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THE EFFECT OF LONGITUDINAL SPACE CHARGE ON MULTI-TURN CAPTURE IN THE AGS BOOSTER

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Computer simulation of longitudinal phase-space dynamics shows that space-charge-induced fields contribute significantly to the efficiency of capture, as well as the distribution, of heavy ions and protons. Statistical analysis of the distribution of the space charge voltage shows that the number of representative particles per bin should be increased with the square of the bin number to ensure a certain statistical accuracy.

With a 10-ms variation of the rf peak voltage $\hat{V}$ and synchronous phase angle ϕ_s, 94% of the 20 turns of the multi-turn-injected Au^{+33} heavy-ion beam can be captured with a longitudinal phase space area of 0.06 eV-s/amu. If this rf variation is performed in the same way for proton injection, 96% of the proton beam can be effectively captured in the Brookhaven Alternating Gradiant Synchrotron (AGS) Booster with a longitudinal-phase-space area of 1.0 eV-s.

1. INTRODUCTION

The Coulomb space-charge field induced by a circulating beam during injection of heavy ions and protons may significantly influence beam capture in low-energy boosters.[1] The simulation of this space charge force, which is proportional to the derivative of the longitudinal bunch distribution function,[2–5] is accomplished by dividing the bunch into bins of finite length in longitudinal space. To obtain a reliable calculation of the space charge force, two precautions must be taken: the bin size should not be too large or the calculation will not reveal the structure of the bunch distribution, and the number of particles per bin should not be too small or the calculation will not maintain statistical accuracy. The simulation program can also be useful in studying collective phenomena. To investigate collective instability, the bin length used to calculate the space charge voltage should be comparable to the collective wavelength. For a specified bin length, a certain number of representative particles is required so that the calculation is accurate. The reliability of the simulation essentially depends on the statistical accuracy of the calculation of the space charge voltage.

In this paper, we discuss our studies of the longitudinal dynamics of multi-turn capture in the AGS Booster. Section 2 gives the methods and statistical analysis used to calculate the space-charge effect. Section 3 describes the results of our simulation of multi-turn injection and rf capture of beams of heavy ions and protons. The discussion and conclusion are given in Section 4.

2. METHOD FOR CALCULATING THE SPACE CHARGE FORCE

2.1. Equations of Motion

Once the beam has been injected, the motion of each particle in the longitudinal phase space (ϕ, $\Delta E/\Omega_0$) is described by the following difference equations.

$$\delta_{n+1} = \delta_n + \frac{Ze\hat{V}_n}{Am_0c^2\gamma_{s,n}} \cdot (\sin\phi_n - \sin\phi_{s,n}) + \Delta_{s.c.}(\phi_n) + \Delta_Z(\phi_n) \tag{1}$$

$$\phi_{n+1} = \phi_n + \frac{2\pi h\eta_s}{\beta^2_{s,n+1}} \cdot \delta_{n+1}, \tag{2}$$

where

$\eta_s = \dfrac{1}{\gamma_t^2} - \dfrac{1}{\gamma_s^2}$

$\gamma_s = (1-\beta_s^2)^{-1/2}$ (synchronous energy of the particle in m_0c^2 units)

$\delta = \dfrac{\Delta E}{E_s} = \dfrac{\Delta\gamma}{\gamma_s} = \beta_s^2 \dfrac{\Delta p}{p_s}$ (relative deviation of particle energy)

ϕ = deviation of the particle rf phase from the synchronous phase, ϕ_s

h = rf harmonic number

Z = charge carried by the particle

A = atomic number of the particle

$\Delta_{s.c.}$ = relative energy gain per turn due to the space-charge fields

Δ_Z = relative energy gain per turn due to other kinds of impedance

n (subscript) = revolution number.

Assume that the line density $\lambda(\phi)$ of the beam changes slowly[3,4] within a distance comparable to the diameter of the vacuum chamber 2b; in other words:

$$\frac{2b}{\lambda}\frac{\partial\lambda}{\partial s} \ll 1. \tag{3}$$

Then the space-charge voltage can be represented by the mean field expression:

$$\Delta_{s.c.}(\phi_n) = \frac{eV_{s.c.}(\phi_n)}{Am_0c^2\gamma_s} = \frac{Z_0g_0R_0cZe}{2Am_0c^2\gamma_s^3} \cdot \frac{\partial\lambda(\phi_n)}{\partial s} \tag{4}$$

where $Z_0 = (\varepsilon_0 c)^{-1} = 377$ (ohms), R_0 is the average radius of curvature, g_0 is the geometric factor, $g_0 = 1 + 2\ln(b/a)$ for a cylindrical geometry, and a is the beam radius.

The $\Delta_R(\phi_n)$ of the resistive wall coupling impedance can similarly be represented by the mean field expression

$$\Delta_R(\phi_n) = \frac{eV_R(\phi_n)}{Am_0c^2\gamma_s} = \frac{R_w\beta_0R_0cZe}{Am_0c^2\gamma_s} \cdot \lambda(\phi_n), \tag{5}$$

where R_w is the wall coupling resistance.

Note that Eqs. (4) and (5) have different dependences on $\lambda(\phi)$. These differences imply different statistical behavior. Note that also the difference equations, Eqs. (1) and (2), satisfy the symplectic condition; in other words, the mapping preserves the phase space area.

The space charge voltage $V_{s.c.}(\phi)$ is obtained by evaluating the derivative of the density distribution function of the beam in the rf phase space:[5]

$$V_{s.c.}(\phi)=\frac{Z^2h^2g_0Z_0ce}{2R_0\gamma_s^2}\cdot\frac{\partial(N_0f(\phi))}{\partial\phi}, \tag{6}$$

with N_0 the total number of particles per bunch and $N_0f(\phi)=\frac{R_0}{h}\lambda(\phi)$ the number of particles per unit ϕ around the phase ϕ. Letting N_b denote the number of bins of the 2π phase period,

$$N_b=\frac{2\pi R_0}{hl_b}, \tag{7}$$

where l_b is the bin length. When studying the effect of microwave instability, the bin length l_b should be of the same order as the wavelength of the microwave cutoff frequency b.[4–7]

2.2. *The Computer Program*

A computer program was developed to simulate the multi-turn injection and the rf capture process. Here is the scenario on which the program is based. Before the injection into the Booster synchrotron, a dc beam pulse with a specified energy, a specified energy spread, and a certain pulse length is generated from the 200 MeV linac (for protons) or from the tandem accelerator (for heavy ions). During the injection period this beam pulse starts to circulate in the Booster. In longitudinal phase space relative to the injecting dc beam, the rf voltage is gradually increased to capture the particles in the bucket. The beam gradually gets into this moving bucket and occupies a band.

Once a particle gets into the bucket, its motion is determined by the rf acceleration force and the space charge force induced by the particles injected earlier. Because of nonlinear synchrotron oscillation and the increasing space charge, the particle distribution in phase space is no longer uniform at the end of the multi-turn injection. The synchrotron motion of these representative (macro) particles is tracked according to the equations of motion, Eqs. (1)–(4).

Both the space charge and the resistive coupling impedance can be included in the simulation. The three-point formula

$$f_p'=f'(\phi_0+ph)=\frac{1}{h}((p-\tfrac{1}{2})f_{-1}-2pf_0+(p+\tfrac{1}{2})f_1)+R_2' \tag{8}$$

is used to calculate the quantity $\frac{\partial f(\phi)}{\partial\phi}$. Calculating $\frac{\partial f(\phi)}{\partial\phi}$ using only the information in every two nearby bins, i.e., by a two-point formula, leads to a large fluctuation in the phase space diagram. The three-point formula of Eq. (8) greatly improves the accuracy. (A five-point formula was also tried; however, it did not change the result significantly.)

2.3. *Estimation of the Statistical Deviation*

In any numerical simulation, numerical accuracy is an important issue. In the present calculation, accuracy depends on the space charge term. We estimate the statistical fluctuation of $\Delta_{s.c.}$ in the following way.

Let N_t be the total number of the representative particles used in the simulation, and let $\Delta\phi$ be the bin size in ϕ space. Then the density function $f(\phi)$ at the ith bin is

$$f(\phi)_i = \frac{1}{N_t}\left(\frac{\left(\frac{N_t}{N_b} g_i\right)}{\Delta\phi}\right), \tag{9}$$

with g_i the density weighting factor,

$$\sum_{i=1}^{N_b} \frac{g_i}{N_b} = 1, \qquad \sum_{i=1}^{N_b} f(\phi)_i \, \Delta\phi = 1, \tag{10}$$

and

$$\frac{\partial f(\phi)}{\partial \phi_i} = \frac{1}{N_t}\left(\frac{\left(\frac{N_t}{N_b} g_i\right)}{\Delta\phi} - \frac{\left(\frac{N_t}{N_b} g_{i-1}\right)}{\Delta\phi}\right) \cdot \frac{1}{\Delta\phi}. \tag{11}$$

The statistical fluctuation of the number of representative particles at the ith bin is

$$\Delta\left(\frac{N_t}{N_b} g_i\right) = \sqrt{\frac{N_t}{N_b} g_i}. \tag{12}$$

Thus, the statistical fluctuation of $V_{s.c.}$ becomes

$$\Delta(V_{s.c.}) \sim \sqrt{g_i} \cdot \sqrt{\frac{N_b^3}{N_t}}. \tag{13}$$

Statistical analysis shows that the deviation of the space charge voltage is inversely proportional to the square root of the total number of representative particles, multiplied by the cube of the bin length. Therefore, if the bin length is decreased to account for the effect of the higher frequency, the number of particles used in the simulation, N_t, must be increased to the cube of N_b to retain the same statistical accuracy.

The behavior of Eq. (13) is observed by systematically evaluating the average statistical deviation as a function of (N_b, N_t). Figures 1a–1f show the typical space charge voltage for a bunch of 2×10^9 Au^{+33} ions, with $N_b = 50$, 150, 300, 600,

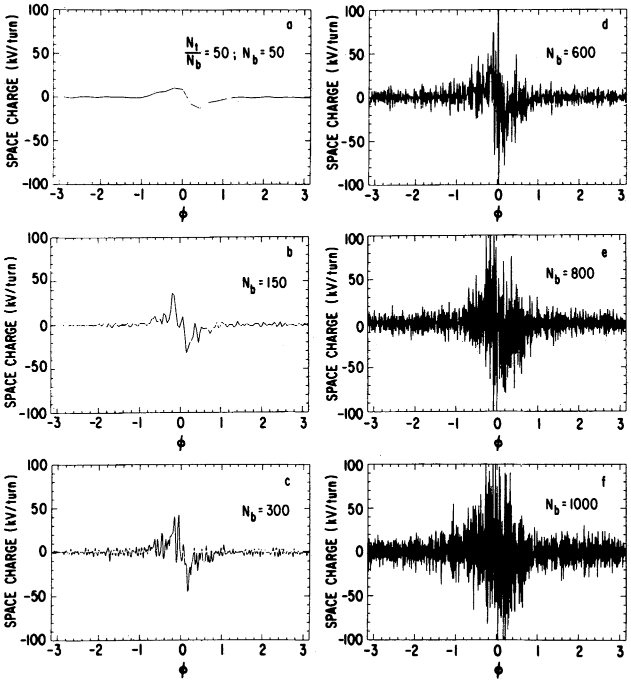

FIGURE 1 The space charge voltage calculated for number of bins $N_b = 50$, 150, 300, 600, 800, and 1000, with $\frac{N_t}{N_b} = 50$.

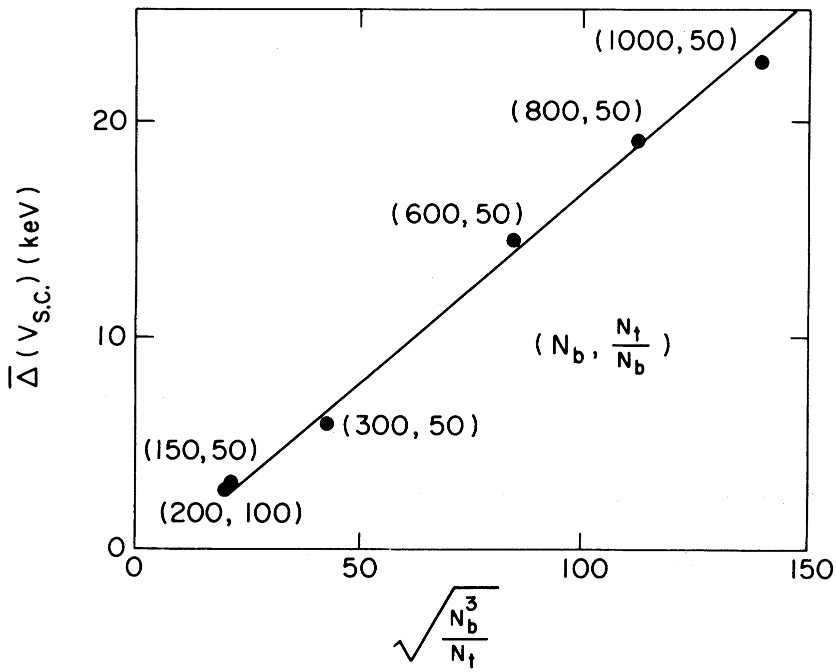

FIGURE 2 The average statistical deviation as a function of $\sqrt{\frac{N_b^3}{N_t}}$.

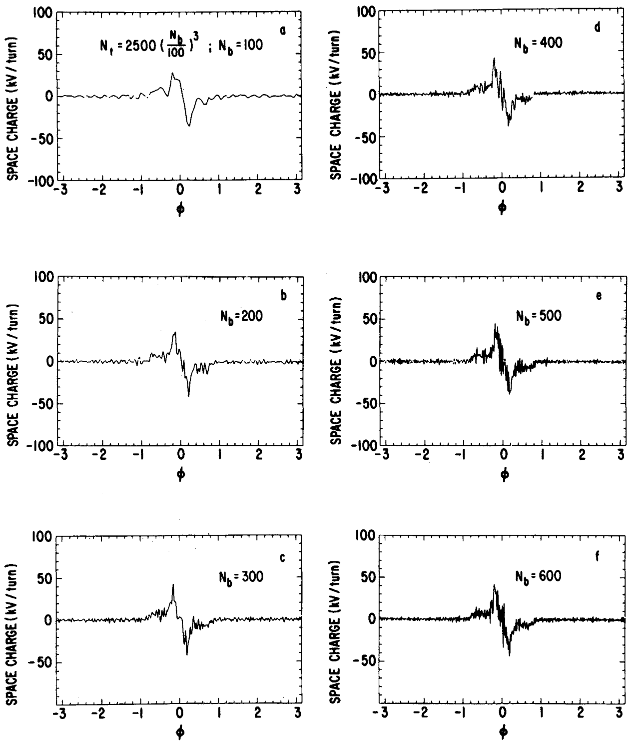

FIGURE 3. The space charge voltage for number of bins $N_b = 100$, 200, 300, 400, 500, and 600. The total number of the representative particles, N_t, is 2500, 20 000, 67 500, 160 000, 312 500, and 540 000, respectively.

800, and 1000, respectively, keeping $\frac{N_t}{N_b} = 50$ as a constant. Note that the statistical fluctuation becomes unreliably large when N_b is large. By statistically analysing the fluctuation component of the space charge voltage, the average statistical deviation can be plotted as a function of $\sqrt{N_b^3/Nt}$, as in Fig. 2. The result agrees with Eq. (13).

Figures 3a–3f give examples of equivalent accuracy ($\bar{\Delta}_{s.c.} \simeq 3.6$ keV/turn) in the calculation of space charge voltage with $N_b = 100$, 200, 300, 400, 500, and 600, respectively, and with $N_t \propto N_b^3$. (Incidentally, the computer time needed for the simulation is proportional to $N_b \times N_t \sim N_b^4$.)

If the dominant impedance is due to the resistive wall, rather than the space-charge field the statistical behavior will be different. In this case, provided that the number of representative particles per bin remains constant, the same statistical accuracy is always achieved. The statistical fluctuation of V_R is

$$\Delta(V_R) \sim \sqrt{g_i} \cdot \sqrt{\frac{N_b}{N_t}}. \tag{14}$$

3. RESULTS OF RF CAPTURE ON THE AGS BOOSTER

3.1 RF Capture of Heavy Ions

The beam of heavy ions comes from the tandem van de Graaff accelerator, with a rms fractional kinetic energy spread of 10^{-3}. Each single 300-μs pulse of $^{197}Au^{+33}$ can provide more than 20 turns of "ribbon" beam. The intensity is 2.2×10^9 particles per bunch.

Capture efficiency is defined here as the ratio of the total number of particles remaining inside the rf bucket when the synchronous energy is far above the injection energy to the total number of particles in the dc beam before injection. During the 10 ms of injection and capture, the rf voltage gradually rises from 1.6 kV to 17 kV, and the synchronous phase angle rises from 0.001 rad to 0.6 rad. Assuming the harmonic number $h = 3$, Figure 4 shows the evolution in

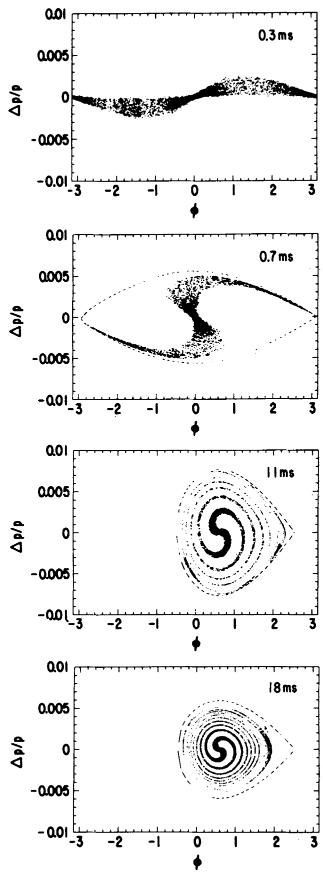

FIGURE 4 The longitudinal-phase-space diagram for an $^{197}Au^{+33}$ beam, without the space charge effect, at different times after injection. The capture efficiency is 94.1%.

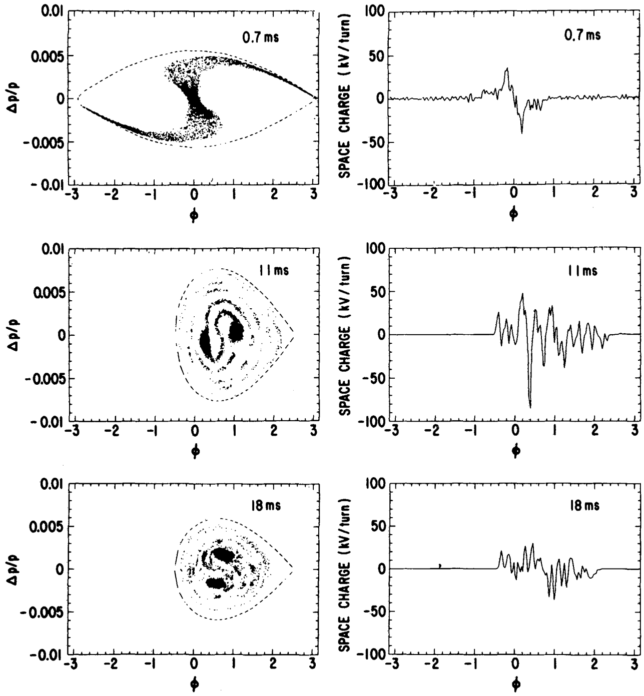

FIGURE 5 The longitudinal-phase-space diagram and the corresponding space-charge voltage of $^{197}Au^{+33}$ beam at different times after injection. 200 bins and 20 000 representative particles are used in the space charge simulation. The capture efficiency is 93.8%.

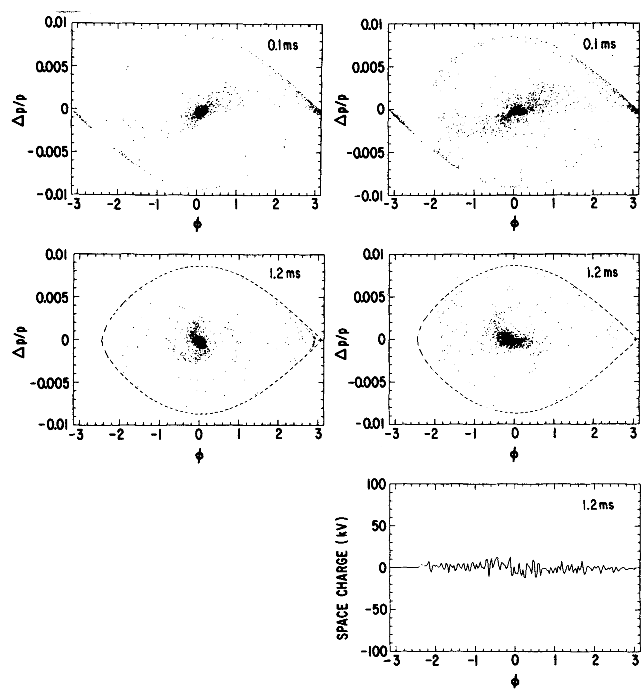

FIGURE 6 The longitudinal-phase-space diagram of the proton beam at different times after injection. $\dot{B}_0 = 1.5$ T/sec. The space-charge effect is not included in the left column; the capture efficiency is 86%. The space-charge effect is included in the right column, where 200 bins and 20 000 representative particles are used in the simulation; the capture efficiency is 82%.

longitudinal phase space, without considering the space charge effect. More than 94% of the Au^{+33} beam is captured in a phase-space area of 0.05 eV-s/amu. Figure 5 shows the effects due to space charge. 200 bins and 20 000 representative particles are used in the simulation, and 93.8% of the beam is finally captured. The bunch area is 0.06 eV-s/amu.

3.2. RF Capture of the Proton

Two different rf capture schemes are explored here: constant $\dot{B}_0$ and the programmed $\dot{B}_0$. In each case, a dc beam of protons of 100-μs pulse length is injected into the AGS Booster. The harmonic number $h = 3$ and the bunched beam intensity is 5×10^{12} particles per bunch.

In the constant-$\dot{B}_0$ scheme, we used $\dot{B}_0 = 1.5$ T/sec and $\hat{V} = 90$ kV, $\phi_s = 0.05$. Figure 6 shows the phase space distribution without the space-charge effect (a

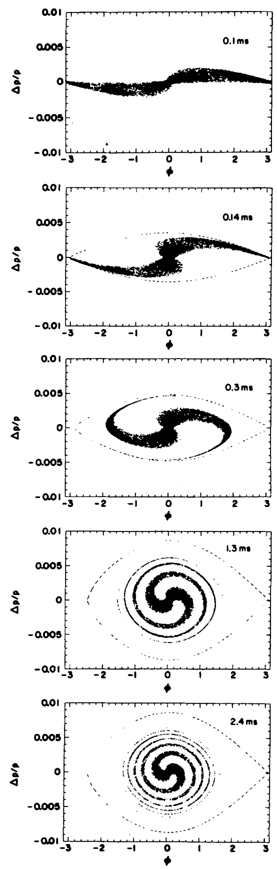

FIGURE 7 The longitudinal-phase-space diagrams for the evolution of the multiturn-injected "ribbon" beam of protons, where $\dot{B}_0$ is varied. When the space-charge effect is not considered, the capture efficiency is 99%.

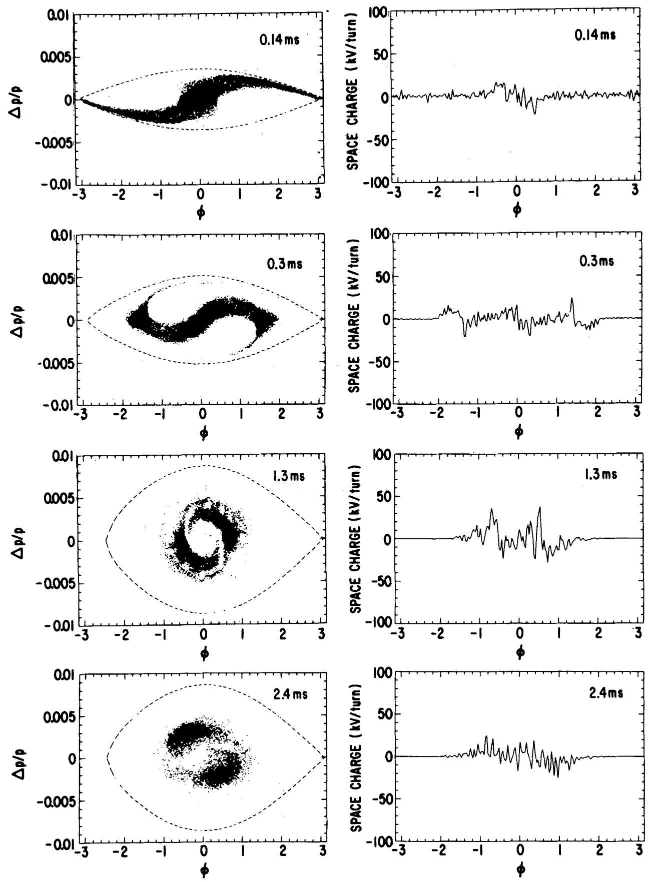

FIGURE 8 Similar to Fig. 7, but with the space-charge effect. 200 bins and 20 000 representative particles are used in the simulation; the capture efficiency is 96%.

capture efficiency of 86%) and with the space-charge effect (82%). The final phase space area is about 1.3 eV-s. The capture efficiency increases with decreasing ϕ_s. To minimize the beam loss, $\dot{B}_0$ was varied starting with $\phi_s \simeq 0$. When the rf voltage and the synchronous phase angle were varied adiabatically, calculations indicated that 96% of the proton beam could be captured in a phase space area of 0.9 eV-s (see Figs. 7–8). The adiabatic nature of the transition between the injected ribbon beam to the bunched beam in the rf bucket is preserved in this latter scheme.

4. DISCUSSION

During rf capture in the AGS Booster, the effects of space charge are not negligible. The defocusing force of the space charge inevitably causes non-uniformity and diffusion in longitudinal phase space. However, if the injection

TABLE I

RF Capture Efficiency of the Multi-Turn-Injected Beams of Au^{+33} Ions and of Protons

	RF Variation	Space† Charge	Phase Space Area	Capture Efficiency
Au^{+33}	$\hat{V}$: 1.6–17 kV	None	0.05 eV-s/amu	94% (Fig. 4)
	ϕ_s: 0.00–0.6 rad 10 ms	†2.2×10^9 per bunch	0.06 eV-s/amu	94% (Fig. 5)
p	$\hat{V}$: 90 kV ϕ_s: 0.05	None	1.3 eV-s	86% (Fig. 6)
	$\dot{B}_0$: 1.5 T/s	5×10^{12} per bunch	1.3 eV-s	82% (Fig. 6)
	$\hat{V}$: 10-90 kV	None	0.9 eV-s	99% (Fig. 7)
	ϕ_s: 0.00–0.05 rad 1 ms	†5×10^{12} per bunch	1.0 eV-s	95% (Fig. 8)

† Calculation is performed with 200 bins and 20 000 representative particles, using the three-point formula.

pulse is long enough, the rf bucket becomes filled with particles; thus the space charge effects average out and become weaker. The ability to vary the rf voltage and the synchronous phase angle is very helpful in achieving a high capture efficiency and a small bunch area. Table I summarizes the capture efficiency for the AGS Booster.

A simple criterion shows how to achieve the required accuracy of numerical simulation. The analysis shows that 200 bins are needed in the calculation to achieve good resolution in the voltage distribution of the space charge. A total of 20 000 representative particles is needed to achieve good statistical accuracy. Although the contribution from high frequency near the microwave cutoff is not included in the calculation, Fig. 4 implies that this contribution is not important, which agrees with an earlier analysis[7].

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