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ON THE TOTAL RECOMBINATION BETWEEN COOLING ELECTRONS AND HEAVY IONS

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A model is presented that enables the determination of the toal electron bare-ion recombination-rate coefficient for various values of the electron density and temperature and for all values of the ionic charge.

1. INTRODUCTION

During the process of electron cooling of heavy-ion beams, cooling electrons may recombine with ions, leading to a change of their charge state. Obviously, it is important to know the rates for these recombination processes, because they determine to a large extent the total loss of stored ions per second. The loss becomes critical if the total recombination rate becomes comparable to the cooling rate. Investigations of recombination processes in laboratory and astrophysical plasmas can serve as a guideline in estimating the required rate coefficient. However, these studies have been almost exclusively limited to hydrogenic plasmas, and it is not obvious how these results can be applied to heavy ions of both high nuclear and ionic charge. In the present paper a model is developed that allows an easy estimate of the total recombination-rate coefficient α and of its physical constituents for various values of the relevant parameters, such as electron density n_e and electron temperature T, and for all values of the nuclear charge Z.

For simplicity, we consider only the recombination between electrons and cold bare heavy ions, which generally is a complicated sequence of events. If n and n' denote the principal quantum numbers of the ionic levels concerned, and A^{+Z} a bare ion, then these events may be classified as
radiative electron capture (REC)

$$A^{+Z} + e^- \rightarrow A^{+(Z-1)}(n) + \hbar\omega, \tag{1}$$

three-body recombination

$$A^{+Z} + e^- + e^- \rightarrow A^{+(Z-1)}(n) + e^-, \tag{2}$$

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re-ionization [the inverse process of Eq. (2)]

$$A^{+(Z-1)}(n) + e^- \rightarrow A^{+Z} + e^- + e^-, \tag{3}$$

collisional de-excitation

$$A^{+(Z-1)}(n) + e^- \rightarrow A^{+(Z-1)}(n') + e^-(n' < n), \tag{4}$$

collisional excitation

$$A^{+(Z-1)}(n) + e^- \rightarrow A^{+(Z-1)}(n') + e^-(n' > n), \tag{5}$$

and radiative de-excitation

$$A^{+(Z-1)}(n) \rightarrow A^{+(Z-1)}(n') + \hbar\omega(n' < n). \tag{6}$$

Bates *et al.*[1] suggested that the net loss mechanism after a chain of interacting processes like Eqs. (1)–(6) be called collisional-radiative decay. In the following we shall summarize the processes of Eqs. (2), (3), (4), and (5) by the expression collisional transitions and ascribe to the net loss the total recombination coefficient.

The radiative electron capture in electron coolers has been treated in detail elsewhere;[2] therefore, we can omit a detailed investigation here by using the results of Ref. 2.

Bates *et al.*[1] were the first to calculate, in a statistical theory, the collisional radiative recombination for hydrogen-ion plasmas by a numerical solution of the rate equations for the processes of Eqs. (1)–(6). Additionally, they showed that their results could be scaled to bare nuclei of charge Z by means of a reduced electron density and temperature and of a reduced recombination coefficient. Unfortunately, their scaling prescriptions are only of limited profit for the parameters that are typical for the cooling of heavy ions ($n_e \approx 10^8\,\mathrm{cm}^{-3}$, $kT \leq 1$ eV and $Z > 10$).

A more refined calculation of α for cold hydrogenic plasmas has been performed by Stevefelt *et al.*[3] The improvement of these calculations consists of the use of more-reliable expressions for the collisional-transition kernels[4] that enter the coupled-state equations and the use of an extension (to about 100) of the number of bound states taken into account. Stevefelt *et al.* succeeded in fitting their numerical results by a simple analytical formula. We will use this formula for an extrapolation (certainly not stringent) to nuclear charges $Z > 1$ to compare with the results of our model.

Since, to our knowledge, appropriate data for the collisional-transition kernels in hydrogenlike, high-Z ions are not available at the moment, we developed a model based on the simplifying considerations of Byron *et al.*[5] rather than performing coupled-state calculations. This model will be outlined in Section 2, and its results will be presented in Section 3 together with a comparison with extrapolations of hydrogen-ion data to high Z.

2. CALCULATIONS

In our calculations we follow the line of arguments given by Byron *et al.*[5] They realized that under equilibrium conditions a pronounced minimum exists in the total rate of de-excitation of atoms as a function of the principal quantum number n of the excited states. This minimum occurs because, on the one hand, the collisional de-excitation probability strongly increases with the principal quantum number, whereas, on the other hand, the radiative-transition probability rapidly decreases with n, and the equilibrium population of excited states passes through a minimum.

Let n^* be the principal quantum number of the excited state at which the minimum appears. The net rate of the total recombination is limited to the rate of de-excitation of the level n^* or of a bottleneck of levels around n^*. The sharper the minimum, the narrower the bottleneck. It is obvious that n^* is a function of the electron temperature T, because in collisional excitation and de-excitation processes the exchange of kinetic energy between the electrons concerned will be on the order of kT. For a given temperature, the minimum will be sharp for small n^*—as sharp as for light ions, which have small values of n^* corresponding to binding energies around kT. Due to the strong increase of the density of bound states with increasing principal quantum number n, the minimum will become shallow for high-Z ions at a given value of kT. The position of the minimum depends also on the number density n_e of the electrons.[3,4] This is because, in addition to collisional excitation and de-excitation, radiative processes also must be considered, processes which populate and depopulate the states around n^* by transitions $n \to n^*$ $(n > n^*)$ and $n^* \to n$ $(n < n^*)$, respectively.

The total transition probability for the radiative decay of a state with principal quantum number n^* to all states with $n < n^*$ is proportional to $1/n^{*4.5}$.[6] Thus, one can expect a shift of the position of the minimum towards larger values of n^* if radiative transitions are added to collisional transitions. The ratio of collisional transition rates to radiative transition rates depends on the electron density. Consequently, the value of n^* will be a function of n_e. This dependence has been found in detailed numerical calculations of the collisional-radiative recombination in cold hydrogenic plasmas.[3]

The most extensive computations of collisional transition rates for hydrogenlike atoms have been performed by Mansbach and Keck[4] based on Monte Carlo trajectory calculations. Their result for the net recombination coefficient [Eq. (IV.18) of Ref. 4] may be scaled to hydrogenlike ions with nuclear charge Z by noting that the characteristic three-body collision rate R_0 scales as

$$R_0(Z, n_e, kT) = Z^3 R_0(1, n_e, kT). \tag{7}$$

where $R_0(1, n_e, kT)$ is given by

$$R_0(1, n_e, kT) = [N_Z]_e [n_e]_e^2 (kT/m)^{1/2} (e^2/kT)^5. \tag{8}$$

In Eq. (8), $[N_Z]_e$, $[n_e]_e$, m, and e denote the Saha-equilibrium number densities of ions and electrons and the mass and charge of the electron, respectively.

The Z^3 scaling in Eq. (8) can easily be verified using the calculation of Hinnov and Hirschberg[7] for the three-body recombination rate, if the hydrogenic energy eigenvalues $E_n(Z=1)=-R/n^2$ are replaced by $E_n(Z)=-Z^2R/n^2$, where R is the Rydberg unit of energy. The same result is obtained from simple classical[8] and dimensional[2] arguments. Inserting Eq. (7) into Eq. (IV.18) of Ref. 4, one finds for the collisional-recombination coefficient

$$\alpha_{\mathrm{coll}} = 2.0 \cdot 10^{-27} \frac{n_e Z^3}{(kT)^{4.5}} [\mathrm{cm}^3 \mathrm{s}^{-1}], \tag{9}$$

with n_e in cm^{-3} and kT in eV.

An estimate of the total radiative decay probability of the states involved in the recombination process is obtained under the following assumption: The net radiative transition rate is determined by the decay of the level n^* plus the sum of radiative transitions of all states n above n^* to all states below n^*. The average probability for the radiative decay of the state n^* is given by (Ref. 6)

$$A_{n^*} = \sum_{n<n^*} A_{n^*n} = 1.66 \cdot 10^{10} \frac{Z^4}{n^{*4.5}} [\mathrm{s}^{-1}]. \tag{10}$$

Correspondingly, the average radiative decay rate of a state n to all states below n^* is obtained from

$$A_n^{n^*} = \sum_{n'=1}^{n^*-1} A_{nn'} = 1.58 \cdot 10^{10} Z^4 \sum_{n'=1}^{n^*-1} \frac{1}{n^3 n'(n^2 - n'^2)}. \tag{11}$$

In Eq. (11) the terms $A_{nn'}$ are expressed in the Kramers approximation.[9] Since for large n^* the sum in Eq. (11) may be replaced by an integral, one finds

$$A_n^{n^*} \approx 1.58 \cdot 10^{10} \frac{Z^4}{n^5} \ln \left\{ \left[\frac{(n^*-1)^2 (n^2-1)}{n^2 - (n^*-1)^2} \right]^{1/2} \right\}. \tag{12}$$

From Eqs. (10) and (12) one finally gets the radiative recombination coefficient

$$\alpha_{\mathrm{rad}} \approx \left[A_{n^*} + \frac{1}{N_e(n^*)} \sum_{n=n^*+1}^{\infty} A_n^{n^*} N_e(n) \right] \frac{N_e(n^*)}{[N_Z]_e [n_e]_e}. \tag{13}$$

In Eq. (13) it is assumed that the population of the state n^* is close to the Saha-equilibrium population given by

$$N_e(n^*) = n^{*2} [N_Z]_e [n_e]_e \left(\frac{2\pi\hbar^2}{mkT} \right)^{3/2} \exp(-E_{n^*}/kT), \tag{14}$$

with

$$E_{n^*} = -\frac{RZ^2}{n^{*2}}. \tag{15}$$

Inserting Eqs. (10), (12), (14), and (15) into Eq. (13) and again replacing the sum by an integral one finds for the radiative recombination coefficient

$$\alpha_{\rm rad} = 2.1 \cdot 10^{-13} \frac{Z^{1.5}}{(kT)^{0.25}} (-\epsilon^*)^{1.25} e^{-\epsilon^*} + 9.6 \cdot 10^{-14} \frac{Z^2}{(kT)^{0.5}} \int_{-\epsilon^*}^{0} e^{-\epsilon} \ln \left\{ \frac{(n^* - 1)^2 [n(\epsilon)^2 - 1]}{n(\epsilon)^2 - (n^* - 1)^2} \right\} d\epsilon, \tag{16}$$

where we have used the reduced energy

$$\epsilon = -\frac{RZ^2}{n^2 kT}. \tag{17}$$

The third contribution to the total recombination coefficient α stems from the process of radiative electron capture (REC) of continuum electrons into bound states. The recombination coefficient for this channel is given by[2]

$$\alpha_{\rm REC} = \frac{1.92 \cdot 10^{-13} Z^2}{\sqrt{kT}} \left[\ln \frac{5.66 Z}{\sqrt{kT}} + 0.196 \left(\frac{kT}{Z^2} \right)^{1/3} \right] [\mathrm{cm^3 s^{-1}}]. \tag{18}$$

Thus, one obtains for the total collisional-radiative recombination coefficient α for a bare nucleus of charge Z

$$\alpha = \alpha_{\rm coll} + \alpha_{\rm rad} + \alpha_{\rm REC}, \tag{19}$$

with $\alpha_{\rm coll}$, $\alpha_{\rm rad}$, and $\alpha_{\rm REC}$ given by Eqs. (9), (16), and (18), respectively.

The problem still to be solved is the determination of the energy ϵ^* of the position of the minimum that enters into Eq. (13). Following the suggestion of Byron *et al.*,[5] ϵ^* corresponds to the energy of the state at which the total transition rate has a minimum (i.e., to the position of the bottleneck). For the calculation of ϵ^*, the REC can be neglected because this process populates predominantly the deepest-lying bound states.[2]

The total equilibrium transition rate is the sum of the collisional de-excitation rate $R(\epsilon)$ per energy interval ϵ and of the radiative transition rate A_n times the equilibrium density $[\partial N_{Z-1}(\epsilon)/\partial \epsilon]_e$ of ions per unit ϵ. An appropriate expression for $R(\epsilon)$ may be obtained from Eq. (III.7) of Ref. 4, if R_0 is scaled according to Eq. (7) of the present paper:

$$R(\epsilon) = 7.8 Z^3 R_0(1, n_e, kT) e^{-\epsilon} (-\epsilon)^{-3.83}. \tag{20}$$

$R(\epsilon)$ has a minimum at $\epsilon = -3.83$ and approaches infinity at the ionization limit $\epsilon = 0$. Because of the strong, almost-unscreened Coulomb field of the nucleus it certainly is not obvious that Eq. (III.7) of Ref. 4, which holds for hydrogenlike atoms, may be scaled to hydrogenlike ions. One should note, however, that the position of the maximum of the cross section for electron impact ionization as a function of the electron energy (the time-reversed process of the three-body recombination) is almost the same for atoms[10] and for hydrogenlike ions.[11] Thus, the position of the minimum at $\epsilon = -3.83$, which corresponds to the cross-section maximum, should not change very much going from neutral atoms to ions. In addition, the equilibrium collisional transition rate between two highly excited levels ϵ_i, ϵ_f of an ion calculated in the Born approximation[9] agrees rather well with the rate calculated in Ref. 4, if the Z^3 scaling of Eq. (7) is applied. In the present paper we use the expression of Mansbach and Keck, because it overcomes the difficulty in the Born approximation for small energy transfers $|\epsilon_i - \epsilon_f|$.

The radiative transition probability is given by Eq. (10), and, since the influence of radiative transitions of the type $n \to n^*$ on the equilibrium number density $[\partial N_{Z-1}(\epsilon)/\partial\epsilon]_e$ is negligibly small, this quantity can be expressed via the Saha-equilibrium density

$$\left[\frac{\partial N_{Z-1}(\epsilon)}{\partial \epsilon}\right]_e = Z^3\left[\frac{\partial N_Z(\epsilon)}{\partial \epsilon}\right]_e = Z^3[N_Z]_e[n_e]_e\frac{\pi^{3/2}}{2}\left(\frac{e^2}{kT}\right)\frac{e^{-\epsilon}}{(-\epsilon)^{5/2}}. \tag{21}$$

Thus, one finally must solve the equation

$$\frac{\partial}{\partial\epsilon}\left\{R(\epsilon)+\left[\frac{\partial N_{Z-1}(\epsilon)}{\partial\epsilon}\right]_e A(\epsilon)\right\}=0, \tag{22}$$

with $R(\epsilon)$ given by Eqs (8) and (20), $[\partial N_{Z-1}(\epsilon)/\partial\epsilon]_e$ by Eq. (21), $A(\epsilon)$ by Eq. (10), and ϵ by Eq. (17).

From Eq. (22) one finds the position of the minimum at an energy ϵ^* that satisfies the equation

$$-(\epsilon^*+3.83)+1.87\cdot 10^{13}\frac{(kT)^{3.75}}{n_e\cdot Z^{0.5}}(-\epsilon^*)^{3.58}(-\epsilon^*-0.25)=0, \tag{23}$$

with n_e in cm^{-3} and kT in eV.

In Eq. (23) $[n_e]_e$ has been replaced by n_e, because usually in an electron cooler $n_e - [n_e]_e$ is negligibly small for Saha-equilibrium, as may easily be verified using Eq. (14). In this context one should note that the expression [Eq. (23)] for the

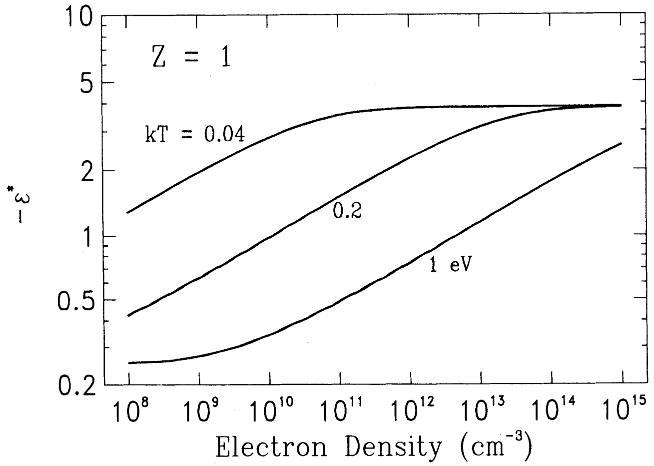

FIGURE 1 Reduced binding energy $-\epsilon^* = RZ^2/(n^{*2}kT)$ for protons as obtained from Eq. (23) as a function of the electron density for three values of the electron temperature.

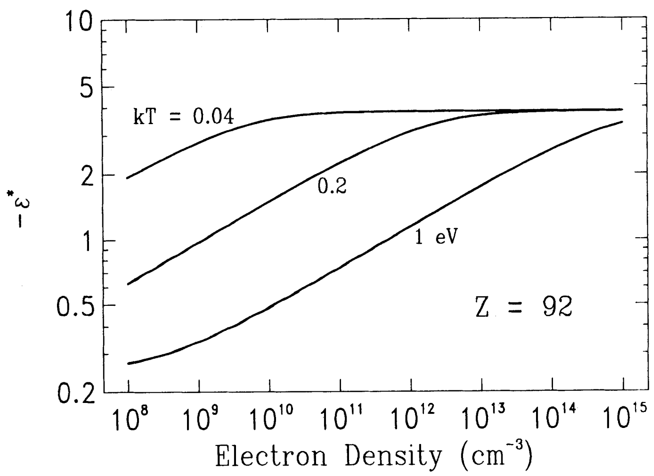

FIGURE 2 Same as Fig. 1 but for uranium ions.

minimum corresponds to Eq. (26) of Ref. 3. In the collisional limit $n_e \rightarrow \infty$ ϵ^* approaches -3.83, independent of Z. This can be considered as a support for the argument expressed in Ref. 5, which means that it is not the details of the wavefunctions that are essential for the position of the minimum but the energies of the bound states involved.

Obviously, Eq. (23) must be evaluated numerically. The results are plotted in Fig. 1 for $Z = 1$ and in Fig. 2 for $Z = 92$ as a function of the electron density in cm^{-3}. The parameter of the curves in both figures is the electron temperature in eV. As expected, the dependence of ϵ^* on Z is rather weak. The collisional limit -3.83 is reached for high Z at smaller electron densities than for low Z at a given

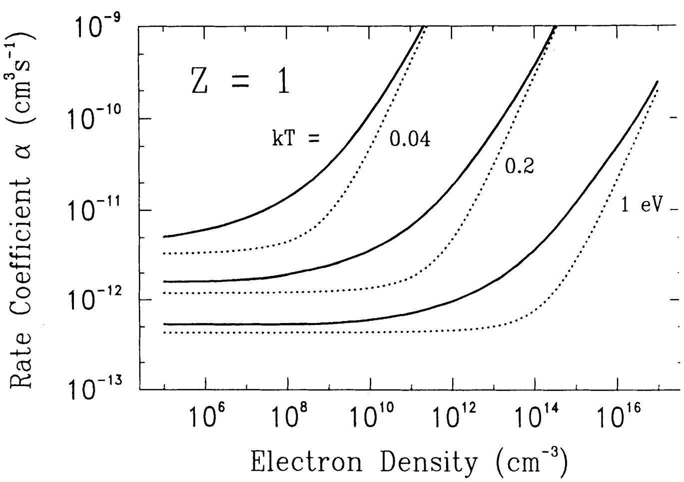

FIGURE 3 The total collisional-radiative rate coefficient for protons as a function of electron density for three values of the electron temperature. Solid lines represent our results and dotted lines the coupled-state calculations by Stevefeldt *et al.*,[3] respectively.

value of the electron temperature. The qualitative expectation expressed above that the position of the minimum will shift towards larger values of n^* (thus, smaller values of $-\epsilon^*$) is confirmed by the results shown in Figs. 1 and 2: The lower the electron densities, the more important the radiative transitions are and the smaller $-\epsilon^*$ becomes.

A comparison between rather extensive numerical calculations by Stevefelt *et al.*[3] and our model for the total collisional-radiative rate coefficient α for $Z = 1$ is shown in Fig. 3.

At fixed values of kT both calculations agree rather well in the collisional limit, whereas for medium densities—depending on the electron temperature—our model predicts values of α about a factor of 3 larger. This is because in our calculation the second term of α_{rad} in Eq. (16), which is proportional to $(kT)^{-1/2}$, is computed up to the series limit, whereas Stevefelt *et al.* take only a limited number of states into account. Since we use $N_e(n) \sim n^2$ up to the series limit, our model probably overestimates this contribution. For smaller values of the electron density, the discrepancy between the two sets of data becomes smaller than a factor of two.

3. DISCUSSION OF THE RESULTS

Figures 4 and 5 show the total collisional-radiative coefficient α according to Eq. (19) for $Z = 1$ and $Z = 92$, respectively, as a function of the electron temperature kT for an electron density of $n_e = 10^8\ \text{cm}^{-3}$. The dashed curves represent α_{coll}, the dashed-dotted curves α_{REC}, and the dotted curves α_{rad}. At low temperatures, the collisional recombination dominates, whereas for high temperatures the REC

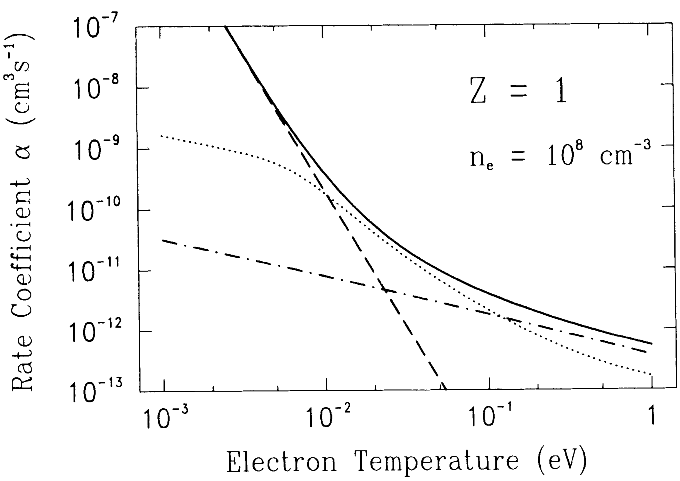

FIGURE 4 Rate coefficients for recombination of protons with electrons of density $n_e = 10^8\ \text{cm}^{-3}$. The dashed line represents the collisional contribution α_{coll}, the dotted curve the radiative contribution α_{rad}, and the dashed-dotted curve the contribution due to radiative electron capture α_{REC}, whereas the solid curve represents the sum.

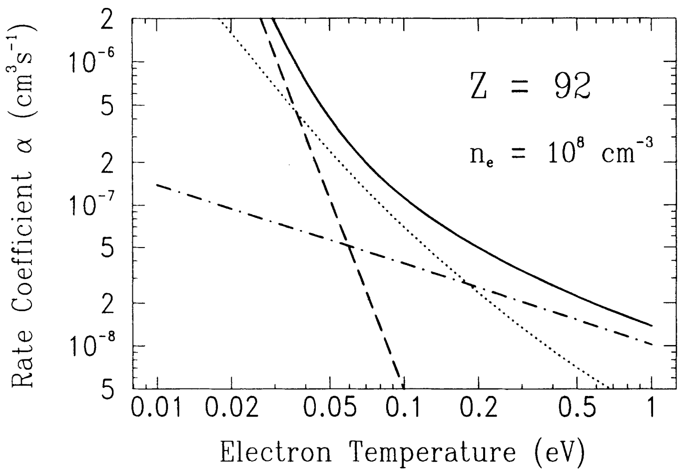

FIGURE 5 Rate coefficients for recombination of bare uranium ions with electrons of density $n_e = 10^8\,\mathrm{cm}^{-3}$. Meaning of the lines is the same as in Fig. 4.

becomes the most important process. In between, the total rate coefficient α is determined by the radiative recombination coefficient α_{rad}.

The variation of the total collisional-radiative recombination coefficient with electron temperature and density as a function of the nuclear charge Z is shown in Figs. 6 and 7. Although for small temperatures and high densities $\alpha \approx \alpha_{\mathrm{coll}}$ for $Z \geq 5$ due to the $n_e Z^3/(kT)^{4.5}$ dependence (Fig. 6), the REC α_{REC} and the radiative recombination α_{rad} are the dominant processes at smaller densities and higher temperatures for all Z. Thus, for the latter parameters n_e and kT, which are typical for most of the cooler rings planned at the moment, the collisional recombination is of minor importance.

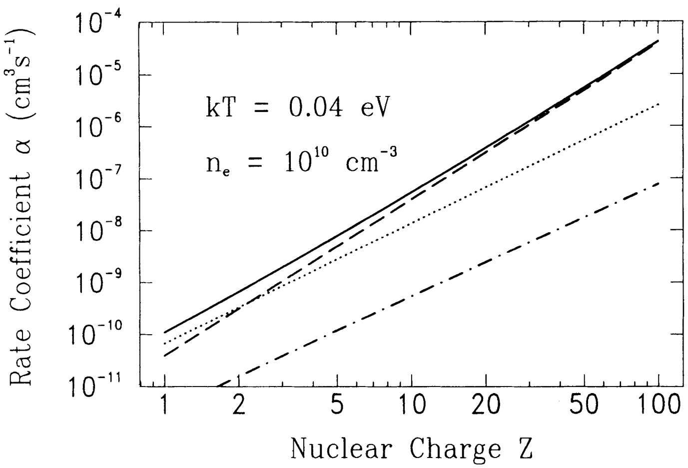

FIGURE 6 The total recombination-rate coefficient and its contributions as a function of the nuclear charge for an electron temperature of 0.04 eV and a density of $10^{10}\,\mathrm{cm}^{-3}$. Meaning of the lines is the same as in Fig. 4.

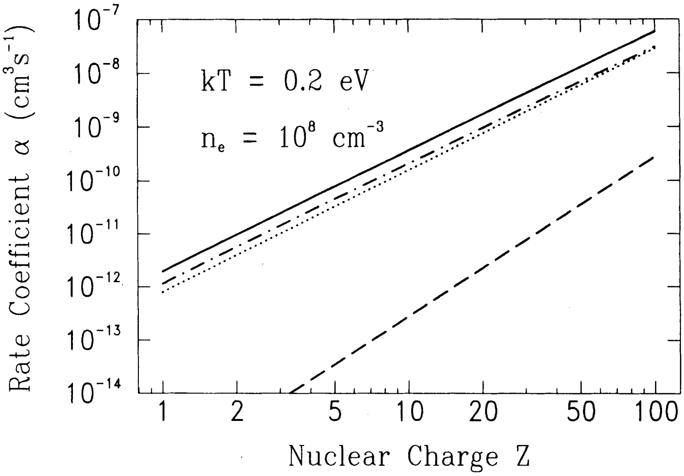

FIGURE 7 Same as in Fig. 6 but for an electron temperature of 0.2 eV and a density of $10^8\,cm^{-3}$.

The dependence of α on the electron temperature for a nuclear charge $Z = 92$ is plotted in Fig. 8 with the electron density as parameter. At temperatures $kT > 0.4\,eV$ the rate coefficient becomes almost independent of the density, because the REC (whose recombination coefficient is independent of n_e) dominates. At lower temperatures ($kT < 0.05\,eV$), the influence of the collisional recombination can be clearly seen. Obviously, for very small electron temperatures the electron-ion recombination in an electron cooler may become very critical.

Figure 9 shows α as a function of the nuclear charge at two electron

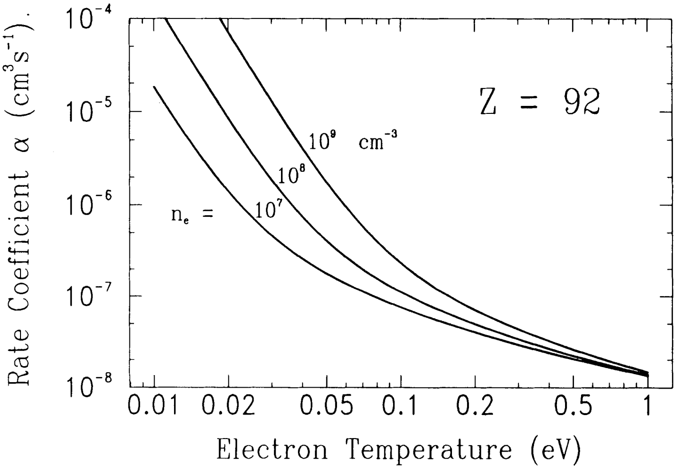

FIGURE 8 Total rate coefficient for recombination of uranium ions with electrons at three densities as a function of temperature. At high temperature the three curves merge because REC, which is independent of density, dominates.

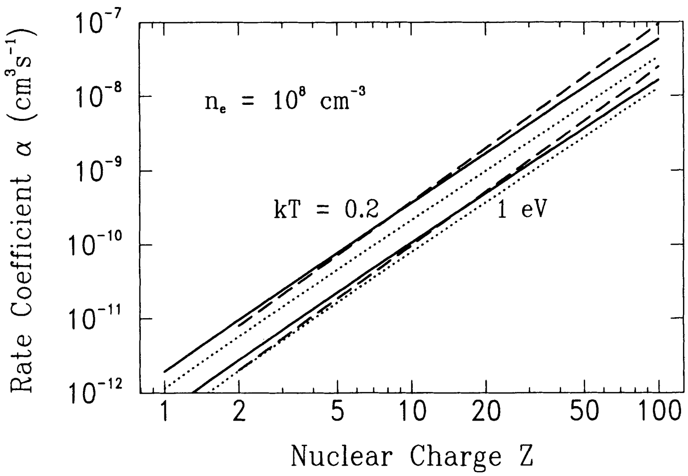

FIGURE 9 Comparison of the present results (solid curves) for the total recombination-rate coefficient with extrapolations using either a $Z^{2.4}$ scaling (dashed curves) of the collisional-radiative rate coefficient calculated by Bates *et al.* or applying the Z scaling of temperature and density to the α_{rad} term calculated by Stevefelt *et al.* (dotted curves). The REC contribution is the same in all cases.

temperatures; the electron density is $10^8\,\mathrm{cm}^{-3}$ (solid lines). The strong increase of the total recombination coefficient with decreasing temperature is obvious.

The result of our model of the Z dependence of the total recombination coefficient may be compared to scaling prescriptions based on rather sophisticated numerical calculations of α in H and H-like plasmas.[1] Realizing that α/Z, kT/Z^2, and n_e/Z^7 are appropriate reduced quantities, Bates *et al.*[1] found from their calculations that for electron densities that are typical for an electron cooler, α scales approximately as $Z^{2.4}$. The corresponding results are given by the dashed lines in Fig. 9 for $kT = 0.2$ and 1 eV at $n_e = 10^8\,\mathrm{cm}^{-3}$.

However, one has to keep in mind that the $Z^{2.4}$ scaling strictly applies only for values of charge Z and temperature kT that are far beyond those plotted in Fig. 9.

One can also try to scale Eq. (29) of Ref. 3, which applies to hydrogen plasmas, to higher Z using these reduced quantities. However, one finds that the expression adopted for the REC is not precise enough for a large extrapolation. To remove this drawback, this contribution to α has been fixed according to Eq. (18) (See Ref. 2.). The two other terms in Eq. (29) of Ref. 3, which both contain the electron density and temperature, have been scaled by n_e/Z^7 and kT/Z^2, respectively, to calculate α/Z. Note that this scaling gives for the collisional-recombination coefficient the proper Z^3 behaviour. The results are plotted as dotted lines in Fig. 9. The rather good agreement between our simple model and the extrapolations of detailed numerical calculations is quite satisfying.

Based on our results for the recombination coefficient α, an estimate of the time constant τ_{loss} can be obtained determining the lifetime of a stored ion beam. If η denotes the ratio of the length of the cooler to the circumference of the

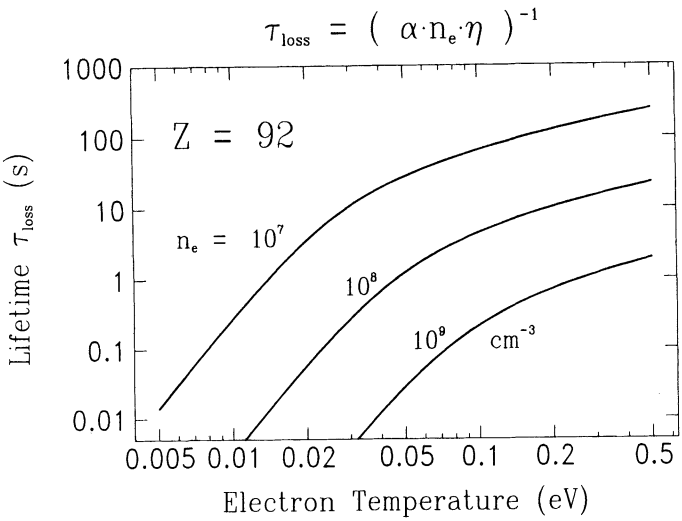

FIGURE 10 The time constant τ_{loss} of a uranium beam as determined by recombination losses as a function of electron temperature. At high ion-beam velocities, the time constant shown must be multipled by the relativistic parameter γ^2.

storage ring, and γ denotes the relativistic variable, then

$$\tau_{\mathrm{loss}} = \gamma^2(\alpha n_e \eta)^{-1} \approx (\alpha n_e \eta)^{-1}, \tag{24}$$

because γ is of the order of 1 for present heavy-ion cooler rings.

Figure 10 shows the liftime τ_{loss} of an uranium beam as a function of the electron temperature, where a value of 0.02 has been adopted for η. The temperature dependence divides into two regions, with the radiative electron capture dominating at high temperatures ($\tau \propto kT^{0.5}$) and the collisional recombination dominating at low temperatures ($\tau \propto kT^{4.5}$). For an electron density of $10^8\,\mathrm{cm}^{-3}$ and temperatures of $kT > 0.2\,\mathrm{eV}$, beam lifetimes can exceed 10 s. However, Fig. 10 clearly shows the increase of problems encountered when electron cooling of heavy ions is envisaged at substantially lower electron temperatures.

Finally, we mention a critical point, the discussion of which is far beyond the scope of this paper, namely, the assumption that in Saha-equilibrium the statistical weight of the excited state is simply n^2 [*cf* Eq. (12)]. In the presence of the magnetic guiding field in the cooler and of the strong electric field due to the electronic and ionic space charge, the electronic structure of the ions will be changed, especially for large principal quantum numbers n. Obviously, it is inevitable that this problem must be treated theoretically in more detail. Some information about these states can be obtained from experiments in which the radiation emitted during the recombination process will be studied. Intensity ratios, line shifts, etc., will probably tell something about these exotic states.

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