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NONLINEAR THEORY OF CHARGE COMPENSATION OF HIGH-CURRENT ION BEAMS IN MAGNETOISOLATED ACCELERATING SYSTEMS

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A nonlinear analytical theory of space-charge compensation of a high-current ion beam in a magnetoinsulated accelerating gap is presented. The problem is described by the set of equations for the charge separation potential and for the self-consistent azimuthal magnetic field. This set is obtained from nonlinear electron motion equations in a drift approximation, the Maxwell equations, the continuity equation, and the energy, and particle-flux conservation laws. The beam thickness is assumed to be much smaller than the beam radius. Also formulated are system-parameter requirements.

1 INTRODUCTION

One of the promising ways to produce high-current heavy ion beams for ICF pulses is the induction accelerating machine. An increase in current at low initial ion energies is achieved through space-charge neutralization of the ion beam by electrons. In this case, the electron current in the accelerating gap is suppressed with the help of magnetic insulation. In the axially symmetric system, the well-known mechanism for transporting the ion beam (which has only limited compensation) through the magnetic barrier[1] cannot take effect because in this system there is no azimuthal-polarization electric field. Therefore, it was earlier suggested[2] that the ion beam in the accelerating gap should be charge-compensated by a specially injected electron beam. The physical meaning of this mechanism is that the injected electron beam drifts through the gap because of both the magnetic self-field of the ion beam and the radial electric field resulting from the radial separation of the beams.

Consider a thin-walled, tubular ion beam moving through a magnetically insulated gap together with a moving electron beam of the same current density as the ion beam. In the approximation $\Delta \ll a$, where a is the inside radius and Δ is the wall thickness, one can use a Cartesian system to describe the ion dynamics. Let the x axis be directed along the radius, and the z axis along the direction of beam propagation. Then the beam density n_i may be written as

$$n_i(x) = n_{i0}\begin{cases}1, & a \leq x \leq a + \Delta \\ 0, & x < a, x > a + \Delta\end{cases}. \tag{1}$$

In this approximation we assume the beam to be unlimited in the OY-axis. The external magnetic field has OX and OZ components; that is, $H_0(H_{OX}, 0, H_{OZ})$. Note that the H_0 value is such that the Larmor radius of the ions r_{Hi} is considerably greater than the accelerating gap length L ($r_{Hi} \gg L$). Accordingly, for electrons, $r_{Hi} \ll L$; that is, the conditions for magnetic insulation of the electrons are fulfilled. The external accelerating electric field $\mathbf{E}_0$ is directed along the OZ axis: $\mathbf{E}_0 = E_0\mathbf{e}_z$. The ion beam is injected into the system along the accelerating field at an initial velocity $\mathbf{v}_{oi} = v_{oi}\mathbf{e}_z$. We assume the gain in the ion velocity, Δv_i, in the process of acceleration to be small as compared with the injection velocity v_{oi}; that is, $eE_0L/mv_{oi}^2 \ll 1$. Then the increase in the ion velocity along the OZ axis may be neglected. We also assume that there are no limits along the OY axis, i.e., we consider that all the quantities are functions only of the X and Z coordinates. The magnetic self-field excited by this ion beam has the form

$$H_y = \begin{cases} \dfrac{4\pi}{c} en_{oi}\Delta, & x > a + \Delta \\ \dfrac{4\pi}{c} en_0 V_{Oi}(x - a), & a \leq x \leq a + \Delta \\ 0, & x < a \end{cases} \tag{2}$$

We assume that the characteristic time of the problem is small compared to ω_i^{-1} and Ω_i^{-1} (ω_α, and Ω_α correspond to the plasma and cyclotron frequencies, respectively, and $\alpha = e,i$). And we ignore the ion velocity increase in the acceleration gap. Thus this problem can be described by a set of nonlinear equations: hydrodynamic drift equations for electrons, a continuity equation for electrons, and equations for the radial decompensation electric field and for the azimuthal magnetic field.

2 ELECTRON BEAM DYNAMICS

Let us consider the dynamics of the compensating electrons. The electrons moving in the cusp field $\mathbf{H}_0$, the external electric field $E_0\mathbf{e}_z$, and the field of charge division $\nabla\boldsymbol{\varphi}$ (φ is the potential of the charge division field) are involved in the drift motion. We restrict our consideration to the electric drift of the electrons:

$$V_{dx} = -\frac{c}{H_0^2}\left(E_0 - \frac{\partial\varphi}{\partial z}\right)H_y, \qquad V_{dz} = -\frac{c}{H_0^2}\frac{\partial\varphi}{\partial x}H_y,$$

$$V_{dy} = -\frac{c}{H_0^2}\left\{-\frac{\partial\varphi}{\partial x}H_{0z} - \left(E_0 - \frac{\partial\varphi}{\partial z}\right)H_{0x}\right\}. \tag{3}$$

In the drift approximation, the set of equations can be written as

$$(V_x + V_{dx})\frac{\partial}{\partial x}(V_x + V_{dx}) + (V_z + V_{dz})\frac{\partial}{\partial z}(V_x + V_{dx})$$

$$= \frac{e}{mH_0^2}\left\{H_x\frac{\partial\varphi}{\partial x} + H_z\left(\frac{\partial\varphi}{\partial z} - E_0\right)\right\} + \frac{e}{mc}V_zH_y, \tag{4}$$

$$(V_x + V_{dx})\frac{\partial}{\partial x}(V_z + V_{dz}) + (V_z + V_{dz})\frac{\partial}{\partial z}(V_z + V_{dz})$$

$$= \frac{e}{mH_0^2}\left\{H_x\frac{\partial\varphi}{\partial x} + H_z\left(\frac{\partial\varphi}{\partial z} = E_0\right)\right\} - \frac{e}{mc}V_xH_y, \tag{5}$$

$$(V_x + V_{dx})\frac{\partial}{\partial x}(V_y + V_{dy}) + (V_z + V_{dz})\frac{\partial}{\partial z}(V_y + V_{dy}) = 0, \tag{6}$$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}\right)\varphi = -4\pi e(n_i - n_e), \tag{7}$$

$$\frac{\partial}{\partial x}\{n_e(V_x + V_{dx})\} + \frac{\partial}{\partial z}\{n_e(V_z + V_{dz})\} = 0, \tag{8}$$

and

$$\frac{\partial H_y}{\partial x} = \frac{4\pi e}{c}\{n_i V_i - n_e(V_z + V_{dz})\} \tag{9}$$

In the general case, the investigation of the set Eqs. (3)–(9) is rather complicated. We take $(\partial/\partial z) \sim (1/L) \ll (1/\Delta) \sim (\partial/\partial x)$ and keep only the terms comprising the derivatives with respect to Equation (8). It follows that

$$n_e(V_x + V_{dx}) = n_{oe}V_{ox}. \tag{10}$$

If $H_z \ll H_x$, then $v_z \ll v_{dz}$, and from Eq. (4) we have

$$\frac{(V_x + V_{dx})^2}{2} = \frac{\varepsilon_{ox}}{m} + \frac{e}{m}\{\varphi(x) - \varphi(a)\}, \tag{11}$$

Using Eqs. (7), (10) and (11) we obtain the equation for φ,

$$\frac{d^2\varphi}{dx^2} - \frac{4\pi e n_{oe} v_{ox}}{\sqrt{2\frac{\varepsilon_{ox}}{m} + 2\frac{e}{m}[\varphi(x) - \varphi(a)]}} = -4\pi e n_i(x), \tag{12}$$

where n_{oe} is the non-perturbed electron density, $\varepsilon_{ox}(z) = mv_{ox}^2/2$, v_{ox} is the initial component of electron velocity (perpendicular to the ion-beam motion), and φ(a) is the potential at the inner beam boundary.

The equation describing the azimuthal magnetic field has the form

$$\frac{dH_y}{dx} - \frac{4\pi e n_{oe} v_{ox} H_y \dfrac{d\varphi}{dx}}{H_0^2 \sqrt{2\dfrac{\varepsilon_{ox}}{m} + 2\dfrac{e}{m}[\varphi(x) - \varphi(a)]}} = 4\pi e \frac{v_i}{c} n_i(x), \tag{13}$$

where v_i is the ion beam velocity.

The set of Equations (12)–(13), taking into account the energy and particle flux conservation laws, has been investigated analytically and numerically. The areas being investigated are divided into two parts:

— at $x > a + \Delta$, where there is no ion beam but the electrons are present; and

— at $a \leq x \leq a + \Delta$ where account must simultaneously be taken of the electrons and ions. It was shown previously[3] that the electron drift velocity approaches the v_i value provided that the system parameters satisfy the following conditions:

$$\varepsilon_0 > eE_0 L,\ 4\pi n_{oi}\varepsilon_0/H_0^2(a) > 1,\ 1 \gg \Delta/a \geq c\Omega_e \omega_e^{-2} a^{-1}, \tag{14}$$

where $H_0(a)$ is the external magnetic field amplitude at $x = a$, and L is the accelerating gap length. The set of equations was analyzed for really existing case[2]. Where $a = 1$ cm, $\Delta = 0.03$ cm, $\varphi(a) = 0$, $H_y(a) = 0$. The values of ε_{ox}, n_{oi} were chosen so that the condition of Eq. (14) was satisfied and, therefore, the drift electron velocity in the direction of ion beam propagation (OZ axis) would be maximally close to the ion velocity v_i.

Consider the behavior of the quantities being studied: the charge separation potential $\varphi(x)$, the electron density $n_e(x)$, the intrinsic magnetic field $H_y(x)$, and the electron drift velocity $v_{dz}(x)$, in summarized fields. This analysis clarifies the behavior of these functions by changing the boundary value $\tilde{E}(a) = (eE_z(a)a/\varepsilon_{ox})$ for different ratios of initial densities of electrons and ions with $n_{oi} = 5.2 \cdot 10^{13}\ \text{cm}^{-3}$ and $\varepsilon_{ox} = 7.2 \cdot 10^5$ eV. At rather large boundary fields ($\tilde{E}(a) = 100$) the separation charge potential rapidly drops from 0 to $-(\varepsilon_{ox}/e)$ over the range well below Δ. So, the compensating electrons are trapped in a narrow region in the vicinity of $x = a$, and their density sharply—explosively—approaches infinity (see Figure 1, curve 1). The azimuthal magnetic field of the beam is, in this case, small, and in the main is also concentrated not far from $x = a$. Figure 2 (curve 1) presents the distribution of electron drift velocity.

As $\tilde{E}(a)$ decreases, the picture of field distribution flattens. Already at $\tilde{E}(a) \approx 34$ the charge separation potential has diminished to $-(\varepsilon_{ox}/e)$ in the range $\simeq \Delta$. Figure 2 (curve 2) presents the electron drift velocity. The electron drift region extends over the range Δ, and the maximum drift velocity attains the value v_i.

When $\tilde{E}(a)$ ($\tilde{E} = 20$) decreases, the charge separation potential diminishes to its minimum over the range $\simeq 2\Delta$. The electron density (see Fig. 1, curve 3) in the region $x = 1.06a$ increases up to $5.88 \cdot 10^{14}\ \text{cm}^{-3}$. The electric field from energy separation in the layer Δ remains nearly constant and then rapidly drops over the range $\simeq 2\Delta$. The magnetic field is concentrated in the beam layer and then rapidly drops outside

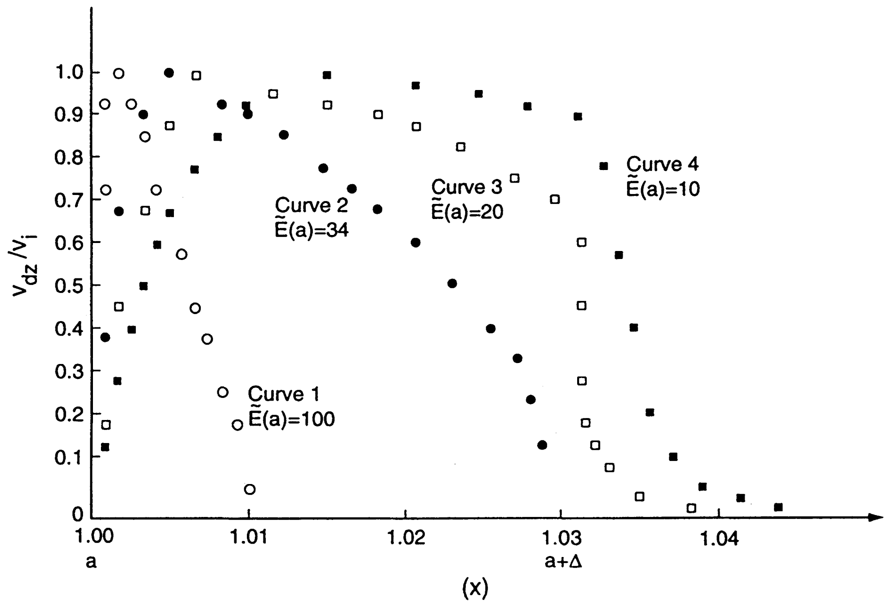

FIGURE 1 Distribution of the electron density versus the transverse coordinate x at different values of $\tilde{E}(a)$. Curve 1 (hollow circles): $\tilde{E}(a) = 100$. Curve 2 (filled circles): $\tilde{E}(a) = 34$. Curve 3 (hollow squares): $\tilde{E}(a) = 20$. Curve 4 (filled squares): $\tilde{E}(a) = 10$.

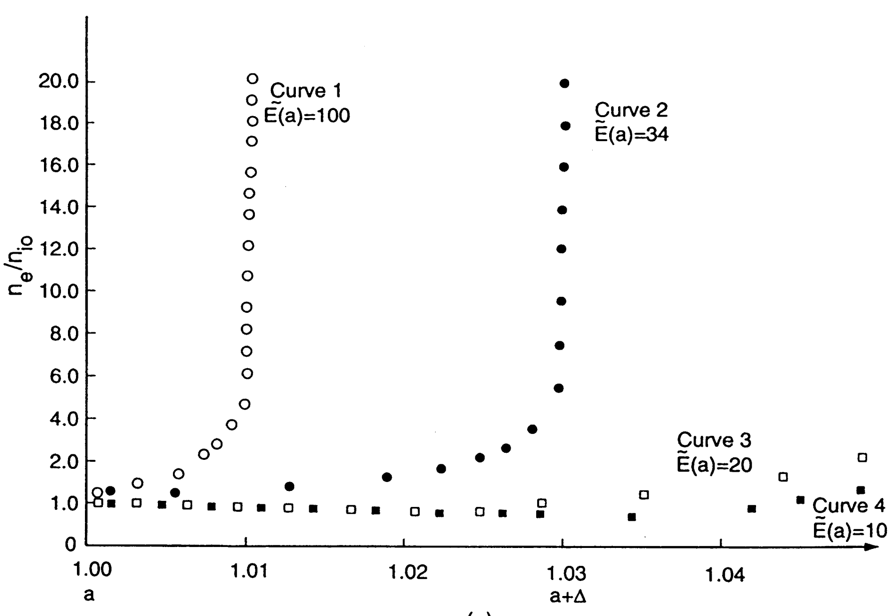

FIGURE 2 Distribution of the electron drift velocity versus the transverse coordinate x at different values of $\tilde{E}(a)$: Curve 1 (hollow circles): $\tilde{E}(a) = 100$. Curve 2 (filled circles): $\tilde{E}(a) = 34$. Curve 3 (hollow squares): $\tilde{E}(a) = 20$. Curve 4 (filled squares): $\tilde{E}(a) = 10$.

of it. The electron drift velocity V_{dz} (Figure 2, curve 3) has a plateau in the region of the beam layer with average value $\simeq V_i$ and rapidly diminishes just behind the layer. It is a classical case of the virtual cathode; the electron cloud being formed is confined near the beam surface by electron escape from the beam region.

The case of $\tilde{E}(a) = 10$ is similar to the foregoing case (Figure 1,2; curve 4), but in this case the electron cloud has no sharply defined localization region.

The analysis allows one to conclude that charge compensation may be achieved by using an electron beam accompanying the ion beam in the accelerating system (with magnetic isolation for the electron beam). However, the ion beam must have sufficiently high current, and the charge separation field must increase to high values. Note that the total numerical integration has been performed.[4,5]

3 CONCLUSIONS

For a high-current ion beam in an acceleration gap to be neutralized by the co-moving electron beam, the electron beam energy must satisfy certain conditions.

The transported ion beam must have a sufficiently high current. Since the injected electron beam drifts through the magnetically insulated acceleration gap, the electron beam current density need not exceed that of the ion beam to provide the charge compensation.

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