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THE PARAMAGNETIC ELECTRON RING

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The motion of electrons (electron rings), that encircle the axis in a cylindrically symmetric magnetic field, is investigated in the presence of an azimuthal field component B_φ for the case of axial acceleration in an expanding magnetic field. As long as the expansion of the field is small, the particle motion is well described by the analytical calculations of Merkel[4] and even for large expansions with the axial field approaching zero, the deviations are not very large. The motion of the particles and the electron ring as a whole is strongly influenced by the sign of B_φ. Chosen correctly, the Lorentz-force of the azimuthal field and the axial ring velocity, that is caused by the expanding field, provides enough radial focussing that the electrons continue to encircle the axis, if the axial field is zero or even increases with opposite sign (cusp field arrangement). The electrons then behave paramagnetic with respect to the axial magnetic field.

INTRODUCTION

Electron rings have been proposed[1] and investigated[2] as a vehicle for acceleration of heavy ions. In the simplest form of an electron ring accelerator, the original transverse energy of the relativistic electrons is transformed into axial energy in an expanding magnetic field. In most experiments that have been performed following this concept electron rings were formed at modest energies at large radii in low fields and were brought to their final state with high energy and small radius, suitable for acceleration, by compression in a mirror type magnetic field. During compression and in the compressed state during the shift of the ring from the mirror field to the expanding field the ring crossed dangerous betatron resonances that could enlarge the minor radius of the electron ring and thus reduce the internal electric field, (the holding power of the ring), which should be maximized for a good performance of an accelerator.

One proposal to avoid the resonances and their deleterious influence was to apply an azimuthal magnetic field B_φ, in addition to the expanding field. One of the first to show the beneficial consequences of this proposal was A. Schlüter[3]. He showed that main resonances could be avoided by a suitable application of B_φ and effects on acceleration seemed to be tolerable. This proposal was investigated in more detail by P. Merkel[4]. With regard to acceleration the result of his paper was, that the B_φ-field "reduces acceleration, just as if the mass of the ring were increased by a factor of $1 + \alpha^2$, where $\alpha = B_\varphi/B_z$ is the ratio of the B_φ-field to the main field B_z" the values of B_φ and B_z taken at the starting point of the expansion acceleration. As long as this linear theory was applicable this reduction of acceleration was of no concern because it could easily be compensated by a faster expanding field. What however could be the effect on a real accelerator that intends to convert the rotational energy fully into axial energy? Would the increase of the ring mass also exist in the nonlinear regime and would this mean, that the final axial energy in the expanding field would be reduced, e.g. by a factor of 2, if α was chosen to be 1? How would the ring behave—or at least the single electrons in the ring-, when the ring- or the single electrons—reaches the area, where the expanding field has decreased to zero value?

Answers to these questions were sought with the help of computer calculations that the author performed during his visit to the electron ring group in the Lawrence-Berkeley-Laboratory in 1972. The results of these calculations were summarized in a comment in an internal report[5] that, in its main part, deals with a different subject (the nonexistence of an instability that had been predicted).

This paper briefly summarizes the effects of the B_φ-field on the particle motion in linear approximation, following the paper by P. Merkel. It then outlines the main components of a computer programme for calculation of the particle orbits and finally describes and discusses the somewhat surprising result.

RING ACCELERATION IN AN EXPANDING FIELD WITH SUPERIMPOSED B_φ-FIELD

An expanding field obeying Maxwell's equations can be represented in cylindrical geometry by the following two equations (following Merkel's assumptions[4])

$$B_r = B_0 \cdot \varepsilon \cdot \frac{r}{2}$$

$$B_z = B_0(1 - \varepsilon \cdot z)$$

The azimuthal field is assumed to be produced by a current, flowing in a rod on the cylinder axis:

$$B_\varphi = \frac{B_{\varphi 0} \cdot R_0}{r}$$

Here ε is a measure of the non-uniformity of the field, B_0 is the value of the axial field at $z = 0$, and $B_{\varphi 0}$ is the value of the azimuthal field at the ring position R_0 for $z = 0$.

Choosing suitable initial conditions and neglecting radial velocities and acceleration Merkel solves the equation of motion and finds for the axial velocity, v_z, as a function of z for small values of $\varepsilon \cdot z$ an equation which can be written in the following form:

$$E_z = \frac{m}{2} \cdot v_z^2 = \frac{1}{2} \frac{m}{1 + \alpha^2} \cdot v_{\varphi 0}^2 \cdot \varepsilon \cdot z$$

This equation implies that the energy of the axial motion increases linearly with the motion in the expanding field. The energy at a certain point, that is for a certain reduction of the magnetic field decreases in inverse proportion to $1 + \alpha^2$.

From this equation it follows that for a pure expanding field accelerator the beneficial action of the superimposed B_φ- field for stabilizing the ring against betatron oscillations has to be paid for by a loss in final axial energy, that can be gained.

But apart from the regrettable loss of efficiency of an expanding field accelerator predicted by the linear theory it seems to be an interesting question to investigate how far the ring follows the linear assumptions and how the motion of the particles looks like in the area of the vanishing field. The following calculations will show that a consequent application of the expanding field method can avoid the loss in acceleration energy and also show how the particle motion is affected.

CALCULATION OF PARTICLE MOTION

For the calculation of the particle motion relativistic equations of motion have been used in cylindrical geometry with the components r, φ and z. The full set of equations has been used unlike in the analytical calculation of Merkel and the energy equation has been used only to check the accuracy of the calculations, as in the case without electric field the energy of the particle should not change.

For the components of the magnetic field the expressions given in the last chapter have been used. The electric field is zero. For the main case of interest here the following initial conditions have been chosen:

$$
\begin{aligned}
\gamma &= 20 \\
r_0 &= \frac{m_e \cdot \gamma \cdot v_{\varphi 0}}{e \cdot B_{z_0}} \\
\varphi_0 &= z_0 = v_{z_0} = v_{r_0} = 0 \\
B_{z_0} &= 2[T] \\
\varepsilon &= 1[m^{-1}] \\
\alpha &= +1; 0; -1
\end{aligned}
$$

With the value of $\varepsilon = 1$ the axial magnetic field B_z is zero at $z = 1.0$ m. m_e and e are the rest mass and charge of the electron and γ the relativistic mass factor.

For the content of this paper it is not at all necessary that the electrons are relativistic. The same calculations have been performed for low energy electrons of 5 keV ($\gamma = 1.01$) and give similar results. The initial and general conditions are chosen here in accordance with the conditions of the electron ring accelerator as the starting point of this discussion.

COMPARISON WITH MERKEL'S CALCULATIONS

Merkel's calculations of the ring acceleration are only valid for $\varepsilon \cdot z \ll 1$, but as Figure 1 shows, the differences between the analytical and the computer calculated quantities are not too large even with $\varepsilon \cdot z$ approaching unity. In Figure 1 a few quantities are plotted as a function of the axial dimension z for different values of α. As one expects, the originally transverse velocity of the ring is almost fully transformed into longitudinal velocity for $\alpha = 0$ at $z = 1$ m, where B_z goes to zero (Figure 1a). The small difference between v_z and v_{φ_0} is due to radial velocity which is connected with the radial expansion of the ring.

With $\alpha = \pm 1$ the azimuthal velocity stays well above and the axial velocity well below the corresponding values with $\alpha = 0$. (The oscillations that can be seen on the figures are due to the initial condition $v_{r_0} = 0$, which is not well adapted to the problem.) The strong decrease in v_φ for $\alpha = +1$, when approaching $z = 1$ m, is again due to the large radial velocity, corresponding to the strong radial expansion, as can be seen from Figure 1c. For $\alpha = -1$ the ring is even compressed during acceleration. Figure 1b compares the axial velocity v_z with Merkel's calculations (dashed curves). The small deviation in the curve for $\alpha = 0$ is again due to a large radial velocity of the expanding ring. For $\alpha = -1$ Merkel's calculation is very good for $\varepsilon \cdot z < 0.2$. For larger values $\varepsilon \cdot z$ the axial velocity stays well below the Merkel-value, that is, the conversion from azimuthal to axial energy is even poorer than calculated by Merkel.

The most remarkable point in Figure 1a, however, is the fact that for $\alpha = -1$ the azimuthal velocity v_φ is still large at $z = 1$ m where B_z becomes zero. On the one hand this means that indeed only part of the original azimuthal energy of the ring has been used for acceleration (even less than was expected from the linear calculation of Merkel) but on the other hand the question arises of what will happen to the azimuthal velocity when B_z remains zero beyond $z = 1$ m or if it even changes sign.

THE "PARAMAGNETIC" MOTION OF THE ELECTRON RING

To investigate the question posed in the last section the calculations of the particle motion were continued beyond $z = 1$ m. For the magnetic fields chosen B_r continues with the same function of r and does not change the sign. B_z changes the sign and its absolute value increases with z. Topologically the new arrangement corresponds to a cusp field with two coils, the axial field direction of which are opposite, instead of a single expanding coil used up to now.

In Figure 2 the calculated results of the ring motion are plotted for an acceleration length of 3 m. Shown are the values of B_φ and B_z (Figure 2c) at the position of the electron and its velocities v_φ and v_z (Figure 2a). Figure 2b gives the radial dependance on the axial position. The most important result is that v_z is not limited now to the value $\delta_\alpha = v_{\varphi_0}/\sqrt{1 + \alpha^2}$ found by Merkel when considering the pure expanding field with a superimposed B_φ, but continues to increase and eventually approaches v_{φ_0}. v_φ does not change sign when B_z does. The particle continues to encircle the axis in

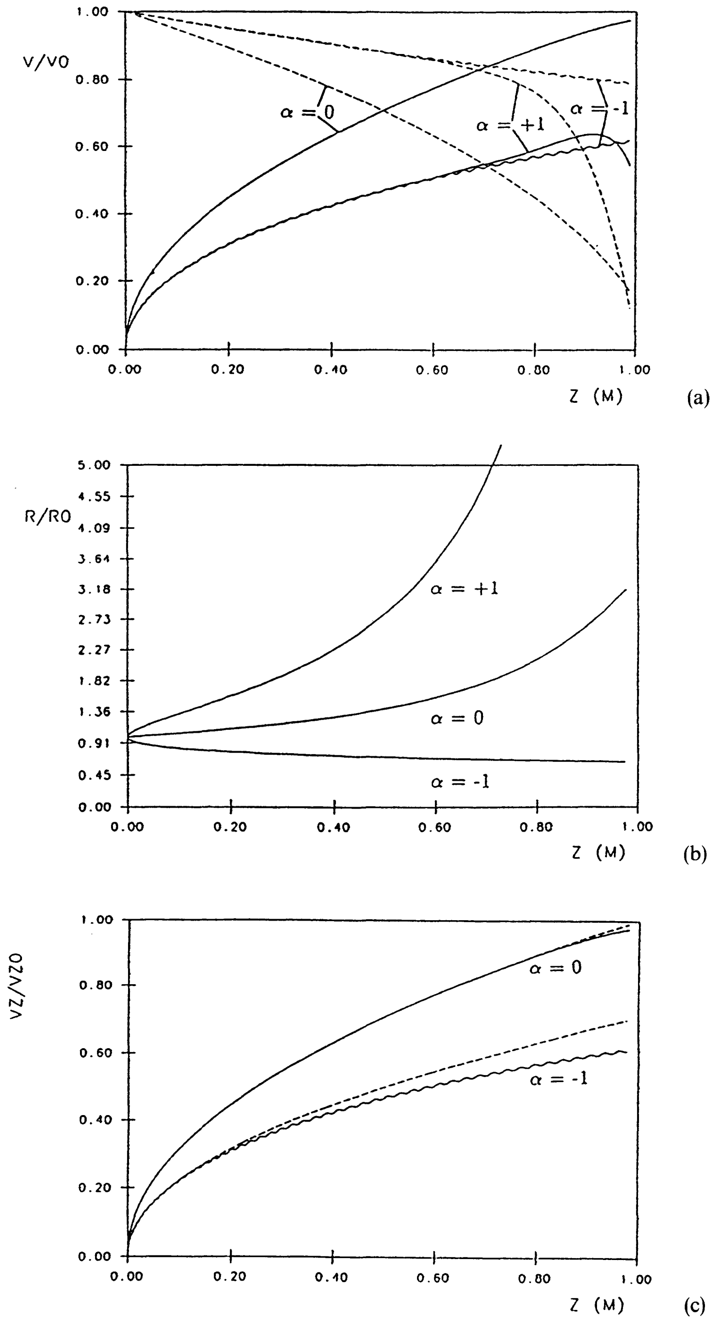

FIGURE 1 Particle motion in the expanding field: (a) azimuthal (dashed curves) and axial (full curve) velocities as a function of distance in the direction of the expanding field for $\alpha = +1/0/-1$; (b) the radius variation of the particle for $\alpha = +1/0/-1$; (c) comparison of the numerical calculation of the axial velocity with Merkel's approximation (dashed curves) for $\alpha = 0/-1$

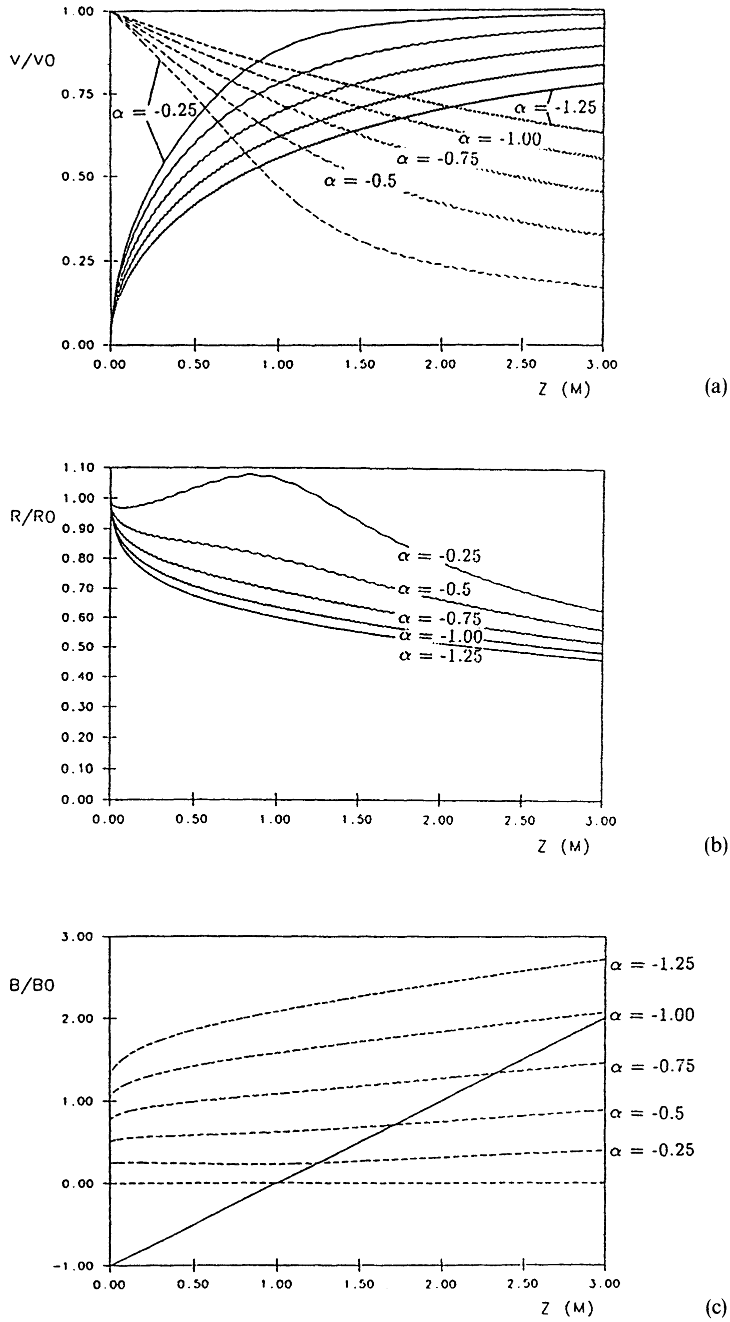

FIGURE 2 Particle motion in the cusp field arrangement for negative α: (a) azimuthal (dashed curves) and axial (full curves) velocities for $\alpha = -0.25/-0.5/-0.75/-1.0/-1.25$. The axial field changes sign at $z = 1$m. (b) the radius variation for different α values. (c) the axial (full curves) and azimuthal (dashed curves) fields at the particle position for different values of α.

the same direction as it did before the axial field changed its sign. The electrons, the motion of which is generally diamagnetic in a given field, now behave in a paramagnetic manner with respect to the axial field. As the plot (Figure 2c) shows the effect is not small. The electron that starts diamagnetically in an axial field of 2 T is found in paramagnetic motion at $z = 3$ m in an axial field of 4 T (for $\alpha = -1$). The azimuthal field is of the same order in this case. It does not change the sign at $z = 1$ m and is the dominant field there.

The actual motion of the electrons is certainly influenced by the choice of B_φ; its radial dependence, especially the dependence of the radial velocity on z can be varied drastically, even the sign of the radial motion can be changed (see Figure 1b). This, however, does not influence the possibility of "paramagnetic" motion of the particles and the possibility to fully convert the rotational energy to axial energy. Calculations have been performed for low energy particles in B_φ-fields that increase with the radius like in tokamaks with constant current density. The results are qualitatively the same as the ones discussed here. The internal relation between particle motion and fields ensures that for $\alpha < 0$ the particles have gained enough axial velocity at $B_z = 0$ to provide sufficient radial force in the B_φ-field for continued rotational motion of the particles.

SUMMARY AND DISCUSSION

The advantage of avoiding betatron resonances by the application of a B_φ-field does not have to be paid for (at least in principle, as the stability of the ring has not been discussed here) by a reduction in acceleration gain. With the correct choice of the sign of the azimuthal field all of the original transverse energy can be converted to longitudinal energy, if a field of the form of a cusp field is applied instead of a simple expanding field. For full conversion the fields, however, may be rather large depending especially on the choice of α. The longitudinal motion of the electrons in the azimuthal B_φ-field assures a balancing force to the centripetal force such that the electrons continue to rotate in the same sense in the inverse axial field as they did in the original field. With respect to the axial field the electrons are forced to behave in a paramagnetic manner.

Their rotation frequency around the axis can be expressed in the following form assuming constant radius in the radial equation of motion:

$$\omega_c = +\omega_z - \frac{v_z}{v_\varphi} \cdot \omega_\varphi = +\omega_z\left(1 - \frac{v_z}{v_\varphi} \cdot \frac{\omega_\varphi}{\omega_z}\right)$$

where ω_z and ω_φ are the cyclotron frequencies with the proper sign in the local axial and azimuthal fields respectively. For negative α one has $B_\varphi > 0$ and v_z and v_φ are both positive. B_z starts with negative values and goes through zero to positive values. As long as B_z is negative the diamagnetic effect is enhanced, that is the rotational frequency of the electron is larger than the cyclotron frequency, corresponding to the local axial field. If B_z changes sign, the motion becomes highly paramagnetic with respect to the local axial field. The validity of the equation for the frequency is not restricted to particles with relativistic velocities used as an example in this paper. It will apply also for low energy particles—like ions and electrons in a plasma—as long as radial acceleration and radial velocities can be neglected in the equation of motion.

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