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RELATIVISTIC COLLECTIVE-EFFECT ACCELERATOR FOR THE ATTAINMENT OF ULTRAHIGH ENERGIES

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A new kind of collective circular particle accelerator is proposed which has the prospect to reach ultrahigh particle energies, including large luminosities. In the proposed concept, an electron cloud is produced within a large toroidal magnetic solenoid by inductive charge injection through the action of a traveling magnetic wave running around the torus, where at the same time the traveling magnetic wave accelerates the electron cloud to relativistic energies. By continuous inductive charge injection in the front of the wave, the cloud can then be relativistically densified. This densified cloud can become the source of ultrastrong electric and magnetic fields which can be many times larger than is possible using only externally applied fields. Ions being held by these large fields in a circular orbit can be accelerated to ultrahigh energies. An accelerator with a circumference of 10 km could reach particle energies up to 10^3 TeV with luminosities up to 10^{33} cm^{-2} sec^{-1}.

1. INTRODUCTION

In 1956, an interesting idea was proposed by Budker,[1] who showed that large electric and magnetic fields could theoretically be produced by a partially space charge neutralized intense relativistic electron beam and which could easily become many orders of magnitude larger than the externally applied field confining the beam. If such a beam could be produced within a toroidal magnetic field, it immediately followed that ions, if held in an orbit along the circular torus axis, could be accelerated to ultrahigh energies. However, the idea was abandoned because at that time there was no way in sight by which the required intense electron beam could be produced within the toroidal magnetic field.

An important step towards a solution of this problem was made under Rostoker,[2] through a modification of the HIPAC concept.[3] In the original HIPAC concept, electrons from thermionic emitters attach themselves to a toroidal magnetic field, and move inward as the field rises in time. This process, known as inductive charge injection, was modified by the addition of a transformer, with the yoke of the transformer passing through the center of the torus. The transformer produces an induced toroidal electric field which accelerates the electrons along the toroidal ring axis. In combination with the inductive charge injection, an intense electron beam can then be made. If, simultaneously with the acceleration, more electrons are added by inductive charge injection, large electron densities could in principle be reached. However, because the magnetic flux passing through the transformer yoke cannot become arbitrarily large, it remains doubtful that a relativistically densified electron cloud with a sufficiently large diameter to reach high luminosities for ions accelerated in the field of the cloud could ever be produced. An additional problem is that the induced electric field can penetrate the electron cloud only to the distance c/ω_p, where ω_p is the electron plasma frequency. If c/ω_p is small compared to the diameter of the cloud, only the outer electrons are accelerated. Furthermore, if the toroidal electron beam has a major dimension of many kilometers, as will be necessary for the attainment of ultrahigh particle energies, many transformers placed along the entire circumference of the accelerator would be required.

In the concept proposed here, the acceleration of the electron cloud is done by a traveling magnetic wave running around the large torus. The accelerating force is a magnetic mirror force acting on the orbital magnetic dipole moment, which completely penetrates the entire electron cloud. As the electrons gain relativistic energies, the cloud can be continuously densified by inductive charge injection at the head of the traveling wave, where the magnetic field is rising with time.

The use of a traveling magnetic wave to accelerate an electron cloud and the use of the large electric field at the head of a thusly accelerated cloud was previously proposed for a novel linear accelerator concept.[4,5,6]

2. DESCRIPTION OF THE ACCELERATOR CONCEPT

The principle of the idea is explained in Fig. 1. It shows a large toroidal magnetic solenoid, with the inside serving as the accelerator chamber. Its circumference can be many kilometers long, depending on the particle energy which is to be reached. The magnetic field of this solenoid consists of a constant part and a time-dependent part of the traveling magnetic wave running around the circular axis of the solenoid. The constant magnetic field most simply can be produced by ordinary electromagnets, whereas the field of the traveling magnetic wave can be produced by the programmed magnetization of additional low-inductance field coils. Furthermore, thermionic emitters are positioned along the entire length of the inner wall of the toroidal chamber. Then, as the rising magnetic field of the traveling wave passes by these emitters, electrons are first inductively injected into the torus, and thereafter accelerated by the magnetic-mirror force of the wave. Because this injected electron cloud acquires a large forward velocity along the circular torus axis, it becomes the source of a large self-magnetic field. As a result of this self-magnetic field, the electrons are pushed closer to the center of the toroidal chamber, thereby making room for more electrons to be inductively injected at the periphery of the chamber by the rising magnetic field of the traveling wave. Since the traveling wave can in principle run around the torus many times, the electron cloud can not only reach high energies, but at the same time can be also relativistically densified. This continuous inductive charge injection process is only limited by synchrotron losses which tend to decelerate the cloud. A limit is reached when the synchrotron losses exceed the energy gain from the traveling wave.

To accelerate ions to ultrahigh energies, they must be held in the orbit by action of the strong beam field, as in Budker's original proposal. This requires that the ions move in the opposite direction from the electrons. An additional axial electric field, the purpose of which is to accelerate the ions held in orbit by the field of the electron cloud would therefore not decelerate the electrons.

In high-energy physics, colliding beams are of greater interest than stationary targets. Colliding beams could here be realized either by two intersecting tori or by replacing the torus with an intersecting figure-eight configuration.

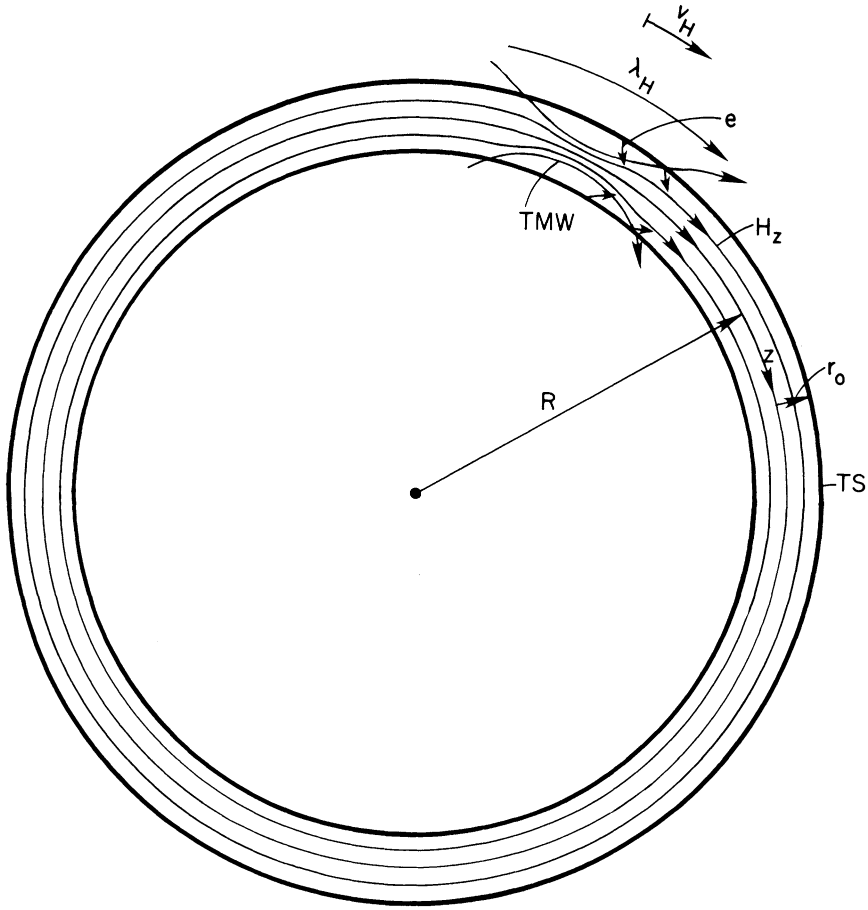

FIGURE 1. Axial cross-section through accelerator. *TS* toroidal magnetic solenoid of major radius R and minor radius r_0; H_z solenoidal field along circular solenoid axis z; *TMW* traveling magnetic wave of width λ_H and moving along the toroidal ring axis with the velocity v_H; e electron injected into torus by inductive charge injection.

3. MAGNETIC CONFINEMENT OF THE ELECTRON CLOUD

An electron cloud, after being produced by inductive charge injection, can be confined by an external magnetic field H_z, directed along the circular torus axis, z, provided

$$E_r < H_z, \tag{3.1}$$

where

$$E_r = 2\pi ner \tag{3.2}$$

is the electric field produced by the electron cloud and directed along the minor torus of radius $r = r_0$. In Eq. (3.2), n is the electron number density and e the electron charge. Inequality (3.1) is the usual magnetic insulation condition.

If the electron cloud is set into motion with velocity v along the circular torus axis z, it produces, in addition to the electric field, an azimuthal magnetic field given by

$$H_\phi = 2\pi nerv/c = \beta E_r, \tag{3.3}$$

where $\beta = v/c$ and c is the velocity of light. With the occurrence of this self-magnetic field, the magnetic insulation condition (3.1) is replaced by

$$E_r^2 < H_z^2 + H_\phi^2 \tag{3.4}$$

Inequality (3.4) can be also viewed as the radial equilibrium condition of the moving electron cloud. By virtue of Eq. (3.3), Inequality (3.4) reduces to

$$E_r < \gamma H_z, \tag{3.5}$$

where $\gamma \equiv (1 - \beta^2)^{-1/2}$.

In a more formal way, this result can be also obtained by a Lorentz transformation between the frame S at rest with the accelerator and the frame S' moving with the electron cloud. One then has the relations

$$\begin{aligned} n &= \gamma n' && \text{(a)} \\ E_r &= \gamma E_r' && \text{(b)} \\ E_z &= E_z' && \text{(c)} \\ H_z &= H_z' && \text{(d)} \\ H_\phi &= \beta\gamma E_r' = \beta E_r && \text{(e).} \end{aligned} \tag{3.6}$$

In the co-moving system S', where $H_\phi' = 0$, one has $E_r' < H_z' = H_z$, but in a system at rest rather $E_r < \gamma H_z$. For this reason, the density of the electron cloud, as seen in the accelerator rest frame, can by γ times larger than would be possible for a cloud at rest and confined by the same external field H_z. The self-fields of the cloud, that is E_r and H_ϕ, can therefore become about γ times larger than the confining magnetic field. As a consequence, ions held in orbit by these fields can acquire energies γ times larger than those which could be reached without using the relativistic magnetic insulation effect, expressed by Inequality (3.5)

Prior to the cloud being set into motion, the maximum electron number density n_0 in the electron cloud is given by

$$n_0 = \frac{E_r}{2\pi er} \simeq \frac{H_z}{2\pi er}. \tag{3.7}$$

After being set into motion, the density in the co-moving system remains the same. We therefore have to put $n' = n_0$, with the maximum density in the accelerator rest frame given by

$$n = \gamma n' = \gamma n_0. \tag{3.8}$$

Let us take as an example $H_z = 3 \times 10^4$ G, and $r = 1$ cm, for which Eq. (3.7) gives $n_0 \simeq 10^{13}$ cm^{-3}. If $\gamma = 100$, the maximum electron density in the accelerator rest frame would be $n \simeq 10^{15}$ cm^{-3}, and for $\gamma = 10^3$ a respectable $n \simeq 10^{16}$ cm^{-3}.

The moving cloud produces a current, which, expressed in amperes, is given by $I_z = 5rH_\phi$, and by using Eq. (3.5) is $I_z \lesssim 5r\gamma H_z$. For the given example $I_z \lesssim 1.5 \times 10^7$ A if $\gamma = 100$, and $I_z \lesssim 1.5 \times 10^8$ A if $\gamma = 10^3$. The magnetic field at the cloud radius $r = 1$ cm is $H_\phi \lesssim \gamma H_z$ which for $\gamma = 100$ is $H_\phi \leq 3 \times 10^6$ G and for $\gamma = 10^3$, $H_\phi \lesssim 3 \times 10^7$ G. The radial electric field is $E_r \lesssim \gamma H_z$ and is for $\gamma = 100$, $E_r \lesssim 10^9$ V/cm and for $\gamma = 10^3$, $E_r \lesssim 10^{10}$ V/cm. Since the maximum electric field at the inner wall of the accelerator tube must be less than 10^8 V/cm, the limit for field ion emission, the wall radius must be larger than 10 cm for $\gamma = 100$ and larger than 1 meter for $\gamma = 10^3$.

To keep the cloud stable, the condition

$$\omega_p < \omega_c \tag{3.11}$$

must be satisfied, where $\omega_p = (4\pi n_0 e^2/m)^{1/2}$ is the plasma frequency and $\omega_c = eH_z/mc$ the electron cyclotron frequency. For $n_0 = 10^{13}\ \mathrm{cm}^{-3}$ and $H_z = 3 \times 10^4$ G, one finds $\omega_p = 1.8 \times 10^{11}\ \mathrm{sec}^{-1}$ and $\omega_c = 5.3 \times 10^{11}\ \mathrm{sec}^{-1}$. These numbers barely satisfy condition (3.11) but if necessary, it would be no problem to make ω_p still smaller than ω_c. All that must be done is to increase the radius of the electron cloud somewhat, which according to Eq. (3.7), would reduce n_0, leaving the value of H_z and ω_c unchanged.

4. RELATIVISTIC DENSIFICATION OF THE ELECTRON CLOUD

After it is set into motion, an electron cloud of initial density $n = n_0$, cannot be relativistically densified γ-fold because the total number of particles in the toroidal magnetic field would remain unchanged. A γ-fold densification by Lorentz contraction is possible only if during the acceleration of the cloud more electrons are added to it.

To decide whether more electrons can be added, we must analyze the inductive charge injection process for accelerated electron clouds. We introduce a cylindrical r, ϕ, z coordinate system along the circular torus axis, neglecting the curvature of the torus.† From Maxwell's equation

$$\operatorname{curl} \mathbf{E} = -\frac{1}{c}\frac{\partial H}{\partial t}, \tag{4.1}$$

we obtain

$$E_\phi = -\frac{r}{2c}\dot{H}_z \tag{4.2}$$

and

$$E_z = \frac{r}{2c}\dot{H}_\phi. \tag{4.3}$$

These induced electrical fields lead to the radial drift motion

$$v_r = \frac{dr}{dt} = -\frac{r}{2}\frac{H_z\dot{H}_z + H_\phi\dot{H}_\phi}{H_z^2 + H_\phi^2}. \tag{4.4}$$

† To compensate for the centrifugal force on the electrons caused by this small curvature, a weak vertical magnetic field has to be applied, which is ignored in this preliminary analysis.

Putting $H^2 = H_z^2 + H_\phi^2$, this can be written as

$$d\ln r^4 = -d\ln \mathrm{H}^2, \tag{4.5}$$

which upon integration yields

$$r/r_0 = [H(0)/H]^{1/2}. \tag{4.6}$$

In Eq. (4.6), $H(0)$ is the total initial magnetic field at the time where the injected particle is positioned at the thermionic emitter evaluated at the inner torus radius $r = r_0$.

The toroidal magnetic field consists of a constant field and the superimposed field of the traveling wave. It follows that $H_z(0)$ is equal to the constant field. The maximum field of the traveling wave is typically about twice as large as the constant field. Therefore, if the circumference of the torus is large compared with the length of the traveling wave, the average of H_z over the entire circumference is about equal to $H_z(0)$.

The shortness of the traveling wave depends on the technical ability to turn on a large magnetic field rapidly. With the length λ_H defined as the width over which the wave field is strong, one has

$$\lambda_H = 2v_H\tau_H, \tag{4.7}$$

where v_H is the velocity of the traveling wave and τ_H the rise time of the magnetic field. In practice, $\tau_H = 10^{-6}$ sec is possible and with some difficulty $\tau_H = 10^{-7}$ sec. For maximum axial acceleration by the traveling magnetic wave, the wave must closely follow the electrons accelerated by it. Since the electrons soon reach relativistic velocities, one therefore can put $v_H \simeq c$. For $\tau_H = 10^{-6}$ sec, one then finds that $\lambda_H \simeq$ 600 meters and for $\lambda_H = 10^{-7}$ sec $\lambda_H \simeq 60$ meters. Therefore, if the circumference of the torus is 10 km, λ_H is small by comparison, and we can put $H_z(0) \simeq H_z$ in Eq. (4.6).

According to Eqs. (3.5) and (3.6), for $\gamma \gg 1$ we can approximately set

$$H \simeq \gamma H_z \tag{4.8}$$

and we therefore can write for Eq. (4.6)

$$r/r_0 \simeq [\gamma(0)/\gamma]^{1/2}, \tag{4.9}$$

where $\gamma = \gamma(0)$ is the initial value of $\gamma(t)$ at $t = 0$. If $\gamma = \gamma(t)$ is known, then Eq. (4.9) gives $r = r(t)$, for a zero-energy electron injected at $t = 0$ at the position $r = r_0$.

The axial equation of motion for the electrons accelerated by the mirror force of the traveling magnetic wave is

$$m\frac{d}{dt}(\gamma v) = -\mu\frac{dH_z}{dz}, \tag{4.10}$$

where

$$\mu = \frac{\gamma m v_\perp{}^2}{2H_z} \tag{4.11}$$

is the orbital magnetic moment of the electron trajectory in the $r - \phi$ plane, which is an adiabatic invariant. To obtain an upper limit for $v_\perp$ and hence for μ, we equate the centrifugal and centripedal forces acting on an electron

$$\frac{\gamma m v_\perp{}^2}{r} = e\left[\frac{v_\perp}{c} H_z + \frac{v}{c} H_\phi - E_r\right]. \tag{4.12}$$

According to Eqs. (3.6), the 2nd and 3rd term on the rhs. of Eq. (4.12) approximately cancel. Then, if $eH_z r > mc^2$, it follows that $v_\perp \simeq c$. We therefore find that

$$\gamma mc^2 \leq eH_z r \tag{4.13}$$

and

$$\mu \leq er/2. \tag{4.14}$$

For $\gamma \gg 1$, we can put $v = c$ and write for Eq. (4.10)

$$mc\frac{d\gamma}{dt} \leq -\frac{er}{2}\frac{dH_z}{dz}. \tag{4.15}$$

Making the approximation $dH_z/dz \simeq 2H_z/\lambda_H$, we have

$$\frac{d\gamma}{dt} \lesssim \frac{erH_z}{mc\lambda_H}. \tag{4.16}$$

For $\gamma \gg 1$ we can also write $d\gamma/dt = cd\gamma/dz$ and have

$$\frac{d\gamma}{dz} \lesssim \frac{erH_z}{mc^2}\frac{1}{\lambda_H}, \tag{4.17}$$

or

$$z \gtrsim (mc^2/erH_z)\lambda_H \gamma. \tag{4.18}$$

In our example, with $r = 1$ cm, $H_z = 3 \times 10^4$ G, we have $erH_z \simeq 10^7$ eV, and hence $z \gtrsim 5 \times 10^{-2}\lambda_H\gamma$. To reach $\gamma = 100$, we therefore find that $z \gtrsim 5\lambda_H$, and for $\gamma = 10^3$, $z \gtrsim 50\lambda_H$. If the torus has a circumference of 10 km and if $\lambda_H \simeq 100$ meters, the traveling wave would have to run around the torus no more than once, but since the wave must also continuously add new electrons to the cloud by inductive charge injection, and which thereafter have to be accelerated to the final large γ-value, several runs are more likely to be required.

The accumulation of electric charge during the inductive injection process is determined by the continuity equation

$$\frac{\partial n}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(rnv_r) = 0. \tag{4.19}$$

According to Eq. (4.9) we have

$$v_r = dr/dt = -(r/2)(\dot{\gamma}/\gamma), \tag{4.20}$$

and hence

$$\frac{\partial n}{\partial t} - \frac{1}{2r}\frac{\dot{\gamma}}{\gamma}\frac{\partial}{\partial r}(nr^2) = 0. \tag{4.21}$$

Equation (4.21) can be solved by the method of characteristics and has the general solution

$$n(r, t) = \gamma f(r^2\gamma), \tag{4.22}$$

where f is an arbitrary function. Since according to Eq. (4.9), $r^2\gamma = \text{const.}$, it follows that $f = \text{const.}$ We thus find that

$$n(r, t) = \gamma n_0(r), \tag{4.23}$$

where $n_0(r)$ is the initial cloud density prior to its acceleration and addition of more electrons. Equation (4.23) is consistent with a γ-fold densified electron cloud, since, as seen from its own frame at rest, $n_0 = n'$ and hence $n = \gamma n'$.

Two additional remarks are in order here. First, it may appear that the traveling magnetic wave accelerating the electron cloud could lead to a velocity distribution where $v_\perp \sim v_\parallel$, and which would lead to a loss-cone instability. This, however, cannot happen because in accelerating the electrons, the wave drives a large electron current. The resulting self-magnetic field H_ϕ thereby pushes the electrons into a lower minor radius r, where their motion is dominated by the large self-field H_ϕ, rather than the much weaker mirror field of the traveling magnetic wave.

The second remark is related to our assumption that the initial electron density is given by Eq. (3.7). In reality, the electron density reached in inductive charge injection experiments has been always much lower and never reached the theoretical upper limit expressed by Eq. (3.7). In our proposed concept, this observed inefficiency of the inductive charge injection process poses no fundamental problem, since here the electron cloud can be slowly fattened to its theoretical limit by the traveling magnetic wave, no matter how inefficient the inductive charge injection process might be. Larger initial electron densities could be reached by replacing the thermionic emitters with electron guns, but for the reasons stated this may be not necessary.

5. SYNCHROTRON LOSSES OF THE ELECTRONS

The synchrotron losses of an electron moving in a circular orbit of radius R are given by[7]

$$I = \frac{2}{3}\frac{e^2 c}{R^2}\gamma^4. \tag{5.1}$$

To obtain the total losses, this number must be multiplied by the number of electrons, given by

$$\begin{aligned} N_e &= 2\pi R \pi r^2 n \\ &= 2\pi^2 R r^2 \gamma n_0. \end{aligned} \tag{5.2}$$

With the help of Eq. (3.7), one finds

$$N_e \simeq (\pi/e)Rr\gamma H_z. \tag{5.3}$$

The synchrotron radiation losses are therefore

$$P = IN_e = (2\pi ec/3)(r/R)\gamma^5 H_z. \tag{5.4}$$

For the example given above, $r = 1$ cm, $R = 1.6$ km, $H_z = 3 \times 10^4$ G, one finds that

$$P \simeq 1.9\gamma^5 \text{ erg/sec} = 1.9 \times 10^{-7}\gamma^5 \text{ Watt}. \tag{5.5}$$

For $\gamma = 10^2$, one has $P \simeq 2 \times 10^3$ Watt, but for $\gamma = 10^3$ already $P \simeq 2 \times 10^8$ Watt. An electron energy corresponding to $\gamma \simeq 10^3$, that is, 500 MeV, therefore appears to be a practical upper limit. Since the power losses grow in proportion to γ^5, a GeV electron cloud would lose energy at the rate of ~ 6 Gigawatt.

The lifetime of the electron cloud due to these radiation losses is given by

$$\tau \simeq \frac{\gamma mc^2}{I} = \frac{3}{2}\frac{R^2 mc}{e^2\gamma^3} = \frac{3}{2}\frac{R^2}{r_e c\gamma^3}, \tag{5.6}$$

where $r_e = e^2/mc^2$ is the classical electron radius. For $\gamma = 10^2$, we find $\tau \simeq 3 \times 10^6$ sec ~ 1 month, and for $\gamma = 10^3$, $\tau \simeq 1$ hour. These rather long lifetimes make it possible to accelerate ions rather leisurely in the strong fields of the cloud, and after the traveling magnetic wave has been turned off.

6. ACCELERATION OF IONS IN THE RELATIVISTICALLY DENSIFIED CLOUD

The electric and magnetic fields of the relativistically densified cloud are given by

$$\left.\begin{aligned} E_r &\simeq \gamma H_z \\ H_\phi &\simeq \gamma H_z \end{aligned}\right\}. \tag{6.1}$$

An ion placed within these fields and accelerated to a velocity v_i is subject to a radial force given by

$$\begin{aligned} F_r &= e(E_r - \beta_i H_\phi) \\ &\simeq e\gamma H_z(1 - \beta_i), \end{aligned} \tag{6.2}$$

where $\beta_i = v_i/c$. To be subject to a large radial force, the ion must move in a direction opposite to those of the electrons. At relativistic energies one therefore has $\beta_i \simeq -1$, and hence

$$F_r \simeq 2e\gamma H_z. \tag{6.3}$$

To keep the ions in orbit, the centrifugal force acting on them must be balanced by F_r.

For relativistic energies, with $\gamma_i \equiv (1 - \beta_i^2)^{-1/2}$, the centrifugal force is given by

$$F_c = \gamma_i Mc^2/R. \tag{6.4}$$

Putting $F_c = F_r$, one finds that

$$\gamma_i = (2e/Mc^2)\gamma H_z R, \tag{6.5}$$

and therefore

$$\gamma_i/\gamma = 2eH_zR/Mc^2. \tag{6.6}$$

Introducing the ion energy $\epsilon_i \simeq \gamma_i \mathrm{Mc}^2$, one has

$$\epsilon_i \simeq 2e\gamma H_z R. \tag{6.7}$$

To obtain ϵ_i in eV, one has to multiply Eq. (6.7) by $300/e$. The result is

$$\epsilon_i = 600\gamma H_z R \quad [\mathrm{eV}]. \tag{6.8}$$

Taking the example $R = 1.6$ km, $H_z = 3 \times 10^4$ G and $\gamma = 100$, one would have $\gamma_i/\gamma = 2 \times 10^3$. For $\gamma = 10^2$ one would have $\epsilon_i \simeq 3 \times 10^{14}$ eV, and for $\gamma = 10^3$, $\epsilon_i \simeq 3 \times 10^{15}$ eV.

The ions could be accelerated to these high energies by microwaves. The electric field of the microwaves would also accelerate the electrons, because they move in the opposite direction. This simultaneous acceleration of the electron cloud in the opposite direction also compensates to some degree the electron synchroton losses. However, because of the long lifetime of the cloud, expressed by Eq. (5.6), no compensation for these losses would be actually needed. Then, if the frequency of the microwaves accelerating the ions is less than the electron plasma frequency of the moving cloud $\omega_p' = \omega_p$ (since $n' = \gamma n_0$ and $m' = \gamma m$) the microwaves cannot penetrate the electron cloud at all and thereby cannot accelerate the electrons. For the example given above, $\omega_p \simeq 1.8 \times 10^{11}\ \mathrm{sec}^{-1}$, and for a microwave frequency less than $\nu \sim 3 \times 10^{10}\ \mathrm{sec}^{-1}$ the electrons draw no energy. At the other hand, the centrifugal force pushes the ions radially outward and they are therefore positioned somewhere outside the electron cloud. Under these circumstances, all the microwave energy can go into the ions.

7. SYNCHROTRON LOSSES BY THE IONS AND THE ATTAINABLE LUMINOSITY

To accelerate ions to high energies, the circulating electron ring is loaded with a relative fraction f_i of ions, with f_i defined by

$$f_i = N_i/N_e. \tag{7.1}$$

This fraction must be kept small, otherwise the space charge and current produced by the ions is going to cancel the electric and magnetic field of the electron cloud which holds the ions in orbit. After the ions are accelerated to high energies, they too can lose their energy by synchrotron radiation. These losses are given by the same formula (5.1) with γ replaced by γ_i. The total losses by the ions are thus given by

$$\begin{aligned} P_i &= \frac{2}{3}\frac{e^2c}{R^2}\gamma_i^4 N_i \\ &= \frac{2}{3}\frac{e^2c}{R^2}\gamma_i^4 f_i N_e \\ &= (\gamma_i/\gamma)^4 f_i P. \end{aligned} \tag{7.2}$$

Assuming that $\gamma_i/\gamma = 2 \times 10^3$, given by the example following Eq. (6.8), and furthermore making use of Eq. (5.5), one finds

$$P_i \simeq 3 \times 10^6 \gamma^5 f_i \quad [\text{Watt}] \tag{7.3}$$

A practical upper limit for P_i is about 3×10^9 Watt, and in this case

$$f_i \sim 10^3/\gamma^5. \tag{7.4}$$

For $\gamma = 10^2$ one finds $f_i \sim 10^{-7}$ and for $\gamma = 10^3$ one finds $f_i \sim 10^{-12}$.

The total number of electrons is given by Eq. (5.3), and for $r = 1$ cm, $R = 1.6$ km and $H_z = 3 \times 10^4$ G, one has $N_e \simeq 6 \times 10^{19}\gamma$. For $\gamma = 10^2$ one has $N_e \simeq 6 \times 10^{21}$ and $N_i \simeq 6 \times 10^{14}$; and for $\gamma = 10^3$ one has $N_e \simeq 6 \times 10^{22}$ and $N_i \simeq 6 \times 10^{10}$.

For high-energy physics experiments, one needs two colliding ion beams, producing a center-of-mass energy equal to $\epsilon_{CM} = 2\epsilon_i$. On the basis of Eq. (6.8), the center-of-mass energy is therefore given by

$$\epsilon_{CM} = 1.2 \times 10^3 \gamma H_z R \quad [\text{eV}]. \tag{7.5}$$

For the example given above $\epsilon_{CM} \simeq 6 \times 10^{14}$ eV, assuming that $\gamma = 10^2$, and $\epsilon_{CM} \simeq 6 \times 10^{15}$ eV for $\gamma = 10^3$.

The luminosity of the colliding beams is given by the usual formula

$$L = N_i^2 \nu/\pi r_b^2, \tag{7.6}$$

where

$$\nu = c/2\pi R \tag{7.7}$$

is the collision frequency and r_b the interaction radius. Optimistically, $r_b \simeq 10^{-4}$ cm may be possible. For $R = 1.6$ km, one finds $\nu = 3 \times 10^4$ sec^{-1}. Then, if $\gamma = 10^2$, with $N_i = 6 \times 10^{14}$, one finds

$$L \simeq 4 \times 10^{33}/r_b^2 \text{ cm}^{-2}\text{ sec}^{-1}, \qquad (\epsilon_{CM} = 600 \text{ TeV}), \tag{7.8}$$

and for $\gamma = 10^3$, with $N_i \simeq 6 \times 10^{10}$,

$$L \simeq 4 \times 10^{25}/r_b^2 \text{ cm}^{-2}\text{ sec}^{-1}, \qquad (\epsilon_{CM} = 6 \times 10^3 \text{ TeV}). \tag{7.9}$$

In the first example, with $\epsilon_{CM} = 600$ TeV, $L \simeq 10^{33}$ cm^{-2} sec^{-1} can be easily reached by equating r_b with the radius of the electron beam. In the second example $L \simeq 10^{33}$ cm^{-2} sec^{-1} can be only reached if $r_b \simeq 10^{-4}$ cm.

The proposed concept therefore has the potential for center-of-mass energies up to $\sim 10^3$ TeV with luminosities up to $\sim 10^{33}$ cm^{-2} sec^{-1}.

8. ADDING A TENUOUS PLASMA TO THE BEAM

If a tenuous plasma is added to the beam, we have the situation originally envisioned by Budker.[1] The question of how the beam could be established in the first place remained unanswered in Budker's proposal.

If a tenuous plasma is added to the beam, it can in part neutralize the space charge of the beam. If the fraction of this space charge neutralization is f, the radial force on a beam electron is

$$F = e[(1 - f)E_r - \beta H_\phi] = [1/\gamma^2 - f]eE_r \tag{8.1}$$

For $\gamma^2 \gg 1$ one has

$$\begin{aligned} F &\simeq -feE_r \\ &\simeq -fe\gamma H_z. \end{aligned} \tag{8.2}$$

If F becomes negative, the beam contracts to a smaller diameter until $F = 0$, or until the buildup of a pressure force by beam heating through Coulomb collisions with the background plasma will produce a repulsive force counteracting F.

The force contracting the beam is a magnetic pinch force. Beam contraction by this pinch force requires that

$$f > 1/\gamma^2. \tag{8.3}$$

For $\gamma \gg 1$, E_r is only slightly reduced, because

$$\begin{aligned} E_r' &= (1 - f)E_r \\ &\leq (1 - 1/\gamma^2)E_r \simeq E_r, \end{aligned} \tag{8.4}$$

where E_r' is the reduced electric field. For $E_r' \simeq E_r$, the magnetic insulation criterion (3.4), and therefore (3.5) is essentially unchanged. However, since the beam now can shrink, $E_r \lesssim \gamma H_z$ is satisfied at a much smaller beam radius. The outer radius of the accelerator tube can therefore be made also smaller, and still make the radial electric field there smaller than $\sim 10^8$ V/cm, the limit for field ion emission. Furthermore, the contraction of the beam also means that a relativistically densified beam can be produced by letting the beam radius shrink rather than by adding new electrons through inductive charge injection. For example, if a cloud of initial density n_0 and radius r_0 is formed, with $n_0 \simeq H_z/2\pi e r_0$, and if this cloud is thereafter accelerated to γ, a relativistically densified beam would be obtained by a contraction down to a radius r given by

$$r \simeq r_0/\sqrt{\gamma}. \tag{8.5}$$

Equation (8.5) has the same form as Eq. (4.9) with the important difference, that Eq. (8.5) applies to the beam as a whole, whereas Eq. (4.9) applied to an individual electron.

The addition of a plasma, even a tenuous one, results in Coulomb collisions. These collisions lead to beam heating and the buildup of a pressure force acting against the magnetic pinch force. However, the beam confinement by the pinch force also leads to electron oscillations perpendicular to the beam axis resulting in large synchrotron losses and which cool the beam. As long as the rate of these synchrotron losses exceeds the Coulomb collision heating rate, the beam contracts.

The Coulomb collisions with the background plasma of density $n_p = fn$, lead to a disordered energy gain per electron given by[8]

$$\frac{dE}{dt} = 4\pi n f \frac{e^4}{mc} \ln \Lambda, \tag{8.6}$$

where $\Lambda = r/r_e, r_e = e^2/mc^2 = 2.8 \times 10^{-13}$ cm. The synchrotron loss per electron is given by

$$P_e = (2/3)(e^2\overline{\dot{v}^2}/c^3)\gamma^4. \tag{8.7}$$

According to Eq. (8.2), the perpendicular equation of motion for a beam electron is

$$\gamma m\ddot{r} = F \simeq -feE_r = -2\pi ne^2 fr, \tag{8.8}$$

or

$$\ddot{r} + \omega^2 r = 0, \tag{8.9}$$

where $\omega^2 = 2\pi ne^2 f/\gamma m$. For an harmonic oscillator one has $\overline{\dot{v}_\perp{}^2} = \overline{\ddot{r}^2} = (1/3)\omega^4 r^2$, and hence

$$P_e = \frac{8\pi^2}{9}\frac{n^2 e^6 f^2 r^2 \gamma^2}{m^2 c^3}. \tag{8.10}$$

For the beam to contract, $P_e > dE/dt$, which leads to the condition

$$eE_r r f\gamma^2 > (9 \ln \Lambda)mc^2. \tag{8.11}$$

Putting $f = k/\gamma^2$, with $k > 1$, and putting $E_r \simeq \gamma H_z$, we find

$$r\gamma > \frac{(9 \ln \Lambda)mc^2}{keH_z}, \tag{8.12}$$

or

$$r\gamma \gtrsim 4.5 \times 10^5/kH_z. \tag{8.13}$$

We can combine inequality (8.13) with Eq. (8.5) to obtain a condition for γ

$$\gamma \gtrsim 2 \times 10^{11}\,(kr_0 H_z)^{-2}. \tag{8.14}$$

Assuming, for example, that $r_0 = 1$ cm, $H_z = 3 \times 10^4$ G and $k = 2$, we find $\gamma \gtrsim 50$. Then, if $\gamma = 100$, one obtains from Eq. (8.5) that $r \simeq 0.1$ cm. Therefore, a tenfold focusing, taking some typical numbers, may be achieved by adding a tenuous background plasma. A tenfold reduction in the beam radius makes possible a tenfold reduction in the outer radius of the accelerator tube, where the radial electric field must be kept below the limit for ion field emission.

The proposed addition of a tenuous background plasma, of course, only makes good sense if the synchrotron losses due to the perpendicular motion, and which were given by Eq. (8.7), do not exceed the synchrotron losses given by Eq. (5.1). The condition $P_e < I$ leads to

$$\overline{\dot{v}_\perp^{\,2}} < c^4/R^2 \tag{8.15}$$

or

$$\frac{eE_r f}{\sqrt{3}\gamma mc^2} < \frac{1}{R}. \tag{8.16}$$

Putting $E_r \simeq \gamma H_z$ and $f = k/\gamma^2$, we find

$$\gamma > 3^{-1/4}\,(eH_z Rk/mc^2)^{1/2} \tag{8.17}$$

or

$$\gamma > 1.8 \times 10^{-2}\,(H_z Rk)^{1/2}. \tag{8.18}$$

For the example $R = 10^5$ cm, $H_z = 3 \times 10^4$ G, $k = 2$ one finds $\gamma > 1400$. This large γ-value ensures beam contraction, but also shows the disadvantage of adding a tenuous plasma, which greatly increases the losses. Nevertheless, the addition of a tenuous background plasma to focus the beam makes good sense at large γ-values. However, large γ-values also mean large synchrotron losses in general, and we had previously estimated that $\gamma \simeq 10^3$ appears to be a practical upper limit, but for a smaller beam radius one can use a less intense beam and still reach the same luminosity. For $\gamma \simeq 10^3$, the beam would shrink down to less than $\simeq 0.03$ cm, assuming as before that $r_0 = 1$ cm.

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