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THE ELECTROSTATIC FIELD OF THE KICKER ELECTRODES IN THE B. N. L. ELECTRON ANALOGUE

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Introduction:

For beam guidance in the B. N. L. Electron analogue, the circuits of the quadrupole elements are adapted to permit these assemblies to serve the additional function of kicker electrodes. The electrode surfaces are basically hyperbolic in shape, being represented [B.N. L. Drg. D01-3-3(B)] by the equations

$$x^2 - y^2 = \pm a^2,$$

with a = 0.440 inch = 1.1176 cm. When functioning as kickers, a pair of opposed electrodes may be raised to potentials $\pm V_o$ while the remaining pair are at ground potential.

The central field of the kicker electrodes has been roughly estimated in B.N.L. Rpt. LJL-6, the result $E_o \cong \frac{V_o}{1.3a}$ being suggested, and this value of the central field employed in the interpretation of certain experimental observations described in LJL-9. For further quantitative work with the

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analogue it may be desirable, however, to have a more accurate value for the central field. In addition a knowledge of the non-linearities present in the field of the kicker electrodes is of interest in connection with the performance of the analogue at certain sub-resonances, as is suggested by a recent B.N.L. report by Lloyd Smith.

A determination of the electrostatic field generated by the kicker electrodes has recently been made by the digital computer at the University of Illinois. The results obtained are given below; in particular the central field is found to be $E_o \doteq 0.8106\ V_o/a = V_o/1.234a$.

Method:

The problem was arranged as a two-dimensional Dirichlet problem with the boundary values as indicated on the figure. The values $0.8846\ V_o$, $0.5000\ V_o$, and $0.1154\ V_o$ for points in the neighborhood of ($\pm$ 2.0 a, 2.0a) were not considered to be critical and were estimated by linear interpolation.

Numerical Results:

The computed values of the potential, at intervals of 0.1a, are given in the following Table for those points in the first quadrant for which $x \leq a$, $y \leq a$.

Potential Characterizing Electrostatic Field of Kicker Electrodes

V/V_o

$\frac{y}{a}$ \ $\frac{x}{a}$	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0	0	0	0	0	0	0	0	0	0	0	0
0.1	0.0813	0.0806	0.0786	0.0753	0.0704	0.0641	0.0559	0.0457	0.0331	0.0177	(0)
0.2	0.1639	0.1626	0.1586	0.1520	0.1425	0.1299	0.1138	0.0938	0.0689	0.0378	(0)
0.3	0.2491	0.2472	0.2414	0.2316	0.2176	0.1991	0.1758	0.1468	0.1107	0.0646	(0)
0.4	0.3381	0.3356	0.3281	0.3153	0.2971	0.2733	0.2434	0.2069	0.1627	0.1100	0.0502
0.5	0.4321	0.4291	0.4200	0.4045	0.3824	0.3535	0.3178	0.2745	0.2231	0.1625	0.0907
0.6	0.5322	0.5288	0.5183	0.5003	0.4743	0.4407	0.3995	0.3504	0.2928	0.2261	0.1500
0.7	0.6391	0.6355	0.6241	0.6039	0.5739	0.5355	0.4892	0.4348	0.3717	0.2991	0.2159
0.8	0.7533	0.7499	0.7388	0.7174	0.6819	0.6383	0.5869	0.5278	0.4601	0.3827	0.2947
0.9	0.8744	0.8721	0.8639	0.8448	0.7980	0.7489	0.6922	0.6294	0.5581	0.4771	0.3847
1.0	1.0000	(1.0000)	(1.0000)	(1.0000)	0.9163	0.8672	0.8037	0.7396	0.6657	0.5828	0.4878

Analytic Representation of Field:

It appears that the electrostatic field in the region of interest may be represented with reasonable adequacy by

$$V/V_o = A\, r \sin\theta + B\, r^3 \sin 3\theta + C\, r^5 \sin 5\theta$$

$$= A\, y + B(3x^2y - y^3) + C(5x^4y - 10x^2y^3 + y^5).$$

We suggest values of the coefficients A, B, and C obtained by matching the computed potential at (0, 0.4a), (0.2a, 0.4a), and (0.4a, 0.4a):

$$A = 0.8106/a$$

$$B = -0.2205/a^3$$

$$C = -0.0260/a^5 \,.$$

Thus, in particular,

$$\left[\frac{-E_y}{V_o/a}\right] = Aa + 3Ba^3\, \frac{x^2-y^2}{a^2} + 5Ca^5\, \frac{x^4 - 6x^2y^2 + y^4}{a^4}$$

$$= 0.8106 - 0.6615\, \frac{x^2 - y^2}{a^2} - 0.13\, \frac{x^4 - 6x^2y^2 + y^4}{a^4} \,;$$

$$\left[\frac{-E_x}{V_o/a}\right] = 6Ba^3\, \frac{xy}{a^2} + 20Ca^5\, \frac{x^3y - xy^3}{a^4}$$

$$= -1.323\, \frac{xy}{a^2} - 0.52\, \frac{x^3y - xy^3}{a^4} \,;$$

and the central field is

$$|E|_o = 0.8106\ V_o/a = V_o/1.234a \,.$$

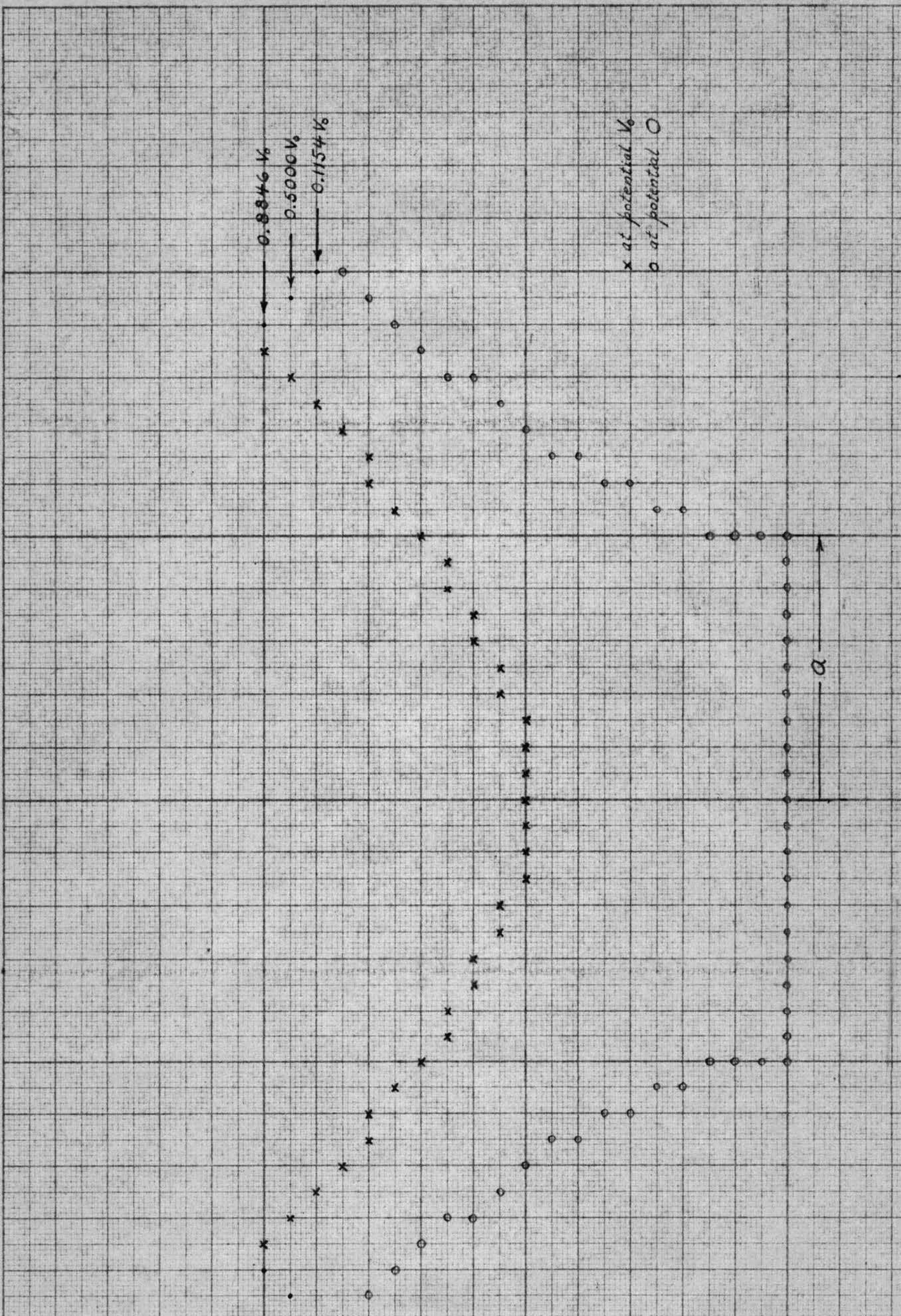

Diagram indicating boundary and assumed potentials for Dirichlet problem characterizing kicker electrodes