MURA 66
MURA/DWK 10

GAP IN A SPIRALLY RIDGED POLE

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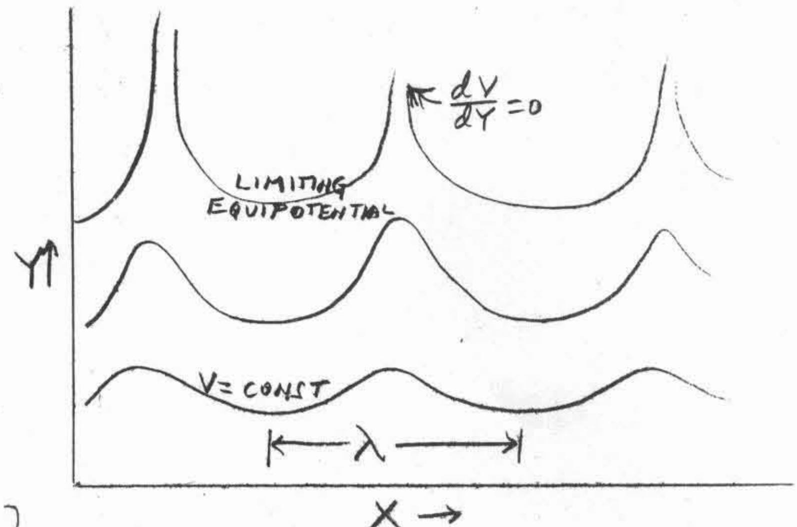
This is a calculation similar to that made by Terwilliger in MURA/KMT-2 but for a sinusoidal distribution of magnetic field in the median plane instead of for a step distribution of field.



Terwilliger has pointed out that for $B = B_o \gamma^{k}(1+f \sin\frac{x}{\lambda})$, the magnetic potential for the case of $k = 0$ is $V = B_o\left\{Y + f\lambda \sin\frac{x}{\lambda} \sinh\frac{Y}{\lambda}\right\}$.

We want to calculate the maximum gap between opposing pole ridge tops, G, for different values of the fractional field variation, f, in the median plane. This maximum G is the separation between ridge tops for the limiting magnetic equipotential which developes an infinite crevice between ridges.

The crevices occur where $\frac{dV}{dY} = 0$ or $0 = 1 + f\sin\frac{x}{\lambda} \cosh\left(\frac{Y_\infty}{\lambda}\right)$ and the value of $\frac{x}{\lambda}$ at crevices is $\frac{x}{\lambda} = -\frac{\pi}{2}$. Thus $\frac{1}{f} = \cosh(Y_\infty/\lambda)$. For this equipotential $V_\infty = B_o\left\{\lambda \cosh^{-1}\frac{1}{f} - \lambda f \sqrt{\left(\frac{1}{f^2} - 1\right)}\right\}$. On this same equipotential surface

Y_{min} = G/2 occurs at $\sin x/\lambda = +1$ and it satisfies

$$V_\infty = B_o\left\{Y_{min} + f\lambda \sin H\left(\frac{Y_{min}}{\lambda}\right)\right\}$$

equating

$$\frac{Y_{min}}{\lambda} + f \sin H\left(\frac{Y_{min}}{\lambda}\right) = \cos H^{-1} \frac{1}{f} - \sqrt{(1+f)(1-f)}.$$

If we now impose the condition that the smooth approximation $\nu_z^2 = \frac{f^2 r^2}{\lambda^2 N^2} = 2A.G.$ be a constant, then, since in cases where $k \neq 0$ N is already chosen by radial motion considerations, we are imposing the condition $\frac{f}{\lambda}$ = constant. This is also requiring that $\left.\frac{dB}{dx}\right|_{max} \equiv$ constant in the orbital plane for different parameters tried.

The graphs show $\frac{G}{\lambda} = 2\frac{Y_{min}}{\lambda(f,\pi)}$ determined by the transcendental equation above and $\frac{Gf}{\lambda}$ which is proportional to the maximum gap attainable for $\frac{f}{\lambda}$ = constant, ν_z = constant, or $\left(\frac{r}{H}\frac{dH}{dr}\right)_{Peak}$ = constant.

The curves show that $f = \frac{1}{4}$ gives about the maximum gap if one is designing for a fixed ν_z, but only 10% of the gap is lost if f = .35 or .15 is used. The biggest usable value of G/λ is .28 which is a good design rule. This result is very closely the same as that given by the Illiac for the solution to the Cauchy problem with $k \sim 150$, that is for $B = B_o (r/r_o)^k (1 + f \sin\frac{x}{\lambda})$ in the orbital plane. Terwilliger found a broad maximum at $f \doteq \frac{1}{3}$ for step shaped field variations.

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PLOT OF MINIMUM GAP IN SPIRAL RIDGED POLES (RIDGES SPACED λ) AS A FUNCTION OF THE FLUTTER f.
$V = H_0 \left(Y + f\bar{\lambda} \sin X/_{\lambda} \sinh Y/_{\lambda}\right)$ IS POTENTIAL FUNCTION.

$B = B_0 \left(1 + f \sin X/_{\lambda}\right)$

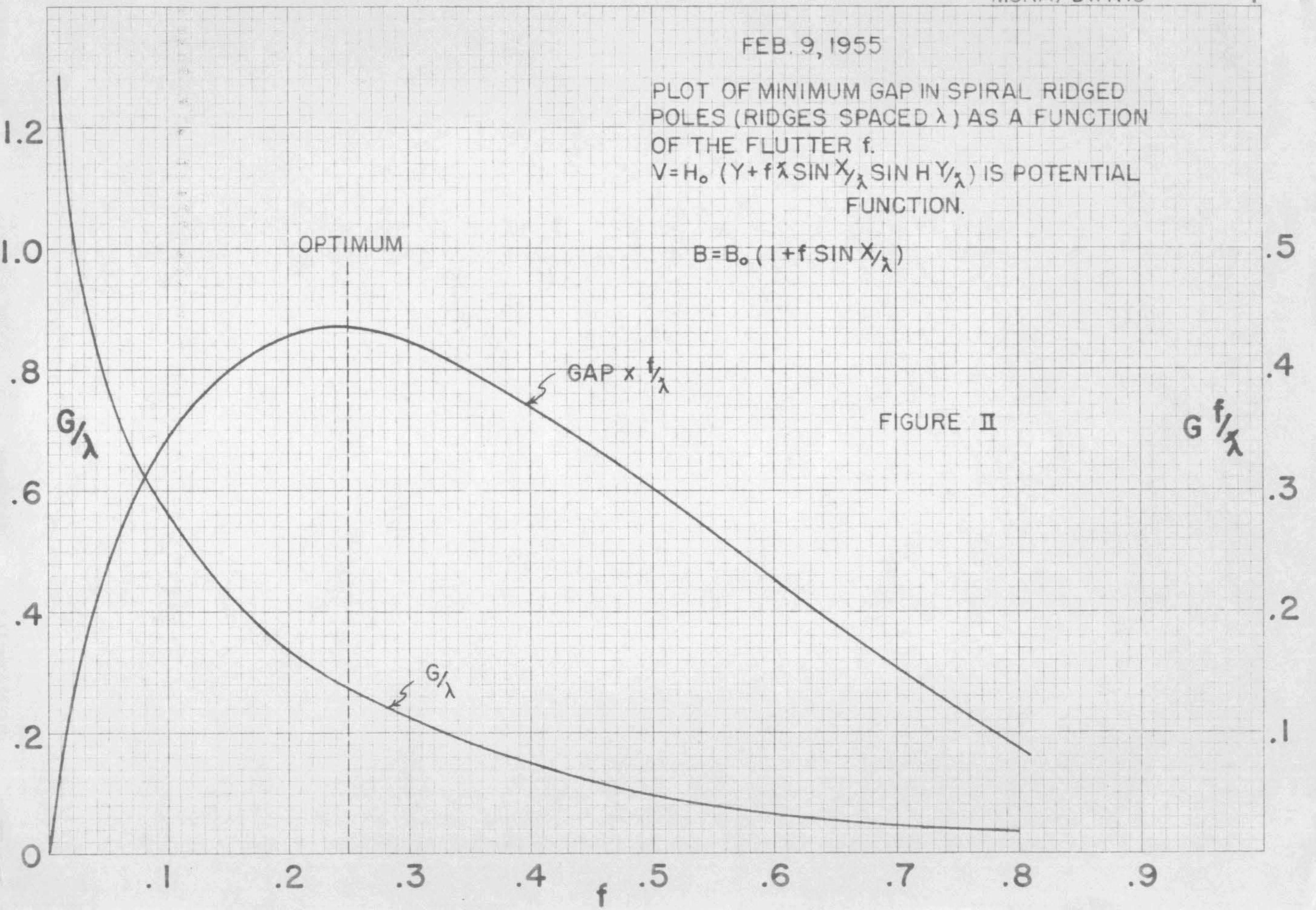

FIGURE II