VI. PLASMA PHYSICS*

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A. INTERACTION BETWEEN THE RADIATION FIELD AND THE STATISTICAL STATE OF A PLASMA

The fact that the electromagnetic radiation generated by a plasma can alter the statistical distribution of the radiating particles is generally not taken into consideration. The effects are expected to be large when, for example, the plasma departs from thermodynamic equilibrium and becomes unstable. The intense radiation that the plasma can now generate[1] reacts on the energy distribution of the particles. And it is not inconceivable that by driving the distribution closer to a Maxwellian, the instability will in time be quenched.

The problem of coupling between the radiation field and the statistical state of the plasma has been discussed recently by Kudryavtsev,[2] Akhiezer et al.,[3] and Dreicer.[4] A self-consistent calculation requires a simultaneous solution of two nonlinear equations and thus far has not been carried out. In this report we make no attempt to solve the problem – we shall merely set up the appropriate equations and present them in a form different from the one given by the above-mentioned authors. Since much of the intense radiation occurs at radio and microwave frequencies and the emission is primarily caused by the free electrons, we shall restrict our considerations to this situation.

We have two equations that describe a system composed of a plasma and its radiation field. The one is the equation of transfer for the flow of radiant energy. If I_ω is the intensity of radiation in the radian frequency interval between ω and $\omega + d\omega$ along a ray $\vec{s}$, then

$$\frac{1}{c}\frac{\partial I_\omega}{\partial t} + \frac{\partial I_\omega}{\partial s} = j_\omega - \alpha_\omega I_\omega \tag{1}$$

where j_ω and α_ω are the emission and absorption coefficients of the plasma, respectively. The foregoing equation can be rewritten[1] in terms of the rate of spontaneous emission of

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radiation, η_ω, and the distribution of electron velocities, f(v):

$$\frac{1}{c}\frac{\partial I_\omega}{\partial t} + \frac{\partial I_\omega}{\partial s} = \int \eta_\omega f(v)\, 4\pi v^2 dv + \left[\frac{8\pi^3 c^2}{\omega^2}\int \eta_\omega(v)\,\frac{\partial f(v)}{\partial u}\,4\pi v^2\, dv\right] I_\omega. \tag{2}$$

Here $u = mv^2/2$ is the electron energy, and f(v) has been assumed to be isotropic (in principle there is no difficulty in extending Eq. 2 to anisotropic distributions).

The second equation is Boltzmann's equation that prescribes the distribution of electron velocities and concentrations in the presence of fields and particle interactions:

$$\frac{\partial f}{\partial t} + \underset{\sim}{v}\cdot\underset{\sim}{\nabla_r} f - \frac{e}{m}\left[\underset{\sim}{E} + \underset{\sim}{v}\times\underset{\sim}{B}\right]\cdot\nabla_v f = C_{\text{collision}} + C_{\text{radiation}}. \tag{3}$$

$\vec{E}$ and $\vec{B}$ are the externally applied fields. Since, however, I_ω can be considered as the electromagnetic radiation composed of externally applied fields, as well as the internally generated radiation, $\vec{E}$, $\vec{H}$ represent only that portion of the external forces not included in I_ω.

The term $C_{\text{radiation}}$ (hereafter C_{rad}) is a way of writing symbolically the difference between the rates at which electrons are scattered into and out of a volume element of velocity space as a result of their emission and absorption of radiation; it is this term that is generally neglected in computations of f and which, as we shall see, makes Eq. 3 a function of I_ω.

We shall now calculate C_{rad}. The quantity $C_{\text{rad}}d^3v$ equals the rate at which electrons enter the volume element d^3v minus the rate at which they leave d^3v, as a result of spontaneous emission, absorption, and stimulated emission. We shall consider, first, the rates at which they enter and leave d^3v as a result of radiative transitions to and from two neighboring elements $d^3(v')$ and $d^3(v'')$, where $v' < v$ and $v'' > v$. The speeds v, v', and v" are such that transitions $v \rightleftarrows v'$, $v \rightleftarrows v''$ are associated with the emission or absorption of a photon $\hbar\omega$ whose frequency is given by

$$\begin{aligned}\hbar\omega &= (1/2)\, mv^2 - (1/2)\, mv'^2 \\ &= (1/2)\, mv''^2 - (1/2)\, mv^2.\end{aligned} \tag{4}$$

Writing $\eta_{\omega A}$ as the rate of absorption and $\eta_{\omega S}$ as the rate of stimulated emission, we obtain

$$\begin{aligned}C_{\text{rad}}(v,v',v'')\, d^3v = &\left[\eta_{\omega A}(v')\, f(v')\, d^3v' - \eta_{\omega A}(v)\, f(v)\, d^3v\right]\frac{I_\omega}{\hbar\omega}\, d\omega\, d\Omega \\ &+ \left[\eta_{\omega S}(v'')\, f(v'')\, d^3v'' - \eta_{\omega S}(v)\, f(v)\, d^3v\right]\frac{I_\omega}{\hbar\omega}\, d\omega\, d\Omega \\ &+ \left[\eta_\omega(v'')\, f(v'')\, d^3v'' - \eta_\omega(v)\, f(v)\, d^3v\right]\frac{d\omega\, d\Omega}{\hbar\omega}\end{aligned} \tag{5}$$

where $\eta_{\omega A}$, $\eta_{\omega S}$, and η_ω are related as in Einstein's A and B coefficients:

$$\left.\begin{aligned} \eta_\omega(v') &= \frac{\hbar\omega^3}{8\pi^3 c^2}\,\eta_{\omega S}(v') \\ v\eta_{\omega A}(v) &= v'\eta_{\omega S}(v') \end{aligned}\right\} \tag{6}$$

We simplify Eq. 5 by proceeding to the low-frequency limit $\hbar\omega \to 0$. By means of Eqs. 6 we first replace $\eta_{\omega A}$ and $\eta_{\omega S}$ by the rate of spontaneous emission, η_ω. We then expand f(v'), f(v"), $\eta_\omega(v')$, and $\eta_\omega(v")$ in Taylor series about the velocity v, retaining three terms of the expansion wherever I_ω appears as a multiplier in Eq. 5, and two terms when I_ω does not enter. After somewhat lengthy algebraic manipulations and use of Eqs. 4 we find that

$$C_{rad}(v,v',v") = \frac{1}{mv^2}\left[\frac{d[v\eta_\omega(v)\,f(v)]}{dv} + \frac{8\pi^3 c^2}{\omega^2}\,I_\omega\,\frac{d}{mdv}\left[\eta_\omega(v)\,\frac{df(v)}{dv}\right]\right] d\omega\, d\Omega. \tag{7}$$

The total value of C_{rad} is obtained by integrating Eq. 7 over all frequencies ω of the low-frequency spectrum and over all angles $d\Omega$ in which radiation flows. Since I_ω and η_ω refer to one characteristic wave only, we must sum over both modes of polarization. Thus,

$$C_{rad} = \sum_{1,2}\int_\omega\int_\Omega \frac{1}{mv^2}\left[\frac{d(v\eta_\omega f)}{dv} + \frac{8\pi^3 c^2}{\omega^2}\,I_\omega\,\frac{d}{mdv}\,\eta_\omega\,\frac{df}{dv}\right] d\omega\, d\Omega. \tag{8}$$

As a check on the algebraic manipulations, we multiply Eq. 8 by $4\pi v^2\,dv$ and integrate over all velocities. We find the correct result, $\int_0^\infty C_{rad}\,4\pi v^2\,dv = 0$, which shows that electrons are conserved. Knowing the form of C_{rad}, we can in principle evaluate I_ω and f from Eqs. 2 and 3 for any radiation mechanism, η_ω, of a plasma subjected to external fields $\vec{E}$, $\vec{B}$. Note that all of our results apply in the nonrelativistic limit. Extension of the results to plasmas with relativistic electrons is straightforward.

The two terms on the right-hand side of Eq. 8 refer to different physical processes. The first term describes the net rate at which particles leave a certain energy range as a result of energy loss by spontaneous emission. The second term represents the rate of "heating" of electrons by the ambient radiation field, I_ω.

Special Cases

(a) Consider a collisionless plasma which is in a steady state in the presence of the radiation field I_ω, which in part is due to the radiation from the plasma, and in part to any external sources that may be present. Then, from Eq. 3,

$$\frac{\partial f}{\partial t} = C_{rad} = 0. \tag{9}$$

Evaluating Eqs. 8 and 9, we obtain

$$f = \text{constant} \times \exp\left[-\int_0^v \frac{\Sigma\int\int \eta_\omega}{\Sigma\int\int \frac{8\pi^3 c^2}{\omega^2} I_\omega \eta_\omega} mv\, dv\right], \tag{10}$$

where the symbols $\Sigma\int\int$ denote summation over two polarizations and integrations over ω and Ω.

(b) Suppose that in addition to C_{rad}, the electrons make elastic collisions with atoms or ions. If $\nu(v)$ is the collision frequency,[5] then

$$C_{coll} = G\,\frac{1}{2v^2}\,\frac{d}{dv}\left[v^2\nu(v)\left\{vf+\frac{kT}{m}\frac{df}{dv}\right\}\right], \tag{11}$$

where $G = 2m/M$ with M as the mass of the atom or ion, and T as their temperature. In the steady state (with $E = B = 0$ as in case (a)),

$$\frac{\partial f}{\partial t} = C_{rad} + C_{coll} = 0. \tag{12}$$

Solving, we obtain

$$f \propto \exp\left[-\int_0^v \frac{\left[1+\frac{\Sigma\int\int \eta_\omega}{uG\nu}\right] mv\, dv}{kT_g + \Sigma\int\int \frac{8\pi^3 c^2}{\omega^2} I_\omega \frac{\eta_\omega}{uG\nu}}\right]. \tag{13}$$

G. Bekefi

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B. ELECTRON TEMPERATURE DECAY IN THE AFTERGLOW OF A PULSED HELIUM DISCHARGE

The Transient Microwave Radiation Pyrometer[1] has been used to study the electron

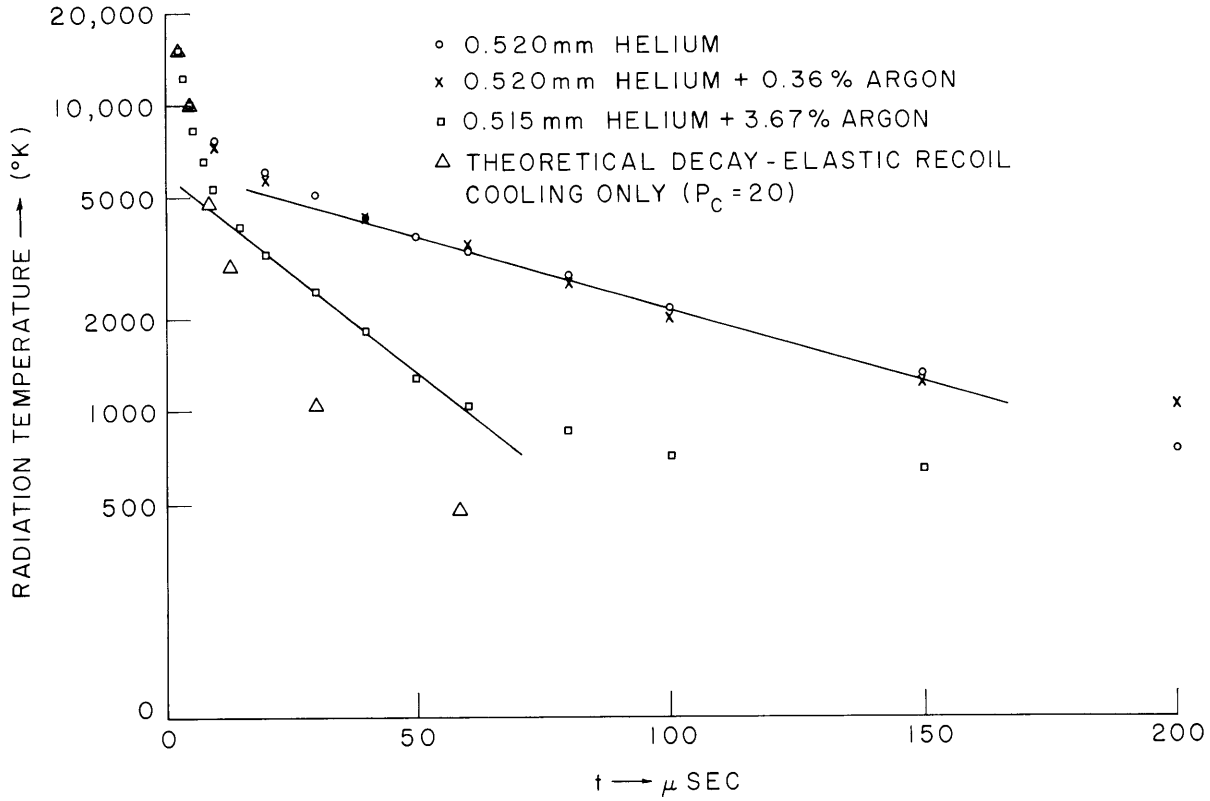

Fig. VI-1. Electron temperature decay for varying amounts of added argon impurity.

temperature decay in the helium afterglow.[2]

Figure VI-1 illustrates the dependence of the temperature decay on fractional additions of argon of zero per cent, 0.36 per cent, and 3.67 per cent at a helium pressure of 0.520 mm Hg. As we have noted, the asymptotic temperature decay rate at this pressure should be equal to the rate of destruction of metastable atoms.[2] The increase in this asymptotic decay rate with increasing argon concentration bears out our hypothesis that metastable atoms are determining the asymptotic behavior of the temperature decay. From the observed asymptotic decay rates in Fig. VI-1, a cross section for the argon-metastable destructive collision can be calculated. A value of 2.6×10^{-16} cm^2 is obtained which compares reasonably with Biondi's[3] measurement of the argon ionization cross section in an argon-metastable collision of 9×10^{-17} cm^2.

Figure VI-2 shows the temperature decay for lower pressures at which, presumably, the metastables are lost by diffusion at such a rapid rate that their heating effect is not felt in the afterglow. The electron temperature decay in this case should now be determined by elastic recoil energy losses. The theoretical curve for such a decay may be readily calculated for helium for which the collision probability is known fairly well to be $P_c = 20p$ cm^{-1}, where p is in mm Hg. The pressure multiplied by the time is the

normalized time variable, so that the curves should superpose if only elastic recoil is operating to cool the electrons. The curve at p = 0.120 mm Hg displays some residual

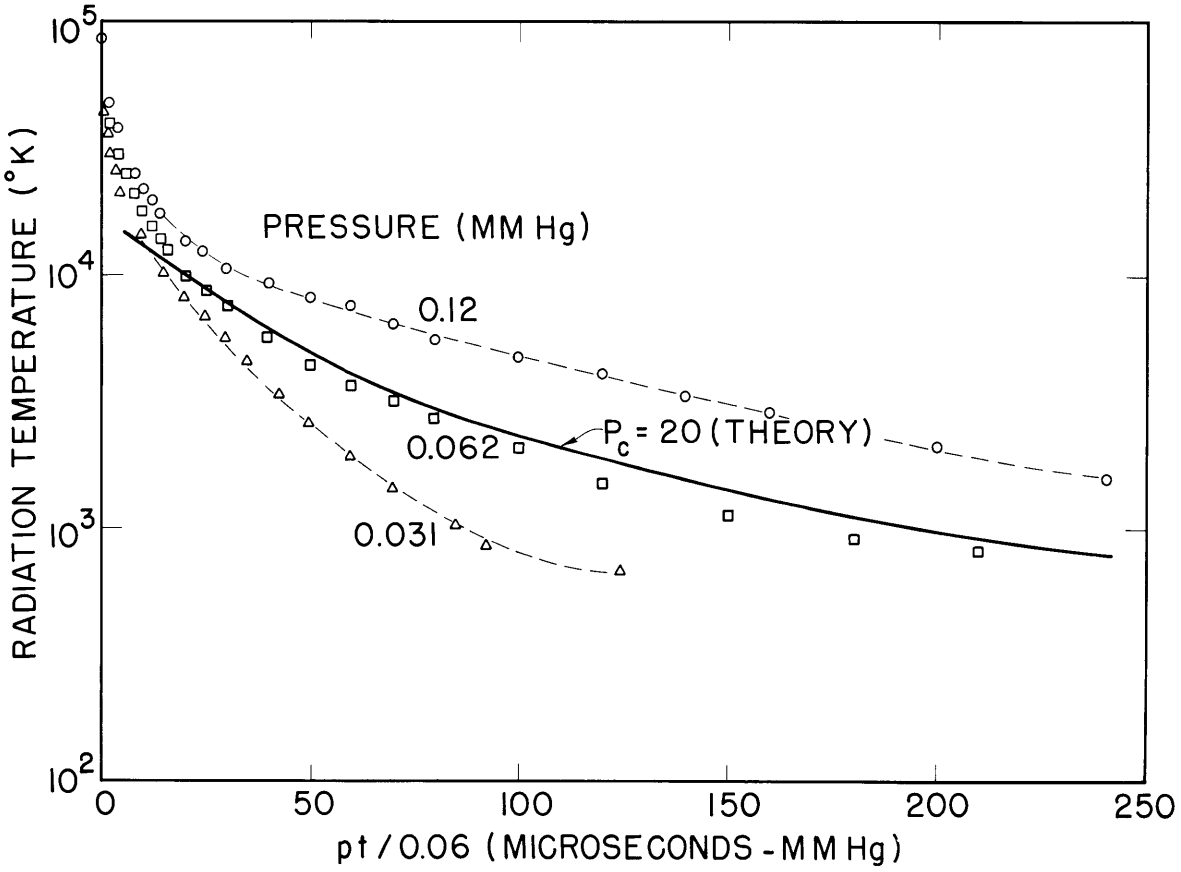

Fig. VI-2. Electron temperature decay for low pressures.

metastable heating, while the curve at p = 0.03 mm Hg displays some apparent additional cooling. Calculations have indicated that diffusion cooling should not contribute significantly in this region.

J. C. Ingraham

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C. MEASUREMENTS OF ELECTRON RELAXATION RATES IN PLASMAS

The thermodynamic state of a plasma can be inferred from its radiation spectrum. In this report we present measurements of the radiation spectrum in the afterglow of a plasma. By observing the approach of the spectrum toward one associated with the equilibrium state, it is possible to infer the relaxation rates to the equilibrium condition. We shall show that in our plasma two relaxation rates are dominant. At high pressures the relaxation toward a Maxwellian distribution is governed by short-range electron-atom collisions, while at low pressures the dominant relaxation mechanism comes from electron-electron impacts.

In previous reports[1,2] we have shown that the microwave radiation spectrum from weakly ionized plasmas departs significantly from the Kirchhoff-Planck law. The departure has been shown to be consistent with theory, provided that one assumed a non-Maxwellian energy distribution for the plasma electrons. The magnitude of the departure also depends on the energy dependence of the electron-neutral collision frequency for momentum transfer in the energy range where most of the electrons are situated. By comparing the measured and calculated spectra, it has been possible to deduce the electron energy distribution function and the mean energy.

Recent measurements further substantiate the above-mentioned results. We measure the radiation temperature T_r (as defined previously[1,2]) of a positive column in argon, immersed in a dc magnetic field as a function of time in the afterglow of the plasma. A discharge of 1-msec width is pulsed at a frequency of 200 cps, and the radiation temperature is measured as a function of magnetic field ($B = \omega_b \frac{m}{e}$, where ω_b is the electron cyclotron frequency), at a fixed time in the afterglow. The measurements were performed with the transient radiation pyrometer described in a previous report,[3] and the measuring frequency was fixed at $6\pi \times 10^9$ rad/sec.

The pyrometer accepts radiation within a 1-μsec gate, which is movable in time from 10 μsec before the discharge pulse is shut off to 3 msec past this cutoff point. We take this point to be our reference in time (i.e., t=0). so that when t is negative we refer to results obtained from a "going" discharge, and when t is positive we refer to the results obtained in the afterglow.

Some of the results are shown in Fig. VI-3. In all cases the radiation temperature is shown as a function of frequency or magnetic field for various times in the afterglow; p_o is the pressure reduced to 0°C, and I is the discharge current. Figure VI-3a and VI-3b gives the results at a low pressure for two discharge currents. Figure VI-3c and VI-3d shows the results for the same currents but at a higher pressure. The departure of the radiation temperature from the Kirchhoff-Planck law (i.e., the peaks at cyclotron resonance, $\omega = \omega_b$) is seen to be decreasing as a function of time in the afterglow. This is consistent with the interpretation that these peaks are due to non-Maxwellian

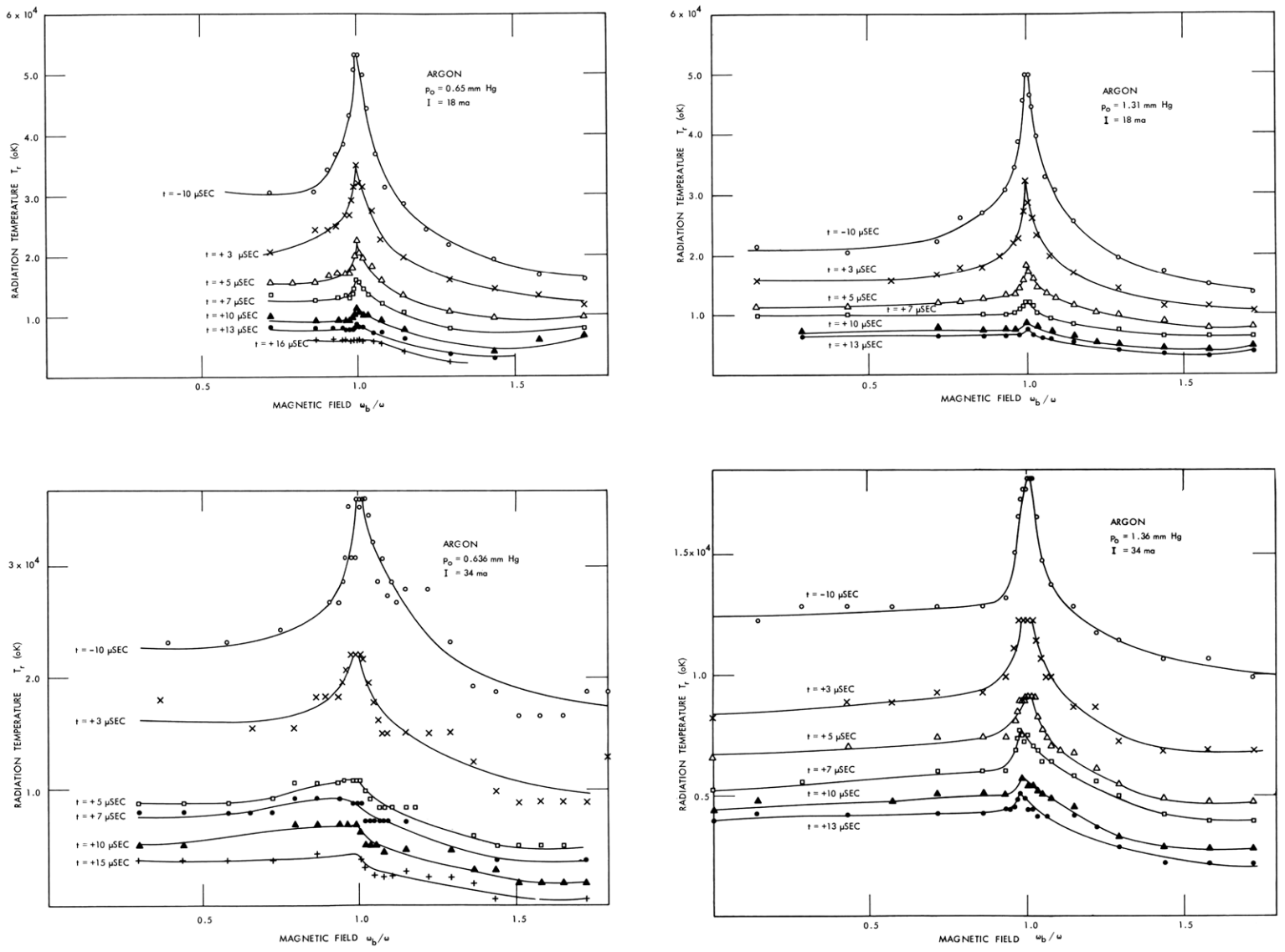

Fig. VI-3. Radiation temperature as a function of magnetic field for various times in the afterglow.

electron velocity distributions. As the discharge pulse is cut off, the electrons relax to a Maxwellian distribution and the peaks disappear. The approach to a Maxwellian distribution may be due to electron-electron collisions or electron-neutral collisions. At low pressures it is the electron-electron collisions that dominate the relaxation process. Here we would expect the rate of relaxation to be strongly dependent on the electron density.[4] This is borne out by the results shown in Fig. VI-3a and VI-3b, in which the peak in T_r is seen to decrease much faster for a discharge current of 34 ma than for a current of 18 ma. Figure VI-3c and VI-3d shows that the relaxation rate (as interpreted by the decrease of the peaks in T_r) at a higher pressure, for which electron-neutral collisions dominate the process, is relatively insensitive to discharge current.

The results can only be interpreted qualitatively, at this time, for two reasons. First, the radiation temperature does not relate linearly to the distribution function, and second, we did not measure the electron density in the afterglow. We are now conducting calculations that will enable us to determine the distribution function and the mean energy of the electrons as a function of time in the afterglow by the methods previously described.[1,2] We have installed a probe in our discharge tube in order to measure the electron density in the afterglow. With these results we hope to determine a time constant for the approach of the electrons to a Maxwellian distribution.

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D. VISCOUS DAMPING OF PLASMA WAVES

An attempt will be made to explain the experimental results on propagation and damping of ion acoustic waves reported by Wong, D'Angelo, and Motley.[1] The momentum transport equations will be modified by the introduction of a viscous force. This procedure was suggested by a comparison of the wavelength and Coulomb mean-free path. For a frequency of 10^5 cps and a phase velocity of 10^5 cm/sec, the associated wavelength is 1 cm. For a particle density of $6 \times 10^{10}/cm^3$ and a temperature of

2300°K, the Debye length squared is approximately 1.8×10^{-6} cm^2. Hence, taking the magnitude of the mean-free path as $L \approx \left(n\lambda_D^3\right)\lambda_D$, where n is the density, and λ_D is the Debye length, we obtain $L \approx 0.2$ cm. Because of the similarity of the magnitudes of wavelength and mean-free path, we suspect that the waves will be strongly viscous-damped. For the magnitude of the viscosity coefficient we take $\eta \approx LV_T\rho$, where V_T is the thermal velocity, and ρ is the mass density. Hence the kinematic viscosity will be $\nu = \eta/\rho \approx LV_T$.

The equations that will be used to obtain the dispersion relation are the linearized force and continuity equations and Poisson's equation.

$$\frac{\partial \vec{\Gamma}_\pm}{\partial t} = -\mu_{s_\pm}^2 \nabla n_\pm \pm \frac{en_o}{m_\pm}\vec{E} + \nu_\pm \nabla^2 \vec{\Gamma}_\pm \tag{1}$$

$$\frac{\partial n_\pm}{\partial t} + \nabla \cdot \bar{\Gamma}_\pm = 0 \tag{2}$$

$$\nabla \cdot \bar{E} = 4\pi e(n_+ - n_-). \tag{3}$$

In these equations the particle flux $\vec{\Gamma}_\pm$ is defined as $n_o\vec{V}_\pm$, where $\vec{V}_\pm$ is the ion or electron velocity, and n_o is the unperturbed particle density; $n_\pm$ is the perturbed particle density; and $\mu_{s_\pm}$ is the ion or electron sound velocity. For equal electron and ion temperatures we have $\mu_{s_-}^2 = m_+/m_-\mu_{s_+}^2$ and $\nu_-^2 = m_+/m_-\nu_+^2$. By using Poisson's equation and the continuity equations, the force equations may be written in terms of the perturbed densities.

$$\frac{\partial^2 n_\pm}{\partial t^2} = \mu_{s_\pm}^2 \nabla^2 n_\pm \mp \omega_{p_\pm}^2 (n_+ - n_-) + \nu_\pm \nabla^2 \frac{\partial n_+}{\partial t}, \tag{4}$$

where ω_p is the plasma frequency, and

$$\omega_{p_\pm}^2 = \frac{4\pi n_o e^2}{m_\pm}.$$

By allowing $n_\pm$ to vary as $e^{j(\bar{k}\cdot\bar{r} - \omega t)}$ in Eq. 4, the following equations are obtained:

$$\begin{aligned}\left[\omega^2 - \mu_{s_+}^2 k^2 - \omega_{p_+}^2 + j\nu_+\omega k^2\right] n_+ + \omega_{p_+}^2 n_- &= 0 \\ \omega_{p_-}^2 n_+ + \left[\omega^2 - \mu_{s_-}^2 k^2 - \omega_{p_-}^2 + j\nu_-\omega k^2\right] n_- &= 0.\end{aligned} \tag{5}$$

Setting the determinant of Eq. 5 equal to zero, the following dispersion relation results:

$$\frac{\omega_{p_+}^2}{\omega^2 - k^2\left(\mu_{s_+}^2 - j\nu_+\omega\right)} + \frac{\omega_{p_-}^2}{\omega^2 - k^2\left(\mu_{s_-}^2 - j\nu_-\omega\right)} = 1. \tag{6}$$

In the experiment of Wong and others,[1] ω^2 varies from $\sim 4 \times 10^7$ to 4×10^{11}. For $n_o = 6 \times 10^{10}/cm^3$, $\omega_{p_-}^2$ will be approximately 1.8×10^{20}, and $\omega_{p_+}^2$, for $m_+/m_- = 10^5$, will be 1.8×10^{15}. By using this information, as well as the order of magnitude of the sound velocity and the kinematic viscosity, an approximate equation for the ion waves can be written from Eq. 6.

$$k^2 = \frac{\omega^2}{2\mu_{s_+}^2 - j\omega\nu_+}. \tag{7}$$

Setting $k = k_o + i\mathcal{K}$, where k_o indicates the propagation, and $\mathcal{K}$ the damping, Eq. 7 yields

$$k_o = \frac{\omega}{2\mu_{s_+}} X(t)$$

$$\mathcal{K} = \frac{\omega}{2\mu_{s_+}} Y(t), \tag{8}$$

where

$$t = \left(\frac{\omega\nu_+}{2\mu_{s_+}^2}\right)$$

and

$$X(t) = \left[\frac{(1+t^2)^{1/2} + 1}{1 + t^2}\right]^{1/2}$$

$$Y(t) = \left[\frac{(1+t^2)^{1/2} - 1}{1 + t^2}\right]^{1/2}.$$

In Fig. VI-4 we show the quantities $\left(\nu_+/\mu_{s_+}\right)\mathcal{K}$ and $\left(1/2\mu_{s_+}\right)\omega/k = V_{phase}/2\mu_{s_+}$ as functions of $\left(\nu_+\omega/2\mu_{s_+}^2\right)$. It should be mentioned that the phase velocity, from simple Navier-Stokes theory, rises much more rapidly with increasing ω than higher order theory or experiments indicate, while the damping remains adequately described by the simple theory.[2,3]

In Fig. VI-5 we present the first attempt to fit the damping theory to the experimental

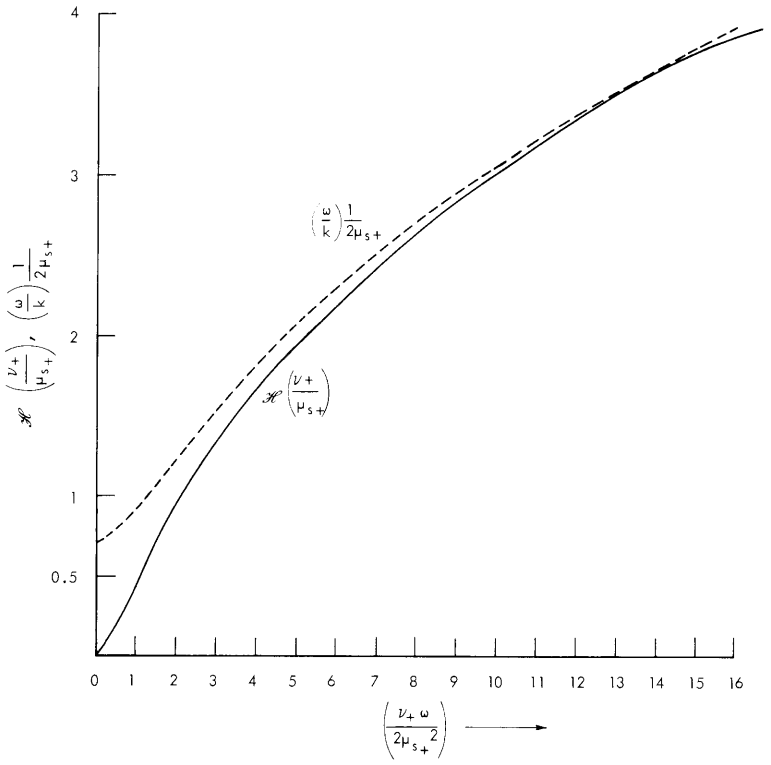

Fig. VI-4. Theoretical propagation and damping functions.

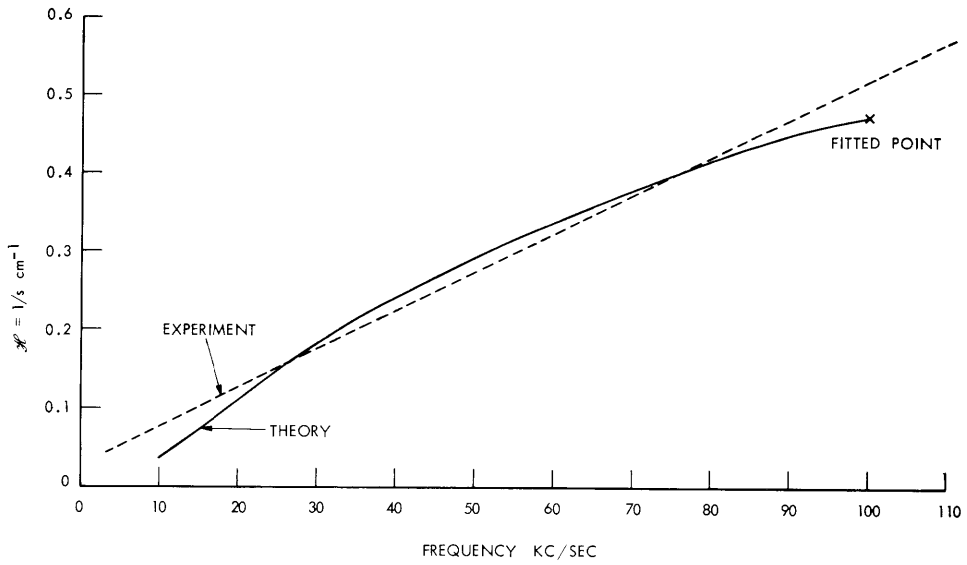

Fig. VI-5. $\mathcal{H}$ as a function of frequency.

data (Fig. 2 of Wong et al.[1]). The data give $\mathcal{K} \approx 0.47$ for potassium at a frequency of 10^5 cps. At this frequency we choose $\left(\nu_+ \omega/2\mu_{s_+}^2\right) = 5$. Since, from Fig. VI-4, $\nu_+/\mu_{s_+}\mathcal{K} = 1.96$ at this point, we have $\mu_{s_+}/\nu_+ = 0.24$. This guess is seen to give a fairly good representation of the experimentally observed damping.

The phase velocity obtained from our fitted data is high because for μ_{s_+} itself we obtain 2.6×10^5 cm/sec, which is approximately the observed phase velocity.[1] Our computed phase velocity will vary from 4×10^5 cm/sec at 10 kc/sec almost linearly to 11×10^5 cm/sec at 100 kc/sec. The fitted point also gives a value of $\nu_+ \approx 11 \times 10^5$ cm^2/sec, which is a correct order of magnitude.

Wong, D'Angelo, and Motley, in their Figure 1, give the experimentally observed phase velocity as a function of frequency.[1] These data appear to indicate slightly higher phase velocities for higher frequencies, as well as higher phase velocities for lower densities. Figure VI-4 shows an increasing phase velocity with frequency and also, if the frequency is held fixed, an increasing phase velocity with decreasing density, since $\nu_+ \sim n\lambda_D^4 \sim 1/n$.

For the sake of completeness, we shall compute the viscosity from Boltzmann theory,

$$\nu_+ = \frac{4}{3}\frac{\tau kT}{m}. \tag{9}$$

In Eq. 9 T is the temperature, k is Boltzmann's constant, m is the ion mass, and τ is the time constant given by Spitzer,[5]

$$\tau = \frac{m^{1/2}(3kT)^{3/2}}{8 \times .714\,\pi n e^4 \ln \Lambda}, \tag{10}$$

where $\Lambda = 12\pi^{3/2} n\lambda_D^3$.

For potassium, with $n = 6 \times 10^{10}/\text{cm}^3$ and $T = 2300°K$, we obtain

$$\nu_+ = 0.883 \times 10^5 \text{ cm}^2/\text{sec}. \tag{11}$$

Defining $\sqrt{2}\,\mu_{s_+}$ as 2.5×10^5 cm/sec, the measured value of the phase velocity, we obtain at 100 kc/sec,

$$\frac{\nu_+ \omega}{(\sqrt{2}\,\mu_s)^2} = 0.886. \tag{12}$$

The theoretical curve and the experimental points for $\mathcal{K}$, the damping constant, as a function of frequency are shown in Fig. VI-6. It is clear that the viscous theory gives the correct order of magnitude of the damping.

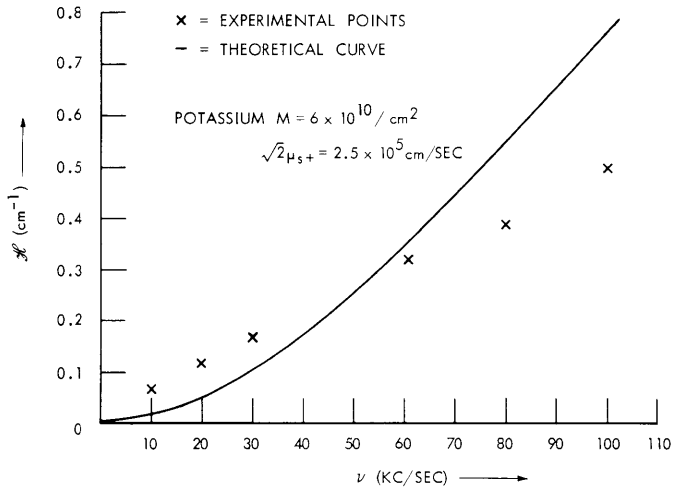

Fig. VI-6. Damping constant as a function of frequency.

Therefore, at present, it seems that a simple viscous theory can explain the observations on ion waves without recourse to Landau damping as discussed by Fried and Gould.[5]

H. R. Radoski

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