part always the case as 1/5 of the dt fusion energy reheats mostly the plasma electrons, and not nuclei (we assume that due to the small size of the target the fusion neutron escapes). We note here that the size of the "pellet" must be chosen such that only a tiny fraction of the fuel is burned up by MuCF—else the helium produced will scavenge muons. If we want 5,000 fusions, we must dimension for this fraction of fuel to be burned. But then the stopped energy yield of fusion is 3.5 MeV/5,000 $\simeq$ keV per hydrogen. This energy may influence the time evolution of the target.

It is fair to say that in IC-MuCF we are facing a set of problems which are at least as involved as were faced in last 30 years in conventional MuCF. So the question arises if it is worth devoting much effort to resolve the issue! It seems to us that this must be done for two reasons:

• Both HI-IF and MuCF require the use of particle beams, and it is likely that the technology could be shared.

• IC-MuCF may, aside from having its independent merits as a fusion system, also form a synergetic system with HI-IF. All depends on many parameters of IC-MuCF hardly yet known.

In conclusion: we have shown that due to enhanced rate of three body direct fusion reactions, muon catalyzed fusion at high density and modest plasma temperatures may be a viable path to nuclear fusion.

ACKNOWLEDGEMENT

This work was in part supported by the U.S. Department of Energy, Division of Advanced Energy Projects

REFERENCES

1. F. C. Frank, *Nature* **160**, 525 (1947).
2. A. D. Sakharov (1948), in Ref. 3.
3. Ya. B. Zel'dovich, *Dokl, Akad, Nauk S. S. R.* **95**, 493 (1954).
4. L. W. Alvarez, H. Bradner, F. S. Crawford Jr., J. A. Crawford, P. Falk-Vairant, M. L. Good, J. D. Gow, A. H. Rosenfeld, F. Solmitz, M. L. Stevenson, H. K. Ticho and R. D. Tripp, *Phys. Rev.* **105**, 1127 (1957).
5. J. D. Jackson, *Phys. Rev.* **106**, 330 (1957).
6. S. E. Jones, *Nature* **321**, 127 (1986).
7. J. Rafelski and S. E. Jones, *Scientific American* **257**, 84 (1987).
8. J. Rafelski and H. E. Rafelski, prepared for: *Advances in Atomic, Molecular and Optical Physics* (Academic Press, New York, 1991), D. R. Bates and B. Bederson, editors.
9. W. P. S. Tan, *Nature* **263**, 656 (1976).
10. E. P. Hincks, M. K. Sundaresan and P. J. S. Watson, *Nature* **269**, 584 (1977).
11. H. Takahashi, H. J. C. Kouts, P. Grand, J. R. Powell and M. Steinberg, *Atomkernenergie-Kerntechnik* **36**, 195 (1980).
12. Yu. V. Petrov, *Nature* **285**, 466 (1980).
13. M. Jändel, M. Danos and J. Rafelski, *Phys. Rev. C* **37**, 403 (1988).
14. G. R. Shin, and J. Rafelski, *Nucl. Instrum. Meth. A* **287**, 565 (1990).
15. L. I. Men'shikov and L. I. Ponomarev, *JETP Lett.* **46**, 312 (1987).
16. L. I. Men'shikov: "Muon catalytic processes in a dense low-temperature plasma; (Institut Atomnoj Energii im. I. B. Kurchatova, March 1988), preprint IAE-4589/2, unpublished.
17. H. E. Rafelski, B. Müller, J. Rafelski, D,. Trautmann and R. D. Viollier, *Advances in Particle and Nuclear Physics* **22**, 279 (1989).
18 M. Jändel, P. Froelich, G. Larson and C. D. Stodden, *Phys. Rev. A* **40**, 2799 (1989).
19. E. A. Vesman, *JETP Lett.* **5**, 91 (1967).
20. D. Harley, B. Müller and J. Rafelski, *Z. Phys.* **336A**, 303 (1990).
21. D. Harley, B. Müller and J. Rafelski: in *Proceedings of An International Symposium on Muon Catalyzed Fusion* (*μCF-89*), J. D. Davies, editor (Rutherford Appleton Laboratory, Oxon, 1990), pp. 56–58.
22. D. Harley, "Nuclear and Molecular Processes in Muon Catalyzed Fusion," University of Arizona PhD thesis (1991).
23. D. Harley and J. Rafelski: Non-resonant fusion reactions in the $t\mu + d$ system, μCF '90 *International Conference on Muon Catalyzed Fusion*, (Vienna, Austria, 1990).
24. C.-Y. Hu, *Phys. Rev. A* **36**, 4135 (1987).