Particle Accelerators, 1988, Vol. 23, pp. 265–277
Reprints available directly from the publisher
Photocopying permitted by license only

A SUPERCONDUCTING RFQ FOR AN ECR INJECTOR

I. BEN-ZVI†

Physics Department, The State University of New York at Stony Brook, Stony Brook, NY 11794, USA

(Received October 10, 1987; in final form January 6, 1988)

The beam dynamics and resonator properties of a superconducting radio-frequency quadrupole (RFQ) for heavy ions are discussed. The motivation is its use as a very low velocity section following an electron cyclotron resonance (ECR) source for injection into a superconducting heavy-ion linac. The constraints on the design and performance of this accelerating structure are presented. Expressions for a limiting stable phase angle and longitudinal and transverse acceptance are derived. A numerical example is given, using the SUNYLAC linac at SUNY Stony Brook. Beam-dynamics calculations with PARMTEQ are reported, verifying the theoretical beam-dynamics calculations.

1. INTRODUCTION

The performance of electron-cyclotron-resonance (ECR) sources today makes it very attractive to use an ECR source and a very low β linac section to replace the tandem accelerator as the injector to superconducting linacs.

What we would like to review here is the possible application of a superconducting RFQ (radio-frequency quadrupole) accelerating structure as this very low β structure.

The RFQ[1,2] is an accelerating structure comprising four parallel conductors in a quadrupole geometry. A particle beam transported along the central axis of this structure will be focused by the quadrupole rf electric field, which is imposed on the structure by a suitable resonator structure. This focusing is particularly effective for slow ions since it is electric focusing. Acceleration is obtained by spatially modulating the conductors' diameters out of phase on orthogonal pairs.

The RFQ has been applied with great success to a variety of low-velocity ion accelerators. The superconducting RFQ presents a special design challenge due to the combination of low charge-to-mass ratio of heavy ions and the low surface electric fields of the superconductor relative to normal conducting resonators.

Optimizing the superconducting RFQ (SRFQ) for heavy ions produces an unusual design. We shall follow the various beam-dynamics calculations and analyze the implications for this particular choice of constraints. This study presents also a numerical example, based on the Stony Brook linac (SUNYLAC),

† Permanent address: Nuclear Physics Department, The Weizmann Institute of Science, Rehovot 76100 Israel.

showing that it is possible to design an attractive SRFQ that will take all the beams up to uranium from an ECR source on a 400-kV platform and accelerate them to the velocity acceptable by the SUNYLAC, ($\beta = 0.05$).

The SRQF is complicated to manufacture. Precise CNC milling of large sections is required because all the adjustments we find in a regular array of resonators and lenses (amplitudes, phases, lens currents) are "built into" the RFQ geometry, which must be calculated precisely before manufacturing.

The SRQF accelerating structure is made up of very few components, since the structure is multigap and continuous and includes a built-in focusing system. The very low β section we are discussing will consist of just three resonators.

The RFQ is very well suited for superconducting use because many resonator forms (like the four-vane) have a high mechanical stability, low stored energy, and high degree of field symmetry that should make them easy to stabilize against acoustical vibrations. Furthermore, such resonators are very well suited to the technology of lead (or lead-tin) plating on copper due to their structure and the very low surface magnetic fields.

Unlike the situation with most other resonators used for heavy-ion acceleration, with an RFQ it is impossible to consider separately the "resonator" and the beam dynamics. It is necesary to understand well dynamics of the beam as well as the resonator and cryogenics and blend them all into a practical design.

For the purpose of the numerical example, as well as for guidance in making approximations, we are going to make the following assumptions:

The RFQ will be injected by an ECR source on a 400-kV platform. Such platforms have demonstrated a voltage stability of 10 V.

The RFQ will inject into the existing superconducting linac, which operates at 150.4 MHz and a minimum $\beta = 0.05$.

The current, for the purpose of space-charge considerations, is negligible.

The linac frequency and the size of four-vane resonators limit us to a choice of either 75.2 MHz or 150.4 MHz. Although the use of a subharmonic buncher enables one to use any frequency below 150.4 MHz, a harmonic of the buncher, the above choice retains the option of a quasi-dc beam by bunching at, e.g., 75.2 MHz.

The charge-to-mass ratio of the source for the heaviest ion will be taken as 0.18. Then we inject the RFQ at $\beta = 0.0124$ (72 keV/amu for all ions).

The peak surface electric field is an important consideration for a superconducting structure and has important consequences for the RFQ design. The performance of superconducting Pb–Sn or pure Pb is 15 MV/m (realistic) up to 18 MV/m (optimistic). The 15 MV/m, which we shall use in numerical examples, corresponds to the high-β split-ring resonators of the SUNYLAC operating at about 2.7 MV/m. This is a value that is obtained regularly for a large number of resonators operating in the beam line for extended periods.

The beam dynamics and the structure of the RFQ are determined solely by the product of the peak surface electric field and the charge-to-mass ratio. Thus, extra performance in the field can be traded for a low charge-to-mass performance.

It is important to know the emittance of the ECR source. We shall use the following numbers[3]: The energy spread is better than 10q eV, where q is the charge of the ion. Typical normalized emittance of light ions is 0.1 mm mrad for extraction with 10 kV and an aperture 8 mm in diameter. The emittance improves as one goes to heavier ions. For uranium the normalized emittance might be an order of magnitude better.

We shall depart from the traditional RFQ design even further by using a separate bunching system. The cost of the SRFQ does not justify sacrificing valuable acceleration space for bunching by the RFQ,[4] especially since bunchers for nuclear-physics applications have demonstrated good efficiencies and performance in similar situations.

2. BEAM DYNAMICS CONSIDERATIONS

2.1. *Equations for the RFQ*

The work of Los Alamos[5] provides us with the basic equations. Let m be the modulation constant. The distance between the axis and any of the four electrodes (vanes) is modulated between a and ma.

The acceleration constant A is given by

$$A = \frac{m^2 - 1}{m^2 I_0(ka) + I_0(mka)}, \tag{1}$$

$$k = \frac{2\pi}{\beta\lambda}, \tag{2}$$

λ is the free-space wavelength of the rf mode, and β is the beam velocity (in units of c).

The focusing parameter χ is given by

$$\chi = 1 - A I_0(ka). \tag{3}$$

The energy-gain gradient dW/dz for an ion at a radius r is

$$\frac{dW}{dZ} = \frac{\pi q V A \cos\phi_s}{2\beta\lambda} I_0(kr), \tag{4}$$

where q is the ion charge, V is the inter-vane peak voltage, and ϕ_s is the stable particle phase. From this we get the accelerating field (on the axis and including the transit time factor, but not including $\cos\phi_s$)

$$E_a = \frac{\pi A V}{2\beta\lambda}. \tag{5}$$

2.2. *The Transverse Stability Problem*

From the K–T work[2] we have the equation of transverse motion,

$$\frac{d^2x}{d\tau^2} + (\Delta - B\cos 2\pi\tau)x = 0, \tag{6}$$

where

$$\tau = (\omega t + \phi_s)/2\pi, \tag{7}$$

$$B = \frac{qV}{Mc^2}\frac{\lambda^2}{a^2}\chi, \tag{8}$$

$$\Delta = \frac{\pi^2}{2}\frac{qV}{Mc^2}\frac{A}{\beta^2}\sin\phi_s, \tag{9}$$

and M is the mass of the ions.

This is the familiar Mathieu equation. In the plane (Δ, B) we have to confine these parameters to the region of stability. Since we must have $\sin\phi_s < 0$ for longitudinal stability and since we do not have to worry about too much radial focusing (B is quite small for the charge-to-mass ratio and surface fields we expect) the stability condition will be met for points above the stability line, which is defined to first order in Δ by

$$B^2 = -8\pi^2\Delta, \tag{10}$$

and to the left of the $\Delta = 0$ axis (negative Δ). By substituting the values of Δ and B from above, we get an equation for $\sin\phi_s$:

$$-\sin\phi_s \leq \frac{1}{4\pi^4}\frac{qV}{Mc^2}\frac{\lambda^4}{a^4}\frac{\chi^2}{A}\beta^2. \tag{11}$$

This is the value of ϕ_s on the transverse instability line. To be inside the stability region ϕ_s has to be smaller than this limit.

The value of the charge-to-mass ratio is given by the set of initial assumptions as well as the peak surface electric field, which is roughly V/a. Thus it leaves only λ and, to some extent, a as free parameters. Since λ can be changed only between 2 and 4 meters (again by the initial assumptions) it is clear that there is a very limited control over ϕ_s. The values calculated for ϕ_s by this expression using the set of parameters given in Section 1 are equal or smaller than 15 degrees in absolute value. This is a small value, since typical stable phase angles are -20 to -30 degrees. Therefore we should next look carefully at the longitudinal acceptance and see if it is still reasonable with the stable phase-angle values imposed by the transverse motion.

2.3. *The Longitudinal Acceptance*

The particles that are trapped by the accelerating field (move in stable longitudinal oscillations about the phase-stable particle) are moving inside ellipses in longitudinal phase space. The axis ratio for these ellipses, which is also the optimal injection ratio, is given by

$$\frac{\delta W}{\Psi}=\left(-\frac{\lambda}{2\pi}qE_a\beta^3\gamma^3Mc^2\sin\phi_s\right)^{0.5}, \tag{12}$$

where δW and ψ are the excursions of the energy and phase of a given particle from those of the stable particle.

Since the width of the acceptance in phase is $3\phi_s$, we can write for the total longitudinal acceptance:

$$\frac{\delta W}{Mc^2}\Psi=\frac{1}{2}\sqrt{-\frac{qV}{Mc^2}A\beta^2\sin\phi_s(3\phi_s)^2}, \tag{13}$$

where $W=0.5Mc^2\beta^2$ is the particle energy, and $\gamma\sim1$. The acceptance is given here as the product of the full energy spread by the full phase spread. The ideal buncher will be able to produce a bunch with the correct axis ratio and the emittance

$$\frac{\delta W}{W}\Psi=2\pi\frac{\delta W_s}{W}, \tag{14}$$

where δW_s is the energy spread of the source. For a nonideal buncher there will be an increase in the emittance due to the nonlinearity of its waveform. Since this emittance has to fit into the acceptance fo the RFQ, we equate the two and get an upper limit on the source energy spread:

$$\frac{\delta W_s}{Mc^2}\leq\frac{9}{4\pi}\phi_s^2\sqrt{-\frac{qV}{Mc^2}A\beta^2\sin\phi_s}. \tag{15}$$

Now we can combine the constraint on ϕ_s we have from the transverse-stability considerations with the last equation and obtain a limit on the source performance.

For small values of ϕ_s we can approximate $\sin\phi_s$ by ϕ_s. Also we can now replace ϕ_s by the upper limit we have on it from the transverse-stability diagram, [Eq. (11)], and we obtain the condition on the source energy spread in terms of the RFQ parameters:

$$\frac{\delta W_s}{W}\leq4.77\times10^{-7}\left(\frac{qV}{aMc^2}\right)^3\frac{\chi^5}{A^2}\beta^4\frac{\lambda^{10}}{a^7}. \tag{16}$$

For the basic set of parameters (Section 1) a frequency of 75 MHz, $\beta=0.0124$, $a=7$ mm, and $\phi_s=-10$ degrees we get $\delta W_s/W=0.018$. Although this number is very large, it is well to remember that it indicates the maximum longitudinal acceptance at the limit of the ϕ_s for transverse stability, so that an order of magnitude should be taken off this for a realistic limit.

How does it compare to the longitudinal emittance of the ion source? The voltage stability of a 400-kV platform is about 10 V. The energy spread of the ECR source is also equivalent to a 10-V spread. Thus if we assume an energy spread of 1 part in 10^4 it will cover both major contributors and leave some margin of safety. Half the current (or π radians) at the fractional energy spread of 10^{-4} results in an emittance of $\pi\ 10^{-4}$. The acceptance of the RFQ operating at $\phi_s = -10$ degrees and the conditions described above is 2.3×10^{-2}, a factor of 72 times larger. This is just as well, since this acceptance applies when the beam fills the separatrix. The emittance of the source should enable us to operate with a phase spread of just 3.5 degrees, or about 125 ps. This type of performance can be obtained by multielement bunching systems like the ones in operation at ANL and at Stony Brook. However, there is no need for such a performance, since the acceptance is large enough to admit a 0.5-ns bunch.

2.4. *Transverse Acceptance*

The transverse acceptance of the RFQ is large because of its short focal period, $\beta\lambda$. The acceptance can be defined by the physical aperture of radius a as

$$\varepsilon = \frac{a^2}{\beta_+ \beta\lambda}, \tag{17}$$

where β_+ is a dimensionless variable determined by the operating point in the (B, Δ) plane. To normalize the acceptance we have to multiply it by $\beta\gamma$. However, $\gamma \sim 1$ for our beams; thus

$$\varepsilon_n = \frac{a^2}{\beta_+ \lambda}. \tag{18}$$

However, this acceptance does not consider the effect of coupling between the transverse and longitudinal motions. A better estimate is obtained by considering the nonlinearity of the Bessel functions in the accelerating field. A particle moving at a radius r will gain more energy than a particle on the axis [see Eq. (4)]. At small kr values we can approximate the Bessel function by $1 + (kr/2)^2$. We can set an upper limit to the radius r by requiring that this extra energy gain, which is proportional to $(kr/2)^2$, is much smaller than the excursions in energy due to the longitudinal motion.

The distance over which this process takes place is approximately half a longitudinal period, that is

$$\sqrt{\frac{2\pi\beta^3 Mc^2\lambda}{-kqAV \sin\phi_s}}.$$

This assumes that the betatron and synchrotron periods are not very different (as is usual in RFQs).

Let us require that the extra energy gain at the maximum radius r, as given by the $(kr/2)^2$ term in Eq. (4), over half the longitudinal period is equal to $(\delta W/\psi)\phi_s$, the longitudinal energy excursion [(the expression for $\delta W/\psi$ is Eq. (12)]. This yields the following expression for the normalized acceptance:

$$\varepsilon_n = \frac{1}{\pi^3} \frac{\beta^2\lambda}{\beta_+} \phi_s \sin\phi_s. \tag{19}$$

The acceptance given by this expression does not represent an exact aperture. It is an approximate threshold for the onset of nonlinear cross coupling between the transverse and longitudinal motions, which leads first to longitudinal emittance degradation, then to actual particle loss.

This expression yields $\varepsilon_n = 0.12$ mm mrad for 75 MHz, $\beta = 0.0124$, and $\beta_+ = 5$. This conservative estimate of the transverse acceptance is still large enough for the emittance of the ECR source, particularly for heavy ions for which the normalized emittance of the ECR source is much better.

To get a good estimate of β_+ we have to construct the two-by-two beam-transport matrix P, from the center of a focusing cell to the center of the next focussing cell. We will approximate Mathieu's equation [Eq. (6)] by a "square wave", where the value of the focusing term $B \cos 2\pi\tau$ alternates between $B/\sqrt{2}$ and $-B/\sqrt{2}$. Then we construct the transfer matrix, as described in Ref. 6, and we get

$$P_{1,1} = P_{2,2} = \cos \Omega \tag{20}$$

$$P_{1,2} = \beta_+ \sin \Omega \tag{21}$$

$$P_{2,1} = -\frac{1}{\beta_+} \sin \Omega, \tag{22}$$

where

$$\cos \Omega = \cosh \Theta - \cos \Theta_+ + \frac{l_+}{2l_-} \sinh \Theta_- \sin \Theta_+ \left[1 - \left(\frac{l_-}{l_+}\right)^2\right] \tag{23}$$

and

$$\beta_+ = \frac{l_+}{\sin \Omega}\left[\cosh \Theta_- \sin \Theta_+ + \sinh \Theta_- \left(\frac{l_-}{l_+} \cos^2 \frac{\Theta_+}{2} + \frac{l_+}{l_-} \sin^2 \frac{\Theta_+}{2}\right)\right]. \tag{24}$$

We relate the variables to the already known B and Δ through the following:

$$\Theta_\pm^2 = \frac{1}{4}\left(\frac{B}{\sqrt{2}} \pm \Delta\right) \tag{25}$$

$$l_\pm^2 = \frac{1}{\dfrac{B}{\sqrt{2}} \pm \Delta}. \tag{26}$$

For a 75-MHz RFQ at $\beta = 0.012$ we get $\beta_+ \sim 5$, and for a 150-MHz RFQ at $\beta = 0.031$ we get $\beta_+ \sim 11$.

Since β_+ is also the axis ratio of the matched beam ellipse, it is useful to note that in units of meters per radians we have to multiply the given value of β_+ by $\beta\lambda$.

3. OPTIMIZING THE ACCELERATION

In order to achieve a reasonable size for the large accelerations required here, one must optimize the E_a/E_p (ratio of average accelerating field to peak surface electric field).

The peak surface field, as given then by Ref. 7, is

$$E_p = \frac{1.45V}{r_0}. \tag{27}$$

For certain RFQ vane geometries the numerical constant can be reduced from 1.45 to 1.35 (Ref. 7). However we shall continue to use the more conservative value of 1.45.

Since

$$r_0 = a/\sqrt{\chi}, \tag{28}$$

then

$$E_p = 1.45\sqrt{\chi}\frac{V}{a}. \tag{29}$$

Thus we obtain the ratio:

$$E_a/E_p = \frac{kaA}{5.8\sqrt{\chi}}. \tag{30}$$

(E_a does not include the cos ϕ but includes the transit-time factor). This function has a broad maximum at $ka \sim 1$ (for $m \sim 2$).

This places an unusual constraint on a. It is unusual in the sense of the traditional RFQ design in which long resonators are made, usually with constant values of V and r_0. These designs may operate in the constant-surface-field regime, but they can not keep ka near its optimum and have a constant surface electric field at the same time; ka should be kept nearly constant even though β increases. This can be done by changing either a or λ, or both, and changing V. This makes it necessary to break the structure into a few separate resonators (tanks in the usual jargon) with a nearly constant surface electric field in all of them, but different values of V, a.

A calculation of the peak surface electric field along the structure for a constant a and constant m shows that, for the range of parameters of interest, E_p changes somewhat, mostly in the first resonator. Thus perhaps a slight ramp in V could be introduced in this resonator to level the peak surface field.

Now we know that we may not take any value of a due to beam-dynamics considerations. So we have to optimize a and m considering both the acceleration and beam dynamics simultaneously. In other words, the longitudinal acceptance and the accelerating fields can present conflicting demands on the parameters m and a. However, a reasonable compromise can be made. The choice of $m = 2.5$ and $a = 7$ mm results in, for the first resonator, an accelerating field that starts at better than 1.8 MV/m at the entrance to the resonator and ends at 1 MV/m, accelerating the beam from $\beta = 0.0124$ to $\beta = 0.031$ in 1.5 meters.

4. CHANGING THE FREQUENCY

Since the diameter of a 75-MHz RFQ is rather large if a four-vane resonator is used, there is interest in going to a higher frequency for the next resonators, even if this may require another reduction in the accelerating field. The particular choice of frequency depends also on the buncher system. For example, a 25-MHz buncher would enable one to select 100, 125, or 150 MHz as the next frequency, whereas a 75-MHz buncher leaves just 150 MHz as a possible option.

A frequency change will be possible if the acceptances (both transverse and longitudinal) are not reduced (and preferably increased), and the beam can be optimally matched to the new resonator.

The normalized transverse acceptance is given by Eq. (19). It can be seen that the acceptance grows as β^2, so that halving λ is acceptable even with the noted increase of β_+ by a factor of 2. However, we have a problem with the stability diagram of the transverse motion, as can be seen from Eq. (8), which we can rewrite as follows:

$$B = \frac{q\sqrt{\chi}V/a}{Mc^2}\frac{\lambda^2}{a}\sqrt{\chi}. \tag{31}$$

This emphasizes the fact that the first term on the RHS is a constant, and thus, without a change in a, B is just proportional to λ^2. This is the core of the difficulty in designing a superconducting heavy-ion RFQ. If we want to increase the frequency there is no choice but to reduce a somewhat and also operate at a lower B value. That explains why it was stated that β_+ increases by a factor of 2 when the frequency is doubled. However, when we look at the limit on ϕ_s resulting from the stability-diagram problem, we see that we are helped there by the decrease in Δ due to the velocity change [Eq. (9)]. It seems that a will have to be reduced by less than a factor of two to maintain the same stable phase angle.

Similarly, the longitudinal acceptance, [Eq. (13)] can be written as follows:

$$\frac{\delta W}{Mc^2}\delta t = \frac{\beta}{\omega}\frac{(3\phi_s)^2}{2}\sqrt{-\frac{q\sqrt{\chi}V/a}{Mc^2}\frac{Aa\sin\phi_s}{\sqrt{\chi}}}. \tag{32}$$

It is easy to see that the canonical longitudinal acceptance is proportional to β/ω and thus will present no obstacle to an increase in frequency.

The next consideration is beam matching. The change in β_+ does not create an acceptance problem, but it makes it necessary to change the axis ratio of the transverse phase-space ellipse if we want to maintain a matched beam. A lens will be required between the first and second resonator to change the axis ratio.

The beam matching in the longitudinal motion is aided by adiabatic phase damping because the motion of the beam maintains the ratio given by Eq. (12). For a constant E_a this would have produced a reduction in the phase spread in proportion to $\beta^{0.75}$. However, E_a is proportional to β^{-1}; therefore, the phase will be damped as $\beta^{0.5}$. For a frequency doubling (which doubles the values of Ψ) some matching scheme for an additional reduction of the phase spread should be provided.

5. THE RESONATOR

The resonator most suitable for the SRFQ is the so-called "four-vane" or "four-chamber" resonator. This is essentially a distorted (capacitively loaded)

TE210 resonator. Due to the strong capacitive loading of the "vanes" the resonance frequency is about four times lower than the unloaded TE210 resonator, thus making for a compact resonator.

This resonator has been studied extensively due to the intensive work which has been done on the RFQ.[8,9] There is considerable experience in the construction, adjustment, and coupling of the four-vane resonator.

For superconducting work the four-vane resonator has the advantage of being a very rigid structure. It has a high degree of symmetry and is expected to have a relatively low mechanical Q. All of these properties make it stable against acoustical noise, a most important requisite for a superconducting resonator. Naturally it will be important to test this experimentally with a prototype.

More particular properties of interest to the designer of a superconducting accelerating structure are E_p/E_a, H_p/E_a, P/V^2, U/V^2, Q_0, and R (the radius of the resonator at the given frequency f).

The parameter E_p/E_a has been evaluated previously. Even with the compromises that we have been forced to make we find that just three resonators, 1.5, 1.7, and 1.8 meters long, will do the job of accelerating the ions from 72 keV/A to 1.17 MeV/A with the given charge-to-mass ratio of 0.18. This nice performance is due to the economy of space characteristic of a continuous accelerating structure with internal focussing.

We shall present some of the expressions of Refs. 8 and 9 in the following.

The important parameter H_p/E_a is given by[8]

$$\frac{H_p}{V} = 2 \times 10^4 f C_l \left(\frac{\text{gauss}}{\text{MV}}\right), \tag{33}$$

where f is the resonator frequency in Hz, C_l is the vane load capacitance in F/m,

$$C_l = 48 \times 10^{-12} \left(\frac{r_0}{\lambda}\right)^{-1/6}, \text{ and} \tag{34}$$

r_0 is the average radius of the vane tip, which is modulated between a and ma. For the range of interest we can take C_l to be 120 pF/m. Thus we get for our resonator

$$\frac{H_p}{V} = 2.4 f(\text{MHz}) \left(\frac{\text{gauss}}{\text{MV}}\right). \tag{35}$$

Substituting 75 MHz and a voltage of 0.125 MV, we obtain 22.5 gauss for the peak surface magnetic field, an unusually low value for a superconducting resonator. It means that with respect to magnetic losses, the resonator will always operate in a region of very low field; thus the Q_0 vs. E_a curve should be flat up to the field-emission level.

Furthermore, conditioning of field emission may be done at a very high power level without the problem of magnetic breakdown.

Finally, this number makes it possible to use easily machineable telurium copper for the resonator rather than OFHC copper and also to use conduction cooling for large sections of the resonator because the specific heat dissipation will be low, being proportional to H_p^2.

Another expression can be obtained for an RFQ with simple vanes[9] that does not involve the intervane capacitance,

$$\frac{H_p}{V} = 10.6\frac{\lambda}{R^2}\frac{\log R/r_0}{\left[1-\left(\frac{r_0}{R}\right)^2\right]\log 0.1417\lambda/r_0}\left(\frac{\text{gauss}}{\text{MV}}\right). \tag{36}$$

The power dissipation for a copper resonator at room temperature is

$$\frac{P}{V^2} = 1.3\times 10^9(fC_l)^{1.5}L\left(\frac{\text{watt}}{(\text{MV})^2}\right), \tag{37}$$

where L is the length of the resonator. For a 1.5-m resonator and $V = 0.125$ MV, we get $P = 26$ kW. With the improvement factors that we have, not counting of course the field emission losses, we expect to have 1 watt for the superconducting resonator. Naturally the losses will be increased by the field emission, depending on how hard one has to push the accelerating field. Thus in actuality we should be ready to spend a few watts.

The low power consumption is another reason why lead plating on copper is such a suitable technology for the SRFQ. The marginal benefit of improving the surface resistance by the use of niobium is negligible in this case.

The stored energy is also important for the control of the frequency of the super-conducting resonator. In particular it is interesting to see what will it be for such a large resonator. From Ref. 8 we have

$$\frac{U}{V^2} = 0.5\times 10^{12}C_lL\left[\frac{\text{joule}}{(\text{MV}^2)}\right]. \tag{38}$$

Again using the same numbers for L, C_l and V we get $U = 1.5$ joules. This is only about four times higher than a single high-β SLR of the Stony Brook Superconducting Linac.

Q_0 is not an independent parameter once P, U, and ω are given, but it is interesting to have it too. For the room temperature copper resonator we have[8]

$$Q_0 = 2.4\times 10^3(fC_l)^{-1/2}. \tag{39}$$

For our resonator we expect $Q_0 = 2.5\times 10^4$.

The equation for the resonator radius, from Ref. 9, is

$$R = \frac{\lambda}{2\pi}\left(\frac{2}{\log\frac{\lambda}{\pi r_0} - 0.5772}\right)^{1/2}. \tag{40}$$

For our frequency we get $R = 43.6$ cm. This is a large radius, which is the price we have to pay for all the other nice properties. We can choose another type of resonator for the 75-MHz unit that will reduce the physical dimensions considerably, such as the $\lambda/2$ resonator for each of the "vanes." However, this is beyond the scope of this report.

The machining of the resonator is estimated to take roughly 150 hours per vane

TABLE I

Calculated parameters for a superconducting RFQ for heavy ions from an ECR source (the peak surface field is 15 MV/m, ϕ_s is -10 degrees, and the modulation constant m is 2.5 for all three resonators)

Quantity	Units	Resonator 1	Resonator 2	Resonator 3
Frequency	MHz	75.2	150.4	150.4
β_{in}	—	0.0124	0.031	0.041
β_{out}	—	0.031	0.041	0.050
Length	m	1.5	1.7	1.8
Diameter	cm	87.2	44.2	45.5
a	mm	7	4	5
$\phi_{s,max}$	degrees	16.6	17.0	15.0
V	MV	0.125	0.0759	0.0952
ε_n	mm mrad	0.11	0.16	0.23
β_+	mm/mrad	0.22	0.76	1.22
$\frac{\delta W/W}{\psi}$	$10^{-3}\,\mathrm{deg}^{-1}$	2	1.8	1.5
E_a(average)	MV/m	1.4	1.1	1.1
H_p	gauss	22.5	27.3	34.2
P(Cu)	kW	26	30.7	51.2
U	joules	1.5	0.59	0.98

on a CNC milling machine. Additionally, one expects approximately 100 hours of software effort (preparing the tapes for the CNC mill).

Table I presents the various properties of the RFQ resonators that have been calculated in this work, based on four-vane resonators.

6. COMPUTER CALCULATIONS

The code SUPERFISH has been used to test the numbers quoted.[10,11] A variable-cross-section vane of a practical shape was used with a square outer conductor that fits into a 86 cm by 86 cm square. This is somewhat larger than the 43.6-cm radius because of the straight sections in the cross section. The radius r_0 was 1.58 cm, which is slightly larger than the optimum radius for the heavy-ion application. The frequency was 74.8 MHz, and for a voltage of 0.125 MV we have 32.7 kW power, 1.63 J stored energy, $Q = 23{,}452$ the peak surface magnetic field is 24.7 gauss, and the peak surface electric field is 8 MV/m. The last number is low because it is particularly affected by the large r_0.

The computer program PARMTEQ[5] has been used to check the validity of the design presented here.[12]

Two resonators have been simulated: the 75-MHz unit, which is critical for the acceptance from the source, and the first 150-MHz resonator, which follows it, to check the concept of frequency doubling. The parameters used are the same as given in Table I.

The particles accelerated were U_{238}^{43+}. The emittance of the beam at the input of the RFQ was 0.08 mm mrad (normalized) in x and in y and 0.63 degree-MeV in z; 100 particles were accelerated without loss. The only nonlinear effects observed were a small increase in the longitudinal emittance from 0.63 to 0.82 degree-MeV due to interaction with the radial motion. This effect is as expected and discussed in Section 2.4 of this report. We have observed that small changes introduced in the transverse emittance resulted in the expected changes in the rate of increase of the longitudinal emittance.

Since the normalized emittance of the ECR source for uranium is expected to be much better (we have used the value for oxygen) this increase is of no consequence.

The output beam from the first resonator was then matched to the second resonator, and it was accelerated there too without any loss.

Although more PARMTEQ calculations will be necessary to obtain the optimal resonator parameters, the work done so far is very valuable since it provides a confirmation of the present work by a program proven in tests of working RFQs.

ACKNOWLEDGEMENTS

The author would like to thank the Nuclear Structure Laboratory and the Physics Department at the State University of New York at Stony Brook for their hospitality.

Helpful discussions with Peter Paul, John Sikora, and Mike Cole of SUNY Stony Brook and Jim Rose of Grumman are gratefully acknowledged.

John Staples of LBL, Lawrence Berkeley Laboratory contributed in many ways to this work, by ideas, criticism, and various numerical calculations.

Special thanks go to Carl C. Paulson of Grumman Space Systems for carrying out the PARMTEQ calculations reported here.

REFERENCES

1. I. M. Kapchinskij and V. A. Tepljakov, *Prib. Techn. Eskp.* **2,** 19 (1970).
2. I. M. Kapchinskij and N. V. Lazarev, *IEEE Trans. Nucl. Sci* **NS-26 3,** 3462 (1979).
3. T. Antaya, (Michigan State Univ.), private communication.
4. J. Staples, (Lawrence Berkeley Laboratory), private communication.
5. K. R. Crandall, R. H. Stokes, and T. P. Wangler, "RF Quadrupole Beam Dynamics Design Studies," 1979 Linac Conference, Montauk, NY (1979), LA-UR 79–2499., p. 205.
6. E. E. Chambers and I. Ben-Zvi, *Particle Accelerators* **7,** 137 (1976).
7. K. R. Crandall, R. S. Mills, and T. P. Wangler, RFQ Vane-Tip Geometries, 1983 Particle Accelerator Conference, Santa Fe NM, LA-UR 83-743.
8. T. P. Wangler, "Lumped-Circuit Model of Four-Vane RFQ Resonator," 1984 LINAC conference, Darmstadt, Germany, LA-UR 84-1342.
9. H. Lancaster, "An Analytical Solution for the Electrical Properties of a RFQ with Simple Vanes," 1981 LINAC Conference, Santa Fe, LBL-13009.
10. J. Rose, (Grumman Corp., Bethpage, NY), private communication.
11. K. Halbach, R. F. Holsinger, W. E. Jule, and D. A. Swenson, "Properties of the cylindrical rf cavity evaluation code SUPERFISH," in *Proc. of* 1976 *Proton Linac Conf.,* AECL-5677, 122 (1976).
12. C. C. Paulson, (Grumman Space Systems, Plainsboro, NJ), private communication.