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NONCANONICAL HAMILTONIAN MECHANICS

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Noncanonical variables in Hamiltonian mechanics were first used by Lagrange in 1808.[1] In spite of this, most work in Hamiltonian mechanics has been carried out in canonical variables, up to this day. One reason for this is that noncanonical coordinates are seldom needed for mechanical problems based on Lagrangians of the form $L = T - V$, where T is the kinetic energy and V is the potential energy. Of course, such Lagrangians arise naturally in celestial mechanics, and as a result they form the paradigms of nineteenth-century mechanics and have become enshrined in all the mechanics textbooks.

Certain features of modern problems, however, lead to the use of noncanonical coordinates. Among these are issues of gauge invariance and singular Lagrange and Poisson structures. In addition, certain problems, like the flow of magnetic-field lines in physical space, are naturally formulated in terms of noncanonical coordinates. None of these features is present in the nineteenth-century paradigms of mechanics, but they do arise in problems involving particle motion in the presence of magnetic fields.

For example, the motion of a particle in an electromagnetic wave[2] is an important one in plasma physics, but the usual Hamiltonian formulation is gauge dependent. For this problem, noncanonical approaches based on Lagrangians in phase space[3] lead to powerful computational techniques which are gauge invariant. In the limit of strong magnetic fields, particle motion becomes "guiding-center motion." Guiding-center motion is also best understood in terms of noncanonical coordinates.[4] Finally, the flow of magnetic-field lines through physical space is a Hamiltonian system which is best understood with noncanonical coordinates. No doubt many more systems will arise in the future for which these noncanonical techniques can be applied.

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