Particle Accelerators, 1986, Vol. 19, pp. 37–42
0031-2460/86/1904–0037/$15.00/0

NUMERICAL COMPUTATIONS IN STELLAR DYNAMICS

PIET HUT†

Department of Astronomy, University of California, Berkeley, CA 94720

An outline is given of the main similarities and differences between gravitational dynamics and the other fields of interest at the present conference: accelerator orbital dynamics and plasma physics. Gravitational dynamics can be divided into two rather distinct fields, celestial mechanics and stellar dynamics. The field of stellar dynamics is concerned with astrophysical applications of the general gravitational N-body problem, which addresses the question: for a system of N point masses with given initial conditions, describe its evolution under Newtonian gravity. Major subfields within stellar dynamics are listed and classified according to the collisional or collisionless character of different problems. A number of review papers are mentioned and briefly discussed, to provide a few starting points for exploring the vast recent literature on stellar dynamics.

I. INTRODUCTION

Gravitational dynamics naturally divides into two rather distinct fields, celestial mechanics and stellar dynamics. The former is concerned with the long-term evolution of well-behaved, small, and ordered systems such as the motion of the planets around the sun, and that of natural and artificial satellites around the sun and planets. The latter covers the behavior of much larger systems consisting of many particles, each of which move on random paths constrained only by a globally averaged distribution, just as atoms move in a gas. Celestial mechanics has much in common with accelerator orbital dynamics, while stellar dynamics has close analogies with plasma physics.

The present paper is intended as a guide to the literature on stellar dynamics for those who are working in any of the other areas of orbital dynamics covered at this conference. No attempt is made to be complete, but rather a few useful references are given to reviews which can serve as convenient starting points in the literature. Nor will the contents of these reviews be discussed at any length: the field of stellar dynamics has seen on the order of a thousand or so new publications in the last decade, and no attempt will be made to even summarize the accomplishments in any subfield—that would require a much larger space than allotted here.

II. STELLAR DYNAMICS

Gravitational dynamics in many ways resembles plasma physics as well as accelerator physics. At nonrelativistic volocities at least, the gravitational

† Present address: Institute for Advanced Study, Princeton, NJ 08540.

Newtonian approximation is identical with the Coulomb approximation for electric forces, with the important difference that in electricity charges can carry either sign, while in gravity only positive masses occur.

Both Coulomb's and Newton's laws are scalar approximations to more complicated interactions, which have a vector and tensor character, respectively. Maxwell's equations show how static electricity is but one approximation to the dynamics of the electromagnetic field, and Einstein's equations show how Newtonian gravity neglects the richness of full general relativity, which features velocity-dependant forces more complicated than in electromagnetism. Both Maxwell's and Einstein's equations are characterized by a finite propagation speed, that of the speed of light (which therefore equals the speed of gravitational waves as well).

A most interesting connection exists between the classical field theories of electromagnetism and general relativity, and their quantum field analogons: quantum electrodynamics and quantum gravity. At present the mathematical treatments of both are still plagued by infinities, which have been shown to be renormalizable only in the former case. However, the fundamental character of these quantum field theories is independent of these mathematical (but very important!) problems and involves a fundamental description of each force as arising from the exchange of quanta which "carry" that force. The carrier of electromagnetism is, of course, the photon, a particle with spin one. Similarly, the carrier of gravity is the graviton, a particle with spin two (undetected as yet, and will probably remain so for a long time to come).

It is this difference in spin which is ultimately responsible for electromagnetism being a vector interaction, and general relativity being a tensor interaction. An immediate consequence of the properties under general coordinate transformations of both types of forces is that electric (and possibly magnetic) charge appears with both signs, but that gravitational "charge" (i.e., mass) can appear with positive sign only (if gravity were a vector interaction, gravitational waves would carry negative mass[1]).

Returning to the subject of the present conference, we notice that the intrinsically more complicated spin-two force in most astronomical applications seems the simpler one: Newtonian gravity corresponds to the exchange of scalar force carriers and misses the richness of the interplay of electric and magnetic forces. However, the positivity of all gravitational charges does make gravitational dynamics intrinsically more complicated than plasma physics in at least one aspect: no stable homogeneous distribution of self-gravitating matter can exist on arbitrarily large length scales. This situation aggravates even more the problems we encounter when trying to give a statistical-mechanics description of long-range forces.

In plasma physics the long-range electric forces are shielded beyond the Debye length because of mutual screening of positive and negative charges. In gravitational physics no such luxury exists, and we have to face squarely the breakdown of any standard, formal statistical-mechanics treatment. The story of gravitational dynamics is an ongoing saga of instability: not only are there no solutions admitting infinite homogeneous distributions of matter but also in-homogeneous self-gravitating systems can last only for a finite time before interactions between the individual particles will unbind the system altogether (particles continuously "boil off" at the surface).

Yet another way of looking at these problems is the realization that gravity lacks the freedom of treating instabilities to arbitrary accuracy in a linear-perturbation treatment. In many problems in physics we can analyze the character of instabilities by assigning a small parameter ϵ as a multiplicative factor to the amplitude of the growing instability. At least in the linear regime the growth of the instability is independent of the value of ϵ, and ϵ can be made arbitrarily small to avoid nonlinear effects from complicating the analysis. In gravity, however, the value of ϵ is fixed: both the (pseudo-)equilibrium configuration and the instabilities leading away from it are governed by the same gravitational coupling constant G, which governs the time-averaged background forces as well as the perturbing forces due to interactions between individual stars. How fast the instability grows therefore depends only on the configuration of the system, for a given total number of stars.

This problem of a lack of free parameters, the absence of a laboratory dial to fine-tune the onset of instability, shows up in collisional systems but is absent in the collisionless approximation. A collisional system is defined as a system where one cannot neglect the fluctuating random interactions between individual stars (somewhat misleadingly termed "collisions," although they are mostly close encounters, and not physical collisions). An example of a collisionless system is a galaxy containing $N \sim 10^{11}$ stars, with the time scale for completing one orbital revolution, the dynamical time scale, being of order $t_{\rm dyn} \sim 10^6$ yr. Because of the long-range character of gravitational forces, with such a large number of stars, each individual star feels a gravitational force which is nearly completely determined by the overall distribution of the other stars—the nearest neighbors exert a small erratic force which only slightly perturbs the average motion through the potential well of the smoothed-out system. Averaged over one revolution through the galaxy, a single star is hardly perturbed by chance encounters with passing neighbors.

But even a galaxy will ultimately change its overall structure because of the continuous perturbations of individual stars on each other. The time scale for these effects to be able to completely redistribute the energies of individual stars is called the two-body relaxation time $t_{\rm relax}$. This time scale is larger than the dynamical time scale for any sizable cluster of stars, as can be seen from the relation $t_{\rm relax} \approx \frac{N}{30} t_{\rm dyn}$ (cf. the clear description from a physical point of view given by Hénon[2]). For a typical galaxy, the above numbers imply $t_{\rm relax} \gtrsim 10^{15}$ yr, much larger than the age of the universe, $T \approx 10^{10}$ yr.

While galaxies are collisionless to an excellent approximation, smaller aggregates of stars, such as open clusters and globular clusters, cannot in general be treated as collisionless. With $N = 10^3$–10^6 and $t_{\rm dyn} = 10^4$–10^8, many star clusters are strongly collisional, i.e., $t_{\rm relax} \ll T$ in at least the denser central regions of the clusters. On a larger scale, some clusters of galaxies also show collisional effects, notwithstanding the large time required for a single galaxy to complete an orbital revolution through the cluster of galaxies. With a typical cluster containing some $N = 10^3$ galaxies and with dynamical time scales of order 10^9 yr, the age of a cluster (itself comparable to the age of the universe) is comparable to a relaxation time.

Collisionless stellar dynamics show a wide variety of interesting problems, concerning, e.g., spiral structure, bar formation, warps, merging of galaxies, the general problem of determining consistent distribution functions of stars in phase space given only limited observational information in the form of projected densities and/or velocity dispersions, etc. Collisional stellar dynamics also features many diverse problems, related to, e.g., core-halo-formation instabilities, mass segregation, redistribution of angular momentum, the formation of black holes, and an old problem which has recently seen many exciting new developments: the evolution of globular clusters after core collapse. Since this short outline lacks the space to even define, let alone discuss, these and other problems, instead a list of important references is given below.

III. NUMERICAL COMPUTATIONS

Below are indicated a few starting points into the literature on numerical computations in stellar dynamics. Unfortinately, no new books on stellar dynamics have appeared recently which treat this subject in any detail, on an advanced level. Several such works are being written, however, and are expected to appear within two or three years. Until then, conference proceedings offer the best treatments of specialized topics.

A comparison of some of the numerical techniques used in stellar dynamics with those used in plasma physics and other applications is given by Hockney and Eastwood.[3] In the chapter on stellar dynamics, they concentrate on two fields: the evolution of spiral galaxies and the clustering of galaxies in an expanding universe.

A concise review of computer simulations in stellar dynamics has been given by Aarseth and Lecar.[4] This older work contains an excellent review of the state of the art of the subject in 1975, but by implication does not address the more modern techniques. The most up-to-date description of N-body calculations via direct-force calculations has been given by Aarseth.[5]

At present, several I.A.U. symposia offer the most efficient way to acquire an overview of the vast and rapidly growing body of literature on stellar dynamics. "I.A.U." is an abbreviation for the International Astronomical Union, which sponsors several symposia each year on a variety of topics within astronomy and astrophysics.

I.A.U. Symposium 69 on *Dynamics of Stellar Systems*[6] contains several reviews on fundamental aspects of computations in stellar dynamics. Although ten years old, many of these reviews are still of interest, because they contain descriptions of details which are generally left out in later treatments. Another interesting aspect of this symposium is that it was the last one to address stellar dynamics as a whole. Because of the rapid growth of the subject, later symposia had to be more specialized and were therefore limited to either collisional or collisionless applications.

I.A.U. Symposium 85 on *Star Clusters*[7] is primarily concerned with topics outside stellar dynamics, such as star formation and stellar evolution. However, it also contains a number of interesting reviews on the dynamics of open clusters and globular clusters, both examples of collisional systems.

I.A.U. Symposium 100 on *Internal Kinematics and Dynamics of Galaxies*[8] contains a wealth of information on modeling collisionless systems. This very recent collection of reviews and more specialized papers covers many topics: kinematics of gas and the underlying mass distribution; spiral structure; warps in spiral galaxies; barred galaxies; spheroidal systems; merging of galaxies during close encounters; formation of galaxies and systems of globular clusters around galaxies.

I.A.U. Symposium 113 on *Dynamics of Star Clusters*[9] forms the counterpart to the previous symposium in that it contains an up-to-date collection of reviews on modeling collisional star systems. Especially interesting for a comparison with the other topics of the present conference are the reviews given on a number of new techniques. Examples are several different implementations of Fokker-Planck techniques to approximate globular-cluster evolution as a diffusion of the stellar-distribution function in energy and angular-momentum space; hydrodynamical approximations where the stars are replaced by a gaseous sphere so that techniques of stellar structure and evolution can be used (two-body relaxation being replaced by an effective heat conduction, and binary formation and hardening by an effective energy-production rate); hybrid models which combine a direct-force integration in the dense inner parts of a cluster with a Fokker-Planck description in the outer layers; and a tree-sorting algorithm to limit the number of (pesudo-)star-(pseudo-)star interactions via a recursive center-of-mass reduction.

Many other references could be given here to a number of interesting reviews which are not very easy to find, scattered as they are across the literature of astronomy and astrophysics over the last ten years. However, virtually all of those are referenced in one or more of the I.A.U. symposia mentioned above.

REFERENCES

1. C. W. Misner, K. S. Thorne and J. A. Wheeler, *Gravitation* (Freeman, San Francisco, 1973), Chap. 7.
2. M. Hénon, in *Dynamical Structure and Evolution of Stellar Systems,* Third Advanced Course of the Swiss Society for Astronomy and Astrophysics, edited by L. Martinet and L. Mayor (Geneva Observatory, Geneva, Switzerland, 1973), p. 224.
3. R. W. Hockney and J. W. Eastwood, *Computer Simulation Using Particles* (McGraw-Hill, New York, 1981).
4. S. J. Aarseth and M. Lecar, *Ann. Rev. Aston. Astr.*, **13,** 1 (1975).

5. S. J. Aarseth, in *Multiple Time Scales* edited by J. U. Brackbill and B. I. Cohen (Academic Press, New York, 1985).
6. *Dynamics of Stellar Systems,* edited by A. Hayli, IAU Symposium 69 (Reidel, Dordrecht, 1975).
7. *Star Clusters,* edited by J. E. Hesser, IAU Symposium 85 (Reidel, Dordrecht, 1980).
8. *International Kinematics and Dynamics of Galaxies,* edted by E. Athanassoula, IAU Symposium 100 (Reidel, Dordrecht, 1983).
9. *Dynamics of Star Clusters,* edited by A. Hayli, IAU Symposium 113 (Reidel, Dordrecht, 1975).