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The relationship of fracture toughness coefficients and geophysical characteristics of rocks of hydrocarbon deposits

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This paper contains the results of laboratory tests to determine the fracture toughness coefficient K_{IC} of rocks for terrigenous and carbonate objects by three methods. The tests were carried out by different methods due to the lack of a standard method for determining the fracture toughness characteristics of rocks in Russia. We used the following methods for determining the K_{IC} coefficient: the extension of core specimens with an annular fracture, the action of a concentrated load on a beam specimen with a fracture and the method of bending semi-circular samples with a fracture according to ISRM recommendations.

The paper presents the relationship of the fracture toughness coefficients with the P-wave velocity and porosity. The obtained dependencies characterize the general trend of changing for the studied parameter and can be used in the design of hydraulic fracturing in the fields for which tests were conducted.

***Key words*:** fracture toughness coefficient; extension; bending; terrigenous and carbonate objects; rocks

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Introduction. All theoretical models and software for hydraulic fracturing modeling, which is the main geotechnical measure aimed to increase well productivity, use the concepts of fracture mechanics and, therefore, the values of fracture toughness coefficients K_{IC} [2, 3, 5].

The determination of this rock parameter of oil and gas fields productive objects was practically not carried out, although the K_{IC} parameter is included in all service packages for hydraulic fracturing design. Most often, the K_{IC} coefficient is assumed to be 1 MPa·m$^{1/2}$ and it does not have any connection with the filtration-capacitive and geophysical rocks characteristics. Due to the importance of this parameter for the theoretical justification of the hydraulic fracturing design, of laboratory tests are presented for K_{IC} values determination by three methods for terrigenous and carbonate objects. Also there was considered the relationship of this parameter with geophysical characteristics, primarily the P-wave velocity.

Currently, there are no methods for determining the rock fracture toughness coefficients in the RF standards. At the same time, there are methods for determining K_{IC} recommended for use by the International Society for Rock Mechanics (ISRM). In total, four different methods for determining the static fracture toughness coefficient were proposed by ISRM at different times [8, 10, 11]. The first two were proposed by a group of scientists in 1988 [11]. The first method involves bending a cylindrical core with a V-shaped cut perpendicular to the axis of the cylinder. In the second method [11], it is recommended that special clamps disclose a short cylinder with a V-shaped cut along the core axis. The third method, proposed in 1995 [8], involves the extension of a cylindrical specimens with a V-shaped cut. The fourth method, proposed in 2013 [10], involves bending semi-circular specimens with a cut. The advantage of this method over others recommended by ISRM is that it uses standard equipment. Also specimens preparing for testing and the results processing are relatively simple.

During the tests conducting for the fracture toughness coefficients of composites, construction and other materials, it is a common practice to use standards developed for metals and alloys, in particular GOST 25.506-85. As a rule, they are based on the determination of the K_{IC} coefficient when testing specimens in tensile or bending mode with the initial fatigue fracture. The growth

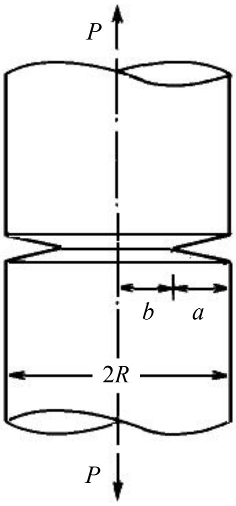

Fig.1. Scheme of loading a specimen with an annular fracture to determine K_{IC} according to [4]
P – extension load; R – radius of the cylinder; a – cut depth, $a = R – b$; b – minimal radius at the place of cut

of the initial fracture on rock specimens seems problematic, since it is a brittle material, and an uncontrolled dynamic fracture growth can occur, up to the destruction of the sample.

According to [1], rock tests can be carried out in two ways: direct extension and bending. When extending, it is desirable to use specimens in the form of thin plates with a cut. Before testing with dynamic loads, a fatigue fracture is created at the tip of the cut. Then the specimen is statically extended and the failure is determined, by which K_{IC} is calculated.

As follows from the data given in [1], K_{IC} coefficients for rocks, which can lie in oil deposit formation (sandstones and siltstones) vary within 0.5-1.7 MPa·m$^{1/2}$.

In this paper, the K_{IC} coefficients were determined by three methods: by extending core specimens with an annular fracture [4], by the action of a concentrated force on a beam specimen with a fracture using a technique similar to that described in [1], and by bending semi-circular specimens with a fracture, according to ISRM recommendations [10]. The main goal of the tests is not only the determination of specific values of K_{IC} coefficients by different methods for hydrocarbon deposits, but also the establishment of a relationship between this parameter and the geophysical characteristics of rocks. There is already such research data in the international literature [6, 7], however, such studies are not presented in the RF literature. There are many known equations that relate the elastic modulus, strength, and other characteristics to geophysical parameters. However, in this article for the first time ever the statistical dependences of fracture toughness coefficients on rock geophysical characteristics have been obtained.

Determination of fracture toughness coefficients on cylindrical core specimens. To determine the K_{IC} value, there were studied the specimens with a diameter of 30 mm and a length of 60 mm, which were made of the core material of well N 118 (Enapaevskoye deposit of LTD LUKOIL-PERM) from the interval of terrigenous rocks 1538-1541 m. Before testing, for individual specimens, the static and dynamic elastic properties of the rocks were determined in formation conditions.

Cylindrical samples with an annular groove were used in the experiments to determine K_{IC} (Fig.1).

Extension tests of rock specimens were carried out on a biaxial electrodynamic test system Instron ElectroPuls E10000. A special tooling was developed and designed to fix the cylindrical specimen in the clamps of the testing machine for extension testing.

The extension test of the specimen was carried out at room temperature. During the test the constant displacement speed of the test machine clamps was set at 0.2 mm/min.

The stress intensity factor K_{IC} was calculated using the formulas given in [4] for a cylindrical specimen with a surface annular fracture while extension (designations according to Fig.1)

$$K_I = P\sqrt{\pi b} f_I(\alpha)/\pi b^2; \quad (1)$$

$$\alpha = b/R;$$

$$f_I = 0,5(1 + 0.5\alpha + 0.375\alpha^2 - 0.363\alpha^3 + 0.731\alpha^4)\sqrt{1-\alpha}.$$

In total, 25 samples were tested according to the described scheme. The average K_{IC} value was 0.108 MPa·m$^{1/2}$, with minimum and maximum values of 0.044 MPa·m$^{1/2}$ and 0.168 MPa·m$^{1/2}$,

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respectively. The tensile strength of the tested specimens varies from 0.55 MPa to 2.34 MPa, which corresponds to the characteristic values for this type of rock.

Determination of fracture toughness coefficients on beam specimens. The determination of K_{IC} coefficients on beam samples was carried out according to the scheme presented in Fig.2. Used core material was taken from terrigenous deposits in the Western Ural (Table 1), represented by mudstones, siltstones and sandstones from carbonate objects of gas condensate deposits of the Uzbekistan Republic (Table 2), and by gray organogenic massive porous strong limestones. Table 2 shows the average deposit values of the parameters.

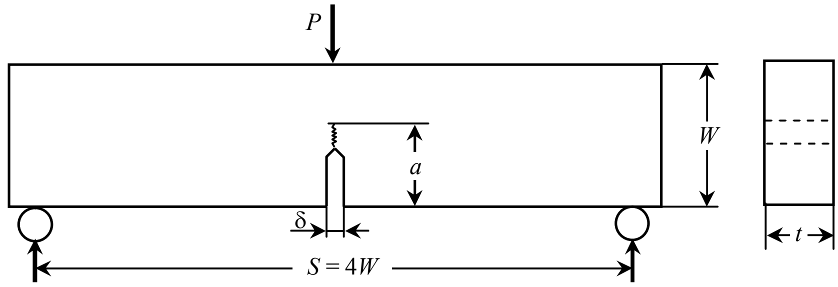

Fig.2. Design scheme of the specimen beam loading to determine K_{IC} according to [1]

P – applied load; δ – width of the cut; S –distance between supports; W – specimen height; t – specimen width; a – cut depth

Table 1

Characteristics of core material for the production of beam specimens and coefficients K_{IC} of the Western Ural oil deposits

Deposit	Number of the well	Specimen quantity	Depth of the sampling, m	K_{IC}, MPa·m$^{1/2}$
Olkhovkoye	164	3	2014.57-2014.72	0.795
Unvinskoye	365, 215	8	2221.76-2221.91	0.413
Kokuiskoye	582	2	1727.43-1727.54	0.462
Sofinskoye	338	3	2553.25-2553.40	0.380
Viktorinskoye	94	4	1649.31-1649.50	0.969
Weighted average				0.604

Table 2

Characteristics of core material for the production of beam specimens and coefficients K_{IC} of the gas condensate deposits of the Uzbekistan Republic

Deposit	Average depth of the sampling, m	Specimen quantity	Open porosity, %	P-wave velocity Vp, m/s	Density ρ, g/cm^3	K_{IC}, MPa·m$^{1/2}$
Akkum	2114.17	5	7.74	5117	2.46	1.460
Alat	2238.52	3	13.27	4490	2.35	0.984
Kandym	2167.12	10	6.87	5227	2.33	0.846
Parsankul	2173.20	5	2.41	5833	2.41	1.038
Khodzhi	2208.18	5	2.54	5814	2.55	1.356
Weighted average						1.137

From each core monolith there were made 2 to 8 specimens for the deposits of Western Ural and 3 to 10 specimens for the deposits of the Uzbekistan Republic. The dimensions of the beam specimens were: W = 2 cm; t = 1 cm; a = (0.3-0.4)W; δ = 0.05W.

The tests were carried out on an Instron 5965 testing machine on specimens in the form of a beam of rectangular cross section with an initial undercut. The initial undercut was made in two stages. At the first stage, cut was applied with a diamond disk with a nominal thickness of 0.6 mm to a depth of 6 mm using a table with a guide. At the second stage, the top of the undercut was formed using two blades of different thicknesses, previously unidirectional teeth were applied on the blades with a pitch of 2-4 mm. The resulting length of the primary fracture was measured using a microscope.

The values of the intensity coefficients were calculated by the formulas (designations in accordance with Fig.2) [4]:

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$$K_{\mathrm{I}} = \frac{3SP}{2tW^2}\sqrt{\pi a}F_{\mathrm{I}}(\alpha); \quad (2)$$

$$\alpha = a/W;$$

$$F_{\mathrm{I}}(\alpha) = \frac{1.99 - \alpha(1-\alpha)(2.15 - 3.93\alpha + 2.7\alpha^2)}{(1+2\alpha)(1-\alpha)^{3/2}}.$$

The K_{IC} values obtained by this method for terrigenous objects of the Western Ural are significantly different from the values obtained by extension of round-shaped specimens with a circular undercut. It is characteristic that even the minimum K_{IC} value determined by the second method significantly exceeds the maximum K_{IC} value determined by the first method. However, this may be due to both the methodology for determining this parameter and the characteristics of the specimens.

The K_{IC} values of carbonate objects of gas condensate deposit of the Uzbekistan Republic, determined by the second method, turned out to be two times higher than the values of this parameter for terrigenous objects of the Western Ural, which can be explained by the difference in strength properties. This experiment does not allow the determination of uniaxial compression strength. However, experiments on other specimens give average values of this parameter – 40-60 MPa for terrigenous objects of the Western Ural and 80-120 MPa for carbonate objects of the Uzbekistan Republic deposits.

Determination of fracture resistance coefficients on semi-circular specimens according to ISRM recommendations. Specimens preparation for the determination of K_{IC} by this method is described down below. A standard cylindrical core is cut in half along the longitudinal axis. In this case, the deviation of the diametrical plane should not exceed 0.2 mm. The thickness of specimen B should be $0.8R$. A cut of length a is made in the middle of the sample, the width of which should not exceed 1.5 mm. The length must satisfy the condition $0.4 \le a/R \le 0.6$. The surface of the cut, located in the direction along the axis of the specimen, must be flat with an accuracy of 0.01 mm. The cut plane should not deviate from the perpendicular direction to the planes along the edges of the specimen by more than 0.5°. The geometry of the prepared specimen and the loading scheme are shown in Fig.3.

The test was performed on standard equipment designed for three-point bending tests. The loading rate should not exceed 0.2 mm/min to avoid dynamic effects. The support points should be located at a distance s from each other, which must satisfy the condition $0.5 \le s/2R \le 0.8$. The diameter of the supports should be 5 mm. The direction of the load must coincide with the plane of the cut.

After failure of the specimen, verification of the type of its failure is required. If the plane of the formed fracture deviates from the plane of the cut by more than $0.05D$, then the results are invalid.

The fracture toughness coefficient is calculated by the formula (designations in accordance with Fig.3)

$$K_{\mathrm{IC}} = Y'\frac{P_{\max}\sqrt{\pi a}}{2RB}, \quad (3)$$

where $Y' = -1.297 + 9.516(s/2R) - (0.47 + 16.457(s/2R))\beta + (1.071 + 34.401(s/2R))\beta^2$; $\beta = a/R$.

The equation determines the fracture toughness coefficient calculated by the finite element method under conditions of plane deformation [9, 10, 12].

Figure 4 shows the position of the specimen in the clamp before testing. The fracture growth process was quasi-equilibrium. In some cases, dynamic areas of sample destruction were noted.

Monoliths up to 0.5 m in size were selected for specimens' production to conduct an experiment to determine the fracture toughness coefficient K_{IC} of terrigenous objects.

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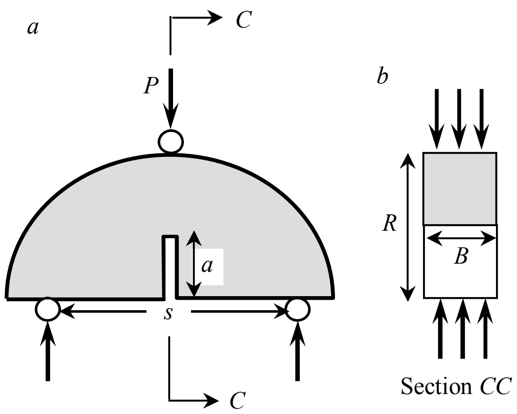

Fig.3. Specimen geometry (*a*) and loading scheme (*b*) according to recommendation of ISRM [10]

P – applied load; *s* – distance between supports; *a* – cut height; *R* – specimen radius; *B* – specimen width

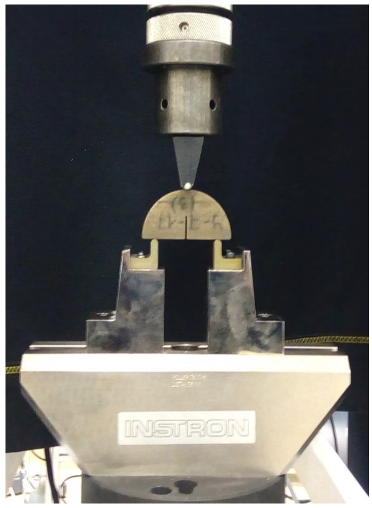

Fig.4. Semi-circular specimen in the clamp before test

Before starting the experiments to determine fracture toughness coefficient, all specimens were extracted, the open porosity and the P-wave velocity were determined under atmospheric conditions (Table 3).

Table 3

Characteristics of core material for the production of semi-circular specimens and coefficients K_{IC} of the Western Ural oil deposits

Deposit	Number of the well	Average depth of the sampling, m	Specimen quantity	Open porosity, %	P-wave velocity *Vp*, m/s	K_{IC}, MPa·m$^{1/2}$
Dorokhovskoye	711	1963.9	15	12.5	4143	1.317
Unvinskoye	531	2411.9	14	17.4	3927	0.852
Batyrbaiskoye	976	1521.7	12	24.0	3720	0.167
Kokuiskoye	976	1984.2	16	15.2	4015	0.865
Weighted average						0.774

A total of 57 specimens were tested, 22 specimens were rejected according to the criterion of deviation of the fracture from the cut plane by more than 0.05D. According to the results of the tests, the fracture toughness coefficients vary over a very wide range: 0.05-1.47 MPa·m$^{1/2}$ with a test average of 0.774 MPa·m$^{1/2}$weighted by the number of specimens.

The average values of K_{IC} coefficients determined by this method are close (with the exception of specimens of the Batyrbaiskoye deposit, characterized by high porosity) to the values of this parameter, determined for terrigenous objects using the test method on a beam specimen. At the same time, a feature is the presence of very low values of the K_{IC} parameter for high values of porosity (24-26 %). Such low values of the fracture toughness coefficient were not noted in the experiments described in the domestic literature.

Relationship of fracture toughness coefficients with geomechanical and geophysical characteristics of the cut. The statistical dependences of the fracture toughness coefficients of K_{IC} rocks on geomechanical parameters, such as elastic modulus, compressive strength, and others are practically not found in the scientific and technical literature, which is primarily due to the difficulty of simultaneously determining the fracture toughness parameter and other geomechanical parameters. In this paper, the porosity and P-wave velocity were previously determined on core specimens, and then either beam specimens or semi-circular specimens, and an experiment was carried out to determine K_{IC}.

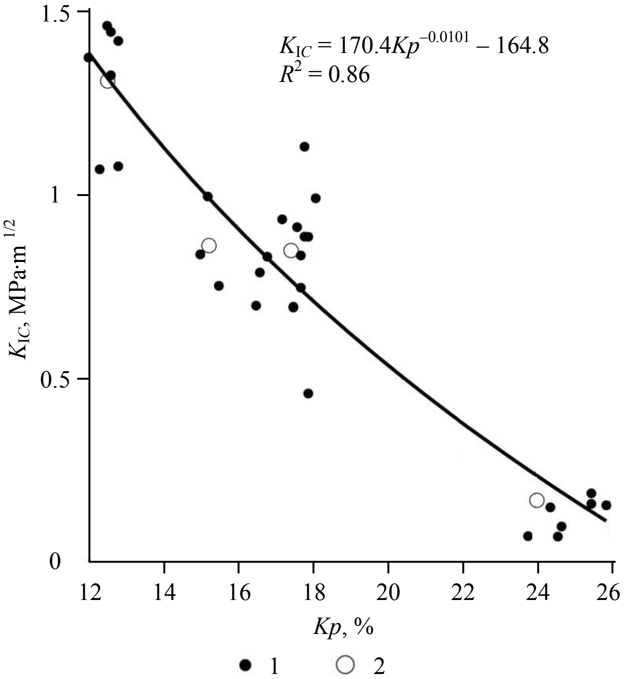

Fig.5. Dependence of the fracture toughness coefficient K_{IC} of oil fields terrigenous objects in the Western Ural, determined according to the ISRM standard, from open porosity Kp

1 – specimen data; 2 – deposit average data

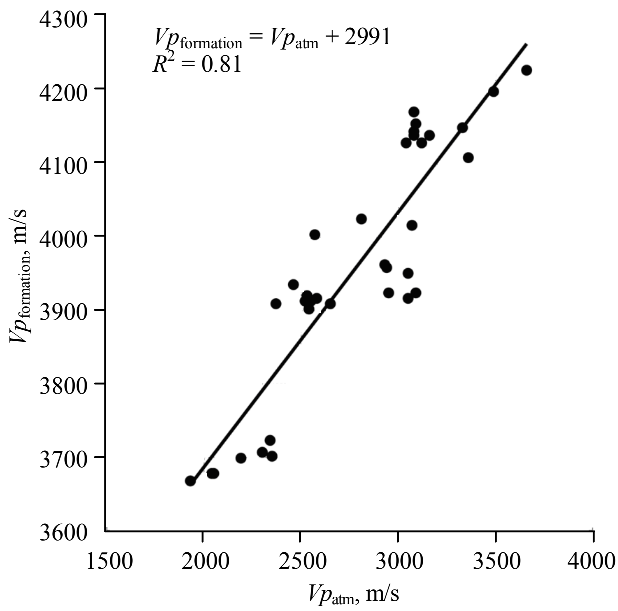

Fig.6. Dependence of the P-wave velocity determined in formation conditions $Vp_{formation}$, from velocity determined in atmosphere conditions Vp_{atm}

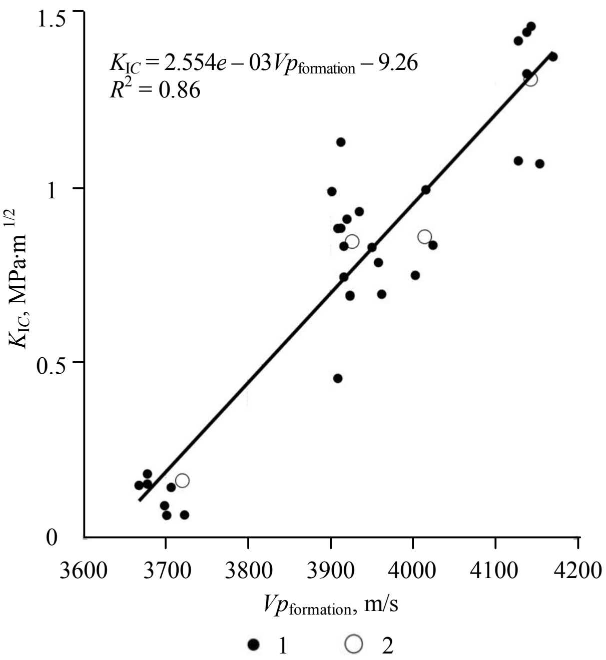

Fig.7. Dependence of the fracture toughness coefficient K_{IC} of oil fields terrigenous objects in the Western Ural, determined according to the ISRM standard, from the P-wave velocity in formation conditions $Vp_{formation}$

1 – specimen data; 2 – deposit average data

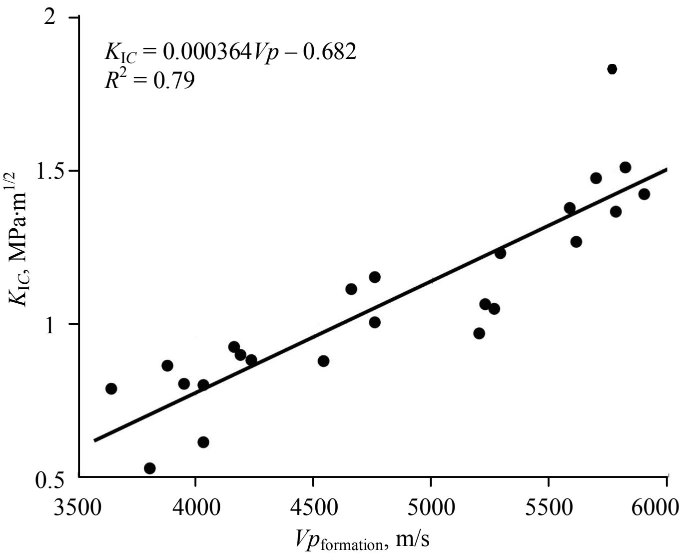

Fig.8. Dependence of the fracture toughness coefficient K_{IC} of the specimens from carbonate gas condensate deposits of the Uzbekistan Republic identified on beam specimens [1] from the P-wave velocity in formation conditions $Vp_{formation}$

The next step was the analysis of the dependence of the fracture toughness coefficient on porosity and P-wave velocity under formation conditions for terrigenous objects of the Western Ural. Dependences for porosity were obtained both for the entire sampling and for averaged values over monoliths (Fig.5). The P-wave velocity and porosity under atmospheric conditions were previously determined for the specimens. However, it is obvious that the P-wave velocity in atmospheric conditions is not equal to the wave velocity in formation conditions, and therefore a transition between these parameters is required. Fig.6 shows the relationship between the P-wave velocity determined in formation conditions and the wave velocity determined in atmospheric conditions also for terrigenous objects of deposits in the Western Ural. Based on this dependence, a relationship was obtained between the fracture toughness coefficient and the P-wave velocity in formation conditions (Fig.7).

A feature of the obtained dependences is the presence of very low values of the K_{IC} parameter for high values of porosity (24-26 %), which were not observed in the experiments described in Russian literature. At the same time, the determination of this parameter was previously carried out mainly for rock massifs, and the results were not correlated with porosity or with any geophysical factors.

For comparison, Fig.8 shows the dependence of the fracture toughness coefficient on the P-wave velocity determined by the authors of this paper for carbonate gas condensate objects of the LLC LUKOIL-Uzbekistan Operating Company deposits. The K_{IC} parameter was determined by the second method, i.e. on beam specimens [1]. A more reliable dependence is noted, which, however, is characterized by a significantly lower angle of inclination to the *Vp* axis.

Conclusion. The results of laboratory tests to determine the fracture toughness K_{IC} of rocks by three methods for terrigenous and carbonate objects reveal its very different values even for the same productive objects. However, it becomes obvious that the determination of this parameter on core samples leads to a deliberately incorrect result – the values of fracture toughness coefficients are much underestimated. The determination of the K_{IC} parameter on beam specimens and semi-circular specimens according to the method recommended by ISRM for terrigenous objects of the Western Ural deposits gives approximately similar results. At the same time, the results of determinations according to the ISRM standard give a very high range of parameter values – 0.05-1.47 MPa·m$^{1/2}$, with a mean value of 0.774 MPa·m$^{1/2}$ weighted by the number of specimens and types of tests. In general, the authors of this paper adhere to the method recommended by ISRM.

The relationship of this parameter with the geophysical characteristics of the productive zone is obtained, primarily with the porosity and P-wave velocity. The obtained dependencies cannot be considered completely reliable due to the limited number of tests and two oil and gas producing regions – terrigenous objects of the Western Ural deposits and carbonate objects of the Uzbekistan Republic. Nevertheless, they characterize the general tendency of this parameter to change depending on the porosity and P-wave velocity and can be used to model hydraulic fracturing based on the acoustic logging of a productive field.

Implementation of the developed methodology for determining fracture toughness coefficients based on correlation equations is planned at the terrigenous objects of the LUKOIL-PERM LLC deposits and at carbonate sites of the Uzbekistan Republic when designing the proppant and acid fracturing. When creating a database using the K_{IC} parameter and its dependence on geomechanical and geological-geophysical parameters, it is planned to use the standard recommended by ISRM.

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