Particle Accelerators
1978, Vol. 8, pp. 161–165

SUGGESTION FOR X-RAY LASER HOLOGRAPHY

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(Received April 5, 1977; in final form July 19, 1977)

It seems that synchrotron radiation from electron storage rings will soon be used to pump soft X-ray lasers. These could serve in holographic studies of microstructures, e.g., biological samples. Storage rings best adapted for such experiments would also be suitable for free electron laser experiments. The two types of experiments might profitably share the same ring. Radiation for X-ray laser pumping should originate in a low-beta section.

It now appears likely that in the near future one will be able to pump soft X-ray lasers with synchrotron radiation emitted by high-energy electron storage rings.[1] In the more distant future, synchrotron-radiation pumping of hard X-ray lasers also seems possible.[2] Electron storage rings would become particularly powerful tools for X-ray laser pumping, if the synchrotron radiation emitted by them were to be enhanced through coherent (between electrons) generation.[3]

Among the numerous applications of a soft X-ray laser, perhaps the most exciting is the possibility of producing holographic pictures of small crystals and biological samples (such as membranes). High-energy accelerators may thus become the front-line research equipment not only in physics, but the biological sciences as well.

Here we intend to study the question of how the various parameters of a storage ring affect its possible application to soft X-ray holography.

Consider the case when the laser volume is rectangular,[1] d_x wide, d_y high, and d_z long. Let ε_{lx}, ε_{ly} be respectively the horizontal and vertical emittance of the coherent photon beam produced by the laser. We have

$$\varepsilon_{li} \approx \frac{1}{4}\frac{d_i^2}{d_z} \qquad (i = x, y) \qquad \text{(1a)}$$

if d_z is chosen (as in Ref. 1) to satisfy $d_z \gtrsim 2l_c$, where l_c is the critical length of the laser.†

Denote by $\varepsilon_{\gamma x}$, $\varepsilon_{\gamma y}$ respectively the horizontal and vertical emittance of the synchrotron radiation which pumps the laser. It turns out,[1] and it is intuitively clear, that the pumping of the laser is efficient, if

$$\varepsilon_{\gamma i} = \sigma_{\gamma i}\sigma'_{\gamma i} \qquad ; \quad (i = x, y) \qquad \text{(1b)}$$

$$\sigma_{\gamma i} = \tfrac{1}{2}d_i, \qquad \sigma'_{\gamma i} \lesssim \frac{1}{2}\frac{d_i}{d_z}; \quad (i = x, y), \qquad \text{(1c)}$$

where $\sigma_{\gamma x}$ is the (rms) horizontal radius of the photon beam at the center of the laser, $\sigma'_{\gamma x}$ is the (rms) spread of the horizontal half angle there, and the quantities with subscript y are defined analogously. Equation (1b) is simply a statement of the fact that both the x and y phase-space ellipse of the pumping X-ray radiation is "upright", i.e., the x axis is a symmetry axis of the first of these ellipses, and so is the y axis for the second. Equation (1c)

† The critical length of the laser is that length which a signal of coherent photons, initiated by a single photon, has to travel in the laser before the signal is amplified so much that thereafter it de-excites (by stimulated emission of coherent photons) essentially all optically inverted atoms.

says that the height and width of the pumping photon beam matches the laser volume at its center and that the angular divergence is small enough so that most of the pumping photons traverse the whole length of the laser.

From Eqs. (1)

$$\varepsilon_{\gamma i} \lesssim \varepsilon_{li}; \quad (i = x, y). \tag{2}$$

The above quantities are related to the parameters characterizing the electron beam at the point where the synchrotron radiation is emitted. Let the horizontal (rms) electron beam radius be σ_x, and the corresponding (rms) spread in the half angle σ'_x, etc. Denote by a superscript 0 quantities referring to the region where the synchrotron radiation is emitted, e.g., σ_x^0 means the electron beam radius where synchrotron radiation originates. Clearly,

$$\sigma_{\gamma i}^0 = \sigma_i^0. \tag{3a}$$

Consider synchrotron radiation with wavelength near λ_0, emitted by circulating electrons of energy E_e. The characteristic angle of emission is θ_c. Then

$$\sigma_{\gamma i}^{0\prime} = [(\sigma_i^{0\prime})^2 + \theta_c^2]^{1/2}. \tag{3b}$$

Assume that at the point where the synchrotron radiation is emitted,†

$$\varepsilon_i = \sigma_i^0 \sigma_i^{0\prime}, \tag{4}$$

i.e., that the x and y phase-space ellipses characterizing the electron beam are "upright." From Eqs. (1b), (3a), (3b), and (4),

$$\varepsilon_{\gamma i} \approx \varepsilon_i \left[1 + \left(\frac{\sigma_i^0 \theta_c}{\varepsilon_i}\right)^2\right]^{1/2} \tag{5}$$

It is well known that ε_i $(i = x, y)$ has the same value everywhere around the ring. The same is not true for $\varepsilon_{\gamma i}$ $(i = x, y)$, which is smaller where σ_i^0 is.

Our aim is to produce a holograph from an object whose horizontal and vertical dimensions are h_x and h_y. The photons emitted by the X-ray laser travel along the z axis (see Figure 1), may be scattered by the object and form an interference pattern with the unscattered photons. The interference pattern at the holographic surface is recorded. The recording medium may be a photographic plate, a polymer, a crystal, or some other material.

† We will find that for our purposes the best place to take synchrotron radiation from the ring is at the center of a low-beta section. Equation (4) is usually satisfied there.

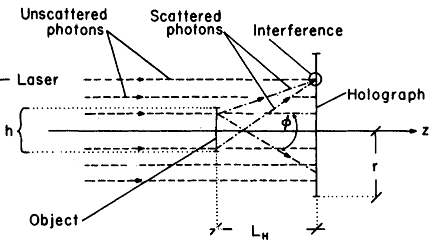

FIGURE 1 Unscattered photons emitted by the X-ray laser (located to the left) travel parallel to the z axis. Their paths are denoted by dashed lines. An object of diameter h scatters some of the photons. The scattering angle is ϕ. After scattering, these photons travel along the dash-dot lines, and form an interference pattern with the unscattered photons. The interference pattern is recorded at a distance L_H from the object, on a holograph of diameter 2r.

We wish to resolve details with dimensions $\geq \Delta a_x$ along the x axis, and with Δa_y along the y axis. By Heisenberg's principle, for this we need photons with momentum component along x and y respectively

$$p_i \gtrsim \frac{\hbar}{\Delta a_i}, \qquad (i = x, y) \tag{6a}$$

or scattering angles

$$\phi_i \gtrsim \text{arc tg} \frac{p_i}{p_0} = \text{arc tg} \frac{\lambda_0}{\Delta a_i}, \tag{6b}$$

where $p_0 \equiv \hbar/\lambda_0$. When $\Delta a_x = \Delta a_y \approx \lambda_0$, then we need to record the interference pattern produced by photons scattered by angles

$$\phi_i \text{ up to } \approx 1 \text{ radian}. \tag{7}$$

On the holographic surface photons will produce an interference pattern of typical dimensions λ_0. The resolution of the recording medium must therefore be of the order of λ_0 or less.

Only very few recording materials are known to have a resolution Δl of the order λ_0, when λ_0 is in the soft X-ray region (and none when λ_0 is in the hard X-ray region). It is, therefore, of interest to know whether an interference pattern produced by features of dimension Δa can be recorded in a medium whose spatial resolution is $\Delta l \gg \Delta a$. In the arrangement shown in Figure 1, that cannot be done. On the other hand, if the geometry is arranged as in Figure 2, it is possible to do this; several recording surfaces are used instead of just one. Each records the interference pattern between scattered photons and unscattered (reference) photons reaching it at grazing angles α_s and α_u respectively. Photons traveling at an angle α_s

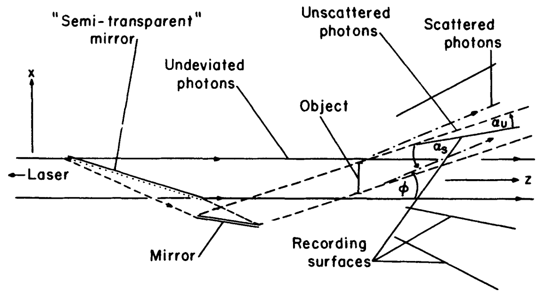

FIGURE 2 X-rays produced by the laser travel parallel to the z axis. Some of them reach the object of diameter h directly (these we call "undeviated photons") and are scattered by it. The scattering angle is ϕ (dash-dot line). The other photons are diverted by a "semitransparent" mirror (actually, it may be a simple mirror intercepting only part of the photons emitted by the laser) and reach the object traveling at an angle $\phi + \alpha_u - \alpha_s$. We call these the "unscattered photons" (dashed line). The recording surface records the interference pattern between the scattered and unscattered photons reaching it at a grazing angle α_s and α_u respectively.

measured from the surface parallel to the (x, z) plane will generate an electromagnetic field which is periodic along the surface, with a periodicity $\Lambda_s = \lambda_0/\cos\alpha_s$. Similarly, photons arriving at the surface with a grazing angle α_u produce a field periodic along the surface with $\Lambda_u = \lambda_0/\cos\alpha_u$. The interference pattern of these two fields will therefore be periodic along the surface with a "beat" period

$$\Lambda = |\Lambda_s - \Lambda_u| = \left|\frac{\cos\alpha_u}{\lambda_0} - \frac{\cos\alpha_s}{\lambda_0}\right|^{-1} \approx \frac{2\lambda_0}{|\alpha_s^2 - \alpha_u^2|}. \quad (8)$$

The last piece of this equation is valid as $\alpha_s, \alpha_u \to 0$. When $\alpha_s, \alpha_u \ll 1$, then $\Lambda \gg \lambda_0$, and the interference pattern can be recorded provided that $\Lambda \geq \Delta l$, even if $\lambda_0 \ll \Delta l$.[5] Rotating the mirrors will direct the unscattered photons successively to the several recording plates, and so record the pattern produced by photons scattered at various angles ϕ.

Holographic reconstruction of the image is possible using the several recording plates: illuminate each with coherent radiation of wavelength λ_0, reaching the surface at a grazing angle α_u. More importantly, enlarged reconstruction is also possible: Enlarge the interference pattern on each surface by a factor ρ, e.g., by geometrical magnification produced by (not necessarily coherent) radiation of wavelength $\lambda \ll \lambda_0$. Arrange the enlarged surface patterns so that they form a ρ-times enlarged version of the original recording plate arrangement. Illuminate each surface with coherent radiation whose wavelength is $\rho\lambda_0$. Then a ρ-times enlarged image of the original object will result. If $\rho\lambda_0$ is in the optical range, then this image can be viewed with optical instruments, including the human eye. The image can also be recorded on a single plate (using a coherent reference beam of wavelength $\rho\lambda_0$). Thereafter the image can be recreated with a single coherent beam of wavelength $\rho\lambda_0$ and viewed as a conventional hologram (i.e., one which is produced as shown in Figure 1). In this manner, one can visualize details of dimension $\approx\lambda_0$, in spite of the fact that the recording medium may have resolution $\Delta l \gg \lambda_0$.

This method could be called "three-wavelength microscopy", because radiation of three different wavelengths is used: radiation with wavelength λ_0 produces the original recorded interference pattern, radiation with wavelength $\lambda \ll \lambda_0$ is used to enlarge this pattern by a factor ρ', and the enlarged pattern is viewed in a radiation with wavelength $\rho\lambda_0$. It is a generalization of the "two-wavelength microscopy" in which radiation of two different wavelengths is employed,[4,6] one with wavelength λ_0 to produce the original recorded interference pattern, the other with wavelength λ to view this pattern. In three-wavelength microscopy $\rho' \neq \rho$ is possible. However, when the original pattern is recorded on several plates (as in Figure 2), reconstruction is easier when $\rho' = \rho$. This is why $\rho' = \rho$ was assumed in the previous paragraph.

The interference pattern at the holograph will be washed out, unless the photon beam produced by the X-ray laser is sufficiently monochromatic:[4]

$$\frac{\delta\lambda}{\lambda_0} \lesssim \frac{1}{2}\,\frac{\lambda_0}{L_H(\alpha_s - \alpha_u)}. \quad (9)$$

At a distance L from the center of the laser as measured along z, the radius of the laser beam will be

$$\sigma_i(L) \approx \sigma_i + L\sigma_i'. \quad (10)$$

Substitute $\approx$ for $\lesssim$ in Eq. (1c). Then with Eqs. (1b), (1c), and (10) we have

$$\sigma_i(L) \approx \tfrac{1}{2}d_i\left(1 + \frac{L}{d_z}\right). \quad (11)$$

Let L_s be the distance of the object from the center of the laser, and L_H the distance of the object from the holographic surface. At the holographic surface $L = L_s + L_H$. The laser beam will cover a holographic surface whose projection on a surface perpendicular to the propagation direction of the photons has a radius R, provided that

$$\tfrac{1}{2}d_i\left(1 + \frac{L_s + L_H}{d_z}\right) \gtrsim R. \tag{12}$$

Those photons which do not reach the holographic surface are wasted. Therefore choose the $\approx$ sign in Eq. (12).

Let A_0 be the undeviated photon amplitude at the object. Then the undeviated photon intensity there is $\sim|A_0|^2$. The amplitude of the unscattered photons at the object is A_u, their intensity $\sim|A_u|^2$. The object scatters a fraction f of all photons reaching it, so that the scattered photon amplitude at the object will be $f^{1/2}(A_0 + A_u)$. Assume that the amplitudes $A_0(H)$ and $A_u(H)$ of undeviated and unscattered photons at the holographic surface will be approximately A_0 and A_u respectively (as when $L_s \gg r_i$). Denote the corresponding intensities by $I_0(H)$ and $I_u(H)$. Assume (see Fig. 2) also $h_i \ll r_i$, and $h_x/r_x = h_y/r_y \equiv h/r$. The absolute value of the amplitude of scattered photons at the holographic surface will be of the order of $|(h/r)f^{1/2}(A_0 + A_u)|$. If, furthermore, $|A_u(H)|/|A_0(H)| \equiv g^{1/2} \ll 1$, this expression can be simplified to $(h/r)f^{1/2}|A_0|$. The photon intensity variation, $I_{int}(H)$, at that surface due to interference between unscattered and scattered photons will be proportional to $A_u^*(H)A_s(H)$, and

$$\frac{I_{int}(H)}{I_0(H)} \approx \frac{h}{r} f^{1/2} g^{1/2}. \tag{13}$$

One needs n_γ photons/cm^2 impinging on the recording surface to imprint a meaningful record. The term "meaningful record" here means a record from which a recognizable image can be reconstructed holographically. The value of n_γ depends on the recording medium.

Suppose that $\Delta l \gg \lambda_0$. Then, by Eq. (8), we need $\alpha_u, \alpha_s \ll 1$. Assuming $\alpha_u \ll \alpha_s$, we require

$$\left(\frac{2\lambda_0}{\Delta l}\right)^{1/2} \gtrsim \alpha_s. \tag{14}$$

If the unscattered beam travels parallel to the (x, z) plane (as in Fig. 2), then the recording surface's projection on the x axis must be $\gtrsim d_x$, and to intercept scattered photons with angle $\leq \alpha_s$, its length must be $\gtrsim d_x/\alpha_s$. One needs about $(\alpha_s^{-1})_{max}$ such surfaces to cover all photons scattered with $|\phi| \leq 1$ radian. Thus one can resolve details of order λ_0 along the x dimension in the object. To resolve also details along the y dimension, a more complicated (cylindrically symmetric) arrangement of recording surfaces may be used. This can be obtained by rotating Figure 2 around the z axis. The total number of recording surfaces will now be of the order of $(\alpha_s^{-2})_{max}$, and the total surface on which interference patterns have to be recorded will be about $h_y h_x(\alpha_s^{-3})_{max}$. The number of photons needed to imprint these patterns is $n_\gamma h_y h_x(\alpha_s^{-3})_{max}$. According to Eq. (13), only a fraction f of all photons will participate in imprinting the interference patterns. Therefore, finally, the total number of photons which have to reach the object in order that the interference patterns be meaningfully imprinted is:

$$N_\gamma \gtrsim \left(\frac{h}{r} f^{1/2} g^{1/2}\right)^{-1} n_\gamma h_x h_y(\alpha_s^{-3})_{max}. \tag{15}$$

As an example, consider the case when $\lambda_0 = 100$Å. The object has diameters $h_x \cdot h_y = 10^{-6}$ cm^2. Assume that the recording medium is a high-resolution film with $\Delta l = 10^{-5}$ cm. Choose $\alpha_s \leq 10^{-1}$ and $\alpha_u \ll \alpha_s$, so that $\Lambda = 2 \cdot 10^{-4}$ cm, more than enough to be recorded on the film. Assume also that the object scatters a fraction $f = 10^{-2}$ of all photons reaching it. (This is a conservative number; for medium heavy and heavy elements, f is higher.) Let the typical h/r be $3.5 \cdot 10^{-2}$, and $n_\gamma = 2.5 \times 10^{14}$ cm^{-2}, $g = 10^{-1}$.† Then from Eq. (15) one finds $N_\gamma \geq 2.5 \cdot 10^{14}$ photons. According to Eq. (9), the photons have to lie within a wavelength band of $\delta\lambda/\lambda_0 \lesssim 1.5 \cdot 10^{-4}$, corresponding to a coherence length of about 2.5×10^{-2} mm. The dimentions specified earlier are consistent with this length: actually, the laser, is expected[1] to put out radiation in a narrower wavelength band than is required by Eq. (9), corresponding to a coherence length of several mm or cm, depending on the lasing material.

Assume that the X-ray laser delivers 10^8 coherent photons per pulse within the desired wavelength band,‡ and that there are 10^6 pulses per second.§

† At $\lambda \approx 100$ Å, a 45° change in photon direction can be obtained in five successive scatterings by grazing incidence mirrors with an overall loss in intensity of < 70%.

‡ These numbers are similar to those which would prevail near $\lambda_0 = 135$ Å (as shown in the second of Ref. 1) provided that the ring radiates photons in an energy interval ≈ 4 eV wide around λ_0 with emittances such that $(\varepsilon_{yx}\varepsilon_{yy}) < 1.2 \cdot 10^{-10}$ cm rad.

§ This is similar to the number prevailing at the Stanford SPEAR ring.

Then the needed number of photons would be delivered by the laser approximately within 2.5 sec.

We conclude that holographic study of microstructures appears feasible.

For purposes of such studies the electron ring does not need high circulating electron energy, E_e. It suffices to have E_e such that the critical energy of synchrotron radiation be $\gtrsim 100$ eV.

The photon emittance should be as small as possible, because the number of pumping photons needed is roughly proportional[1] to $(\varepsilon_{\gamma x}\varepsilon_{\gamma y})^{1/2}$ for fixed λ. The photon beam emittance being related to the electron beam emittance and the latter being generally smaller at lower E_e, this argues in favor of a ring which emits synchrotron radiation with a low critical energy.

To achieve low $(\varepsilon_{\gamma x}\varepsilon_{\gamma y})^{1/2}$, the photons should be taken out from a low-beta section of the ring [Eq. (5)]. This is contrary to common practice today.

Pumping the X-ray laser uses only a relatively narrow energy band of the emitted synchrotron radiation.† Therefore, for purposes of X-ray laser holography, it is desirable to enhance synchrotron radiation in that narrow band. This can be done with wiggler magnets, provided that the momentum spread of the circulating electron beam is small enough, another argument in favor of a small emittance ring.

Thus, for purposes of holography, one would like to have a ring with relatively low energy, low emittance, and high instantaneous current. These characteristics would also make the ring well suited for free electron laser experiments (producing intense coherent radiation mostly in the infrared, and up to the ultraviolet).

It seems reasonable to suggest that if an electron ring were to be built for free electron laser experiments, at least a section of it should be designed for pumping X-ray lasers which could then be used for holographic studies of microstructures, as well as for other purposes.

† Nevertheless, the pumping bandwidth is several orders of magnitude broader than the bandwidth of the emitted coherent X-ray photons.

ACKNOWLEDGEMENTS

I wish to thank B. W. Matthews, V. Rehn, H. Wiedemann, and H. Winick for discussions concerning this subject.

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