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COLLECTIVE ION ACCELERATION MECHANISMS INVOLVING DRIFTING INTENSE RELATIVISTIC ELECTRON BEAMS†

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Several possible mechanisms of collective ion acceleration, involving intense relativistic electron beams injected into low pressure neutral gases, are examined in depth. A brief summary of existing experimental data is given. Possible acceleration mechanisms are categorized as to how the main accelerating electric field is created. These categories include electrostatic space charge fields, induced fields, and collective wave fields. Six existing theories of ion acceleration are examined, including four potential well models, a localized magnetic pinch model, and an inverse Cerenkov radiation model. Each theory is summarized, compared with the data, and commented upon in regard to its validity and applicability. It is concluded that the mechanism responsible for the observed acceleration must be an electrostatic type effect, but that a new theory is needed to explain the existing data with all of their parametric dependences.

1 INTRODUCTION

One of the most intriguing phenomena associated with the propagation of intense relativistic electron beams is that of ion acceleration. Basically, it is observed that when an intense relativistic electron beam is injected into a metallic drift tube filled with a neutral gas at low pressure, a substantial ion bunch is formed which is accelerated in the *same* direction that the beam propagates, and the resultant ion energy is substantially *larger* than the beam electron energy. Experimental observation of the process is well documented.[1–12] The phenomenon was first observed by Graybill *et al.*,[1] then studied in more experiments at Ion Physics Corporation[2–5] (IPC), Physics International Company[6–11] (PI), and Sandia Laboratories.[12]

There is considerable interest in trying to understand the acceleration mechanism, its scaling laws, and its potential for use in future high energy proton and heavy ion accelerators. In this paper, possible acceleration mechanisms are categorized as to how the main accelerating electric field is created. General results are given for mechanisms employing electrostatic space charge fields, inductive fields, and collective wave fields. Six existing ion acceleration theories are then examined. These include the potential well models of Rostoker,[13–14] Uglum *et al.*,[3,15,16] Rosinskii *et al.*,[17] and Poukey and Rostoker;[18] the localized pinch model of Putnam;[19–22] and the inverse coherent Cerenkov radiation model of Wachtel and Eastlund.[23] All of these theories are substantially different, yet four of them[13–17,19–22] have claimed agreement with the data. In concept, each of these theories represents a possible ion acceleration mechanism. Here each theory is summarized, compared with the existing data, and commented upon in regard to its validity and applicability. It is shown that serious questions arise concerning the validity of some of the theories, and in all cases, major difficulties are encountered in trying to explain the data with each of these theories. It is concluded that the mechanism responsible for the observed acceleration must be an electrostatic-type effect, but that a new theory is needed to explain the data with all of their complicated parametric dependences.

Several reviews[22,24–27] have already been made of the data, and of some of the theories, but in these no in-depth study was made to determine if any of the existing theories could indeed account for the existing data. The work presented here should help clarify the status of these theories in regard to the existing data, and it should also provide a useful foundation for future theoretical work.

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For clarity, we point out that all of the work in this paper refers to ion acceleration by *drifting* intense beams—which is to be distinguished from the related phenomenon of ion acceleration *in the diode* that generates these beams. The latter type phenomenon has enjoyed substantial experimental investigation by Plyutto *et al.*,[28–30] Korop *et al.*,[31] Mkheidze *et al.*,[32] Suladze,[33] and Bradley and Kuswa,[34] but it will not be considered here.

Our presentation is as follows. In Sec. 2, a concise summary of the basic data is given. From this, it is established what a theory should be able to explain, and what quantities it should be able to predict analytically. In Sec. 3, some general moving potential well results are given, and four potential well models[13–18] are examined. In Sec. 4, inductive field effects are considered, and a localized pinch model[19–22] is examined. In Sec. 5, collective wave fields are discussed, and an inverse coherent Cerenkov radiation model[23] is examined. Conclusions are given in Sec. 6.

2 GOALS OF A THEORY

To establish what a theory should be able to explain, we briefly review the experimental data[1–12] in Table I. Basically, Table I shows that ion acceleration occurs at low pressures (e.g., 0.1 Torr H_2), it produces ions with energy $\mathscr{E}_i$ that scales as the actual ion charge number Z_i (at least at low pressures) and the ion energy per charge $\mathscr{E}_i/Z_i$ is substantially larger than the electron beam energy $\mathscr{E}_e$. Also given in Table I are typical values of the number of ions N, the ion bunch pulse length T, the observed acceleration length l_a, and the acceleration time t_a. The dependence of $\mathscr{E}_i$ on the pressure p was first thought to be negligible,[1,2,7] but more recent work[3–5,11] has shown a definite pressure effect; the ion energy typically first remains constant as the pressure increases, then it increases with pressure until a pressure is reached above which no accelerated ions are seen. The data have been interpreted to show that no acceleration takes place until after a time equal to the force neutralization time τ_{FN}. The basic differences in the data interpretations include (i) the acceleration being *with* or *behind* the beam front, (ii) scaling of $\mathscr{E}_i$ with respect to the beam current, and (iii) multiple ion pulses.

Based on these data, we believe that an acceptable theory should explain, at least,

a) how the bunch forms,
b) what produces the accelerating field,
c) why the ion energy scales as Z_i at low pressures, and
d) what determines the cutoff, or peak, $\mathscr{E}_i$.

TABLE I

Summary of ion acceleration data

	IPC data[1–5]	PI data[6–11]	Sandia data[12]
Beam	$\sim$ 1.5 MeV $\sim$ 40 kA $\sim$ 50 ns ($\nu/\gamma \sim 0.6$)	$\sim$ 0.25 – 1 MeV $\sim$ 200 – 110 kA $\sim$ 50 ns ($\nu/\gamma \sim 2 - 10$)	$\sim$ 1.8 MeV $\sim$ 80 kA $\sim$ 60 ns ($\nu/\gamma \sim 1$)
Gas	$\sim$ 0.1 – 0.3 Torr H_2 $\sim$ .03 Torr N_2	$\sim$ 0.1 – 0.6 Torr H_2 $\sim$ .03 Torr N_2	$\sim$ 0.15 Torr H_2
Similar Features	$\mathscr{E}_i \sim Z_i(H, D, H_e, N, A)$ $\mathscr{E}_i/(Z_i\mathscr{E}_e) \sim 3$ $N \sim 10^{13}$ $T \sim 3$ ns $l_a \sim 30$ cm $t_a \sim 25$ ns $\mathscr{E}_i$ depends on p $t > \tau_{FN}$ required	$\mathscr{E}_i \sim Z_i(H, D, N, A)$ $\mathscr{E}_i/(Z_i\mathscr{E}_e) \sim 2 - 10$ $N \sim 10^{13}$ $T \sim$ 3–5 ns $l_a \sim$ 5–10 cm $t_a \sim 10$ ns $\mathscr{E}_i$ depends on p $t > \tau_{FN}$ required	$\mathscr{E}_i/(Z_i\mathscr{E}_e) \sim 0.1 - 2$ $N \sim 10^{11} - 10^{14}$ $l_a \sim 6$ cm $t > \tau_{FN}$ required
Differences	acceleration BEHIND beam front $\mathscr{E}_i \sim I^2$ monoenergetic	acceleration WITH beam front multiple pulses monoenergetic, or spread	acceleration WITH beam front large ion energy spread

The emphasis in all of the theories considered here has been on (b), i.e., in establishing an E field of the required order of magnitude. Essentially all of the theories have ignored (a), except for Putnam,[19–22] and all have been unable to account for (c), except for Rostoker's theory[13,14] in a limited parameter range. Point (d) has been considered in some of the theories. In addition, we believe that a theory should predict, at least, the quantities

$$\mathscr{E}_i,\ N,\ T,\ l_a,\ t_a. \tag{1}$$

In this paper, we shall concentrate our efforts on obtaining expressions for the quantities in (1) for each of the theories, and then comparing these with the similar features of the data as listed in Table I. Since we shall encounter substantial difficulties in so doing, we shall refrain, in general, from even considering the theories in the light of the differences in the data as listed in Table I.

3 ELECTROSTATIC FIELD MECHANISMS

Since all of the electrostatic field theories[13–18] involve 1-D potential well models, we consider first a 1-D potential well of width l and depth φ_0 ($\varphi_0 > 0$), with electric field E given by

$$E = \begin{cases} \varphi_0/l & 0 \le z \le l, \\ 0 & z < 0, z > l. \end{cases} \tag{2}$$

This represents an open-ended well as shown in Figure 1a. We now ask what energy $\mathscr{E}_i$ an initially stationary ion will attain, if the well given by (2) is stationary, translating with velocity $v_0(>0)$, or accelerating with acceleration $a_0(>0)$. The ion has mass M and charge $Z_i e$, where e is the magnitude of the charge of an electron. Taking the ion at $t = 0$ to be at $z = 0$ for the stationary case, and at $z = l$ for the other cases, we have computed the $\mathscr{E}_i$ attained as summarized in Table II. Depending

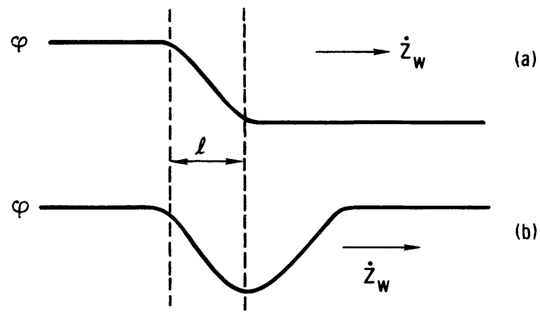

FIGURE 1 (a) An open-ended, moving, potential well. (b) A closed, moving, potential well that permits true trapping.

TABLE II

Potential well ion acceleration

Well	Restriction	$\mathscr{E}_i$	$\mathscr{E}_i$ Scaling
Stationary	None	$Z_i e\varphi_0$	$\sim Z_i$
Translating	(i) Ion lags	$\frac{1}{2}Mv_0^2\left[1-\left(1-\frac{Z_i e\varphi_0}{\frac{1}{2}Mv_0^2}\right)^{1/2}\right]^2$	$\sim Z_i^2/M$
($\dot{z}_w = v_0$)	$Z_i e\varphi_0 < \frac{1}{2}Mv_0^2$		(For $Z_i e\varphi_0 \ll \frac{1}{2}Mv_0^2$)
	(ii) Ion trapped $Z_i e\varphi_0 = \frac{1}{2}Mv_0^2$	$\frac{1}{2}Mv_0^2$	$\sim M$
	(iii) Ion shot ahead $Z_i e\varphi_0 > \frac{1}{2}Mv_0^2$	$4(\frac{1}{2}Mv_0^2)$	$\sim M$
Accelerating	(i) Ion lags	$\frac{Z_i e\varphi_0}{[(Ma_0 l)/(Z_i e\varphi_0)]-1}$	$\sim Z_i^2/M$
($\ddot{z}_w = a_0$)	$Z_i e\varphi_0 < Ma_0 l$		(For $Z_i e\varphi_0 \ll Ma_0 l$)
	(ii) Ion trapped $Z_i e\varphi_0 = Ma_0 l$	$\frac{1}{2}M(a_0 t)^2$	$\sim M$
	(iii) Ion shot ahead $Z_i e\varphi_0 > Ma_0 l$	$\frac{1}{2}M(a_0 t)^2$ (Well always catches)	$\sim M$

on the size of $Z_i e\varphi_0$, an ion may lag behind, be trapped, or be shot ahead of the well. The most interesting result is that only a stationary well has the scaling $\mathscr{E}_i \sim Z_i$; all other cases show a grossly different scaling behavior. With these comments, we proceed to discuss the potential well models of Rostoker,[13–14] Uglum *et al.*,[3,15,16] Rosinskii *et al.*[17] and Poukey and Rostoker.[18]

3.1 *Theory of Rostoker*[13–14]

This is a 1-*D* accelerating potential well model. The well has a depth $e\varphi_0 \approx (\gamma_b - 1)mc^2$ where $\gamma_b = (1 - \beta^2)^{-1/2}$, $\beta = v_b/c$, v_b is the injected beam electron velocity, m is the mass of an electron, and c is the speed of light. If there is no charge neutralization, beam electrons are turned back due to electrostatic forces in a characteristic distance c/ω_b, where $\omega_b = [4\pi n_b e^2/(\gamma_b m)]^{1/2}$ and n_b is the beam density. Charge neutralization is taken to occur on the time scale $\tau_N = \tau_i + \tau_e$, where τ_i refers to collisional ionization by the beam electrons, and τ_e is a radial escape time. Roughly, $\tau_e \approx (1/\omega_b)[r_b/(c/\omega_b)] = r_b/c$, where r_b is the beam radius. As charge neutralization occurs, the well advances with velocity L/τ_N. The characteristic moving well length is $L \approx 2c\omega_b^{-1}$, which follows from a calculation involving Poisson's equation.[13] The well accelerates if τ_N decreases in time, and it is argued that τ_N does decrease in time due to ionization ahead of the well caused by radiation, beam temperature effects, and finite risetime effects. To model these effects, it is assumed that there is a beam of density n_b^* preceding the main beam of density n_b. The "precursor" beam exists at all z and is switched on at time $t = 0$. Including this beam, it is found that

$$\tau_i(t) = \tau_i\{1 - (n_b^*/n_b)[(t/\tau_i) - 1]\}, \tag{3}$$

and, letting z_w denote the well position,

$$\ddot{z}_w(t) = \frac{L(n_b^*/n_b)}{\{\tau_i(1 - (n_b^*/n_b)[(t/\tau_i) - 1]) + \tau_e\}^2} \tag{4}$$

where the superscript dot ($\cdot$) denotes a time derivative (d/dt). Thus the well acceleration is even time dependent. The well acceleration proceeds up to the z where $\tau_i(t) = 0$, after which the well continues to translate with velocity $\dot{z}_w \approx L/\tau_e \sim c$.

Ion acceleration occurs in the well only up to the z where the well acceleration $\ddot{z}_w$ equals the electric field acceleration on the ion due to the well electric field E_0. Beyond this z, the well slips ahead of the ion. Thus the ion is trapped if

$$\frac{\mathrm{d}}{\mathrm{d}t}(\gamma_w M\dot{z}_w) \leq Z_i e E_0 \tag{5}$$

where $\gamma_w = [1 - (\dot{z}_w/c)^2]^{-1/2}$. By assuming that the beam density is constant in the beam front region [see Eqs. (3) and (4) of Ref. 13], the expression

$$E_0 = 4\pi n_b e(2^{1/2} - 1)c\omega_b^{-1} \tag{6}$$

was obtained. Then, apart from a numerical factor of order 2, (4), (5), and (6) were combined to give[13]

$$\mathscr{E}_i = \frac{2mc^2 Z_i n_b}{\gamma_w^3 n_b^*} \approx \frac{n_b}{n_b^*} Z_i(\mathrm{MeV}), \tag{7}$$

by assuming slip-out occurs precisely when equality holds in (5). We note that for typical parameters, slip-out always occurs in this model. It was also found that[13]

$$l_a = L(n_b/n_b^*)\ln[1 + (n_b/n_b^*)] \tag{8}$$

$$t_a \approx \tau_i n_b/n_b^* \tag{9}$$

$$N \approx \pi r_b^2(c/\omega_b)n_b \tag{10}$$

$$T \approx (c/\omega_b)/(\beta_i c) \tag{11}$$

where $\beta_i c$ is the final ion velocity.

We have obtained different results for this model starting with essentially the same initial assumptions. For example, results (8) and (9) were derived from (4). However, from (4), we find exactly that

$$l_a = L\left(\frac{n_b}{n_b^*}\right)\ln\left\{\left[1 + \frac{n_b^*}{n_b} + \frac{\tau_e}{\tau_i}\right]\left[\frac{\beta_i c\tau_i}{L}\right]\right\} \tag{12}$$

$$t_a = \tau_i\left[1 + \frac{n_b}{n_b^*}\left\{1 + \frac{\tau_e}{\tau_i} - \frac{L}{\beta_i c\tau_i}\right\}\right], \tag{13}$$

which may differ substantially from (8) and (9). Also, the field E_0 in (6) does not agree with the desired expression $E_0 \approx \mathscr{E}_e(eL)^{-1}$. Using the later expression (and assuming that the beam density adjusts to justify this choice), we find

$$\mathscr{E}_i = \frac{n_b Z_i}{n_b^* 2\gamma_w^3}\mathscr{E}_e \tag{14}$$

in place of (7).

For comparison with experiment, we use parameter values typical of the PI data;[6–10] $Z_i = 1$, $\gamma_b = 3$, $n_b = 5 \times 10^{12}$ cm^{-3}, $r_b = 1.25$ cm, and pulse length $\tau_b = 50$ ns. Using data[35] for 1-MeV electrons in H_2 at a pressure of 0.1 Torr, we find

$\tau_i \approx 50$ ns. Then choosing $n_b/n_b^* = 2$ (to make $\mathscr{E}_i$ agree with the data), we obtain for protons, using (7)–(11),

$$\mathscr{E}_i = 2\ \text{MeV} \quad (\sim 2\ \text{MeV}) \qquad (7')$$

$$l_a \approx 2\ \text{cm} \quad (\sim 5\text{–}10\ \text{cm}) \qquad (8')$$

$$t_a = 100\ \text{ns} \quad (\sim 5\text{–}10\ \text{ns}) \qquad (9')$$

$$N \approx 10^{13} \quad (\sim 10^{13}) \qquad (10')$$

$$T = 0.2\ \text{ns} \quad (\sim 3\text{–}5\ \text{ns}), \qquad (11')$$

where the observed values for each quantity are as indicated in the parentheses. There are large discrepancies for t_a and T and these discrepancies are even larger for data from IPC.[1–5]

The principal problems one encounters in trying to explain the data[1–12] with this model are as follows.

a) First, at neutral gas pressures where ion acceleration is observed, τ_i may be greater than or equal to the beam pulse length t_b.[35] This means that the well would not even start to move before the beam expired. Also, since the predicted t_a is always larger than τ_i, we typically find the predicted value of t_a to be larger than that observed, even if τ_i could be reduced somewhat.

b) The sudden beam front acceleration when $\tau_i(t) \to 0$ as predicted by this theory [and required to obtain slip-out, and the Z_i scaling in (7)] is not seen in the data. In the theory, the well velocity $\beta_f c$ increases from $\beta_i c$ to about c during the time interval $\Delta t = t_1 - t_a$, where t_1 is the time at which $\tau_i(t_1) = 0$. Using (3) and (13), we find

$$\Delta t = (n_b/n_b^*)[L(\beta_i c)^{-1} - \tau_e]. \qquad (15)$$

For the parameters used in (7′)–(11′), the theory predicts $\beta_f c$ increases from $\beta_f c \approx 0.06\,c$ to about c over the time interval $\Delta t \approx 0.8$ ns. No such sudden jumps in the beam front motion are seen in the data.[9,11,12]

c) The theory predicts a moving, beam-front, well length of order c/ω_b, but beam-front lengths of this order are not seen in the data. For example, since the full beam current propagates behind the well, the final net current risetime t_r^* associated with the beam front would be predicted to be $t_r^* \approx (2c/\omega_b)(\beta_e c)^{-1} \approx 2\omega_b^{-1}$. For the parameters used in (7′)–(11′), this gives $t_r^* \approx 0.03$ ns whereas typical observed values are of the order of $t_r^* \approx 10$ ns.[9,11,12] Further evidence that moving well lengths of order c/ω_b do not occur is that the predicted ion pulse length T is more than an order of magnitude smaller than what is observed. In the theory,[13] it was argued that the ion pulse spreads in the evacuated drift region of the ion detection apparatus. However, consider the IPC experiments[1–3] where the ion detection drift tube had current monitors at distances 10 cm, 40 cm, and 70 cm from the point of injection. The data for H_2, for example, shows that although the total ion current decreases as the ion pulse moves along the tube, the ion pulse *shape* stays remarkably the same, with essentially no axial spreading. Substantial axial spreading would be expected if the pulse had expanded axially from ~ 0.4 cm to ~ 7 cm during its traversal of the first 10 cm of the tube.[36]

d) The theory gives no pressure dependence to $\mathscr{E}_i$, whereas a definite pressure dependence is seen in the data.[4,5,11] Once the ratio n_b^*/n_b is chosen, and p is varied, only t_a and l_a will vary, while $\mathscr{E}_i$ will remain unchanged. This disagrees, e.g., with the PI data[11] in which $\mathscr{E}_i$ increased from 3 MeV to 10 MeV as p increased from 0.3 Torr to 0.6 Torr H_2.

e) Finite well length effects, not included in (7)–(11), can alter the results adversely. In obtaining (7), instantaneous slip-out was assumed when equality held in (5). Here we note that slip-out would really occur in the finite well length ($\sim 2c/\omega_b$) rather than in an infinitesimally small well length as assumed. At the true slip-out time, the inequality in (5) would actually be reversed. Calculations we have done show that if the true slip-out occurs at a time less than the time at which $\tau_i(t) \to 0$, then the scaling $\mathscr{E}_i \sim Z_i$ is preserved; if slip-out occurs at a time later than this, then $\mathscr{E}_i$ does not scale as Z_i. In addition, finite well length effects raise the minimum predicted energy [i.e., $\mathscr{E}_i$ in the limit $n_b^* \to n_b$] to above $\mathscr{E}_i = Z_i$(MeV). This is somewhat disturbing since ion energies less than Z_i(MeV) have been reported.[8] [Note that our expression (14) for $\mathscr{E}_i$ does not exhibit this problem.]

f) Since slip-out always occurs in this theory, the ions would have to drift in the charge neutral region behind the well. There the ions would see the net magnetic field of the beam B_θ and be deflected out to the guide tube walls. (At typical ion acceleration pressures, substantial current neutralization does not occur, so B_θ can be large.) Only a few ions (those almost exactly on axis) would reach the ion detection apparatus.

g) A free parameter (n_b/n_b^*) is employed so the ion energy cannot be determined from first principles. Also, note that if there is no ionization ahead of the beam ($n_b^*/n_b \to 0$), the theory predicts $\mathscr{E}_i \to \infty$, but that $l_a \to \infty$ and $t_a \to \infty$ also, so that this case would not be physically realizable. Actually if $n_b^* = 0$, a constant beam front velocity $2c(\omega_b \tau_i)^{-1}$ would result and the ion energy would be $\mathscr{E}_i \approx 2Mc^2(\omega_b \tau_i)^{-2}$. In this case, $\mathscr{E}_i$ would be $\ll \mathscr{E}_e$ typically, and $\mathscr{E}_i \sim M$, both of which disagree with the data.

Thus although this model is intuitively attractive, it is apparently not able to account for the existing data.[1–12]

3.2 *Theory of Uglum, Graybill, and McNeill*[3,15,16]

This is a different potential well model in which the well accelerates due to a neutral gas avalanche breakdown mechanism. This model was designed in conjunction with the IPC data which was interpreted as showing that the beam propagates ahead of the ion pulse. Thus at $t = 0$, the beam is taken to be propagating down the entire drift tube and a background charge neutralization fraction $f_e = \gamma_b^{-2}$ (sufficient to provide force neutralization) is assumed. The well is open-ended as in Figure 1a, and has length L. The electric field associated with the well is $E = \varphi_0/L$ where $e\varphi_0 \approx (\gamma_b - 1)\mathrm{mc}^2$. At $t = 0$ the well is at the anode ($z = 0$). Using values of E and p with the data of Felsenthal and Proud,[37] an avalanche breakdown time τ is obtained. With the length L and time τ, an acceleration a_0 is constructed, $a_0 \equiv 2L/\tau^2$. Using a_0 and assuming an ion is at the front of the well ($z = L$) at $t = 0$, the energy attained by the ion as the well passes is computed to be as in the appropriate entry in Table II.

We have verified that this model predicts

$$\mathscr{E}_i = \frac{Z_i e\varphi_0}{[(Ma_0 L)/(Z_i e\varphi_0)] - 1} \tag{16}$$

$$l_a = \frac{Z_i e\varphi_0 L}{Ma_0 L - Z_i e\varphi_0} \tag{17}$$

$$t_a = \left[\frac{2ML^2}{Ma_0 L - Z_i e\varphi_0}\right]^{1/2}. \tag{18}$$

In the theory,[3,16] the quantities N, T, l_a, and t_a were not given, but the values of l_a and t_a (as given above) follow readily from the derivation of (16). Note that we must have $Ma_0 L > Z_i e\varphi_0$ for (16) to be valid, but that we need $Ma_0 L \approx Z_i e\varphi_0$ to make $\mathscr{E}_i > Z_i e\varphi_0$. Thus $\mathscr{E}_i$ will be a sensitive function of $Ma_0 L$. Also note that L is a free parameter to be specified.

For comparison with experiment, we use parameter values relevant to the IPC data ($e\varphi_0 =$ 1 MeV, $p = 0.2$ Torr H_e, H_e^{+2} so $Z_i = 2$ and $M = 4\, M_{\text{proton}}$). We choose $L = 5$ cm as in the examples in the references.[3,16] Then if we calculate E/p, find τ and compute a_0, we find $\mathscr{E}_i$ from (16) is negative because a_0 is slightly too small. Actually we have $Ma_0 L < Z_i e\varphi_0$ so (16) does not apply and, from Table II, $\mathscr{E}_i \sim \frac{1}{2}M(a_0 t)^2$ until the well acceleration terminates. In Ref. 16, in comparing the theory with the data, the approach used is to set $\mathscr{E}_i$ equal to the observed ion energy in (16) and then solve for a_0. Doing this with $\mathscr{E}_i = 10$ MeV gives $a_0 = 0.115$ cm/ns^2, whereas the breakdown data give $a_0 = 0.044$ cm/ns^2.[38] Using $a_0 = 0.115$ cm/ns^2, we find

$$l_a = 25 \text{ cm} \qquad (20\text{–}30 \text{ cm}) \tag{17'}$$

$$t_a = 22.8 \text{ ns} \qquad (\sim 25 \text{ ns}) \tag{18'}$$

where the experimental values are as in the parentheses. The agreement in (17′) and (18′) is somewhat fortuitous since, in effect, the results (17′) and (18′) refer to nothing more than the acceleration of an ion in the field $E = \varphi_0/L$ up to an energy $\mathscr{E}_i$—and here, φ_0, L, and $\mathscr{E}_i$ were chosen.

Some fundamental problems concerning the validity of this model are as follows:

a) The initial assumption of a propagating force neutral beam violates the limiting current requirement as discussed by Olson and Poukey.[39] Briefly, if a force neutral beam could propagate, the potential depression at the center of the beam would be roughly three times greater than $\mathscr{E}_e/e$ for parameters typical of the IPC data.[1–5] Clearly such a beam could not propagate in the first place.

b) The calculation of τ is apparently invalid. To find τ, one is instructed[3,16] to extrapolate several orders of magnitude off the data of Felsenthal and Proud,[37] into a regime where the mean free path between ionizing collisions λ is $\gg L$. For example, for $e\varphi_0 = 1$ MeV and $L = 5$ cm, an electron initially at rest would be accelerated and exit the 5 cm region in a time $t_0 \approx 2L/c = 0.3$ ns. Using the smallest possible collisional ionization time $\tau_i = 5$ ns (for 100 eV electrons in H_2 and $p = 0.1$ Torr)[40] shows that an accelerated electron could, at best, produce only $t_0/\tau_i \approx 0.06$ ion pairs as it traversed the length L. Clearly t_0/τ_i should be $\gg 1$ for an avalanche calculation to be valid, and the avalanche mechanism suggested could not begin to be operative for the parameter values used.

c) No explanation is given as to why the well should accelerate in the first place. Physically this would require τ to decrease as the well moves. For constant τ, the well should simply translate with constant velocity L/τ. (This point was also noted earlier by Putnam.[22])

Even ignoring the above, one encounters other difficulties in trying to explain the data with this model, as follows.

a) The ion energy scaling does not agree with the data. From (16), $\mathscr{E}_i$ depends on Z_i, M, a_0, and L. The scaling $\mathscr{E}_i \sim Z_i$ could occur only if Ma_0L/Z_i were constant, which cannot occur in general since the four parameters M, a_0, L, and Z_i are all independent. For example, suppose $e\varphi_0 = 1$ MeV and $\mathscr{E}_i = 5$ MeV for $Z_i = 1$ in (16). Then increasing M by a factor of 4 and keeping $Z_i = 1$ (which corresponds to H_e^{+1}) gives $\mathscr{E}_i = 0.26$ MeV, instead of 5 MeV as the scaling $\mathscr{E}_i \sim Z_i$ would predict.

b) The pressure dependence does not agree with the data. For constant L, as p increases, τ decreases, a_0 increases and therefore $\mathscr{E}_i$ decreases. Since $\mathscr{E}_i$ is very sensitive to a_0, $\mathscr{E}_i$ falls sharply as p increases. For example, suppose $p = 0.15$ Torr, $e\varphi_0 = 1$ MeV, $Z_i = 1$, and $\mathscr{E}_i = 5$ MeV in (16). Then since $p\tau \approx$ constant, increasing p to only $p = 0.17$ Torr would make $\mathscr{E}_i$ drop to 1.85 MeV. This pressure dependence is not seen in the data.

c) If avalanching could occur in the well E_z, it would also occur simultaneously throughout the beam channel in the E_r of the force neutral beam ($E_r \sim E_z$). Then the well would "disappear" in time τ and no well acceleration would occur.

d) If well acceleration could occur, then wells should accelerate in from *both* ends of the drift tube. Ions would be accelerated in each of them, the wells would meet at the center of the drift tube, and the ion bunches would pass through each other.

e) A free parameter (L) is employed so that the ion energy cannot be determined from first principles.

In summary, this model appears to be physically invalid. Yet, even ignoring this, the model is apparently unable to account for existing data.[1–12]

3.3 *Theory of Rosinskii, Rukhadze, and Rukhlin*[17]

This is another potential well model. The mechanism employed is a traveling ionization front at the head of the beam, very much like that in Rostoker's model. Here, however, the front is taken to move at constant velocity. A strong magnetic field $B = \hat{e}_z B_0$ is imposed on the system, with $B_0 \gg B_\theta$, where B_θ represents the beam self-magnetic field. The well length is now given by the "slowing down length" z_0 for an unneutralized (uncompensated) beam propagating in a strong magnetic field,[41,42]

$$z_0 = [(mc^3)/(2\pi J_0 e)]^{1/2}(\gamma_b^{2/3} - 1)^{3/4}$$
$$= (c/\omega_{pe})(2/\beta)^{1/2}(\gamma_b^{2/3} - 1)^{3/4} \qquad (19)$$

where J_0 is the beam current density and ω_{pe} is the plasma frequency corresponding to the beam density $[\omega_{pe} \equiv (4\pi n_b e^2/m)^{1/2}]$. The well advances a distance z_0 in a collisional ionization time ν_i^{-1}, i.e., it translates with velocity

$$\dot{z}_w = z_0 \nu_i. \qquad (20)$$

To distinguish the theories examined thus far, we list the characteristic well "length" and "time" associated with each of them in Table III. Note that the well velocity of Rosinskii *et al.* is essentially the same as that of Rostoker at $t = 0$.

TABLE III

Characteristic lengths and times associated with potential well motion in three ion acceleration theories.

Theory	Length	Time
Rostoker[13–14]	$\sim c/\omega_b$	$\tau_N(t)$
Uglum *et al.*[3,15,16]	L(chosen)	τ(avalanche)
Rosinskii *et al.*[17]	z_0 [Eq. (19)]	ν_i^{-1}

It is assumed that $z_0 \ll r_b$ so that the beam is current neutralized as well as charge neutralized behind the ionization front. The processes of ion bunch formation, trapping, and acceleration are regarded as "transient processes" and are essentially ignored. In essence, all that is given is a picture of a constant velocity beam front. The well depth is $e\varphi_0 \lesssim (\gamma_b - 1)mc^2$ and since $\mathscr{E}_i$ must be greater than this to explain the data, *some* acceleration mechanism must be employed, although none is mentioned.

The results of this model are:

$$\mathscr{E}_i = \tfrac{1}{2}Mz_0^2\nu_i^2 \qquad (21)$$

$$N = \pi r_b^2 z_0 n_b/Z_i \qquad (22)$$

$$T = z_0/\dot{z}_w = \nu_i^{-1} \qquad (23)$$

The quantities l_a and t_a were not considered. Also, two validity conditions were given. First, $z_0 \ll r_b$ so return current can flow; this amounts to requiring that the beam current I_0 be much larger than the vacuum limiting current.[41,42] Second, the beam current must be less than the critical current for a charge neutralized (compensated) beam in order for the beam to propagate.

To compare with experiment, Rosinskii *et al.* chose the parameters $\mathscr{E}_e = 1$ MeV, $I_0 = 100$ kA, $r_b = 1$ cm, and $\dot{z}_w \equiv 0.06$ c, i.e., they *chose* $\mathscr{E}_i = \frac{1}{2} M\dot{z}_w^2 = 2$ MeV for protons. From (21) they then found $\nu_i \simeq 10^9$ sec^{-1}, which they say is "likely" for pressures near 1 Torr. Actually, accelerated ions are most frequently observed near ~ 0.15 Torr (H_2) for which $\nu_i \simeq 0.03 \times 10^9$ sec^{-1} (for $\gamma \approx 3$),[35] so the "agreement" on ν_i is really off more than an order of magnitude. Using $\nu_i = 10^9$ sec^{-1} and $\dot{z}_w = 0.06$ c in (20) gives $z_0 = 1.8$ cm which is $> r_b$ so the validity condition $z_0 \ll r_b$ is even violated. [Actually, using (19), (20), and the parameters above, we find $z_0 = 0.3$ cm and $\nu_i = 6 \times 10^9$ sec^{-1} so that although z_0 is $< r_b$, ν_i is off more than two orders of magnitude.]

In trying to use this model to explain the existing data, one also encounters the following problems:

a) The theory assumes a strong B_0 whereas no magnetic field was used in any of the experiments.[1–10,12] In fact, recent experiments at PI have shown that with even a modest B_0 no accelerated ions are seen.[11] Thus the mechanism of this theory is directly contradicted by the existing data.

b) The energy scaling disagrees with the data. Since all accelerated ions have the same velocity as the well, $\mathscr{E}_i \sim M$, whereas the data show $\mathscr{E}_i \sim Z_i$.

c) The model is very incomplete, with no explanation of bunch formation, trapping, acceleration, acceleration length or acceleration time.

Thus this model cannot be used to explain the existing data.[1–12]

3.4 *Theory of Poukey and Rostoker*[18]

One-dimensional intense beam propagation into a guide tube filled with vacuum or with neutral gas has been investigated numerically by Poukey and Rostoker.[18] For the vacuum case, they found that a deep potential well was created near the anode. The location of the density maximum and the potential these were found to oscillate about their mean values (by about $\pm 10\%$) in time. The well depth φ_0 was of the order of 2 to 3 times $\mathscr{E}_e/e$. It was suggested[18] that this potential well may form a mechanism for accelerating ions. For intense beam propagation into neutral gas, it was found that the average speed of propagation was similar to that of the Rostoker model, i.e., a velocity of order $(c/\omega_b)/\tau_i$. For $n_b = 10^{11}$ cm^{-3}, $\beta_0 = 0.99$ ($\gamma = 7$), and $\tau_i = 0.95$ ns, an average propagation speed of $(0.06)c$ was obtained from numerical simulation.

This work was not actually developed into a theory of ion acceleration. However some of the problems that one encounters in trying to use this mechanism to explain the data are as follows:

a) The vacuum well is 1-*D* and one-sided as in Figure 1a. It would continually produce ions with energies from zero up to the peak well depth. Ions would be accelerated immediately (even before $t = \tau_{FN}$) and continuously (no bunching effect). Of course, an actual well would be 2-*D* as in Figure 1b. For a 2-*D* stationary well, the ions would oscillate and never escape. For a 2-*D* well oscillating in position and amplitude by $\pm 10\%$, net ion energy gains are possible, but only of order 20% of the peak well depth. For protons, this would give $\mathscr{E}_i \approx 0.4\mathscr{E}_e \rightarrow 0.6\mathscr{E}_e$, whereas $\mathscr{E}_i = 2\mathscr{E}_e \rightarrow 10\mathscr{E}_e$ is observed.[1–12]

b) If the beam propagation into neutral gas is considered as a mechanism for ion acceleration [it was not in Ref. 18], then one encounters problems similar to those that occur in Rostoker's model. These problems are that τ_i is too long, the beam front risetime is too short, and the ion energy scaling is $\mathscr{E}_i \sim M$ for a closed well (all of which disagree with the data).

Thus, this model established the new feature of a potential well depth greater than the electron beam energy for vacuum injection. However, it is still not possible to account for the data[1–12] with this model.

4 INDUCED FIELD MECHANISMS

Induced electric fields caused by a time-varying current or a time-varying inductance, may provide a source of possible ion acceleration. A uniform beam with risetime t_r and radius r_b inside a guide tube of radius R, produces an electric field on axis[43]

$$E = I_0(c^2 t_r)^{-1}[1 + 2\ln(R/r_b)]. \qquad (24)$$

For typical parameters ($I_0 = 40\ kA$, $t_r = 10$ ns, $R/r_b = 2$), (24) gives $E \lesssim 10^4$ V/cm, whereas fields of order 10^6 V/cm are required to explain the observed acceleration.

A different inductive field mechanism occurs if there is time-dependent beam pinching, as in the localized pinch model of Putnam.[19–22] The inductive pinch field for a uniform beam of radius $r_b(t)$ that carries a constant current I_0 is

$$E = \frac{2I_0}{c^2}\frac{(-1)}{r_b(t)}\frac{dr_b(t)}{dt} \tag{25}$$

where $r_b(t)$ represents the beam envelope radius in the laboratory frame. With the assumption

$$r_b(t) = r_b(0)e^{-t/t_p}$$

we find

$$E = \frac{2I_0}{c^2 t_p} \tag{26}$$

where t_p is the characteristic "pinch time." Using (26), we have performed several calculations to determine if this field could accelerate ions efficiently. A brief discussion of these cases follows—the cases are sketched in Figure 2 and the analytic results are summarized in Table IV.

i) *Stationary pinch*—By this, we mean a beam which pinches uniformly during the time $0 \leq t \leq t_p$, as in Figure 2a. An ion would accelerate in the E field of (26) and at time $t = t_p$ attain the energy $\mathscr{E}_i$ listed in Table IV. Note that $\mathscr{E}_i$ does *not* depend on t_p; if t_p is decreased, E increases, but the acceleration time (which is also t_p) decreases, with the result that $\mathscr{E}_i$ remains constant. For $I_0 = 10^5 A$, a proton would attain energy $\mathscr{E}_i = 2I_0^2 e^2/(Mc^4) \approx$ 20 keV. Thus, typically $\mathscr{E}_i$ would be too small to be useful in explaining the data.

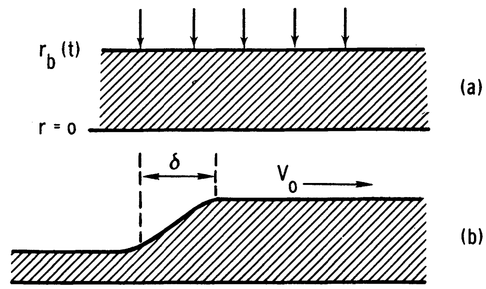

FIGURE 2 Pinching beam configurations, as used to estimate their effectiveness in accelerating ions. (a) Stationary pinch. (b) Translating pinch for v_0 = constant; synchronized pinch for $v_0 = v_i(t)$.

ii) *Translating pinch*—Here we consider a "one-sided" pinch that moves with velocity v_0 = constant, as indicated in Figure 2b. As the constriction passes by in the laboratory frame, it appears as a pinching beam with characteristic pinch time "t_p" $= \delta/v_0$, where δ represents the distance over which r_b decreases by the factor e^{-1}. For $I_0 = 10^5 A$ and $v_0 = 0.05\ c$ (a typical beam front velocity), a proton would attain $\mathscr{E}_i = 22$ keV, which again is typically too small to explain the data.

iii) *Synchronized pinch*—This case represents the true localized pinch mechanism; the pinching velocity is set equal to the ion bunch velocity $v_i(t)$. The characteristic pinch time is now $t_p(t) \approx \delta/v_i(t)$, i.e., the pinch accelerates. In this case, the velocity exponentiates with the characteristic time $\tau \equiv Mc^2\delta/(2eI_0)$, so $\mathscr{E}_i$ increases by the factor $e^2 = 7.39$

TABLE IV

Pinch acceleration.

Pinch type	Pinch time	$\mathscr{E}_i$	$\mathscr{E}_i$ Scaling
Stationary	t_p	$\frac{2I_0^2 Z_i^2 e^2}{Mc^4}$	$\sim \frac{Z_i^2 I_0^2}{M}$
Translating $\dot{z} = v_0$	$\frac{\delta}{v_0}$	$\frac{1}{2}Mv_0^2\left[1 - \left(1 - \frac{4Z_i e I_0}{Mv_0 c^2}\right)^{1/2}\right]^2$	$\sim \frac{Z_i^2 I_0^2}{M}$
		$(4Z_i e I_0 \leq Mv_0 c^2)$	(For $4Z_i e I_0 \ll Mv_0 c^2$)
Synchronized $\dot{z} = v_i(t)$	$\frac{\delta}{v_i(t)}$	$\frac{1}{2}Mv_i^2(0)e^{2t/\tau}$	Depends on M, $v_i(0)$, and time cutoff
		$\tau \equiv Mc^2\delta/(2eI_0 Z_i)$	

in time τ. For $I_0 = 10^5 A$ and $\delta = 2$ cm (i.e., δ of order r_b) a proton would have $\tau = 10$ ns. Thus, typically τ is too large to explain the data. Also the initial velocity $v_i(0)$ would have to be established by some other process before this process would dominate.

Thus these preliminary considerations indicate that the pinch mechanism does not appear to be able to generate a high enough $\mathscr{E}_i$ to begin to explain the data. Also note that none of the cases considered here exhibit the scaling $\mathscr{E}_i \sim Z_i$. With these comments, we proceed to examine the theory of Putnam.

4.1 *Theory of Putnam*[19–22]

This theory is based on the model shown in Figure 3. A propagating beam exists ahead of, and behind,

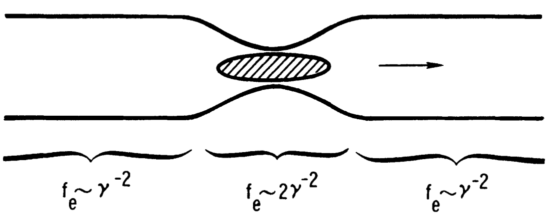

FIGURE 3 Model of localized pinch acceleration proposed by Putnam.[19–22] The ion bunch is indicated by the shaded region.

an ion bunch. The fractional charge neutralization is $f_e \sim \gamma^{-2}$, i.e., the beam is force neutral. The ion bunch enhances f_e locally to $f_e \sim 2\gamma^{-2}$, and the beam pinches in response. The pinch generates an electric field which accelerates the ions, and since the pinch follows the ions, the whole process is synchronized. Acceleration continues until a cutoff (as yet unspecified) is reached.

Most of this theory involves a calculation of the value of the E field, which requires an expression for $r_b(t)$. Starting with the fields $E_r(r, z, t)$ of a uniform rod charge and $B_\theta(r, z, t)$ of a uniform current rod, Putnam uses Faraday's law to find a general expression for $E_z(r = o, z, t)$. For the pinch model, two terms in that expression are used,

$$E_z(r = 0, z) = \frac{2\beta_{L_e}\lambda_e}{ca_e}\frac{\partial a_e}{\partial t} + \frac{2\lambda_e}{a_e}\frac{\partial a_e}{\partial z}. \quad (27)$$

The notation used is as in Refs. 20–22—$a_e(a_i)$ refers to the beam (ion) envelope radius, $\beta_{L_e}(\beta_{L_i})$ refers to the longitudinal β of the beam electrons (ion bunch), and λ_e refers to the linear beam charge density. The first term in (27) is the IdL/dt term, while the second term is an electrostatic force term due to variation of a_e with z. We have also derived (27) and find it valid only if $|E_z| \ll |E_r|$, which requires

$$\frac{\partial a_e}{\partial z} \ll 1 \quad (28a)$$

$$\frac{2\beta_{L_e}}{c}\frac{\partial a_e}{\partial t} \ll 1 \quad (28b)$$

both hold. Transforming to the ion rest frame, $u_i = z - \beta_{L_i}ct$, (27) becomes

$$E_z = -\frac{2\lambda_e\beta_{L_e}\beta_{L_i}}{a_e}\frac{\partial a_e}{\partial u_i} + \frac{2\lambda_e}{a_e}\frac{\partial a_e}{\partial u_i}. \quad (29)$$

Putnam then *drops* the first term, the IdL/dt term, (since it is smaller than the other one) and has

$$E_z \approx \frac{2\lambda_e}{a_e}\frac{\partial a_e}{\partial u_i}. \quad (30)$$

Thus the "pinch field" used is actually *not* an inductive field but an electrostatic field (in either the ion or laboratory frame). Note that to accelerate ions we need $E_z > 0$, which from (30), requires $\partial a_e/\partial u_i < 0$ since $\lambda_e < 0$.

Next, the Kapchinskii-Vladimirskii (KV) envelope equations are used to obtain an expression for $\partial a_e/\partial u_i$. Upstream and downstream of the ion bunch, a special value f_e^0 is derived that corresponds to uniform envelope radii for both the ions and electrons. At the ion bunch, f_e is enhanced and the beam envelope constricts. Using the expression found for $\partial a_e/\partial u_i$ in (30), we have verified the result[22]

$$E_z = \frac{2^{3/2}\lambda_e}{\beta_{L_e}a_e^0}\left(\frac{v}{\gamma}\right)^{1/2}\left(f_e^0 + \frac{\Delta\lambda_i}{|\lambda_e|}\right)^{1/2}\frac{1}{s}\frac{\partial s}{\partial x} \quad (31)$$

where

$$\left(\frac{\partial s}{\partial x}\right)^2 = 1 - s^2 + P\ln s^2 - Q(s^{-2} - 1) \quad (32)$$

$$P = \left[\gamma_{L_e}^2\left(f_e^0 + \frac{\Delta\lambda_i}{|\lambda_e|}\right)\right]^{-1}$$

$$Q = (f_e^0 - \gamma_{L_e}^{-2})\left(f_e^0 + \frac{\Delta\lambda_i}{|\lambda_e|}\right)^{-1}.$$

Here $\Delta\lambda_i$ is the ion bunch linear charge density, and the superscript (^{0}) refers to the equilibrium upstream and downstream from the ion bunch. By neglecting P and Q in (32) the result $s^{-1}\,\partial s/\partial x \approx$

$-\pi^{-1}$ was found.[22] Using this in (31) gives the result reported in references 20 and 21,

$$E_z = 4.5 \times 10^5 \left(\frac{v}{\gamma}\right)^{1/2} \frac{v(1 + v/\gamma)^{1/2}}{a_e^0 \beta} \times \left(f_e^0 + \frac{\Delta\lambda_i}{|\lambda_e|}\right)^{1/2} \text{V/cm}, \quad (33)$$

which we find convenient to write as

$$E_z \approx 0.21 \frac{I_0(A)^{3/2}}{\beta_{L_e}^{5/2} \gamma^{1/2} a_e^0} \left(f_e^0 + \frac{\Delta\lambda_i}{|\lambda_e|}\right)^{1/2} \text{V/cm}. \quad (33')$$

Typically the field (33) is so large that beam electrons would be stopped by it in a distance of order a_e, violating (28a) and assumptions inherent in the use of the KV equations. Noting this, Putnam returned to the first term in (27), the IdL/dt term, and assuming $\partial a_e(t)/\partial t = c$, he obtained an expression for the "saturated" or maximum inductive field possible

$$E_z^{\text{SAT}} = \frac{2I_0}{ca_e} = 60 \frac{I_0(A)}{a_e(\text{cm})} \text{V/cm}. \quad (34)$$

One concludes that (33) is not applicable, although it is still used, and that (34) represents an absolute upper bound on E_z, but even this expression is not applicable because (28b) is violated. In any event, to this electric field, other electrostatic fields are added qualitatively—those due to $\partial\lambda/\partial z$ caused by beam slowdown, and those from the ion bunch itself. The general conclusion is that there are many electric field contributions, and that the peak value of them is of the order given by (33) or (34).

The remainder of this model includes a criterion for ion bunching. This is that E_z near the anode drives ions away from the anode faster than they can be produced by collisions. Also given are many speculations as to why the acceleration should cut off, the favored one being depletion of ions near the anode. It is argued that this would cause the beam to shut off, a new pulse would form, and multiple pulses would occur. Note that the bunching mechanism *requires* ion depletion near the anode (i.e., cutoff), so it has not been explained why the ion bunch would ever leave the anode region in the first place.

Since cutoff has not been determined, the ions are assumed to accelerate in a constant E_z given by (33), through a length l_a chosen to make $\mathscr{E}_i$ equal the observed ion energy $\mathscr{E}_i$. Thus we have

$$\mathscr{E}_i = eZ_i E_z l_a \quad (35)$$

$$t_a \approx \tau_i \gamma^{-2} + 2l_a(\beta_{L_i} c)^{-1} \quad (36)$$

$$N \approx \pi r_b^3 n_b \gamma^{-2} \quad (37)$$

$$T \approx r_b(\beta_{L_i} c)^{-1} \quad (38)$$

In (37) and (38), we have taken the bunch length to be of the order of the beam radius. Results (36)–(38) are not given explicitly in the references,[19–22] but they follow readily.

For comparison with experiments, we use $\gamma = 3$, $v/\gamma = 1$, $I_0 = 50$ kA, $a_e^0 = 1$ cm, $f_e^0 = \gamma^{-2}$, $\Delta\lambda_i/|\lambda_e| = \gamma^{-2}$, and find from (33), $E_z \approx 7.4 \times 10^5$ V/cm whereas (34) gives $E_z^{\text{SAT}} \approx 3 \times 10^6$ V/cm. Using the former, we choose $l_a = 6$ cm so $\mathscr{E}_i \approx 4$ MeV for $Z_i = 1$. Then for $p = 0.1$ Torr in H_2,

$$t_a \approx 10 \text{ ns} \quad (\sim 10 \text{ ns}) \quad (36')$$

$$N \approx 10^{12} \quad (\sim 10^{13}) \quad (37')$$

$$T \approx 0.3 \text{ ns} \quad (\sim 3 \text{ ns}) \quad (38')$$

where typical experimental values are indicated in the parentheses.

Some fundamental problems concerning the validity of this model are as follows:

a) The initial assumption of a propagating force neutral beam violates the limiting current requirement as discussed by Olson and Poukey.[39] This means that the "equilibrium" force neutral beam cannot exist (even before the ion bunch is added) because the space charge potential depression would exceed the electron beam energy. Since the ion acceleration data[1–12] typically has $I_0 > 3I_l$ (where I_l is the limiting current[39]), this means that we would need $0.67 < f_e < 1$ to keep the potential depression less than or equal to the beam energy, and have the propagating beam radius equal to the injected beam radius. A force neutral beam has only $f_e = \gamma^{-2} \approx 0.1$ (for $\mathscr{E}_e \approx 1$ MeV), and it therefore could not exist self-consistently.

(b) A synchronized inductive pinch field calculation was never performed. The field used in (33) is an *electrostatic* field (in either the laboratory or the ion frame) associated with $\partial a_e/\partial z$. The true IdL/dt field depends on the ion bunch speed, while the electrostatic field depends only on the envelope shape $\partial a_e/\partial z$.

c) The direction of the accelerating E field depends on which end of the pinch one is at. The IdL/dt term in (27) would accelerate ions at the front of the pinch ($\partial a_e/\partial u_i > 0$), but would *decelerate* ions at the rear or "unpinch" region ($\partial a_e/\partial u_i < 0$).

Thus the IdL/dt field would spread the ion bunch in time. The electrostatic term in (27) accelerates ions only in the unpinch region ($\partial a_e/\partial u_i < 0$); at the front of the pinch ($\partial a_e/\partial u_i > 0$) this term would decelerate ions. Thus the electrostatic term produces the characteristics of a two-sided potential well, both sides of which would accelerate ions to the center. Net acceleration of the well is difficult to explain. In fact, if $f_e = 0$ ahead of the well, then apparently *deceleration* would dominate.

d) The assumption that f_e^0 has the exact value necessary for a_i^0 and a_e^0 to be constant ($f_e^0 \approx \gamma^{-2}$) outside the pinch region is somewhat idealistic. In general f_e will become larger than this value, and according to the KV equations, envelope oscillations would then occur everywhere.

Even ignoring the above problems, one encounters other difficulties in trying to explain the data with this model:

a) The energy scaling $\mathscr{E}_i \sim Z_i$ has not been demonstrated. For a true synchronized inductive calculation $\mathscr{E}_i$ does not scale as Z_i (see Table IV). With the electrostatic field (33), to obtain $\mathscr{E}_i \sim Z_i$, one must show that (i) E_z is the same for all species, and that (ii) all species have the exact same cutoff length (as argued in Ref. 20). Only if $f_e^0 + (\Delta\lambda_i/|\lambda_e|)$ is the same for all species will E_z, as given by (33), be invariant. In fact, the PI data shows that $\Delta\lambda_i$ is not the same for all species; in one well-documented example,[7,10] the $Z_i = 2$ ion density (H_e^{++}) was at least two orders of magnitude smaller than the $Z_i = 1$ ion density (H^+). In addition, the effect of varying $\Delta\lambda_i$ may be much worse than as indicated by (33), since the turning point approximation $s^{-1}\,\partial s/\partial x \approx -\pi^{-1}$ breaks down in the limit $\Delta\lambda_i \to 0$. Result (31) correctly shows $E_z \to 0$ as $\Delta\lambda_i \to 0$ whereas (33) does not.

b) Even ignoring the fact that (34) is inapplicable because (28b) is violated, the pinch field (34) could not accelerate ions efficiently due to the following. (i) The ion pulse would have to be created almost instantaneously to allow $\partial r_b/\partial t \approx c$; if the pulse were created in time τ, then $\partial r_b/\partial t$ would be of order r_b/τ ($\ll c$ for reasonable values of τ). (ii) Even if $\partial r_b/\partial t \approx c$, the field (34) would only exist for a time $r_b/c \approx 0.03$ ns (for $r_b = 1$ cm), during which time an ion would barely begin to be accelerated.

c) A free parameter (l_a) is employed so the ion energy cannot be determined from first principles.

In summary this model does not validly apply to the existing data[1-12] for which $I_0 > I_l$. Even ignoring this, one apparently is unable to account for the existing data with this model.

5 COLLECTIVE WAVE FIELD MECHANISMS

Collective wave fields may also provide a mechanism for ion acceleration, provided the wave phase velocities are slow enough to permit capture and ion acceleration up to the observed ion speeds $\beta_i c$, where $0.05 < \beta_i < 0.14$.[1-12] The required slow phase velocities immediately restrict the types of waves that one may consider. In addition, any continuous wave phenomena would produce a continuous ion output, whereas a fairly localized ion pulse is always seen in the data.[1-12]

In regard to waves excited by streaming instabilities, we briefly mention the electron-ion two-stream instability. The relevant particle species would be fast beam electrons and cold ions. In one dimension, and assuming the ion density $n_i Z_i$ equals the beam density n_b, this instability[44] produces a wave with phase velocity V_{PH},

$$V_{\text{PH}} = (0.40)(Z_i \gamma^3 m/M_i)^{1/3} V_0 \tag{39}$$

where $\gamma = (1 - \beta_0^2)^{-1/2}$, $\beta_0 = V_0/c$ and V_0 is the injected electron velocity. Even if we assume that the wave E field were large enough to permit ion trapping, and that V_{PH} were such as to give the right order of magnitude for the ion energy $\mathscr{E}_i$, we would find the scaling

$$\mathscr{E}_i \sim Z_i^{2/3} M_i^{1/3} \gamma^2. \tag{40}$$

This scaling disagrees with the data, which shows $\mathscr{E}_i \sim Z_i$ (at least at low pressure). Also, if $n_i Z_i \neq n_b$, V_{PH} scales as $(n_i/n_b)^{1/3}$. This means that V_{PH} and therefore the velocity of any trapped ions, would *decrease* in the direction toward the beam front. In addition, Putnam[22] estimates that only very low ion energies could be produced by streaming instabilities. Thus it is not too surprising that no instability-driven, wave-trapping, ion acceleration mechanisms have been seriously proposed to explain the existing data.[1-12]

However, a novel collective wave mechanism that employs Cerenkov radiation, and that was first proposed by Veksler,[45,46] has been considered by Wachtel and Eastlund[23] as a possible explanation of the observed acceleration.[1-12] To introduce this mechanism, we consider briefly the Cerenkov effect for a single particle in a plasma, the coherent Cerenkov effect for a cluster "particle," and finally cluster acceleration by inverse coherent Cerenkov radiation.

Cerenkov radiation was first observed in the context of light (EM) waves.[47,48] The radiation occurs when a charged "point" particle moves with velocity V in a medium which allows EM wave propagation of waves with phase velocity $\omega/k < V$. The radiation is emitted at the angle $\theta = \cos^{-1}[\omega/(kV)]$, and accordingly, the necessary condition for Cerenkov radiation is $\beta n > 1$ where $\beta = V/c$ and the index of refraction $n = kc/\omega$. Since $\beta < 1$ always, $n > 1$ is required for the radiation. For EM waves propagating in a plasma with no external magnetic field, $\omega/k > c$ always and Cerenkov radiation is not possible, as is well known. If an external magnetic field is present, Cerenkov radiation is allowed in many regions of a CMA diagram, and these have been studied extensively, especially for propagation along the magnetic field.[49–53]

Since no external magnetic field was employed in the ion acceleration experiments,[1–10,12] it might first appear that Cerenkov radiation would never occur. However, it happens that the Cerenkov effect can occur for longitudinal waves in a plasma, if thermal effects are included.[54–58] Longitudinal plasma waves have phase velocities in the range $3V_{TH} \lesssim V_{PH} < \infty$ [where $V_{TH}(V_{PH})$ is the thermal (phase) velocity], so Cerenkov radiation becomes possible for $V > 3V_{TH}$. The electric field produced that slows the particle down due to its wave emission is

$$E = \frac{q\omega_p^2}{2V^2} \ln\left(1 + \frac{2V^2}{V_{TH}^2}\right) \tag{41}$$

Many authors have obtained or quoted this result, but usually with some minor variation (e.g., the 1 in the ln factor is frequently missing).[45,46,55–62] We have derived (41) by considering a particle with charge density

$$\rho(\underline{x}, t) = q\delta(z - vt)\delta(x)\delta(y),$$

using Poisson's equation (i.e., considering plasma waves in the electrostatic approximation), and calculating $E_z(\underline{x}, t)$ by using standard Fourier–Laplace transform techniques. The result (41) follows when the wave number (k) integrals that occur are limited to the range $0 < k < k_d$, where the Debye wave number $k_d \approx 2^{1/2}\omega_p/V_{TH}$.

If we replace the single charged particle by a "cluster" composed of N particles, each with charge $Z_i e$, then the E field in (41) will be enhanced by the factor NZ_i. This will occur provided the bunch has a dimension l in each wave propagation direction that is much smaller than the wavelength of waves that propagate in that direction. The shortest wavelength allowed is of order $\lambda = 2\pi/k_d$, so we must have $l \ll \lambda$ is the cluster is to act as a point particle and produce coherent Cerenkov radiation. If $l \gtrsim \lambda$, waves will still be emitted by different particles in the cluster, but they will, in general, be out of phase and mix to produce an E much less than that given by (41). Thus, for $l \ll 2\pi/k_d$,

$$E = NZ_i \frac{e\omega_p^2}{2V^2} \ln\left(1 + \frac{2V^2}{V_{TH}^2}\right) \tag{42}$$

and the energy loss per centimeter for a particle in the cluster is

$$W = NZ_i^2 \frac{e^2\omega_p^2}{2V^2} \ln\left(1 + \frac{2V^2}{V_{TH}^2}\right) \tag{43}$$

This result has been used to explain the plasma heating observed in several bunched-beam/plasma experiments,[59–61] and ion acceleration in diode experiments.[30]

The ion acceleration mechanism is now explained by considering an ion bunch placed in an intense beam as shown in Figure 4. In the beam frame, the bunch slows due to Cerenkov wave emission, and this appears as acceleration in the laboratory frame. However, it should be noted that the results (41)–(43) are based on the implicit assumptions that (i) in the beam frame, the background "plasma" (the beam particles with thermal velocity) may be treated as a uniform infinite plasma that supports

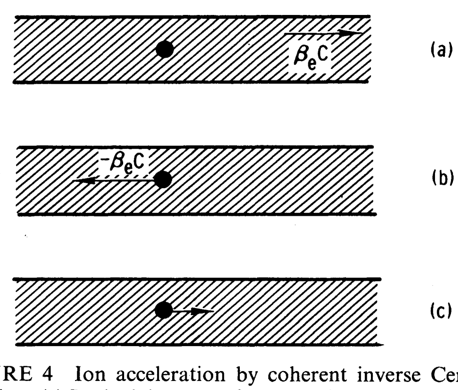

FIGURE 4 Ion acceleration by coherent inverse Cerenkov radiation. (a) In the laboratory frame, an ion bunch (dark dot) is at rest and the beam has velocity $\beta_e c$. (b) In the beam rest frame, the ion bunch has velocity $-\beta_e c$. If coherent Cerenkov radiation is possible in this frame, the bunch slows down, which appears in the laboratory frame in (c) as a net acceleration.

longitudinal plasma waves, and that (ii) the bunch is treated as a small perturbation to the plasma background.

5.1 *Theory of Wachtel and Eastlund*[23]

Wachtel and Eastlund[23] have applied the above inverse Cerenkov acceleration mechanism to the case of ion acceleration by intense beams. From the above results, we see that in either the beam frame or the laboratory frame, E is given by (42), since longitudinal E fields transform relativistically unchanged. However, (42) is to be evaluated in the beam frame, and since $|V| \sim c$ there, a full relativistic calculation of E should be made.[62] In regard to the bunch dimension, Eastlund reports that (42) should be modified by a factor of the form[63]

$$F = [\sin^2(\pi l/\lambda_d)]/(\pi l/\lambda_d)^2$$

where $\lambda_d = 2\pi/k_d$. For $l \ll \lambda_d$, $F \approx 1$, whereas for $l = \lambda_d/3$, $F \approx 0.7$.

As discussed above, validity of (42) requires, at least,

$$l \ll \lambda_d \quad (44a)$$

$$l \ll r_b \quad (44b)$$

$$n_i \ll n_b \quad (44c)$$

$$\lambda_d \ll r_b \quad (44d)$$

These conditions represent, respectively, the requirements that (a) the radiation be coherent, (b) the bunch be immersed in the beam, (c) the bunch be a small perturbation (and that gross space charge spreading is avoided), and (d) that the wave picture is applicable. Also note that since $N \sim \lambda_d^3 n_b$ (but $N \ll \lambda_d^3 n_b$), and that $\lambda_d \sim n_b^{-1/2}$, it follows from (42) that

$$E \sim n_b^{1/2}. \quad (45)$$

Thus large E requires very high beam density.

As an example, consider $Z_i = 1$ and $n_b = 10^{12}$ cm^{-3} in the beam frame. Also, we assume V_{TH} is of the order of c in the beam frame. This crude assumption follows from the facts that (i) beam electrons are injected with a finite and significant energy spread and that (ii) beam electrons attain significant transverse velocities as they oscillate in the beam self-magnetic field. Thus $\lambda_d \approx \sqrt{2}\,\pi c/\omega_p$. To evaluate (42) we assume l is the smaller of $\lambda_d/3$ or $r_b/3$, $n_i = 0.1\ n_b$, $F = 1$, and that the ln term in (42) equals unity. The number of ions is $N \approx (4/3)\pi l^3 n_i$ where n_i is the ion density in the laboratory frame. Then for $r_b = 1$ cm, and $\gamma = 3$, we find

$$E \approx 10^4 \text{ V/cm}$$
$$N \approx 1.5 \times 10^{10}$$

If we assume $V_{\text{TH}} \ll c$, E would increase by the factor $\sim 2 \ln(c/V_{\text{TH}})$ whereas N would decrease (considerably) by the factor $(V_{\text{TH}}/c)^3$.

In trying to use this model to explain the present data,[1–12] one encounters the following difficulties:

a) Assuming cutoff at some fixed distance, $\mathscr{E}_i \sim Z_i^2$ which does not agree with the data.

b) Both N and E are too small. To get $E \sim 10^6$ V/cm and $N \sim 10^{13}$ requires using $l > 2\lambda$ which strongly violates both the condition for coherent radiation (44a) and the assumption that the ion bunch is fully immersed in a plasma (44b).

c) The mechanism produces continuous acceleration, with the obvious cutoff being that the beam runs out—but the observed acceleration occurs in a time much less than the beam pulse length.

d) Linearized theory for an infinite plasma was used to calculate E. In general both the finite perturbation effect of the ion cluster and the finite transverse dimensions of the plasma would necessitate a nonlinear, bounded plasma, calculation.

e) It has been assumed that the ion bunch will stay together during the acceleration. Provided $n_i \ll n_b$, at least gross space charge spreading of the pulse is avoided. However, net charge density gradients may indeed pose a severe bunch spreading problem.

Thus, apparently, one cannot use this model to explain the existing data.[1–12]

6 CONCLUSIONS

Ion acceleration mechanisms involving electrostatic space charge fields, inductive fields, and collective wave fields have been investigated. Six theories of ion acceleration[13–23] have been examined. Each theory was discussed in detail and compared with the existing data[1–12] in regard to (1) ion energy, (2) acceleration length, (3) acceleration time, (4) number of ions, and (5) ion pulse length. It was concluded that none of these theories could be used to explain the existing data. Also, some new and general results concerning potential wells and pinch acceleration were given.

Based on the results presented above, we have concluded that the observed ion acceleration must involve only electrostatic space charge fields, at least to lowest order, but that a new theory is needed to account for the existing data with all of their parametric dependence. Indeed, we have recently developed a new theory[64–66] of ion acceleration in which ions are accelerated only in collective space charge fields. In this theory, inductive fields and collective wave fields are assumed to represent higher-order effects, as can be verified a posteriori. This theory is in substantially good agreement[64–66] with the data.[1–12]

In any event, it is hoped that the work presented here constitutes a useful investigation of possible acceleration mechanisms, that it clarifies the status of several existing theories,[13–23] and that it will provide a foundation for future research.

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