98

MURA 98 Laslett

MURA NOTES
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RF ACCELERATION SCHEMES

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ACCELERATION SCHEMES

I Bucket lift.

A FFAG accelerator can accommodate at one time particles circulating at all energies between the injector and output energies. In a synchrotron particles of various energies circulate with various corresponding frequencies. If a radio frequency voltage is applied to an accelerator gap, then in the neighborhood of each energy for which the frequency of revolution of the particles is equal to the radio frequency or any of its subharmonics, there is a region of particle energies and phases ("bucket") within which particles execute stable phase oscillations around the synchronous energies. If the radio frequency is modulated, "buckets" move up or down the energy scale. Under suitable conditions particles in any of the buckets can be accelerated by this system. Thus it is possible to accelerate a number of buckets of particles at a number of different energies simultaneously, with a single accelerator frequency. As the frequency is modulated, each bucket can be filled by the injector when the energy corresponding to that bucket coincides with the injector energy. There are two general bucket schemes.

la. Scheduled bucket lifts.

In this case the radio frequency is modulated from a frequency f_1 to a frequency f_2 in such a way that the buckets carry particles to increasing energy during the modulation cycle. The frequency is then changed back to f_1 and the cycle repeated.

The frequency f_1 and f_2 are so chosen that the buckets corresponding to f_1 coincide in energy as nearly as possible with the buckets corresponding to frequency f_2. Thus, for example, if $f_2 = 2f_1$ a bucket at the h order subharmonic of f_1 will have the same energy as a bucket of the 2 h order subharmonic f_2. If the frequencies f_1 and f_2 are properly chosen, a particle may be accelerated during many successive radio frequency cycles by being pushed up at the beginning of each cycle by a bucket which coincides in energy with the final energy of the bucket in which the particles were carried on the previous cycles.

1b. Nonscheduled bucket lift.

In this case, the frequencies f_1, f_2 above, have no particular relation to each other or they may even vary from cycle to cycle. Particles which happen to land in a bucket at the beginning of a cycle are carried up in energy and left at a higher energy at the end of that cycle. Particles not in buckets are displaced in energies, on the average downward, by the action of the radio frequency voltage during the cycle. Thus, each particle executes a random walk in energies sometimes increasing and sometimes decreasing its energies until it reaches the output energy or is lost by collision with the injector, with the walls of the donut, or with a gas molecule. Various partial schemes are also possible.

II Stacking Schemes.

Particles may be accelerated by a radio frequency cycle as described above, until they reach an energy E_2 which corresponds to the final frequency f_2 of the radio frequency voltage. On successive cycles buckets full of particles are deposited at the energy E_2. The particles already there are displaced by successive buckets, on the average downward in energy, to make room in phase space according to Liouville's theorem for the newly arriving particles. When a suitable number of buckets of particles have been stacked near the energy E_2, a second radio frequency accelerator may accelerate the particles on to a new energy E_3. If the bucket size for the second cycle is n times the bucket size for the first cycle, then n buckets can be stacked at E_2 during the first cycle. These can then be picked up in a final bucket and carried to E_3 in a single cycle of the second type. The advantage of this system is that the radio frequency's schedule can be chosen in the most efficient way to capitalize upon the bucket size vs. energy relation which in turn depends upon the frequency of revolution vs. energy curve. Thus usually $\frac{d\Omega}{dE}$, where Ω is the frequency of revolution, decreases with energy. This has two consequences:

1. For a given radio frequency voltage, the bucket size increases with energy and, hence, in the usual accelerator schemes the buckets are nearly empty when they arrive at the transition energy, but by stacking at intermediate energies this can be corrected.
2. For a given radio frequency voltage, the allowable rate of frequency modulation noticeably decreases and hence the repetition rate is limited. By stacking, one can use a higher repetition rate with small buckets at lower energies; and a smaller repetition rate at higher energies, but with larger buckets so that the output current corresponds to the total injected current at the higher repe-

tition rate.

3. One might find that the extent of frequency modulation required during any modulation cycle is reduced.

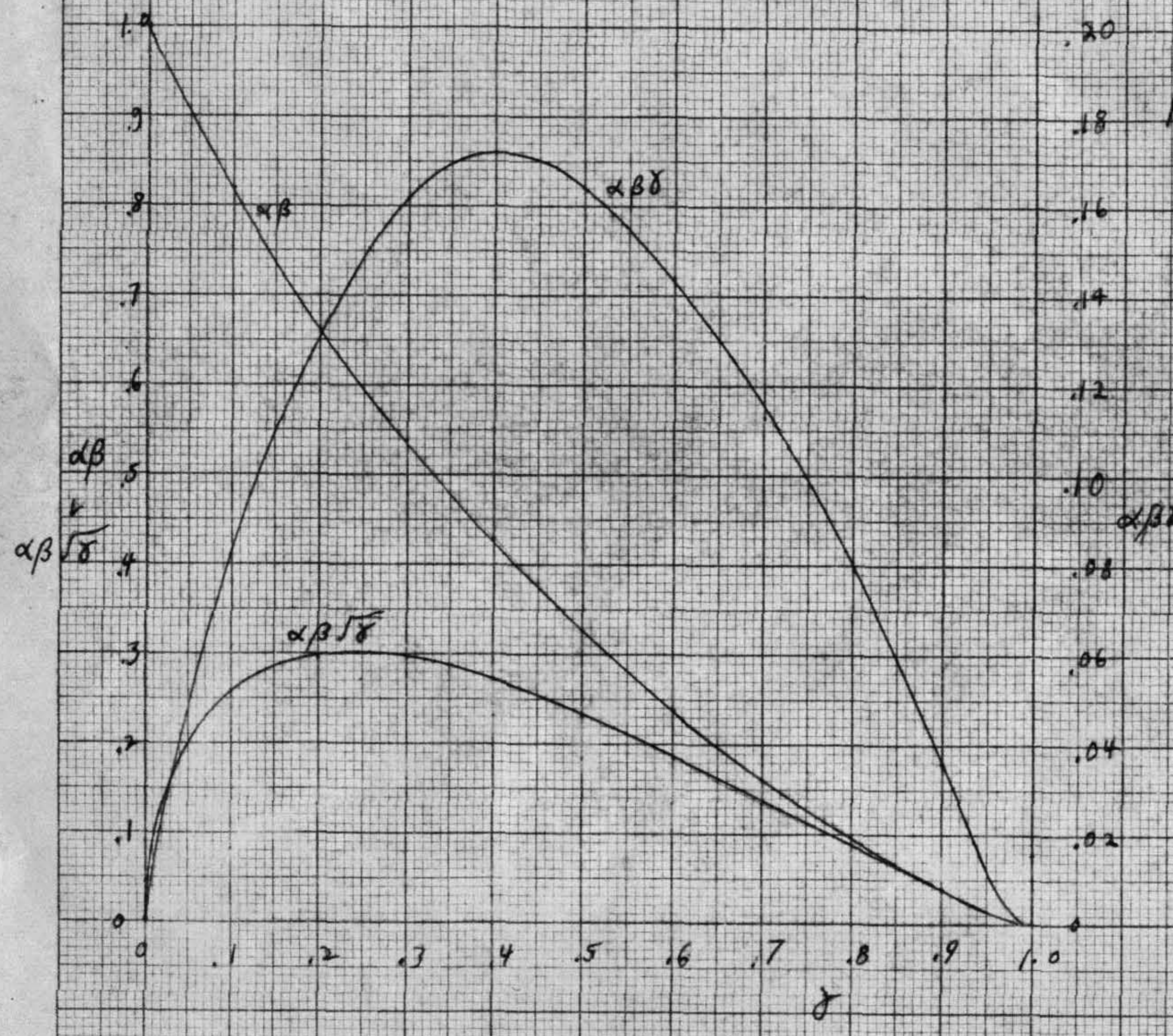
III Phase Displacement Schemes

These are based on the observation that particles are accelerated if subject to a radio frequency gap which is initially at a frequency corresponding to an energy higher than that of the particles, and then the oscillator frequency is modulated to a frequency corresponding to an energy lower then that of the particles. Note that in this scheme the frequency is modulated in just the reverse direction from that used in the bucket lift. The mechanism may be readily understood, for the oscillator carries virtual particles down in energy, and thus by Liouville's Theorem must force real particles upward in energy.

One can readily develop a hydrodynamic theory which can be made to include the effect of the variation of bucket size with energy, rate of modulation, and cavity voltage; as well as the possibility of both carrying particles in the buckets, as well as phase displacing those particles not in buckets.

In general, since the current accelerated by phase displacement equal the virtual current carried down by the oscillator, phase displacement and bucket lifts are about equally efficient. The methods vary in the length of time necessary for transit of a given energy internal by any single particle, and as such each scheme has distinct advantages or disadvantages. It should be clear that the carrying of particles in buckets, and the phase displacement of particles not in buckets is complimentary. For any proposed scheme involving particles in buckets, one can envision a complimentary scheme involving phase displacement, which is equally efficient if loss of particles to the walls, injector, or gas scattering is neglected.

Just as in the bucket lift, one may have scheduled or unscheduled programs. In particular, one can envision the use of many unphased oscillators, all modulating in frequency such as to continually phase displace particles upward in energy.



$$\gamma = \frac{\dot{w}_s}{V} = \frac{\dot{\nu}}{h\omega\frac{d\omega}{dE}V}$$

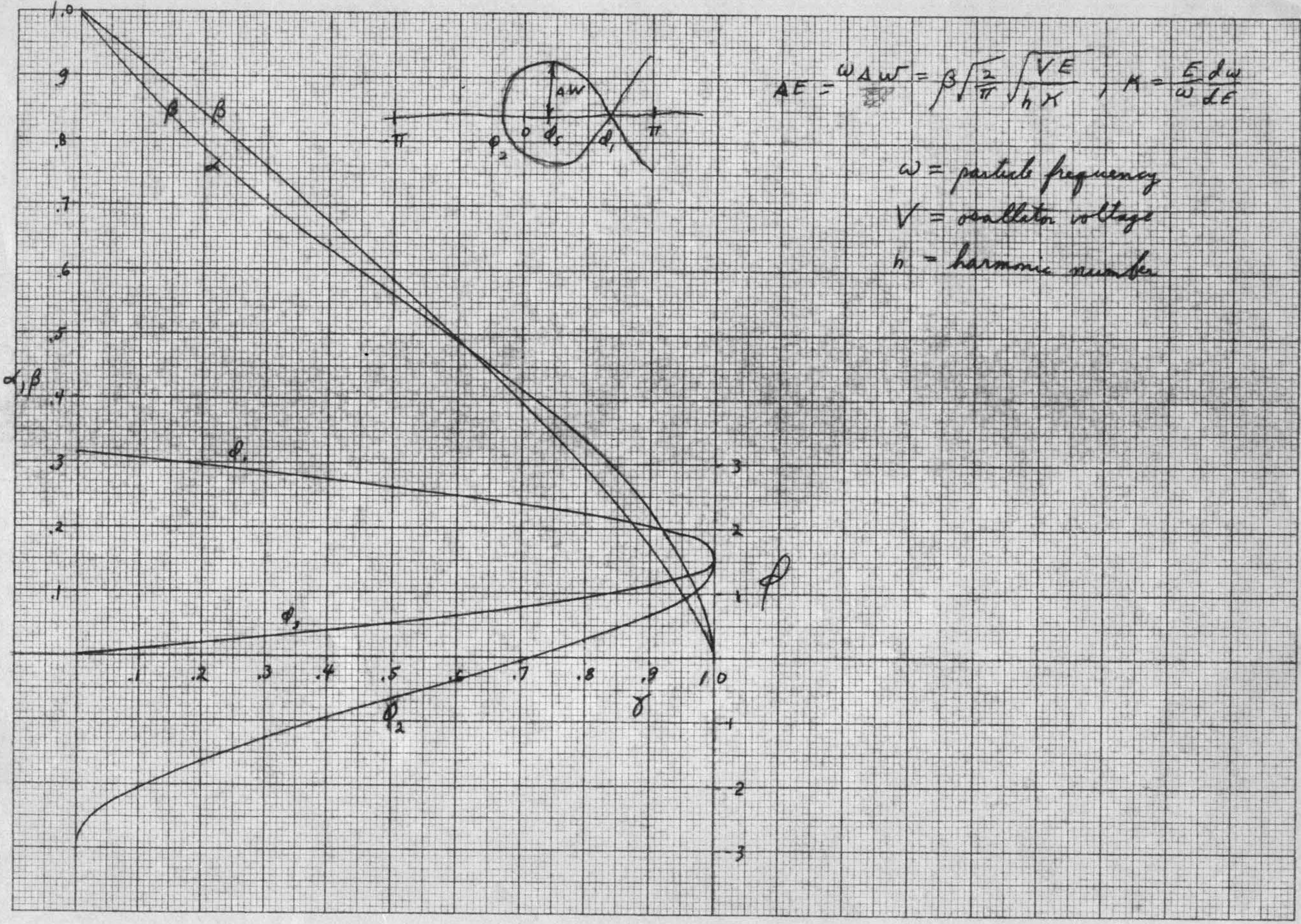
$$\text{Bucket Area} = 8\sqrt{\frac{2V}{\pi h\omega\frac{d\omega}{dE}}}\,\alpha\beta = 8\sqrt{\frac{2}{\pi}}\sqrt{V}\sqrt{\dot{\nu}}\,\alpha\beta\sqrt{\gamma}$$

$$\text{Phase Flux} = 4\sqrt{\frac{2}{\pi}}\sqrt{\frac{(h+\frac{1}{2})^2}{h}}\left(\frac{1}{\omega}\frac{d\omega}{dE}\right)^{\frac{1}{2}}V^{\frac{3}{2}}\alpha\beta\gamma \qquad (a)$$

$$= 8\sqrt{\frac{2}{\pi}}\sqrt{\frac{1}{h}}\left(\frac{1}{\omega}\frac{d\omega}{dE}\right)^{-\frac{1}{2}}\frac{V^{\frac{3}{2}}}{\Delta E}\alpha\beta\gamma \qquad (b)$$

(a). $\Delta E \to 0$, small barrier buckets

(b) one bucket over distance ΔE.



$$\Delta E = \omega \Delta W = \beta\sqrt{\frac{2}{\pi}}\sqrt{\frac{VE}{hK}}, \quad K = \frac{E}{\omega}\frac{d\omega}{dE}$$

ω = particle frequency

V = oscillator voltage

h = harmonic number